Transmit Strategies for Multiuser MIMO MAC

Aylin Yener

Wireless Communications and Networking Laboratory

EE Dept. Penn State

yener@ee.psu.edu

http://labs.ee.psu.edu/faculty/yener

May 7, 2004

Joint work with Semih Serbetli

Introduction

- Multiple transmit and receive antennas increase capacity [Telatar]
- Diversity
- Spatial Multiplexing (Multiple Symbol Transmission)
- Spatial Processing
- Coding over Space and Time
- The substantial potential capacity of the MIMO link motivates the use in multiple access channels
  $\Rightarrow$ Multiuser MIMO Systems

What is a Multiuser MIMO System?

- MIMO System
- Each user has multiple transmit antennas
- Each user can only utilize its own resources
- Users interfere with each other
- Common receiver with multiple antennas

Motivation and Setting

- CSI at the transmit side can improve performance significantly
- Transmit shaping should be employed in accordance with available transmit side feedback
- **Transceivers of all users should be jointly optimized**
- Model assumptions:
  - Uplink (MAC)
  - Perfect feedback
  - Static channel

Objective

- Find the **jointly optimum** transceiver structures that maximize the performance metric of choice: Sum capacity, MSE,...
  - The multiaccess "structure" of the system
    - Multiuser MIMO systems $\Rightarrow$ precoder/decoder design
    - Time slotted multiuser MIMO systems $\Rightarrow$ scheduling and beamformer design
    - Multiple antenna CDMA systems $\Rightarrow$ signature and beamformer design
  - Feedback at the transmitter side
  - The accuracy of the channel state information

Efficient Transmit Strategies for Multiuser MIMO Systems

- Multiuser MIMO System
- Multi symbol transmission
- Transmit Power Constraint for each user
- Channel known at the transmitter and receiver
- Error-free and low delay feedback
- Find the linear transmitter and receivers that will minimize the **system-wide MSE** of all users

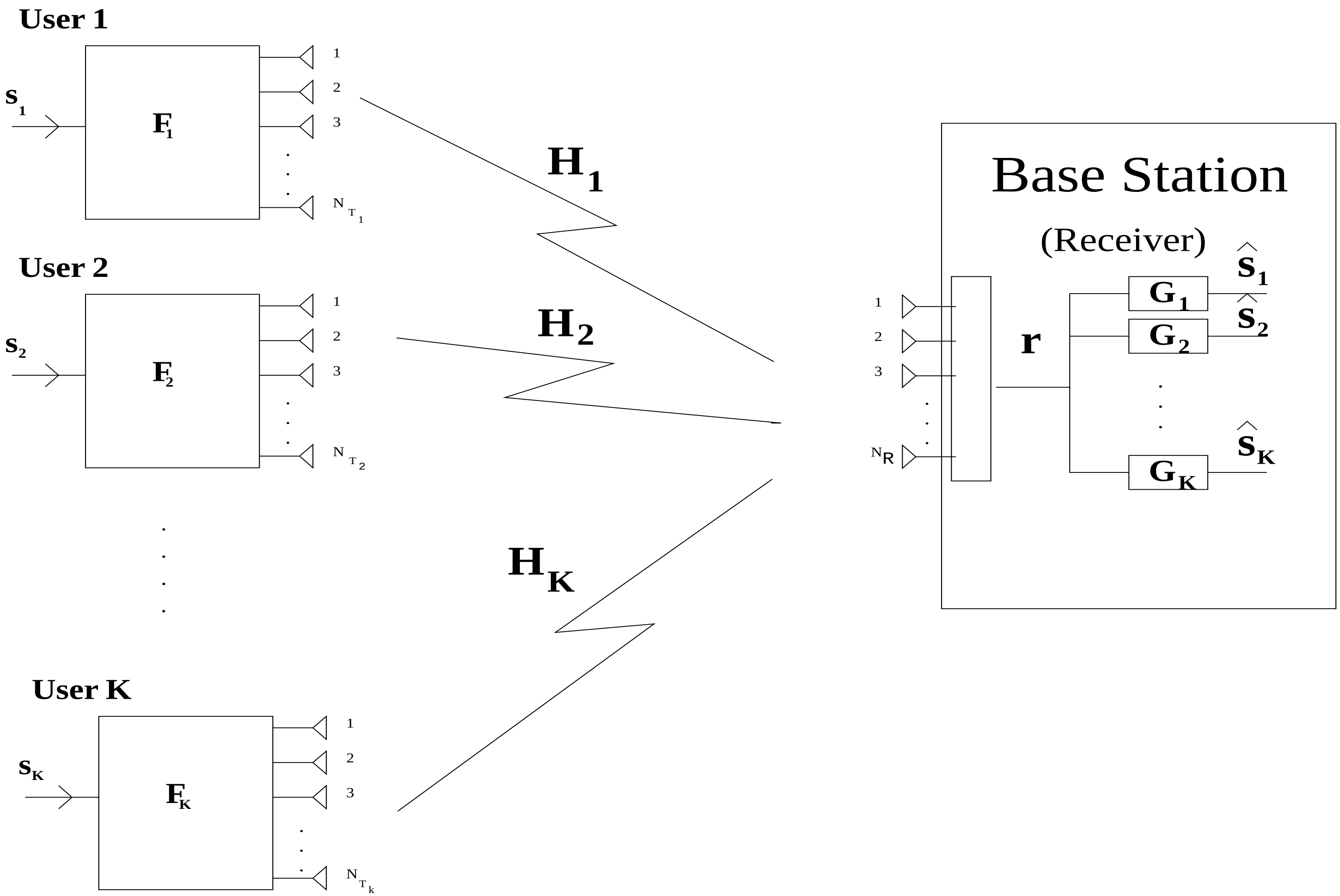
Previous work on single-user MIMO

- The availability of and the content of channel state information affects transmitter design (spatial transmit shaping)
  - No CSI at the transmitter
    - BLAST [Bell-Labs]
    - Space-time coding [Tarokh et.al.]
  - Limited feedback: Antenna selection [e.g. Blum, Molisch]
  - CSI at the transmitter: Linear precoding for single MIMO link
    - Precoder/Decoder design [Sampath et.al.]
    - Space-Time Linear Precoder/Decoders [Scaglione et.al.]

Previous Work on Multiuser MIMO

- Capacity
  - Iterative Waterfilling [Yu et. al.(Stanford)]
  - Interference Avoidance [Popescu, Rose (WINLAB)]
  - Downlink Multiuser MIMO Decomposition [Choi et. al. (HKUST)] [Spencer, Haardt]
- Target SIR
  - Single Symbol SIR Target SDMA Modeling [Chang et.al.(UMD)]
- System-wide MSE
  - Transmitter-Receiver Design for ISI Channels (Matrix Constraints) [Luo et.al. (McMaster)]

Multiuser MIMO System Model



Notation

- M_k: Number of symbols transmitted by user k
- Total transmit power constraint for user k

$$tr\{\mathbf{F}_k \mathbf{F}_k{}^{\dagger}\} \leq P_k$$

- Channel Model
  - Particular channel realization (slow fading); independent gains between antennas
  - Channel is perfectly known by the receiver and transmitter

Multiuser MIMO Communication Model

- Each user precodes its symbol vector $\mathbf{s}_k$ with $\mathbf{F}_k$. The received vector is

$$\mathbf{r} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{F}_k \mathbf{s}_k + \mathbf{N}$$

- $\{\mathbf{G}_k\}_{k=1}^{K}$'s are the linear receivers. System-wide MSE of all users is

$$\text{MSE} \quad = \quad \text{tr}\left\{ \sum_{i=1}^{K} \left\{ \sum_{j=1}^{K} \mathbf{F}_j{}^\dagger \mathbf{H}_j{}^\dagger \mathbf{G}_i{}^\dagger \mathbf{G}_i \mathbf{H}_j \mathbf{F}_j - \mathbf{F}_i{}^\dagger \mathbf{H}_i{}^\dagger \mathbf{G}_i{}^\dagger - \mathbf{G}_i \mathbf{H}_i \mathbf{F}_i + \mathbf{I} + \sigma^2 \mathbf{G}_i \mathbf{G}_i{}^\dagger \right\} \right\}$$

- The optimization problem

$$\begin{array}{lll} \min & \underset{\{\mathbf{F}_k, \mathbf{G}_k\}_{k=1,\cdots,K}}{\text{MSE}} & \\ \text{s.t.} & \text{tr}(\mathbf{F}_k{}^\dagger \mathbf{F}_k) \leq P_k & k = 1, \cdots, K \end{array}$$

Algorithm

- MSE not jointly convex over $\{\mathbf{F}_k, \mathbf{G}_k\}$
- MSE convex over $\mathbf{F}_k$ (or $\mathbf{G}_k$) when all other variables are fixed
- Construct an iterative algorithm
- First order optimality conditions yield the updates.
- Receiver and Transmitter of each user is updated as

$$\mathbf{G}_k = \mathbf{F}_k^\dagger \mathbf{H}_k^\dagger \left(\sigma^2 \mathbf{I} + \sum_{i=1}^{K} \mathbf{H}_i \mathbf{F}_i \mathbf{F}_i^\dagger \mathbf{H}_i^\dagger \right)^{-1}$$

$$\mathbf{F}_k = \left(\mu_k \mathbf{I} + \sum_{i=1}^{K} \mathbf{H}_k^\dagger \mathbf{G}_i^\dagger \mathbf{G}_i \mathbf{H}_k \right)^{-1} \mathbf{H}_k^\dagger \mathbf{G}_k^\dagger$$

- μ_k is the Lagrange multiplier associated with user k's transmit power constraint.

