Signature and Beamformer Design for MIMO-CDMA with Various Levels of Feedback

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***Abstract*— We investigate the signature and beamformer design problem for multiple antenna CDMA (MIMO-CDMA) systems with sum capacity as the performance metric. We first find upper bounds for the sum capacity by relaxing a set of structural constraints. We construct an iterative algorithm to find the jointly optimum temporal signatures and transmit beamformers under the assumption of perfect temporal signature and transmit beamforming feedback. Next, motivated by milder feedback requirements, a low complexity orthogonal temporal signature assignment algorithm is presented that aims to approach the capacity upper bound for given transmit beamformers. The transmit beamformers can be shaped depending on the channel state information available at the transmitter. We next investigate the cases of various different levels of feedback, and combine the proposed orthogonal temporal signature assignment algorithm with antenna selection, eigen transmit beamforming and perfectly controlled transmit beamforming models. We observe that as the available feedback level is increased, the performance of the algorithms approach the upper bounds developed. In particular, a substantial sum capacity gain is attained when the individual channel state information is available at the transmitter side.**

I. Introduction

Considering the rapidly increasing demand for high data rate and reliable wireless communications, spectrally efficient multiuser transmission schemes are of great importance for next generation wireless multiaccess systems. CDMA is a promising candidate to meet these challenges. However, CDMA systems are known to suffer from multiaccess interference that degrades the performance. Recent studies indicate that using multiple antennas at the transmitter and receiver can dramatically increase the performance of the wireless communication systems [1]. While existing interference cancellation methods such as multiuser detection and interference avoidance exploit the temporal structure of the system [2], the use of multiple antennas provides additional substantial gain to suppress interference by exploiting the spatial structure of the system [3].

Performance of a multiuser system is highly dependent on the transmission scheme of each user. Thus, feedback to the transmitter side is an important factor that can be exploited to improve the performance of a multiuser system by coordinating the transmission of the users. For single antenna CDMA systems, there have been a number of transmitter design methods proposed to date, e.g. [4], [5]. Similarly, for multiuser multiple input multiple output (multiuser MIMO) systems, feedback to the transmitter side can be used to coordinate the transmission of each user [6]. Most of previously proposed transmission schemes for multiuser multi-antenna CDMA (MIMO-CDMA) systems rely on assigning orthogonal signature sequences to the antennas/symbols without reusing the signatures [7], [8]. This model is not appropriate for overloaded systems, i.e., when there are more symbols than the processing gain. In the case of a MIMO-CDMA system where users' transmissions interfere with each other, jointly optimum signatures and beamformers are recently investigated with the system-wide MSE as the performance metric [9].

The premise of this paper is that, as in the case of single antenna CDMA systems, and multiuser MIMO systems, we can utilize the channel state feedback at the transmitter side to design appropriate temporal and spatial transmitters and improve the sum capacity of a MIMO-CDMA system. We first find an upper bound for the sum capacity of the MIMO-CDMA system via relaxing a set of structural constraints considering the level of feedback available at the transmitter side. Next, algorithms to find the signature sequences for given transmit beamformers are proposed. We investigate how the transmit beamformers should be chosen with different levels of feedback, and combine the proposed signature selection algorithms with antenna selection, eigen transmit beamforming and perfect feedback transmit beamforming models. Numerical results to support the analysis and conclusions are presented.

II. System Model

We consider the uplink of a single cell synchronous MIMO-CDMA system with K users and processing gain N. The common receiver is equipped with N_R receive antennas and user i has N_{T_i} transmit antennas, and signature sequence $\mathbf{s}_i$. We assume user i transmits its symbol by precoding it with an $N_{T_i} \times 1$ unit norm transmit beamforming vector, $\mathbf{f}_i$. We assume that the transmit beamforming vector for each user is given and fixed first. We relax this assumption later and investigate the effect of transmit beamforming on the system. Similar to the notation in [9], the received vector in the mth chip interval is

$$\mathbf{r}_m = \sum_{j=1}^{K} \sqrt{P_j}\mathbf{H}_j \mathbf{f}_j (\mathbf{s}_j)_m b_j + \mathbf{n}_m \qquad (1)$$

where P_j, b_j and $\mathbf{H}_j$ are the transmit power, symbol, and the $N_R \times N_{T_j}$ complex MIMO channel matrix of user j, respectively and $\mathbf{n}_m$ is the zero mean complex Gaussian noise vector in the mth chip interval with $E\left[\mathbf{n}_m \mathbf{n}_m^\dagger\right] = \sigma^2 \mathbf{I}$ where $(\cdot)^\dagger$ denotes the hermitian of a vector or matrix. We assume that the channel realizations are constant and perfectly known at the receiver side. For clarity of notation, we denote the joint effect of the transmit power, channel matrix and the transmit beamforming vector of user j as $\mathbf{a}_j = \sqrt{P_j}\mathbf{H}_j \mathbf{f}_j$. The received vector in each chip interval can be represented as:

$$\mathbf{r}_m = \sum_{j=1}^{K} \mathbf{a}_j (\mathbf{s}_j)_m b_j + \mathbf{n}_m \tag{2}$$

