Transmitter Optimization for Multiuser MIMO Systems

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Multiple transmit and receive antennas are a means to increase wireless capacity [1, 2]. Recently, there has been considerable research in exploiting the space dimension through transmit diversity, and spatial multiplexing which is used to transmit multiple data streams that can be separated using receiver signal processing, e.g., [3–5].

Spatial multiplexing can significantly benefit from transmit precoding when the channel is known at the transmitter side in addition to the receiver side. In such cases, designing the appropriate precoding strategy has been studied under a variety of system objectives, see [5] and references therein. All of these studies, as most of the MIMO system analysis, have been done for a single user system that transmits multiple data streams. Optimum or near optimum transmit strategies that maximize the information theoretic sum capacity of vector multiple access channels have been investigated recently [6, 7].

Transmit and receiver beamforming for multiuser systems when each user is transmitting a single data stream have also been studied extensively up to date, see [8] and references therein. Algorithms that identify transmit and receiver beamforming strategies and the corresponding transmit power assignments are investigated in [8] with the aim of maximizing the minimum achievable SIR or providing each user with its SIR target. The algorithms suggested were numerically shown to enhance system performance, but were observed to converge to local optima.

Our aim in this work is to design algorithms that converge to the optimum transmitters and receivers for all users in a multiuser MIMO system, when users transmit possibly multiple data streams. We assume linear precoders and decoders for all users. We work with a system-wide performance measure for the joint optimization of precoders and decoders, namely the system-wide mean squared error (MSE). In contrast to receiver optimization for fixed transmitters, e.g. in [9], optimization of the individual MSEs is not equivalent to total MSE optimization. However, one can construct iterative algorithms that monotonically decrease the total MSE under the given system constraints. The proposed algorithms converge to the best precoder-decoder pairs under the given power constraints, and result in enhanced performance for all users.

We consider the uplink of a single cell synchronous system with K users. The receiver employs N_R antennas. We assume that the ith user multiplexes M_i data streams through its N_{T_i} transmit antennas employing a linear precoder. Similar to the notation in [5], the received vector is

$$\mathbf{r} = \sum_{j=1}^{K} \mathbf{H}_j \mathbf{F}_j \mathbf{s}_j + \mathbf{n} \tag{1}$$

where, $\mathbf{s}_i$ is the $M_i \times 1$ symbol vector, $\mathbf{H}_i$ is the $N_R \times N_{T_i}$ matrix of channel gains, $\mathbf{F}_i$ is the linear precoder and $\mathrm{tr}\{\mathbf{F}_i^\dagger \mathbf{F}_i\} \leq p_i$ is the transmit power constraint for user i; $\mathbf{n}$ is the zero mean Gaussian noise vector with $E[\mathbf{n}\mathbf{n}^\top] = \sigma^2 \mathbf{I}$. The linear receiver (decoder) for user i is denoted by $\mathbf{G}_i$. The decision statistic $\mathbf{y}_i$ is given by

$$\mathbf{y}_i = \mathbf{G}_i \left(\sum_{j=1}^{K} \mathbf{H}_j \mathbf{F}_j \mathbf{s}_j + \mathbf{n} \right) \tag{2}$$

In this work, we aim to design precoder-decoder pairs that minimize the system-wide MSE. The MSE incurred by user i, $\mathrm{MSE}_i = E\left[\|\mathbf{y}_i - \mathbf{s}_i\|^2\right]$ is given by

$$\mathrm{tr}\left\{ \sum_{j=1}^{K} \mathbf{F}_j^\dagger \mathbf{H}_j^\dagger \mathbf{G}_i^\dagger \mathbf{G}_i \mathbf{H}_j \mathbf{F}_j - \mathbf{F}_i^\dagger \mathbf{H}_i^\dagger \mathbf{G}_i^\dagger - \mathbf{G}_i \mathbf{H}_i \mathbf{F}_i + \mathbf{I} + \sigma^2 \mathbf{G}_i \mathbf{G}_i^\dagger \right\} \tag{3}$$

where $\mathrm{tr}(\mathbf{A})$ denotes the trace of matrix $\mathbf{A}$. The total MSE of all users in the system, $\mathrm{MSE} = \sum_{i=1}^{K} \mathrm{MSE}_i$, then becomes

$$\mathrm{tr}\left\{ \sum_{i=1}^{K} \sum_{j=1}^{K} \mathbf{F}_j^\dagger \mathbf{H}_j^\dagger \mathbf{G}_i^\dagger \mathbf{G}_i \mathbf{H}_j \mathbf{F}_j - \mathbf{F}_i^\dagger \mathbf{H}_i^\dagger \mathbf{G}_i^\dagger - \mathbf{G}_i \mathbf{H}_i \mathbf{F}_i + \mathbf{I} + \sigma^2 \mathbf{G}_i \mathbf{G}_i^\dagger \right\} \tag{4}$$

Total MSE minimization by choosing the transmitters and receivers has recently been studied for synchronous CDMA systems with single antennas in the context of CDMA signature optimization [10]. This performance measure is desirable to work with in transmitter (precoder) optimization, in contrast with each user minimizing its own MSE as is adapted in receiver optimization [9]. This is because the choice of the transmitter of a user affects the MSE of each user in the system. In the following, we pose the problem of minimizing the total MSE in the presence of power constraints, and devise iterative algorithms that reach the solution.