Algorithm 1

- For given precoders, receivers (decoders) are the familiar MMSE receivers.
- Substitute for the decoders in the precoder update:

$$\mathbf{F}_k^{\star} = \left(\mu_k \mathbf{I} + \mathbf{H}_k^{\dagger}(\mathbf{T}^{-1} - \sigma^2 \mathbf{T}^{-2})\mathbf{H}_k\right)^{-1} \mathbf{H}_k^{\dagger}\mathbf{T}^{-1}\mathbf{H}_k\mathbf{F}_k$$

- Random starting points
- Parallel updates:
  - Update all precoders simultaneously
  - Update all decoders simultaneously
- Sequential updates:
  - Update precoders one by one, updating **T** after each iteration
  - Faster convergence

Convergence

- Algorithm is convergent
  - Decreases MSE at each iteration
  - MSE lower bounded
- Fixed point of the algorithm satisfies

$$\mathbf{H}_k^\dagger \mathbf{T}^{-2}\mathbf{H}_k\mathbf{F}_k = \mu_k/\sigma^2\mathbf{F}_k$$

where

$$\mathbf{T} = \sigma^2\mathbf{I} + \sum_{i=k}^{K}\mathbf{H}_k\mathbf{F}_k\mathbf{F}_k^\dagger\mathbf{H}_k^\dagger$$

- Optimal $\{\mathbf{F}_k\}$ is not unique
  (Permutations/phase shifted versions of columns of $\mathbf{F}_k$ yields the same MSE as $\mathbf{F}_k$).
- **Is there a way checking the optimality of the fixed point?**

Optimality

- When MMSE receivers are used by each user, the total MSE is

$$\text{MSE} = \sum_{k=1}^{K} M_k - N_R + \sigma^2 tr\left\{\mathbf{T}^{-1}\right\}$$

- Define $\mathbf{R}_k = \mathbf{F}_k\mathbf{F}_k^\dagger$, and the equivalent optimization problem is

$$\begin{aligned}
\min_{\{\mathbf{R}_k\}} \quad & tr\left\{\mathbf{T}^{-1}\right\} \\
\text{s.t.} \quad & \mathbf{T} \leq \sigma^2\mathbf{I} + \sum_{k=1}^{K} \mathbf{H}_k\mathbf{R}_k\mathbf{H}_k^\dagger \\
& tr\{\mathbf{R}_k\} \leq P_k; \quad \mathbf{R}_k \geq 0 \qquad k = 1,\cdots,K \\
& rank(\mathbf{R}_k) \leq \min(N_{T_k}, M_k) \qquad k = 1,\cdots,K
\end{aligned}$$

- Rank constraint is problematic.
- Note: Relaxing the rank constraint yields a convex optimization problem.

Optimality Check

- KKT Conditions for optimality over $\mathbf{R}_k \;\; k = 1, \cdots, K$

$$\begin{aligned} \lambda_k \mathbf{I} &= \mathbf{H}_k^{\dagger} \mathbf{T}^{-2} \mathbf{H}_k + \Psi_k \\ tr\{\mathbf{R}_k\} &= p_k \\ tr\{\Psi_k \mathbf{R}_k\} &= 0 \\ \Psi_k, \mathbf{R}_k, \lambda_k &\geq 0 \end{aligned}$$

- Optimality check: For $k = 1, \cdots, K$, compute $\mathbf{R}_k$ using the $\mathbf{F}_k$ at the fixed point; check for optimality using KKT conditions above.
- If $M_k \geq N_{T_k}$, then the rank constraint is redundant $\Longrightarrow$ Optimality check is exact.
- If $M_k < N_{T_k}$, then the optimality check is "pessimistic".
- Recent work [Rhee et.al. (Stanford)] on upper bounds for $\sum_{k=1}^{K} rank(\mathbf{R}_k)$ on a similar setting suggests that the rank constraint may be redundant in most cases.

Single Symbol Transmission (SDMA)

- Algorithm 1 for special case of $M_k = 1$, $k = 1, \cdots, K$

$$\mathbf{f}_k^\star = \left(\mu_k \mathbf{I} + \mathbf{H}_k^\dagger (\mathbf{T}^{-1} - \sigma^2 \mathbf{T}^{-2}) \mathbf{H}_k\right)^{-1} \mathbf{H}_k^\dagger \mathbf{T}^{-1} \mathbf{H}_k \mathbf{f}_k$$

- Algorithm 1 optimizes the MSE for each user over its receiver and then transmitter
- Is there a more “greedy” approach?
- Faster convergence?

Algorithm 2

- Define

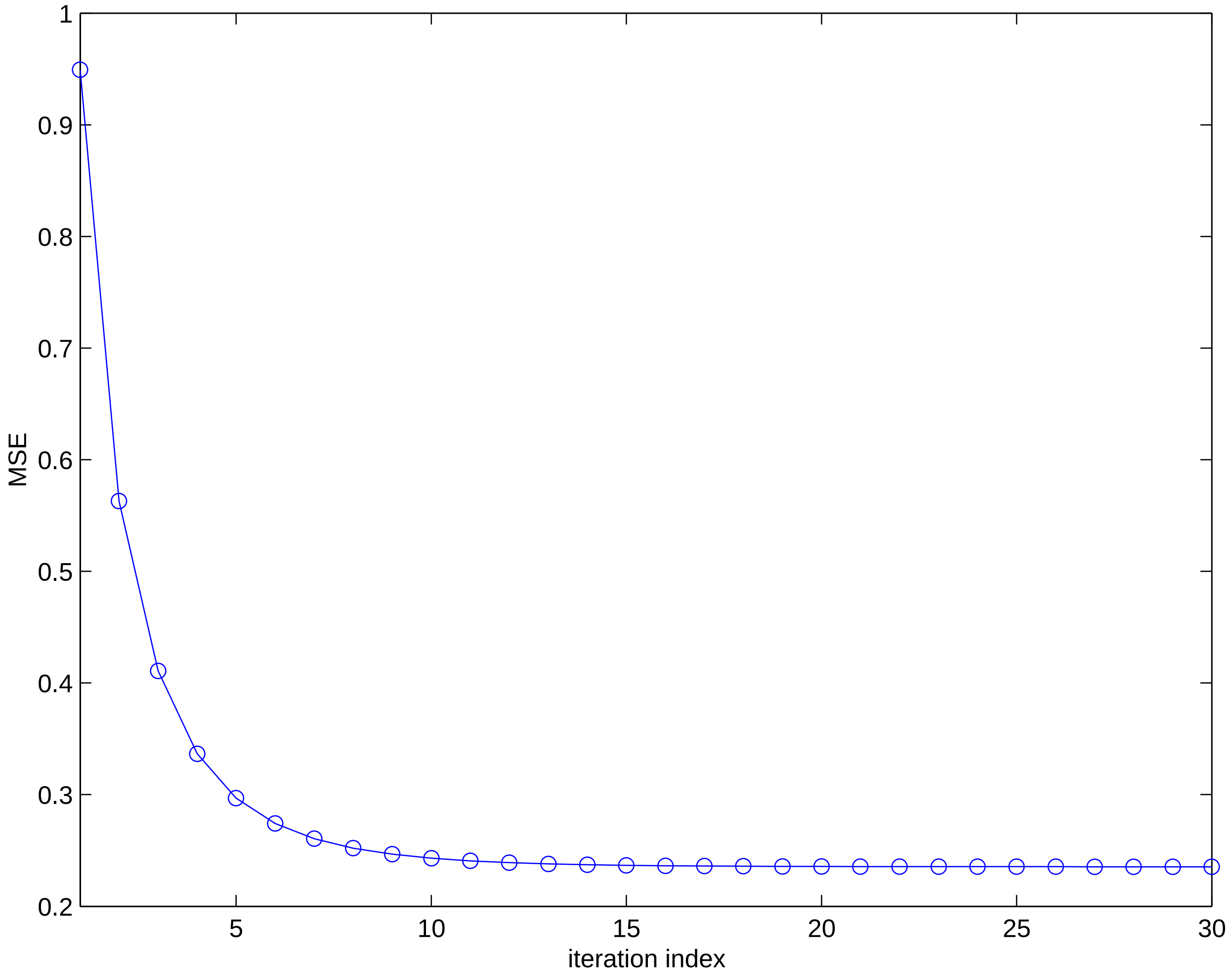
$$\mathbf{E}_k = \sum_{i \neq k} \mathbf{H}_i \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}_i^\dagger + \sigma^2 \mathbf{I} = \mathbf{T} - \mathbf{H}_k \mathbf{f}_k \mathbf{f}_k^\dagger \mathbf{H}_k^\dagger$$

  and rewrite the total MSE as

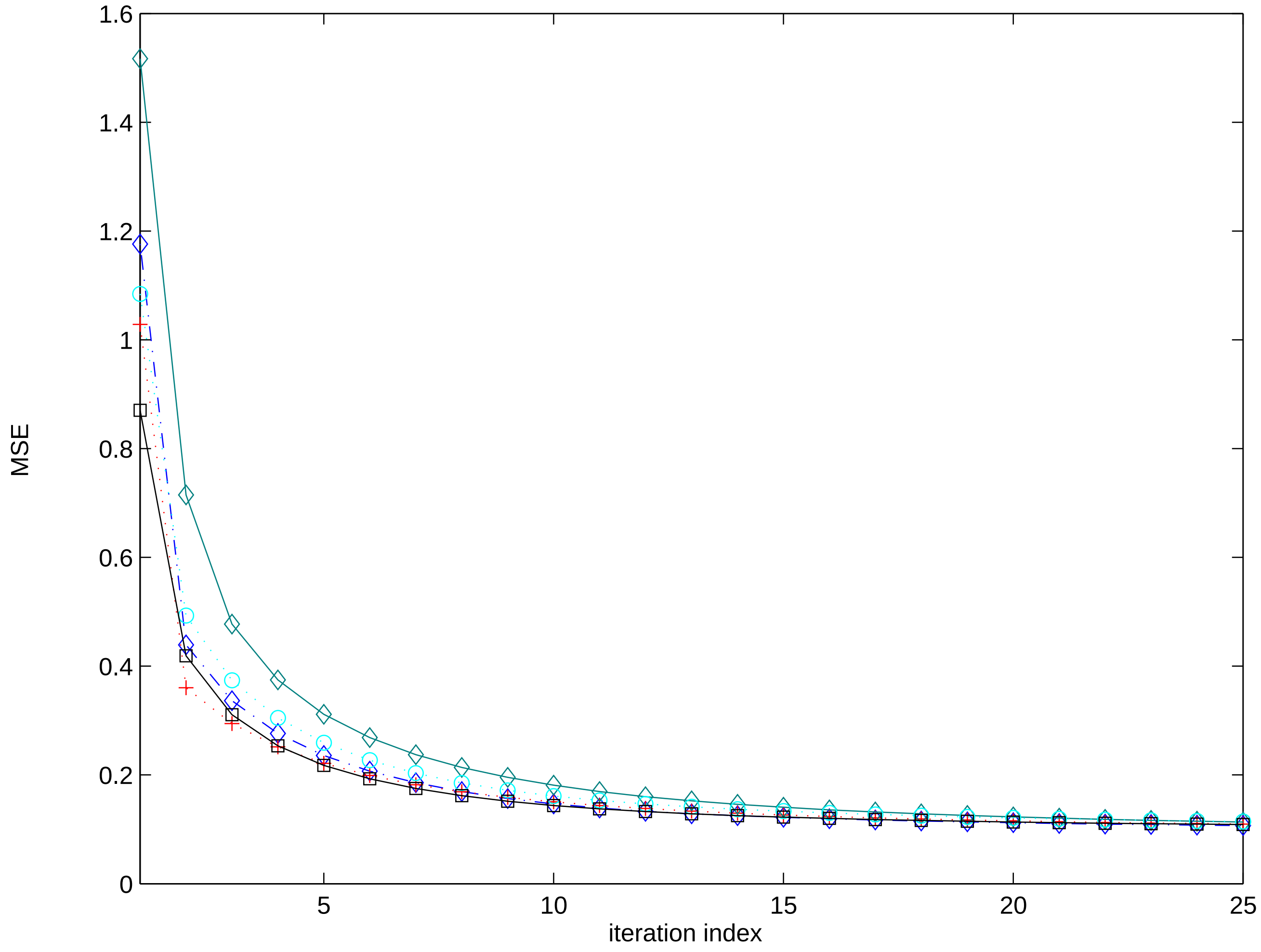
$$\text{MSE} = C_k - \sigma^2 \left(\frac{\mathbf{f}_k^\dagger \mathbf{H}_k^\dagger \mathbf{E}_k^{-2} \mathbf{H}_k \mathbf{f}_k}{1 + \mathbf{f}_k^\dagger \mathbf{H}_k^\dagger \mathbf{E}_k^{-1} \mathbf{H}_k \mathbf{f}_k} \right)$$