Stacking all the received signals at each chip interval and defining $(NN_R) \times N$ block diagonal matrices $\{\mathbf{A}_j\}$'s with $\mathbf{a}_j$'s as the block diagonal entries, the received signal can be represented in a long vector form of

$$\mathbf{r} = \sum_{j=1}^{K} \mathbf{A}_j \mathbf{s}_j b_j + \mathbf{n} \tag{3}$$

Observe that (3) has the same form as the received signal of a multiuser MIMO system where users have channel matrices $\{\mathbf{A}_j\}$'s, and transmit beamforming vectors $\{\mathbf{s}_j\}$'s. Thus, the MIMO-CDMA system can be viewed as a special case of multiuser MIMO system where each user transmits a single symbol and transmit beamforming is employed. Throughout the paper, we consider MIMO-CDMA systems with $K \leq NN_R$ users and develop signature and beamformer design algorithms for such systems.

III. Sum Capacity and Its Upper Bounds

Our aim in this section is to investigate the effect of temporal signatures on the sum capacity of MIMO-CDMA systems and to obtain upper bounds for the sum capacity of MIMO-CDMA systems. The information theoretic sum capacity optimization problem of a multiuser system with effective signatures $\{\mathbf{t}_j = \mathbf{A}_j \mathbf{s}_j\}$'s is given by

$$\begin{aligned} \max_{\{\{\mathbf{s}_j\},\{\mathbf{f}_j\}\}} C_{sum} &= \frac{1}{2}\log[\det(\mathbf{I}_{NN_R} + \sigma^{-2}\sum_{j=1}^{K} \mathbf{t}_j \mathbf{t}_j^\dagger)] \quad (4)\\ \text{s.t. } \|\mathbf{s}_j\| &= 1, \quad \|\mathbf{f}_j\| = 1 \quad j = 1, ..., K \\ \mathbf{t}_j &= \mathbf{A}_j \mathbf{s}_j \end{aligned}$$

Sum capacity maximization for single antenna CDMA systems has been studied in [4], [10] where the optimal sequences are identified. Using multiple antennas, in effect increases the dimension of the users' signature space on the order of the number of receivers. However, each user's signature space is constrained by its channel realization. Transmitter optimization in this case entails optimization of temporal signatures and transmit beamformers given the channels of the users. The resulting optimum transmitters are a function of the particular channel realizations of the users, and do not have a closed form.

Given the hardship of the optimization problem at hand, we note that, it is meaningful to upper bound the sum capacity and try to construct algorithms that would approach the bound, and hence the actual sum capacity. As we will see in the sequel, this proves to be a much simpler task with near-optimum results.

Recall that MIMO-CDMA is a special case of a multiuser MIMO system with constrained transmit beamforming vectors. Consequently, its sum capacity is upper bounded by that of a multiuser MIMO system with exact channel state information (CSI) at the transmitter and the receiver. The optimum transmission schemes for such systems are identified in [11]. We define the upper bound for the sum capacity of MIMO-CDMA system, the actual sum capacity of MIMO-CDMA system and the achieved sum capacity of a MIMO-CDMA system with given temporal signatures and spatial signatures as C_{upper}, C_{actual} and $C_{achieved}$, respectively. It is obvious that

$$C_{achieved} \leq C_{actual} \leq C_{upper} \tag{5}$$

and that as the achieved sum capacity approaches to the upper bound, it also approaches the actual sum capacity of MIMO-CDMA. The upper bounds we will use are as follows.

A. Multiuser MIMO Capacity Upper Bound

The sum capacity optimization problem in (4) can be reformulated as

$$\begin{aligned} &\max_{\{\mathbf{R}_j\}} \quad C_{sum} = \tfrac{1}{2}\log[\det(\mathbf{I}_{NN_R} + \sigma^{-2}\textstyle\sum_{j=1}^{K} \mathbf{A}_j \mathbf{R}_j \mathbf{A}_j^\dagger)] \quad (6)\\ &\text{s.t.} \quad \mathbf{R}_j = \mathbf{s}_j \mathbf{s}_j^\dagger, \quad \text{tr}\{\mathbf{R}_j\} = 1 \quad j = 1, 2, ..., K \end{aligned}$$

where $\mathbf{R}_j = \mathbf{s}_j \mathbf{s}_j^\dagger$ is the transmitter covariance matrix. By relaxing the rank constraints for $j = 1, ..., K$, to simple power constraints, i.e., $tr\{\mathbf{R}_j\} \leq 1$, one can easily obtain an upper bound for the sum capacity of the MIMO-CDMA system, i.e.,

$$C_{actual} \leq \max_{\{\mathbf{R}_j | tr\{\mathbf{R}_j\} \leq 1\}_{j=1,\cdots,K}} C_{sum} = C_{upper1} \tag{7}$$