To solve the optimization problem of minimizing the MSE subject to a *transmit* power constraint for each user, we first formulate the Lagrangian dual objective. Optimum precoder and decoder structures should satisfy the KKT conditions for each user, k, which are easily found. The KKT conditions result in the following structure for the precoder and decoder for user k:

$$\mathbf{G}_k = \mathbf{F}_k^\dagger \mathbf{H}_k^\dagger \left(\sigma^2 \mathbf{I} + \sum_{i=1}^{K} \mathbf{H}_i \mathbf{F}_i \mathbf{F}_i^\dagger \mathbf{H}_i^\dagger \right)^{-1} \quad (5)$$

$$\mathbf{F}_k = \left(\mu_k \mathbf{I} + \sum_{i=1}^{K} \mathbf{H}_k^\dagger \mathbf{G}_i^\dagger \mathbf{G}_i \mathbf{H}_k \right)^{-1} \mathbf{H}_k^\dagger \mathbf{G}_k^\dagger \quad (6)$$

where $\mu_k \geq 0$ is the Lagrange multiplier associated with the transmitter power constraint of user k. Note that, as expected, the optimum decoders (receivers) for a given set of precoders are in the form of the well-known *MMSE receivers* [9]. Note also that the precoders are functions of decoders of all users, while the decoders are functions of precoders of all users. To find the joint optimum set of precoders and decoders, one can devise iterative algorithms that monotonically decrease the total MSE. In particular, alternating minimization where variables are optimized one at a time, keeping all others fixed, proves attractive in the design of iterative algorithms [11]. Equations (5) and (6) describe the precoder-decoder updates we can perform. The algorithm starts with a given set of precoders-decoders and we can update the decoders $\{\mathbf{G}_k\}$ and precoders $\{\mathbf{F}_k\}$ independently in a parallel fashion using (5) and (6). Note that at each iteration, μ_k in (6) should be calculated such that the transmit power constraint is satisfied.

Alternatively, if we assume that each receiver (decoder) is updated instantaneously when the precoder is updated, we can reduce the two step iteration given by (5), (6) to a single iteration. This is accomplished by inserting the resulting decoders of (5) in (6). Define

$$\mathbf{T} = \sigma^2 \mathbf{I} + \sum_{i=1}^{K} \mathbf{H}_i \mathbf{F}_i \mathbf{F}_i^\dagger \mathbf{H}_i^\dagger \quad (7)$$

Then, following some straightforward algebra, we arrive at the following iteration:

$$\mathbf{F}_k^\star = \left(\mu_k \mathbf{I} + \mathbf{H}_k^\dagger (\mathbf{T}^{-1} - \sigma^2 \mathbf{T}^{-2}) \mathbf{H}_k \right)^{-1} \mathbf{H}_k^\dagger \mathbf{T}^{-1} \mathbf{H}_k \mathbf{F}_k \quad (8)$$

Note that the above update can be executed in a parallel fashion over all the users. The algorithm decreases the MSE converging to the minimum.

Although the algorithms given in this paper are geared towards MSE optimization with transmit power constraints, received power constraints, i.e., $\mathrm{tr}\{\mathbf{F}_i^\dagger \mathbf{H}_i^\dagger \mathbf{H}_i \mathbf{F}_i\} \leq p_i$, as opposed to transmit power constraints, $\mathrm{tr}\{\mathbf{F}_i^\dagger \mathbf{F}_i\} \leq p_i$, for user i, can easily be accommodated using the same methodology.

A sample run of the iterative algorithm described by (8) is given in Figure 1. The figure shows the evolution of the total MSE versus the iteration index of the algorithm, where an iteration signifies updating all users' precoders, i.e., K updates. The sample system is a $K = 2$ user system where each user transmits $M = 2$ data streams. Each user is equipped with $N_{T_i} = 4$, for all i, transmit antennas and the receiver has $N_R = 4$ antennas. It is observed that the total MSE monotonically decreases and converges to its minimum value.

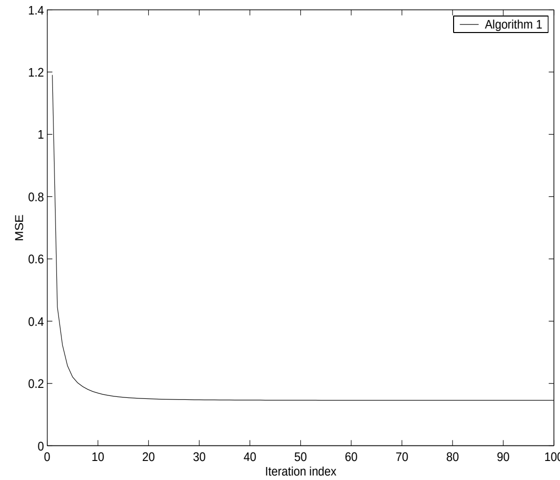

Figure 1: $K = 2$ user MIMO system with $M = 2$ data streams per user. $N_{T_i} = N_R = 4$

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