- C_k represents the terms independent of user k.
- From the perspective of user k, MSE can be minimized by choosing $\mathbf{f}_k$ to minimize the second term.
- Note: We need $\mathbf{f}_k^\dagger \mathbf{f}_k = p_k$ to maximize the second term.
- We need to choose $\mathbf{f}_k$ to be the maximum generalized eigenvalued eigenvector of $\mathbf{H}_k^\dagger \mathbf{E}_k^{-2} \mathbf{H}_k$ and $1/p_k \mathbf{I} + \mathbf{H}_k^\dagger \mathbf{E}_k^{-1} \mathbf{H}_k$.
- Iterate over the users, minimizing the MSE from each user's perspective at each iteration.
- **Extension to multisymbol/user case: each symbol of each user $\Rightarrow$ *virtual user***

$K = 2$ **users,** $M_1 = M_2 = 2$**,** $N_T = 2, N_R = 4$

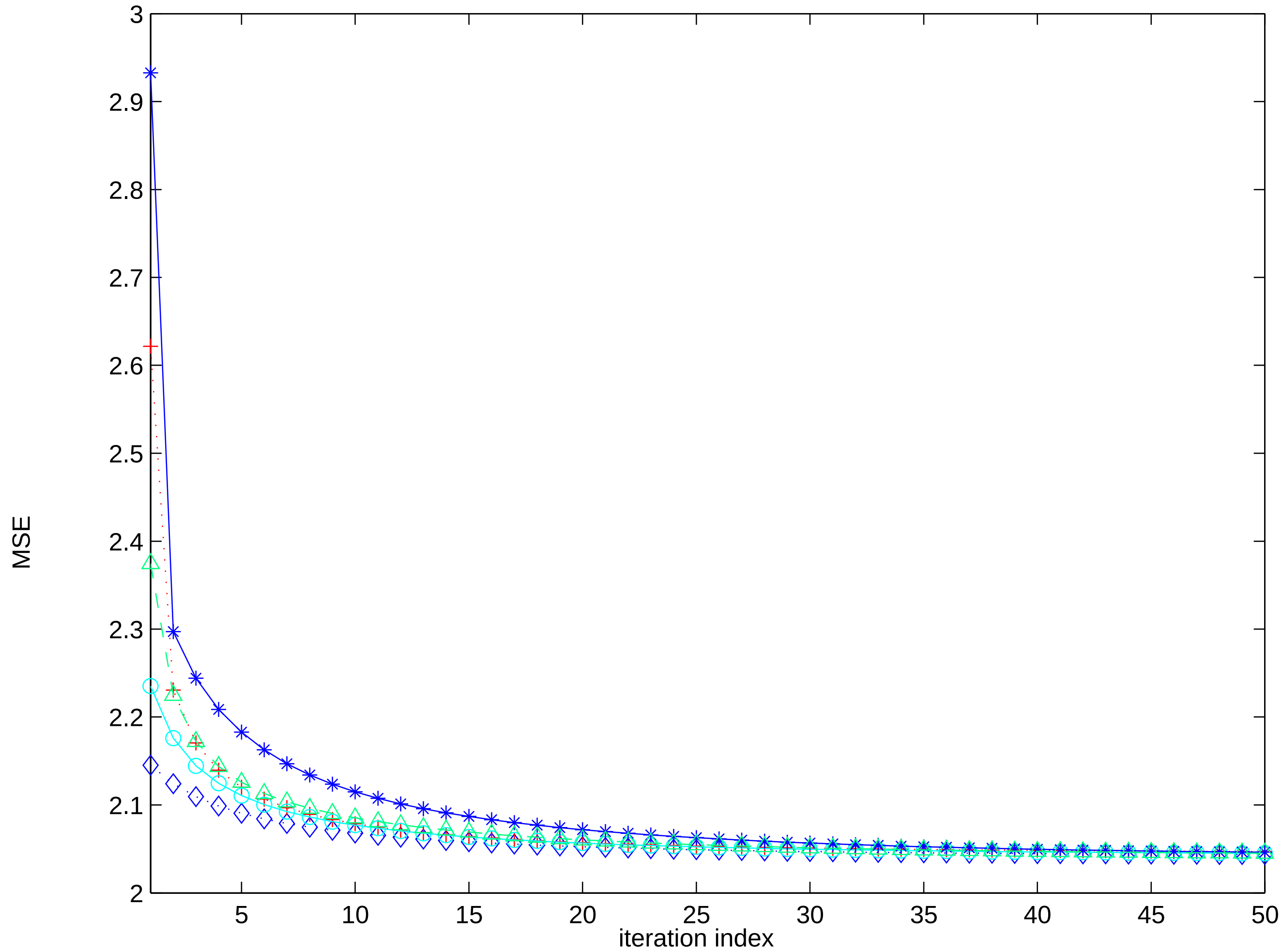


$K = 2$ users, $M_1 = M_2 = 2$, $N_T = N_R = 4$



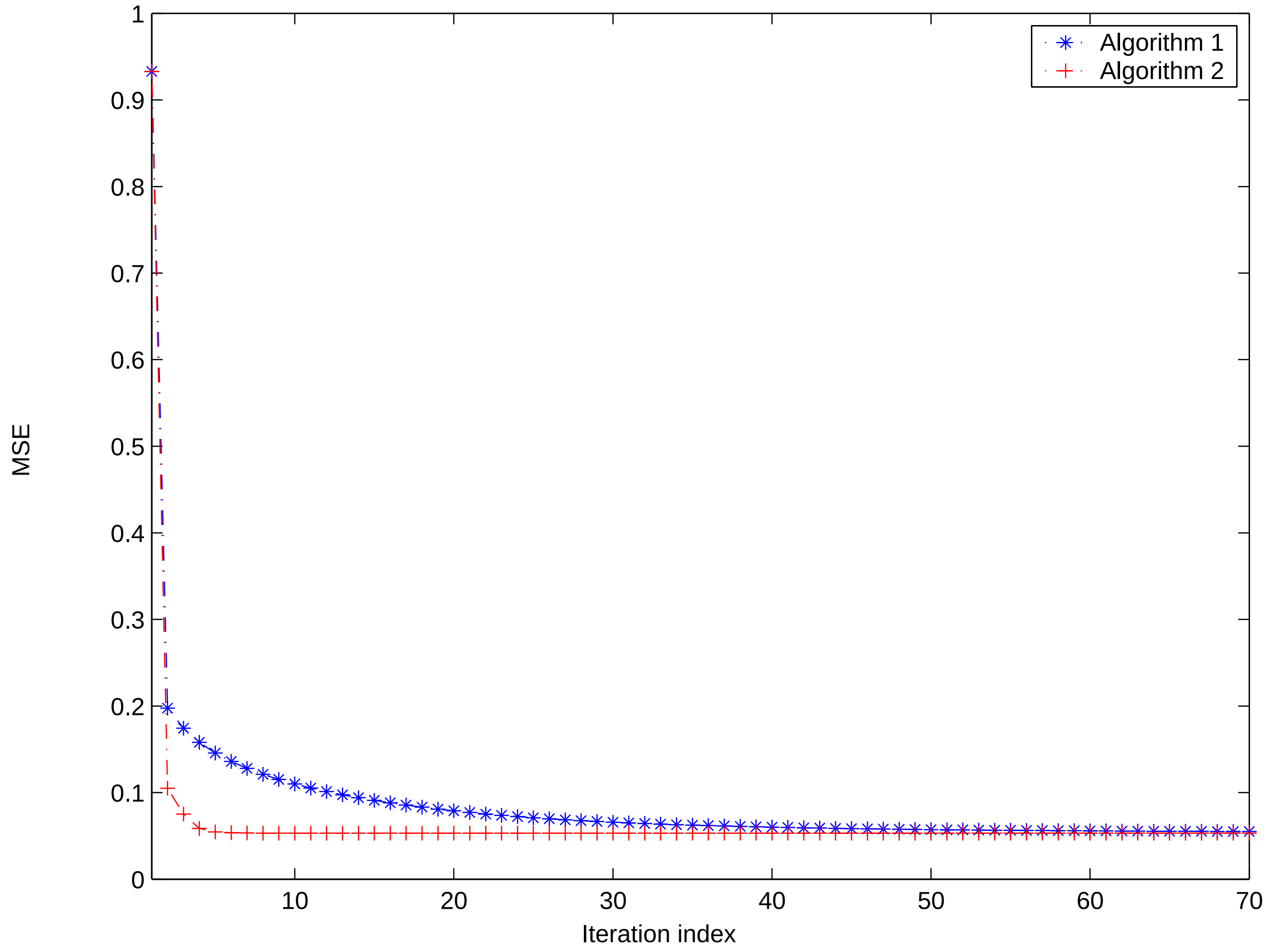
Different starting points

$K = 3$ **users,** $M_1 = M_2 = 2$**,** $N_T = N_R = 4$



Different starting points

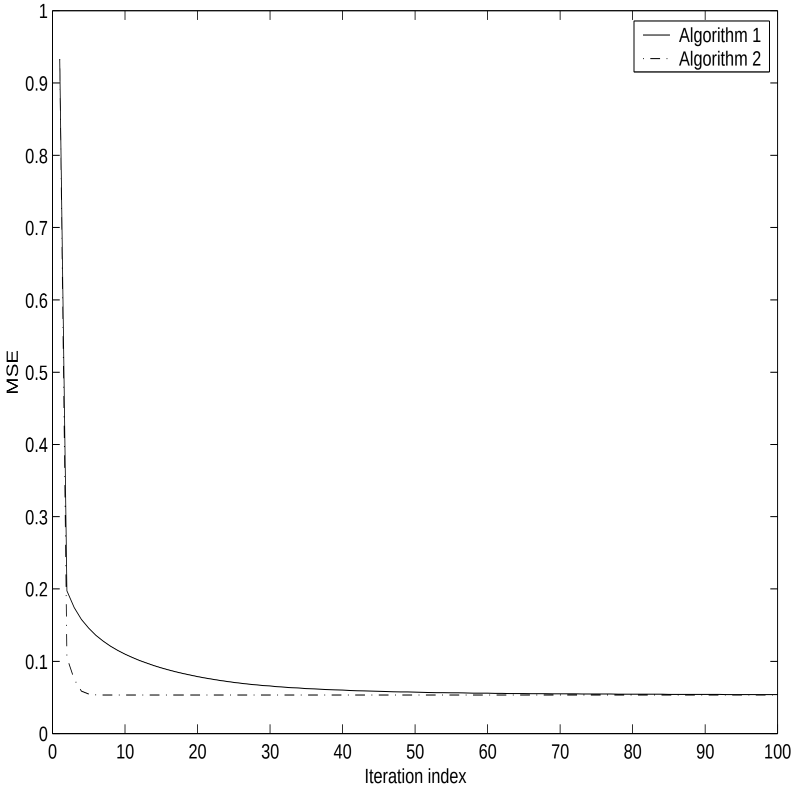
$K = 4$ **users,** $M_1 = M_2 = 1$**,** $N_T = N_R = 4$