The upper bound found by this relaxation is nothing but the sum capacity of the multiuser MIMO system with channel matrices $\{\mathbf{A}_j\}$'s, and unit power constraints. For a given multiuser MIMO system, the sum capacity maximizing transmit covariance matrices, $\{\mathbf{R}_j\}$'s, can be found easily by the iterative waterfilling procedure as defined in [11]. At each step of the waterfilling procedure, the signals of other users are viewed as the noise and the individual transmit covariance matrix of a user is optimized for maximizing the sum capacity.

B. Unconstrained Effective Signature Upper Bound

The sum capacity of the MIMO-CDMA system is a function of the effective signatures $\{\mathbf{t}_j = \mathbf{A}_j \mathbf{s}_j\}$'s and the subspace of the effective signature is defined by the span of the channel matrices $\{\mathbf{A}_j\}$'s. The received power of user j is $\mathbf{t}_j^\dagger \mathbf{t}_j = \mathbf{s}_j^\dagger \mathbf{A}_j^\dagger \mathbf{A}_j \mathbf{s}_j$ which is simply $\|\mathbf{a}_j\|^2 = P_j \|\mathbf{H}_j \mathbf{f}_j\|^2$ for the MIMO-CDMA with unit norm temporal signatures. From a system point of view, the MIMO-CDMA system can be viewed as a CDMA system with a processing gain NN_R and received powers $\{\|\mathbf{a}_j\|^2\}$'s with structural constraints on the effective signatures. Thus, the sum capacity of a CDMA system with a processing gain NN_R and received powers

TABLE I

SIGNATURE AND BEAMFORMER DESIGN WITH PERFECT FEEDBACK

Step	:	Updating signatures and beamformers
For		$k = 1 : K$
		$\mathbf{E}_k(n+1) = \sum_{i<k} \mathbf{t}_i(n+1)\mathbf{t}_i(n+1)^\dagger + \sum_{i>k} \mathbf{t}_i(n)\mathbf{t}_i(n)^\dagger + \sigma^2 \mathbf{I}$
		$\mathbf{s}_k(n+1) =$ max eigenvalued eigenvector of $\mathbf{A}_k(n+1)^\dagger \mathbf{E}_k(n+1)^{-1} \mathbf{A}_k(n+1)$
		Update $\mathbf{Q}_k(n+1)$
		$\mathbf{f}_k(n+1) =$ max eigenvalued eigenvector of $\mathbf{Q}_k(n+1)^\dagger \mathbf{E}_k(n+1)^{-1} \mathbf{Q}_k(n+1)$
		Update $\mathbf{A}_k(n+1)$
		$\mathbf{t}_k(n+1) = \mathbf{A}_k(n+1)\mathbf{s}_k(n+1)$
End		

$\{\|\mathbf{a}_j\|^2\}$'s forms an upper bound for the sum capacity of MIMO-CDMA system. For $K \leq NN_R$, the upper bound, is simply the sum capacity of an underloaded CDMA system which assigns orthogonal effective signature sequences, and results in:

$$C_{actual} \leq C_{upper2} = \frac{1}{2} \sum_{j=1}^{K} \log[1 + \sigma^{-2} \|\mathbf{a}_j\|^2] \quad (8)$$

Both upper bounds are obtained by relaxing different constraints of the actual optimization problem. The tighter of the upper bounds, $C_{upper} = min(C_{upper1}, C_{upper2})$, will be used as a benchmark to evaluate the performance of the algorithms we will construct next.

IV. SIGNATURE AND BEAMFORMER DESIGN WITH PERFECT FEEDBACK

Our aim in this section is to design temporal signature and transmit beamformer sets that achieve sum capacity near the upper bounds developed in the previous section.