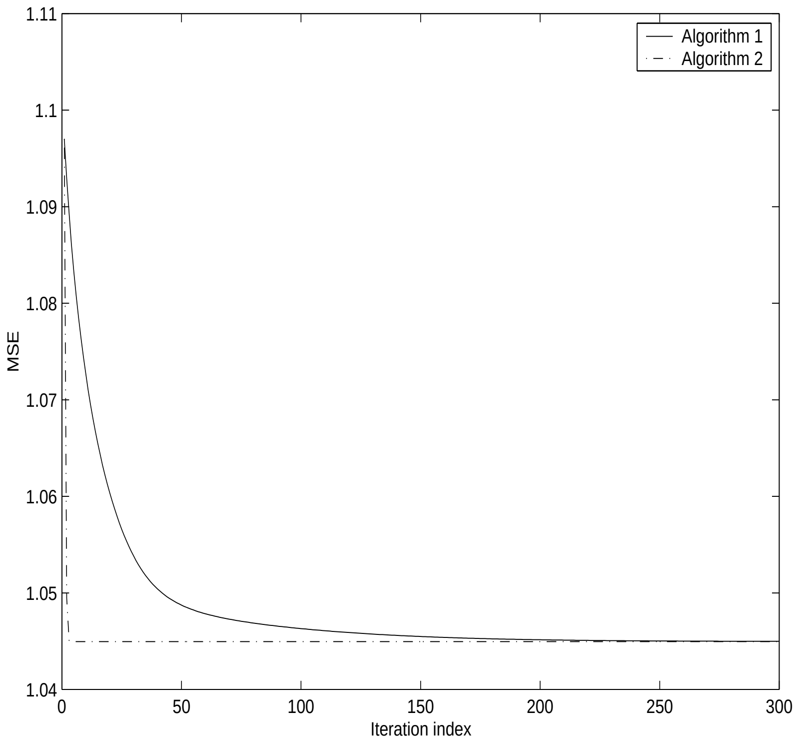
Comparison of the two algorithms

$M_1 = M_2 = 1, N_T = N_R = 4$

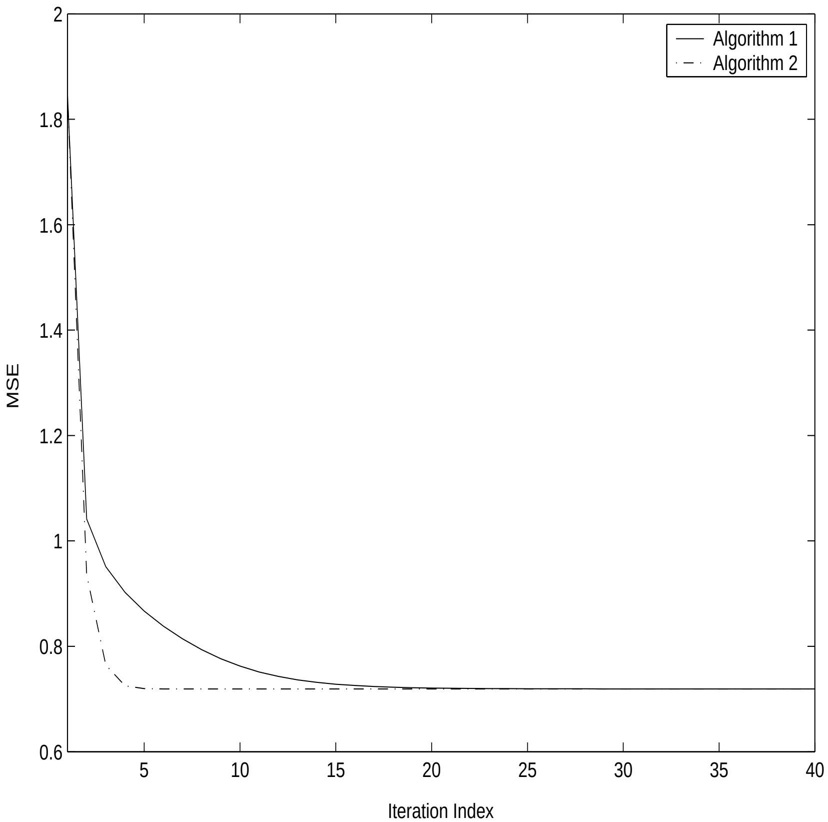
$K = 4$ users



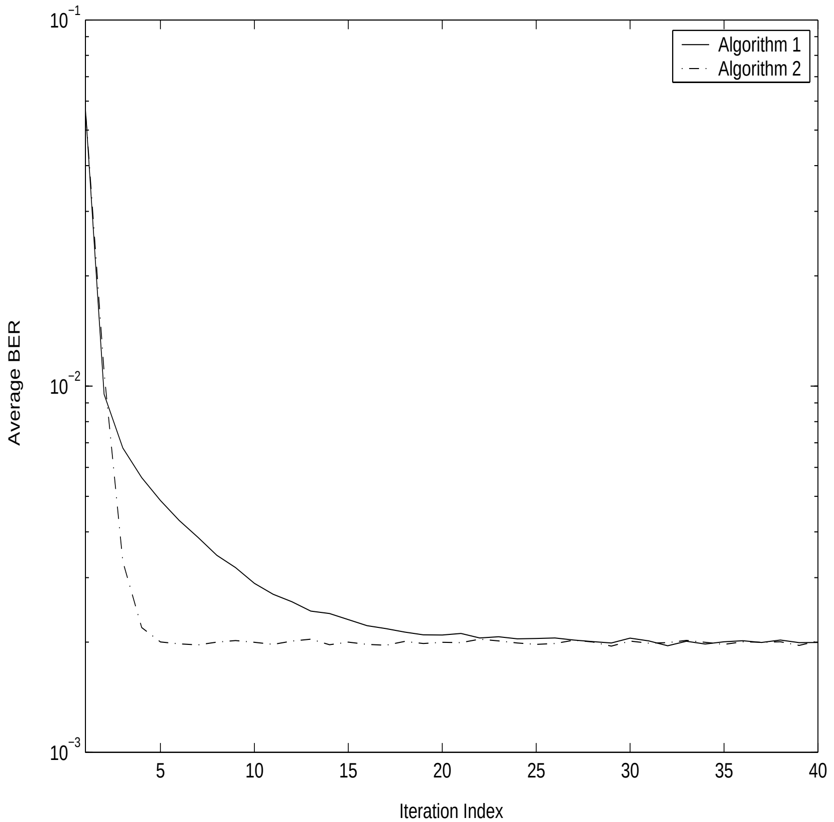
$K = 5$ users



$K = 4$ users, $M_1 = M_2 = 1$, $N_T = N_R = 4$

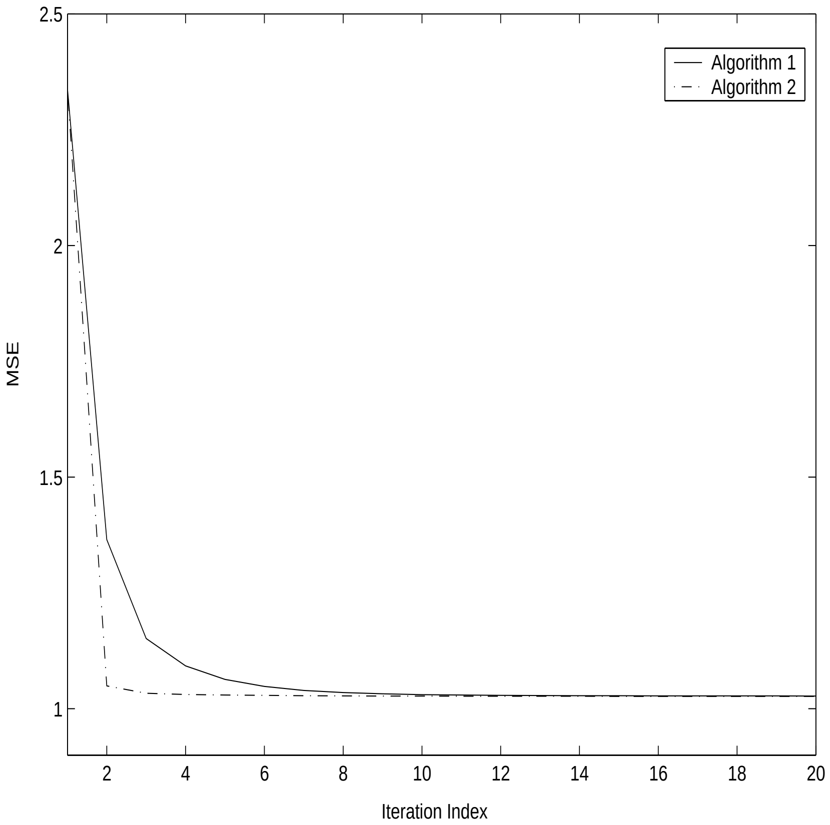


(a) MSE Analysis at each iteration.

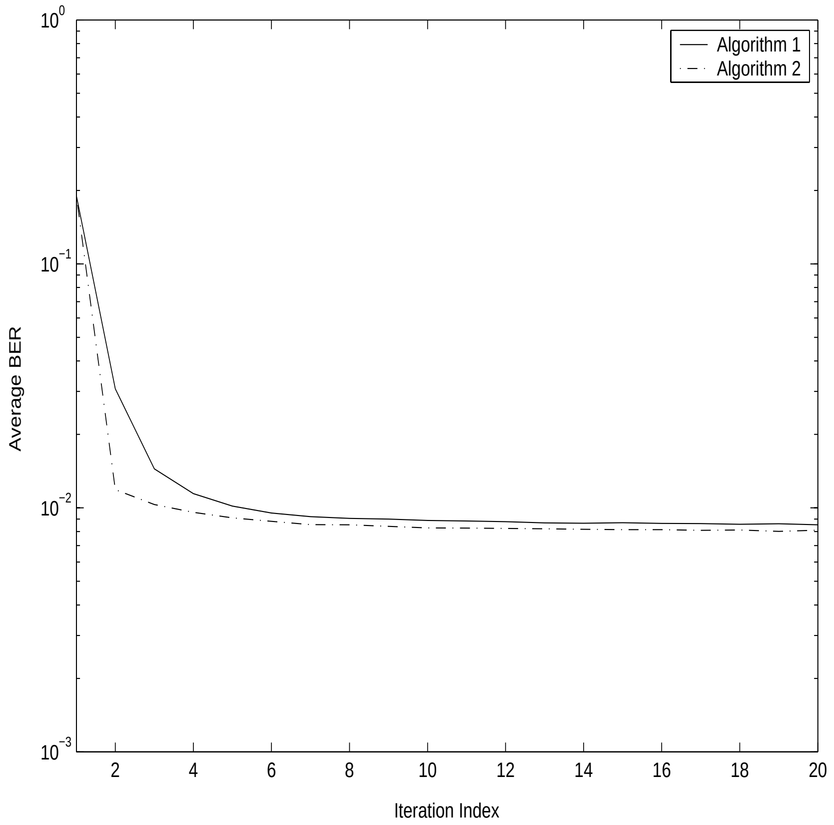


(b) BER Analysis at each iteration.

$K = 2$ users, $M_1 = M_2 = 2$, $N_T = N_R = 4$

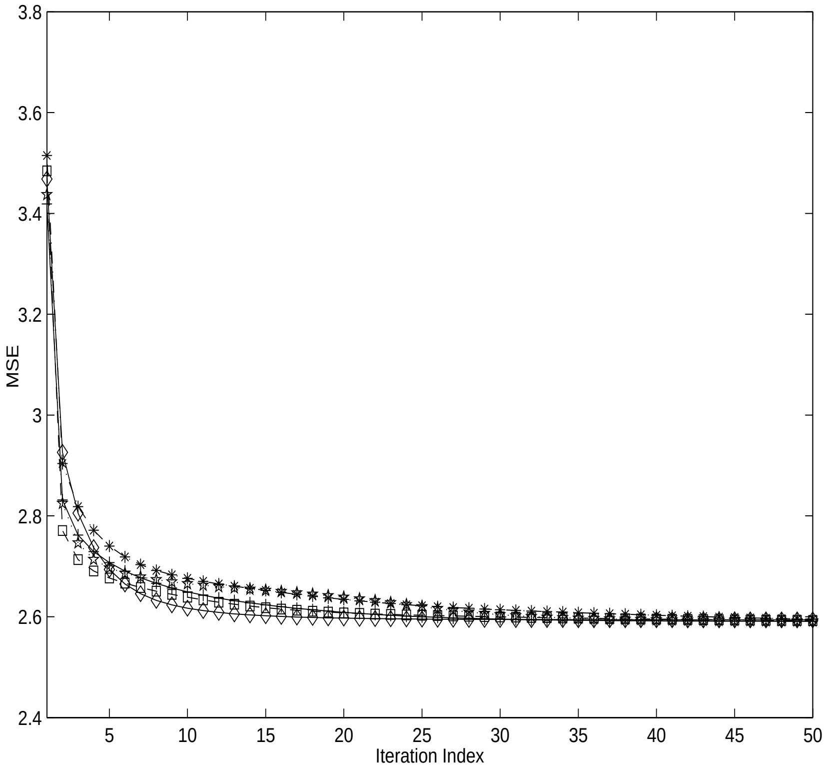


(a) MSE Analysis at each iteration.



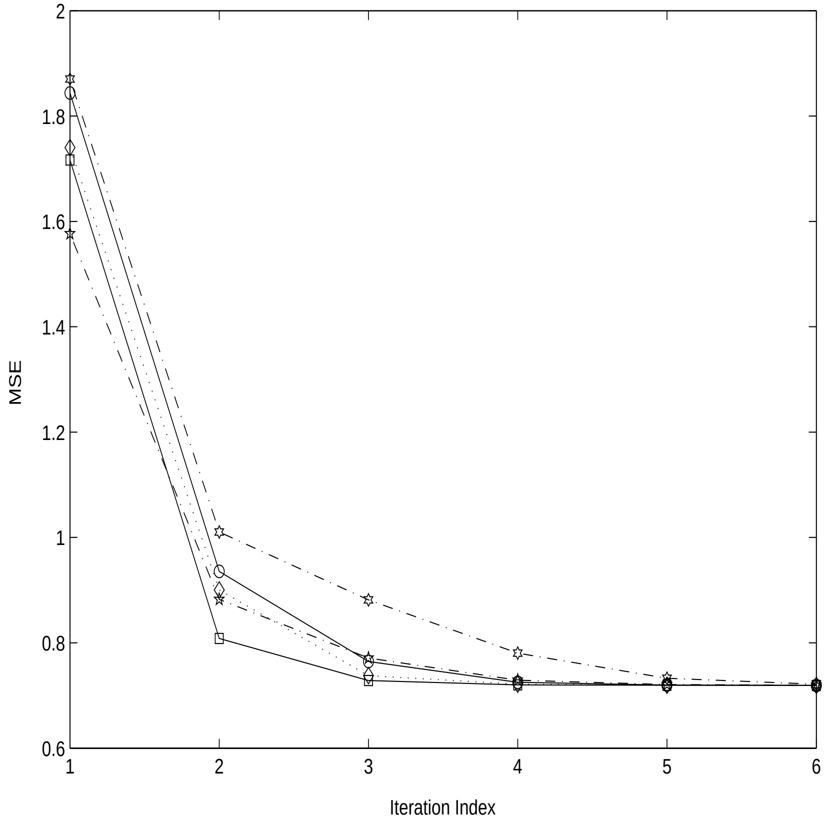
(b) BER Analysis at each iteration.

5 Different Starting Transmitter Sets



(a)$K = 3$, $M_i = 2$, and $N_T = N_R = 4$

Performance of Algorithm 1



(b)$K = 2$, $M_i = 2$, and $N_T = N_R = 4$

Performance of Algorithm 2

Summary

- Iterative transmitter-receiver update algorithms for multiuser MIMO systems
- Algorithms designed to minimize the system-wide MSE
- Fairly accurate optimality check available through convex relaxation of the problem.
- Single symbol case yields a greedy algorithm with faster convergence.
- Perfect feedback and CSI requirements

Simpler, more practical transmit schemes? $\Rightarrow$ Distribute complexity between physical and medium access layers

Time Slotted Multiuser MIMO Systems

- Scheduling and Beamformer Design
- Reuse the time slots
- Suppress interference of co-slot users by appropriate choice of beamformers
- Single symbol transmission
  - Less complex transceiver design
  - Practically implementable on current TDMA structures
  - Scheduling at the medium access layer by interacting with the PHY
  - Suboptimum in the information theoretical sense

Previous Work on Time Slotted Multiuser MIMO Systems

- Handover management, dynamic slot allocation [Shad et.al., Yunjian et.al.]
- Scheduling for maximum capacity in S/TDMA Systems is NP hard [Zhang]

Scheduling and Beamformer Design for Time Slotted Multiuser MIMO Systems

- Find the jointly optimum scheduling and beamformers that will maximize the **sum capacity**
  - Transmit power constraint for each user
  - No channel matrix constraints
  - Perfect CSI at the receiver
  - Various levels of feedback
  - Available feedback is error-free and low-delay
    - Perfect feedback
    - Limited feedback: Suppress interference of co-slot users by appropriate choice of beamformers with the available feedback
      - Antenna selection
      - Eigen mode selection