Recall that the sum capacity of the MIMO-CDMA system for given transmit beamformers is

$$C_{sum} = \frac{1}{2} \log[\det(\mathbf{I}_{NN_R} + \sigma^{-2} \sum_{j=1}^{K} \mathbf{A}_j \mathbf{s}_j \mathbf{s}_j^\dagger \mathbf{A}_j^\dagger)]. \quad (9)$$

When the parameters of all users except k are fixed, the sum capacity can be expressed in terms of user k's parameters as

$$C_{sum} = \frac{1}{2} \log[\det(\sigma^{-2}(\mathbf{A}_k \mathbf{s}_k \mathbf{s}_k^\dagger \mathbf{A}_k^\dagger + \mathbf{E}_k))] \quad (10)$$

$$= \gamma_k + \frac{1}{2} \log[1 + (\mathbf{s}_k^\dagger \mathbf{A}_k^\dagger \mathbf{E}_k^{-1} \mathbf{A}_k \mathbf{s}_k)] \quad (11)$$

where $\mathbf{E}_k = \sum_{i \neq k}^{K} \mathbf{A}_i \mathbf{s}_i \mathbf{s}_i^\dagger \mathbf{A}_i^\dagger + \sigma^2 \mathbf{I}_{NN_R}$ is the interference covariance matrix of user k and γ_k represents the terms independent of user k. Recall that $\mathbf{A}_k \mathbf{s}_k$ is the effective signature of user k where $\mathbf{A}_k$ is the block diagonal matrix with $\mathbf{a}_k = \mathbf{H}_k \mathbf{f}_k$ as the block entries. Thus, the sum capacity can be expressed in terms of only signature sequence, $\mathbf{s}_k$ or transmit beamformer $\mathbf{f}_k$ when all other variables are fixed. For fixed transmit beamformers, the sum capacity expression in terms of the temporal signature of user k is as in (11). Defining $\mathbf{Q}_k = [(\mathbf{s}_k)_1 \mathbf{H}_k; (\mathbf{s}_k)_2 \mathbf{H}_k; ...; (\mathbf{s}_k)_N \mathbf{H}_k]$, the sum capacity can be expressed in terms of the transmit beamformer as

$$C_{sum} = \gamma_k + \frac{1}{2} \log[1 + (\mathbf{f}_k^\dagger \mathbf{Q}_k^\dagger \mathbf{E}_k^{-1} \mathbf{Q}_k \mathbf{f}_k)] \quad (12)$$

TABLE II

ORTHOGONAL TEMPORAL SIGNATURE ASSIGNMENT

System Parameters
$\mathcal{K}_a$: Users that temporal signatures are not assigned
$\mathcal{K}_i$: The users that temporal signature $\mathbf{e}_i$ is assigned, i=1,...N
$\mathcal{N}_a$: Available temporal signatures that are not assigned to users
$\{\mathbf{a}_j\}$: Effective spatial signatures of users
$Avuser$: Av. number of users per remaining temporal signatures
Temporal Signature Assignment Algorithm
$\mathcal{K}_a = \{user-1, user-2, ..., user-K\}$
$\mathcal{N}_a = \{\mathbf{e}_1, \mathbf{e}_2, ..., \mathbf{e}_N\}$
For $i = 1 : N$
User Selection for temporal signature $\mathbf{e}_i$
$Avuser = \lceil \frac{n(\mathcal{K}_a)}{n(\mathcal{N}_a)} \rceil$
For $j = 1 : Avuser$
  $k^\star = \arg\max_{k \epsilon \mathcal{K}_a} z_{ik}$
  $\mathcal{K}_i = \mathcal{K}_i \bigcup \{user - k^\star\}$
  $\mathcal{K}_a = \mathcal{K}_i \backslash \{user - k^\star\}$
End
$\mathcal{N}_a = \mathcal{N}_a \backslash \{\mathbf{e}_i\}$
End

The above construction suggests an iterative way of improving the sum capacity of MIMO-CDMA systems. Specifically, an iterative algorithm that maximizes the sum capacity at each step can be devised by optimizing the temporal signature and transmit beamformer of one user, say user k, $\mathbf{s}_k$ and $\mathbf{f}_k$, at each step. From the perspective of user k, the sum capacity can be improved by choosing $\mathbf{s}_k$ and $\mathbf{f}_k$ to maximize the second term in (11) and (12). This is accomplished by choosing $\mathbf{s}_k$ to be the maximum eigenvalued eigenvector of $\mathbf{A}_k^\dagger \mathbf{E}_k^{-1} \mathbf{A}_k$, and $\mathbf{f}_k$ to be the maximum eigenvalued eigenvector of $\mathbf{Q}_k^\dagger \mathbf{E}_k^{-1} \mathbf{Q}_k$. The algorithm iterates over the users increasing the sum capacity at each step. This implies that the algorithm, which produces a monotonic sequence that is bounded by C_{upper}, is convergent.

The algorithm proposed employ iterative updates of the temporal signature sequences and transmit beamformers. We note that the algorithm relies on the error-free and low-delay feedback channels.