Communication Model

- N time slots, $K \leq NN_R$ users
- Each user transmits a weighted form of its signal
- The received signal at the ith time slot

$$\mathbf{r}_i = \sum_{j \in K_i} \sqrt{P_j}\mathbf{H}_j\mathbf{f}_j s_j + \mathbf{n}_i, \qquad i = 1, \ldots, N$$

- $\mathbf{n}_i$ is the zero mean Gaussian noise vector with $E[\mathbf{n}_i \mathbf{n}_i^{\dagger}] = \sigma^2 \mathbf{I}$
- $\mathbf{a}_j = \sqrt{P_j}\mathbf{H}_j\mathbf{f}_j$
- Received signal vectors at all time slots are represented by a long vector $\mathbf{r}$

$$\mathbf{r} = \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \vdots \\ \mathbf{r}_N \end{bmatrix} = \sum_{j=1}^{K} \begin{bmatrix} \mathbf{a}_j & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{a}_j & \vdots & \vdots \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{a}_j \end{bmatrix} \mathbf{t}_j s_j + \mathbf{n} = \sum_{j=1}^{K} \mathbf{A}_j \mathbf{t}_j s_j + \mathbf{n}$$

  where $\mathbf{t}_j = \mathbf{e}_i$ if $j \in K_i$

- Multiuser MIMO system with channel matrices $\{\mathbf{A}_j\}$ and transmit beamformers $\{\mathbf{t}_j\}$

Sum Capacity

- Sum capacity of K_i is

$$C_{K_i} = \frac{1}{2N}\log[\det(\mathbf{I}_{N_R} + \sigma^{-2}\sum_{j\in K_i}\mathbf{a}_j\mathbf{a}_j^\dagger)]$$

- The sum capacity optimization problem is

$$\max_{K_i} \quad C_{sum} = \sum_{i=1}^N C_{K_i} = \sum_{i=1}^N \frac{1}{2N}\log[\det(\mathbf{I}_{N_R} + \sigma^{-2}\sum_{j\in K_i}\mathbf{a}_j\mathbf{a}_j^\dagger)]$$

$$\text{s.t.} \quad \bigcup_{i=1}^N K_i = \{1,2,\ldots,K\}, \qquad K_i \bigcap K_l = \emptyset, \quad \forall i \neq l$$

- NP hard
- Derive upper bounds and compare the performance
- Define C_{upper}, C_{actual} and $C_{achieved}$

$$C_{achieved} \leq C_{actual} \leq C_{upper}$$

Sum Capacity Upper Bounds

- The sum capacity optimization problem is

$$\max_{\{\mathbf{R}_j\}_{j=1,\cdots,K}} \quad C_{sum} = \frac{1}{2N}\log[\det(\mathbf{I}_{NN_R} + \sigma^{-2}\textstyle\sum_{j=1}^{K}\mathbf{A}_j\mathbf{R}_j\mathbf{A}_j^{\dagger})]$$

$$\text{s.t.} \quad \mathbf{R}_j \in \{\mathbf{e}_1\mathbf{e}_1^{\dagger}, \mathbf{e}_2\mathbf{e}_2^{\dagger}, \ldots, \mathbf{e}_N\mathbf{e}_N^{\dagger}\} \qquad j = 1, 2, \ldots, K$$

- Relaxing different constraints results different upper bounds
  - Relaxing the rank constraint $\Rightarrow$ Sum capacity of multiuser MIMO systems (iterative waterfilling)

$$C_{actual} \leq \max_{\{\mathbf{R}_j | tr\{\mathbf{R}_j\} \leq 1\}_{j=1,\cdots,K}} C_{sum} = C_{upper1}$$

  - Relaxing the signal space constraint $\Rightarrow$ Sum capacity of an underloaded CDMA system

$$C_{actual} \leq C_{upper2} = \frac{1}{2N}\sum_{j=1}^{K}\log[1 + \sigma^{-2}\|\mathbf{a}_j\|^2]$$

- $C_{upper} = min(C_{upper1}, C_{upper2})$

Scheduling Strategy

- **Minimize the gap between the upper bound and the achieved sum capacity at each assignment**
- N step slot assignment algorithm
- Distance from the upper bound (C_{upper2}) from user k's perspective

$$C_{upper2} - C_{achieved} = \frac{1}{2}\log \frac{(1+\sigma^{-2}\|\mathbf{a}_k\|^2)}{(1+\mathbf{a}_k^{\dagger}(\sigma^2\mathbf{I}_{N_R} + \sum_{j\in\bar{K}_i^{(k)}} \mathbf{a}_j\mathbf{a}_j^{\dagger})^{-1}\mathbf{a}_k)} + \gamma_{ik}$$

where γ_{ik} represents the terms independent of user k.

- Choose the user with the highest

$$\frac{(1+\mathbf{a}_k^{\dagger}(\sigma^2\mathbf{I}_{N_R} + \sum_{j\in\bar{K}_i^{(k)}} \mathbf{a}_j\mathbf{a}_j^{\dagger})^{-1}\mathbf{a}_k)}{(1+\sigma^{-2}\|\mathbf{a}_k\|^2)} \approx \frac{\mathbf{a}_k^{\dagger}(\mathbf{I}_{N_R} + \sigma^{-2}\sum_{j\in\bar{K}_i^{(k)}} \mathbf{a}_j\mathbf{a}_j^{\dagger})^{-1}\mathbf{a}_k}{(\|\mathbf{a}_k\|^2)} = z_{ik}$$

- Fairness $\Rightarrow$ Assign no more than $\lceil \frac{\text{Number of users}}{N} \rceil$ users
- Maximum of N_R users can be assigned to the same time slot.

Scheduling Algorithm

System Parameters

K_a : Users that are not assigned to a time slot

K_i : The users that are assigned to time slot i, i=1,...N

N_a : Available time slots that are not assigned to users

$\{\mathbf{a}_j\}$: Effective spatial signatures of users

Avuser : Av. number of users per remaining time slots

Scheduling Algorithm

$K_a = \{user-1, user-2, \ldots, user-K\}$

$N_a = \{1, 2, \ldots, N\}$

For $i = 1 : N$

User Selection for time slot i

$Avuser = \lceil \frac{n(K_a)}{n(N_a)} \rceil$

For $j = 1 : Avuser$

$$k^\star = \arg\max_{k \in K_a} z_{ik}$$

$$K_i = K_i \bigcup \{user - k^\star\}$$

$$K_a = K_a \backslash \{user - k^\star\}$$

End

$N_a = N_a \backslash \{i\}$

End

Combined Beamformer Design and Scheduling

- Performance depends on the choice of the beamformers
- Feedback level
- Antenna selection
  - Maximize the received power of each user

$$\mathbf{a}_k = \underset{m \in \{1,2,\ldots,N_T\}}{\arg\max} \sqrt{P_k} ||\mathbf{h}_{km}||$$

  where $\mathbf{h}_{km}$ is the mth column vector of user k's channel matrix
  - Selection diversity $\Rightarrow$ Choose the best performing transmitter antenna
- Individual CSI
  - Maximize the received power of each user

$$\mathbf{f}_k = \underset{\mathbf{u}_{km} | m \in \{1,2,\ldots,N_T\}}{\arg\max} \mathbf{u}_{km}^{\dagger} \mathbf{H}_k^{\dagger} \mathbf{H}_k \mathbf{u}_{km}; \quad \mathbf{a}_k = \sqrt{P_k} \mathbf{H}_k \mathbf{f}_k$$

  where $\mathbf{u}_{km}$ is the mth eigenvector of $\mathbf{H}_k^{\dagger} \mathbf{H}_k$
  - Selection diversity $\Rightarrow$ Choose the best performing eigenmode

Beamformer Design with Perfect Feedback

- Perfect feedback
- Performance metric in terms of beamformers is

$$z_{ik} = \frac{\mathbf{f}_k^\dagger \mathbf{H}_k^\dagger (\mathbf{I}_{N_R} + \sigma^{-2} \sum_{j \in \bar{K}_i^{(k)}} \mathbf{a}_j \mathbf{a}_j^\dagger)^{-1} \mathbf{H}_k \mathbf{f}_k}{\max_{\{\mathbf{f} | \mathbf{f}^\dagger \mathbf{f} = 1\}} \mathbf{f}^\dagger \mathbf{H}_k^\dagger \mathbf{H}_k \mathbf{f}}$$

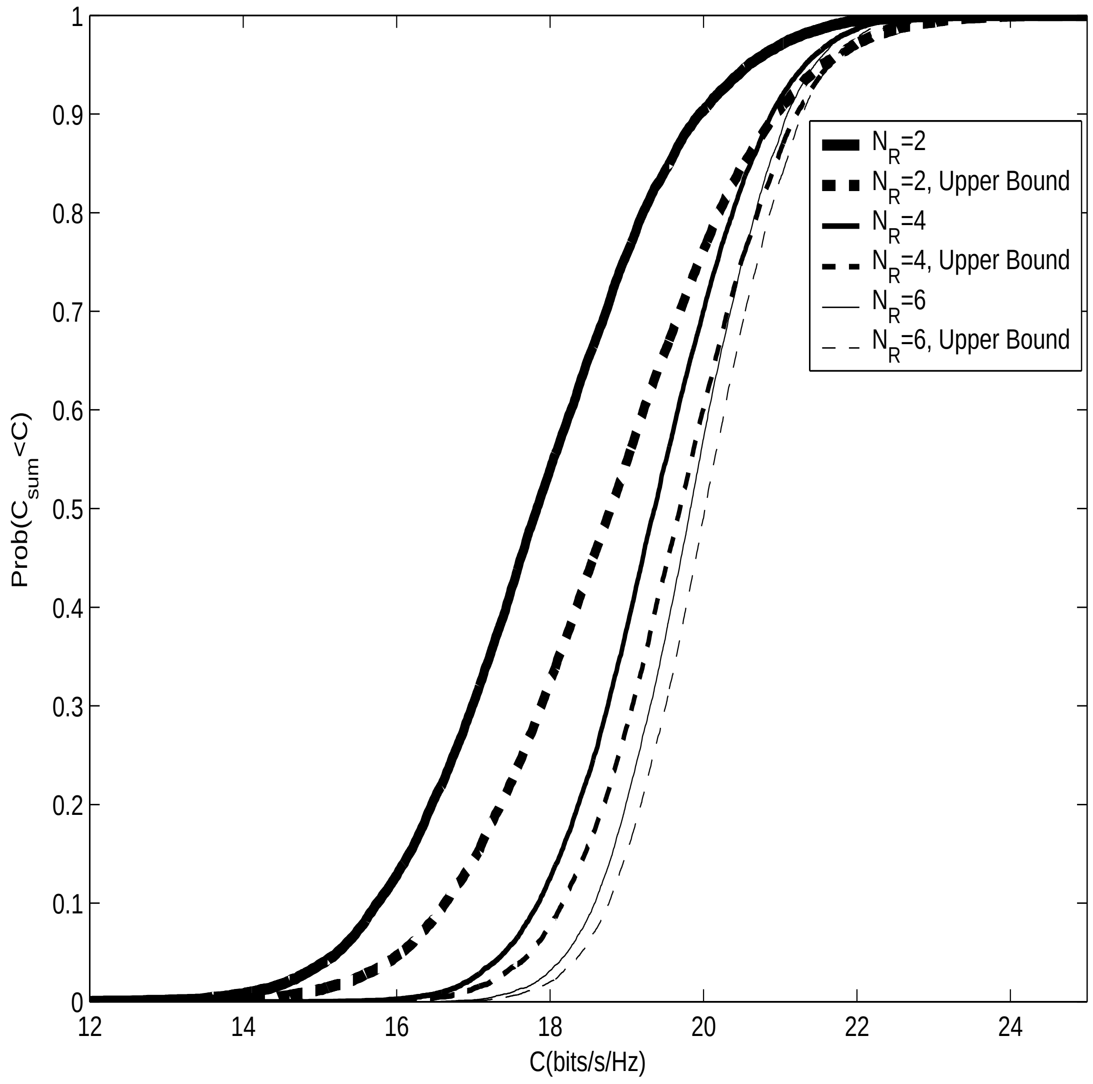
- Compare the performance metric for each user with the best performing beamformers
- Best beamformer for each user is

$$\mathbf{f}_k = \arg\max_{\{\mathbf{f} | \mathbf{f}^\dagger \mathbf{f} = 1\}} \mathbf{f}^\dagger \mathbf{H}_k^\dagger (\mathbf{I}_{N_R} + \sigma^{-2} \sum_{j \in \bar{K}_i^{(k)}} \mathbf{a}_j \mathbf{a}_j^\dagger)^{-1} \mathbf{H}_k \mathbf{f}$$

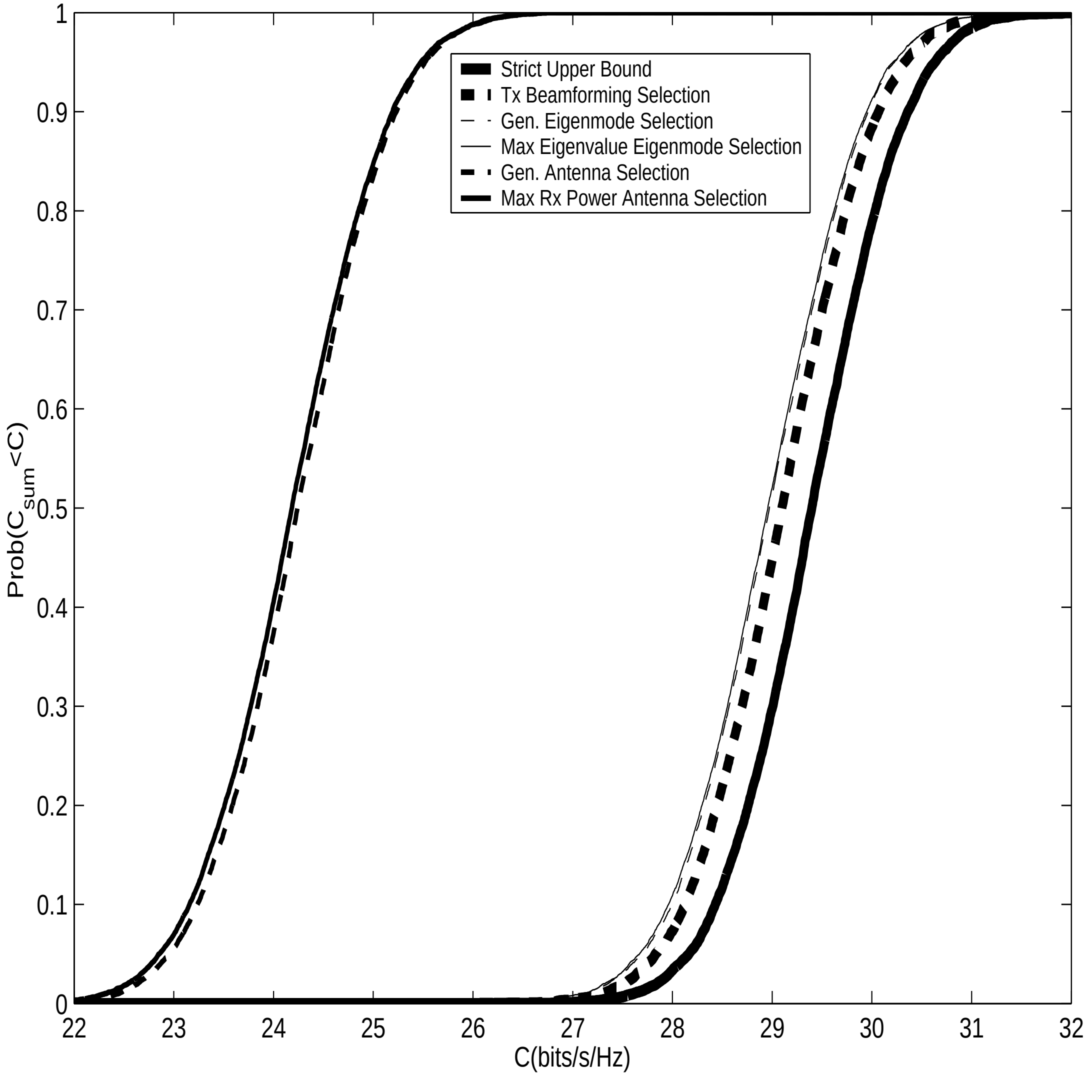
Numerical Results

- Time slotted Multiuser MIMO System with K=16 users and N=8
- Various levels of feedback
- $SNR = 7dB$
- Independent identically distributed complex Gaussian channel realizations
- CDF curves of the sum capacity obtained by 10000 channel realizations

Scheduling Algorithm for $N_T = 1$ and $N_R = 2, 4, 6$



Combined Scheduling and Beamformer Design Schemes for $N_T = 4$ and $N_R = 4$



Summary

- Scheduling and beamformer design algorithms for time slotted multiuser MIMO systems
- Various feedback levels
- Algorithms are designed to maximize the sum capacity
- Scheduling and beamformer design with perfect feedback performs the best
  - High feedback requirements
- **Individual CSI facilitates a substantial gain**

Channel State Information Accuracy

- All algorithms we have presented so far rely on accurate CSI!
- Transceiver (precoder+decoder) design should consider the errors in the channel estimation process
- There is a resource allocation trade-off between channel estimation and symbol transmission.
  - If a limited total power budget is available for the information transfer, how much of it should be allocated to training?
- Our recent results for single MIMO link show that (CISS'04)
  - MMSE transceiver structures that take the statistics of the estimation errors perform better
  - Transceiver structures as well as transmission rate should be designed with the estimation accuracy in mind
  - An optimum power allocation that partitions the total power budget between training and data transmission exists and is a function of the channel coherence time.
- Insights readily generalize to MIMO MAC

Power Allocation Problem

- Limited total power budget
- ML estimate of the channel, $\overline{\mathbf{H}}$, is available at the receiver and the transmitter
- Channel is constant for $N_T + L$ symbols
- Minimizing MSE is equivalent to

$$\min \text{MSE} \equiv \min(\text{tr}\{\left(\mathbf{I} + \frac{\rho^2}{\rho_1}\overline{\mathbf{H}}\mathbf{F}\mathbf{F}^\dagger\overline{\mathbf{H}}^\dagger\right)^{-1}\})$$

- Normalizing the expressions, $\overline{\mathbf{H}}$ and $\mathbf{F}\mathbf{F}^\dagger$, we have the effective SNR:

$$\rho_e = \frac{\rho^2 P_s(\sigma_H^2 + \sigma_e^2)}{\rho_1}$$

- To improve overall performance ρ_e should be maximized.
- $\alpha = \frac{P_s L}{P_{total}}$ and $c = \frac{(N_T - L)P_{total}\sigma_H{}^2}{LN_T\sigma^2 + LP_{total}\sigma_H{}^2}$:

$$\rho_e = \frac{P_{total}^2\sigma_H{}^4}{\sigma^2 L(\sigma_H{}^2 P_{total} + N_T\sigma^2)}\frac{\alpha(1-\alpha)}{c\alpha+1} = Kf(\alpha)$$

Optimum Power Allocation

- Function to be maximized

$$f(\alpha) = \frac{\alpha(1-\alpha)}{c\alpha + 1}$$

- **Theorem 1** *The maximizer of* $f(\alpha)$ *always lies in* $[0,1]$. α_{opt} *and corresponding* ρ_e *is given by*

$$\alpha_{opt} = \begin{cases} \frac{-1+\sqrt{1+c}}{c}, & \text{for } N_T > L; \\ \frac{1}{2}, & \text{for } N_T = L; \\ \frac{-1+\sqrt{1+c}}{c}, & \text{for } N_T < L; \end{cases}$$

$$\rho_e = \begin{cases} \frac{P_{total}^2 {\sigma_H}^4}{4\sigma^2 L({\sigma_H}^2 P_{total} + N_T \sigma^2)}, & \text{for } N_T = L; \\ \frac{P_{total}^2 {\sigma_H}^4}{\sigma^2 L({\sigma_H}^2 P_{total} + N_T \sigma^2)} \left(\frac{\sqrt{1+c}-1}{c}\right)^2, & \text{for } N_T \neq L; \end{cases}$$