V. ORTHOGONAL TEMPORAL SIGNATURE ASSIGNMENT

In the previous section, we proposed an iterative algorithm to design temporal signatures and transmit beamformers that improves the sum capacity of the MIMO-CDMA system at each step. It is important to note that this approach requires perfect feedback of the resulting signatures and transmit beamformers to the transmitter side. The resulting signatures and transmit beamformers are a function of the channel parameters. This implies that the algorithm would have to be run each time the channels change. Thus, the amount of feedback may be overwhelming in practice and, it is necessary to quantize the signature space with an acceptable level of quantization error.

An alternative way to drastically reduce the amount of feedback to the transmitter side is to constrain the temporal signature set to an orthogonal set. In this case, since each orthogonal signature can be labelled with $log_2 N$ bits, the information required to be fedback related to signatures is the label of each signature.

In this case, the problem reduces to multiuser MIMO scheduling problem that is shown to be NP-complete [12].

However, a low complexity orthogonal temporal signature assignment algorithm with good heuristics that perform close to sum capacity upper bound can be derived. Observe that the following development assures a fixed beamformer for each user. The sum capacity maximization problem with orthogonal temporal signatures is

$$\max_{\{\mathbf{s}_j\}} \quad C_{sum} = \tfrac{1}{2}\log[\det(\mathbf{I}_{NN_R} + \sigma^{-2}\textstyle\sum_{j=1}^{K} \mathbf{A}_j \mathbf{s}_j \mathbf{s}_j^\dagger \mathbf{A}_j^\dagger)] \quad (13)$$

$$\text{s.t.} \qquad \mathbf{s}_j \epsilon \{\mathbf{e}_1, \mathbf{e}_2, ..., \mathbf{e}_N\} \qquad j = 1, 2, ..., K \quad (14)$$

where $\{\mathbf{e}_i\}$ is an orthogonal signature set. When temporal signature of $\mathbf{e}_i$ is assigned to user k, its effective signature takes the form $\mathbf{t}_k = \mathbf{A}_k \mathbf{e}_i$. Clearly, user k interferes only with the users that have the temporal signature $\mathbf{e}_i$. When orthogonal temporal signatures are used, the sum capacity can be represented as

$$C_{sum} = \sum_{i=1}^{N} C_{\mathcal{K}_i} = \sum_{i=1}^{N} \frac{1}{2}\log[\det(\mathbf{I}_{N_R} + \sigma^{-2} \sum_{j\epsilon\mathcal{K}_i} \mathbf{a}_j \mathbf{a}_j^\dagger)] \quad (15)$$

where $\mathcal{K}_i$ is the set of users with $\mathbf{e}_i$ as temporal signature and $C_{\mathcal{K}_i}$ is the sum capacity of this set.

Assume that user k is assigned the temporal signature $\mathbf{e}_i$. Let us define the set of users assigned to $\mathbf{e}_i$ excluding user k as $\mathcal{K}_i - user(k) = \bar{\mathcal{K}}_i^{(k)}$. The contribution of user k on the sum capacity is simply

$$\Delta_{ik} = C_{\mathcal{K}_i} - C_{\bar{\mathcal{K}}_i^{(k)}} \quad (16)$$

$$= \frac{1}{2}\log[1 + \mathbf{a}_k^\dagger(\sigma^2 \mathbf{I}_{NN_R} + \sum_{j\epsilon\bar{\mathcal{K}}_i^{(k)}} \mathbf{a}_j \mathbf{a}_j^\dagger)^{-1}\mathbf{a}_k] \quad (17)$$

Recall that the optimum unconstrained effective signatures for $NN_R \geq K$ MIMO-CDMA systems are orthogonal signature sequences. However, the predefined spatial signatures of the users may prevent the effective signatures to be orthogonal to each other. Thus, the near-optimum temporal signature assignment strategy should try to assign the orthogonal temporal signatures to users such that the effective signatures, $\{\mathbf{A}_j \mathbf{s}_j\}$'s are as close to being orthogonal as they can. Observe that assigning more than N_R users to the same temporal signature is likely to cause high correlation among the users. Besides the fact that each temporal signature should not be assigned more than N_R users, intuitively, it is logical to assign no more than $\lceil \frac{\text{Number of users}}{N} \rceil \leq N_R$ users to each temporal signature for the sake of fairness.

The above observations suggest that an N step temporal signature assignment algorithm that tries to select the spatially less correlated users for each temporal signature is a good candidate for near optimum performance. Specifically, at each step, the number of users that will be assigned for each temporal signature is $\lceil \frac{\text{Number of available users}}{N} \rceil$ which is guaranteed not to exceed N_R users for $NN_R \geq K$.

Recall that the contribution of a user to the sum capacity is given by (17). Also, the capacity of the user k with unconstrained effective signature orthogonal to other users, i.e. the single user capacity is

$$C_k = \frac{1}{2}\log(1 + \sigma^{-2}\|\mathbf{a}_k\|^2) \quad (18)$$

Observe that the assignment of user k to temporal signature $\mathbf{e}_i$ will result a difference of $C_k - \Delta_{ik}$ between the unconstrained effective signature sum capacity upper bound, and the achieved sum capacity of MIMO-CDMA system from user k's perspective. This difference can be expressed as

$$C_k - \Delta_{ik} = \frac{1}{2}\log \frac{(1 + \sigma^{-2}\|\mathbf{a}_k\|^2)}{(1 + \mathbf{a}_k^\dagger(\sigma^2 \mathbf{I}_{NN_R} + \sum_{j\epsilon\bar{\mathcal{K}}_i^{(k)}} \mathbf{a}_j \mathbf{a}_j^\dagger)^{-1}\mathbf{a}_k)} \quad (19)$$

Thus, the user with the highest

$$\frac{(1 + \mathbf{a}_k^\dagger(\sigma^2 \mathbf{I}_{NN_R} + \sum_{j\epsilon\bar{\mathcal{K}}_i^{(k)}} \mathbf{a}_j \mathbf{a}_j^\dagger)^{-1}\mathbf{a}_k)}{(1 + \sigma^{-2}\|\mathbf{a}_k\|^2)} \approx \frac{\mathbf{a}_k^\dagger(\mathbf{I}_{NN_R} + \sigma^{-2}\sum_{j\epsilon\bar{\mathcal{K}}_i^{(k)}} \mathbf{a}_j \mathbf{a}_j^\dagger)^{-1}\mathbf{a}_k}{(\|\mathbf{a}_k\|^2)} = z_{ik} \quad (20)$$

will result in the smallest difference from the upper bound from a single user's perspective. The signature assignment algorithm minimizes the difference between the sum capacity of the unconstrained effective signature upper bound, and the achieved sum capacity of the MIMO-CDMA system from a single user's perspective: At each step, we choose the user that has the highest z_{ik} to assign to each orthogonal temporal signature $\mathbf{e}_i$.

This algorithm obviously favors the *earlier* temporal signatures, since these temporal signatures will have more users available to select from that will minimize the difference between the achieved sum capacity of the MIMO-CDMA system from the upper bound from a single user's perspective. However, by limiting the number of users using each orthogonal temporal signature by $\lceil \frac{\text{Number of available users}}{N} \rceil$ the *earlier* temporal signatures may have 1 user more than the later ones and the unfairness is somewhat decreased. An arbitrary user can be chosen from the available users as the first user to start the algorithm.

VI. Beamformer Design with Orthogonal Temporal Signatures

The previous section considered the case where transmit beamforming vectors are fixed. The performance of the MIMO-CDMA system is clearly a function of the choice of the transmit beamformers. In turn, the choice of the transmit beamformers is highly dependent on the feedback level at the transmitter side. In this section, we consider different levels of feedback at the transmitter side, and determine the corresponding transmit beamformers to be employed. Specifically, no feedback case, antenna selection feedback, individual CSI feedback; and the perfect transmit beamforming feedback cases are investigated next.

We note that the orthogonal temporal signature assignment algorithm easily accommodates the beamformer we design in accordance with the feedback requirement by an appropriate definition of the effective signature as explained in each of the following sections.

A. No CSI at the Transmitter Side

In the absence of channel state related feedback, multiple transmitters simply provide diversity [1]. The transmitter distributes its power equally among each transmit antenna, and the transmit beamformer is $1/\sqrt{N_T}[1,1,...,1]^T$. The effective spatial signature is a scaled version of the sum of each column of the channel matrix.

B. Antenna Selection

In the case of limited channel state feedback, a popular approach is antenna selection, where the only required feedback is which antenna(s) should be used [13]. Consider the case where one transmitter antenna is selected. In this case, the spatial signature of the user is the spatial signature of the transmitter antenna to be used, i.e., when user k selects transmitter antenna m, $\mathbf{a}_k = \mathbf{h}_{km}$ where $\mathbf{h}_{km}$ is the mth column vector of user k's channel matrix. It is clear that the received power of the user k is $\|\mathbf{a}_k\|^2 = \|\mathbf{h}_{km}\|^2$. Intuitively, an effective antenna selection method is to choose the transmitter antenna that will maximize the received power of the user, i.e., the transmitter antenna with the highest norm $\|\mathbf{h}_{km}\|$. The orthogonal temporal signature assignment algorithm with maximum received power antenna selection is a modified version of the algorithm presented in Table II with the effective spatial signatures defined as the spatial signatures of the transmitter antennas with the highest norm as

$$\mathbf{a}_k = \underset{m\epsilon\{1,2,\ldots,N_T\}}{\arg\max} \|\mathbf{h}_{km}\| \tag{21}$$