Observations

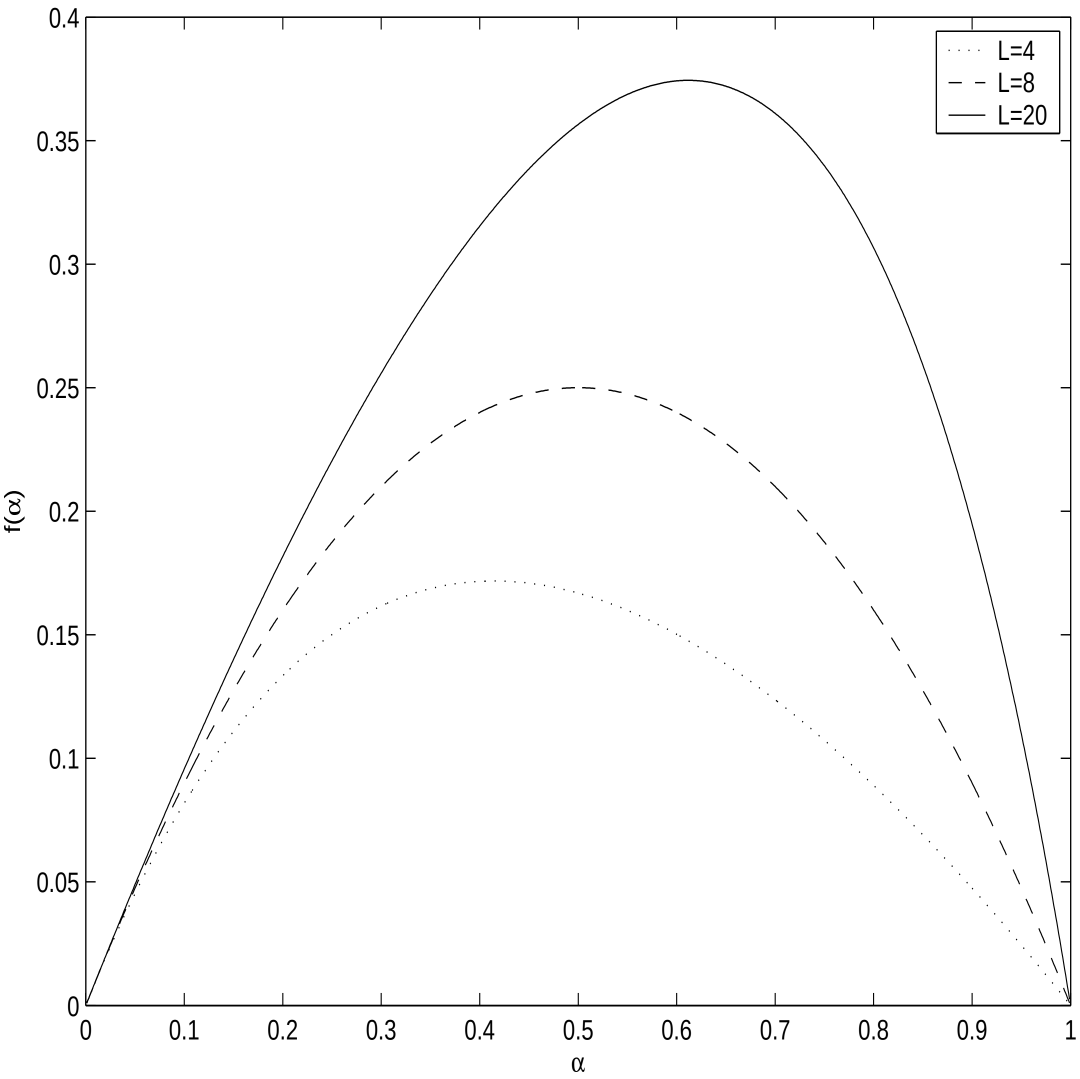
- For $N_T > L$,
  - $\alpha_{opt} \in [0, \frac{1}{2}) \Rightarrow$ More power to the training sequences
  - When $N_T \longrightarrow \infty$, then $c \longrightarrow \frac{P_{total}{\sigma_H}^2}{\sigma^2 L}$
- For $N_T < L$,
  - $\alpha_{opt} \in (\frac{1}{2}, 1] \Rightarrow$ More power to the data transmission
  - When $L \longrightarrow \infty$, then $c \longrightarrow \frac{-P_{total}{\sigma_H}^2}{P_{total}{\sigma_H}^2 + N_T \sigma^2}$
- For high SNR,

$$c = \frac{N_T - L}{L} \Rightarrow \alpha_{opt} = \frac{\sqrt{L}}{\sqrt{N_T} + \sqrt{L}}, \qquad \rho_e = \frac{P_{total}{\sigma_H}^2}{\sigma^2} \frac{1}{(\sqrt{N_T} + \sqrt{L})^2}$$

- For low SNR,

$$c = 0 \Rightarrow \alpha_{opt} = \frac{1}{2}, \qquad \rho_e = \frac{P_{total}^2 {\sigma_H}^4}{4\sigma^4 N_T L}$$

$f(\alpha)$ vs α for 8×8 MIMO System with $L = 4, 8, 20$, $P_{total} = 100$, and $\sigma^2 = 0.05$



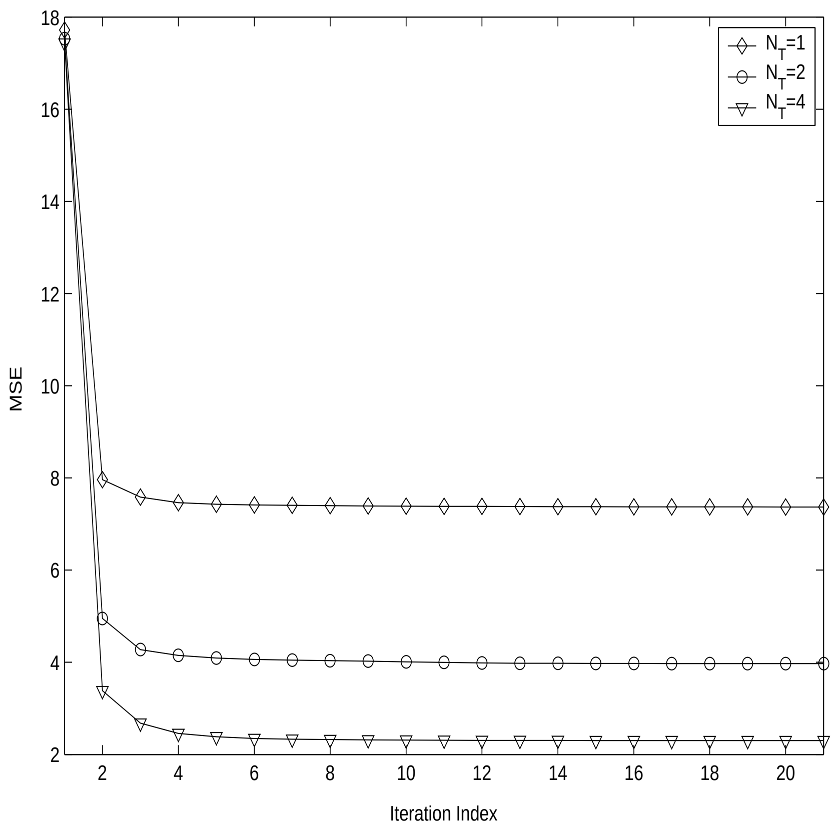
MIMO CDMA Systems: Current Work and Directions

- Transmit shaping helps improve the performance for CDMA
- Temporal-Spatial transmitter design
- Algorithms that iterate over each user's signature and beamformer: similar in sprit to algorithms presented in the first part of this talk (CISS'03)
- Orthogonal signatures can be reused by designing appropriate beamformers: similar in sprit to joint scheduler and beamformer design presented in the second part of this talk (CISS'04)
- The problem of complete characterization of optimum temporal signatures is open (Preliminary results in ICC'04)
- The problem of finding optimum strategies for fading MIMO CDMA is open

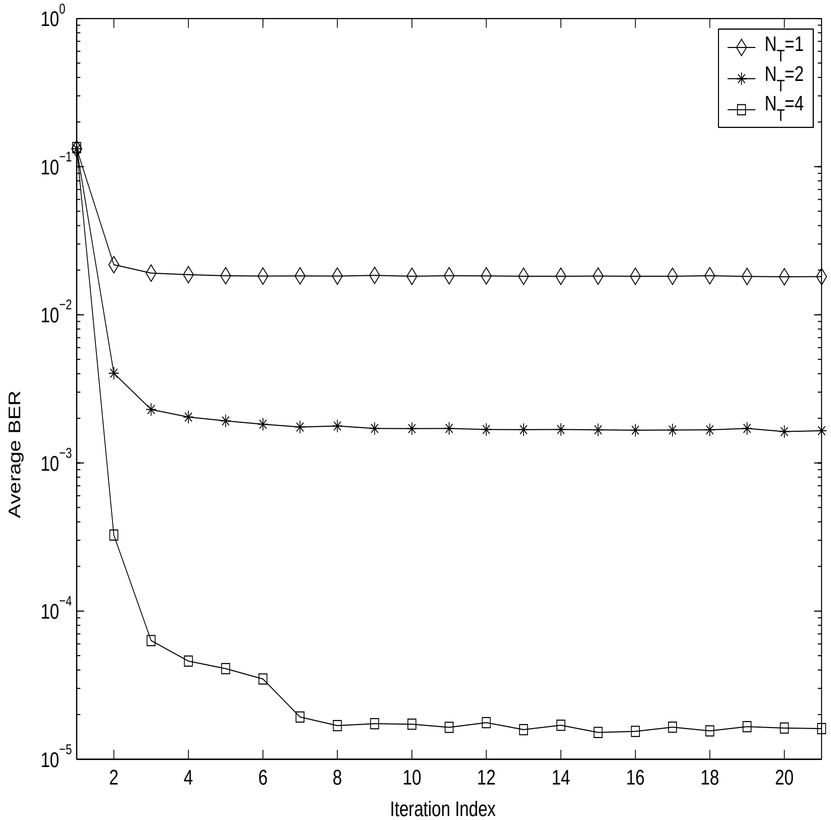
WCAN@Penn State Web Site:

http://labs.ee.psu.edu/labs/wcan

MIMO CDMA System with K=30 N=16 N_R=2 and $N_{T_i} = 1, 2, 4$



(a) MSE analysis at each iteration



(b) Av. BER analysis at each iteration