C. Eigen Mode Selection

An alternative scenario with limited feedback is when each user has its own CSI at the transmitter side. This is a reasonable assumption when the system is operated in time division duplex. The eigenmodes of the channel matrices can be viewed as the transmitter antennas with different spatial signatures. Similar to the antenna selection case described above, the simplest form of transmit beamformer selection is choosing the eigenmode of the channel that will maximize the received power of the user. This approach simply results the maximum eigenvalued eigenvector of $\mathbf{H}_k^\dagger \mathbf{H}_k$ to be selected as a transmit beamformer. Formally, the transmit beamformer and the spatial signature of the user is

$$\mathbf{f}_k = \underset{\mathbf{u}_{km}|m\epsilon\{1,2,\ldots,N_T\}}{\arg\max} \mathbf{u}_{km}^\dagger \mathbf{H}_k^\dagger \mathbf{H}_k \mathbf{u}_{km}; \quad \mathbf{a}_k = \mathbf{H}_k \mathbf{f}_k \tag{22}$$

where $\mathbf{u}_{km}$ is the mth eigenvector of $\mathbf{H}_k^\dagger \mathbf{H}_k$. The orthogonal temporal signature assignment algorithm can be easily combined with maximum received power eigenbeamforming selection by defining the spatial signatures of the users as in (22).

Maximum received power antenna selection and eigenbeamforming selection approach are expected to perform well especially if one transmitter antenna/eigen mode has a spatial signature with a norm significantly larger than the others, and the MIMO channels of the users are independent of each other. If this is not the case, the channels of the users may be highly correlated and the transmitter antenna/eigen mode maximizing the received power may not significantly outperform the remaining transmitter antennas/eigen modes. We note that, in such cases, a transmit beamformer design approach that considers the performance of all transmitter antennas/eigen modes of the users should be used [14].

D. Perfect Transmit Beamformer Feedback

In this section, we assume that the transmit beamformers can be designed without any feedback constraints. The feedback related to the temporal signature assignment is still limited to one of N labels of the orthogonal signatures. The motivation for this design is to obtain a benchmark for the performance of beamformer design with limited feedback.

For a given set of effective signatures, $\{\mathbf{a}_j = \mathbf{H}_j \mathbf{f}_j\}_{j=1}^K$, the metric used in the orthogonal temporal signature assignment, z_{ik} can be reformulated as

$$z_{ik} = \frac{\mathbf{f}_k^\dagger \mathbf{H}_k^\dagger (\mathbf{I}_{NN_R} + \sigma^{-2} \sum_{j\epsilon\bar{\mathcal{K}}_i^{(k)}} \mathbf{a}_j \mathbf{a}_j^\dagger)^{-1} \mathbf{H}_k \mathbf{f}_k}{\max\limits_{\{\mathbf{f}|\mathbf{f}^\dagger\mathbf{f}=1\}} \mathbf{f}^\dagger \mathbf{H}_k^\dagger \mathbf{H}_k \mathbf{f}} \tag{23}$$

where the denominator represents the maximum received power of user k. Following the development in (19) and (20), we observe that transmit beamforming vector that maximizes z_{ik} is the maximum eigenvalued eigenvector of $\mathbf{H}_k^\dagger(\sigma^2 \mathbf{I}_{NN_R} + \sum_{j\epsilon\bar{\mathcal{K}}_i^{(k)}} \mathbf{a}_j \mathbf{a}_j^\dagger)^{-1} \mathbf{H}_k$. Thus, the orthogonal temporal signature assignment algorithm with perfect transmit beamforming feedback should compare the performance of the users with their best transmit beamforming vectors at each step. The resulting signature assignment and beamformer design algorithm is a two-step algorithm where the best transmit beamformers are found at the first step, and the user with the highest z_{ik} is assigned a temporal signature next. After the assignment of the temporal signatures, the users may choose to update their transmit beamformers improving the performance at each step. In the numerical results shown next, we have chosen to perform a single iteration in order to provide a fair comparison with antenna and eigen mode selection.

VII. Numerical Results and Conclusion

In this section, we present numerical results related to the performance of the signature and transmit beamformer design algorithms. We also compare the performance of the transmission design strategies for different levels of feedback to investigate the benefit gained by exploiting the channel state information. The simulations are performed for $K = 12$ user, 2×2 and 4×4 MIMO-CDMA systems with a processing gain of $N = 8$. The channels are realizations of a flat fading channel model where all links are assumed to be independent and identically distributed complex Gaussian random variables. The received SNR of each user is 7dB. CDF curves for sum capacity obtained by simulating 10000 channel realizations are presented.

We compare the performance of the signature and transmit beamformer design strategies for $K = 12$ user 2×2 and 4×4 MIMO-CDMA system with different levels of

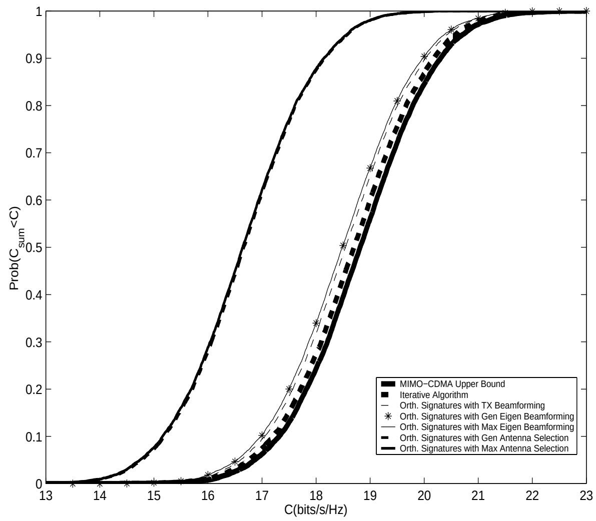

Fig. 1. $K = 12$ user MIMO-CDMA system with 2×2 MIMO model and N=8. Comparison of the CDF curves of the upper bound and the MIMO-CDMA systems with different levels of feedback.

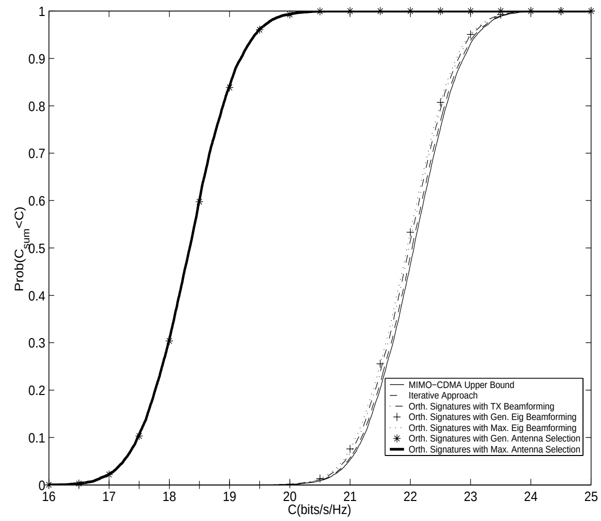

Fig. 2. $K = 12$ user MIMO-CDMA system with 4×4 MIMO model and N=8. Comparison of the CDF curves of the upper bound and the MIMO-CDMA systems with different levels of feedback.

feedback to investigate the benefit gained by exploiting the channel state information in Figures 1 and 2, respectively. For the sake of completeness, the performance of a generalized beamformer design approach that considers the performance of all transmitter antennas/eigen modes with the orthogonal temporal signature assignment is also included. This approach simply increases the complexity of the assignment process without changing the feedback requirements, and results in a low relative gain in performance.

The performance of the algorithms is improved as the level of feedback is increased, and the signature and transmit beamformer design algorithm with iterative updates performs the best. The largest relative gain is due to the feedback related to each user's own CSI combined with the orthogonal temporal signature assignment algorithm, where each user selects its transmit beamforming vector so that its received power is maximized. The results indicate that the performance of this algorithm comes close to the upper bounds, hence the actual capacity. As expected, the gap between the performance of proposed algorithms and the upper bounds decrease as the dimension of the MIMO system is increased.

In conclusion, in this paper, we proposed signature and beamformer design algorithms for MIMO-CDMA systems with various levels of feedback. First, we investigated the problem with perfect feedback and derived an iterative algorithm that is geared towards enhancing the system performance by maximizing the sum capacity at each step. Motivated by limited feedback requirements, we investigated the orthogonal temporal signature assignment problem for given transmit beamformers and proposed a near-optimum signature assignment algorithm with low complexity. Since the performance of the system depends on the transmit beamformers, we have considered transmit beamformer selection next, and combined temporal signature assignment with several beamformers resulting from different levels of feedback at the transmitter side. We have observed that as the feedback level at the transmitter is increased, the performance of the proposed algorithms approaches to the capacity upper bounds, and consequently to the capacity of the jointly optimum signature and beamformer set. Notably, the individual CSI feedback facilitates a substantial gain.

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