Constrained Slowest Descent Detectors for Multiuser CDMA Systems

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Abstract — **Previous work showed that several of the well-known suboptimum detectors correspond to *nonlinear programming relaxations* of the optimum multiuser detection problem. We use the *slowest descent* approach to design multiuser detectors of improved performance. The aim is to achieve a performance closer to the performance of the optimum receiver without significantly compromising the low complexity of the resulting receiver.**

I. Introduction

The capacity of a CDMA system is limited by the interference each user creates to other users in the system. Receivers that utilize the structure of the received signal to reduce the interference enhance system performance. The optimum receiver which performs maximum likelihood detection of users' bits is exponentially complex in the number of users [1].

A number of well-known suboptimum multiuser detectors correspond to quantization of a relaxed joint likelihood maximizer, including the linear decorrelator detector (see [2, 3]), the soft interference canceller (see [4]), and the generalized MMSE detector (see [4]). The slowest descent approach [3] searches for a more likely bit vector by analyzing the slope of the likelihood function in a neighborhood of the relaxed maximizer. The search involves a quantized relaxed maximizer bit vector estimate and a number of other estimates close to lines of the least decrease in the relaxed likelihood function from its (relaxed) maximizer. Here, we formalize the application of the slowest descent approach to general convex sets and corresponding relaxed maximizers.

II. System Model and the Relaxed Solution

We consider a synchronous CDMA system employing BPSK. The received signal K by 1 vector at the output of the matched filters, $\boldsymbol{y}$, is a sufficient statistic for the multiuser detection problem and is given by

$$\boldsymbol{y} = \boldsymbol{\Gamma\Lambda a} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{a}$ is a K by 1 vector of user information bits, $\boldsymbol{\Gamma}$ is the nonnegative definite cross correlation matrix, $\boldsymbol{\Lambda}$ is a diagonal matrix containing the users' received amplitudes, $\boldsymbol{a}$ is the vector containing the information bits of the users and $\boldsymbol{n}$ is a zero mean Gaussian random vector with auto covariance matrix $E[\boldsymbol{nn}^\top] = \sigma^2\boldsymbol{\Gamma}$.

The maximum likelihood (ML) multiuser detection problem [5] solves for

$$\bar{\boldsymbol{a}} = \arg \min_{\boldsymbol{a}\in\{-1,1\}^K} \underbrace{\boldsymbol{a}^\top \boldsymbol{R}\boldsymbol{a} - 2\boldsymbol{a}^\top\boldsymbol{\Lambda}\boldsymbol{y}}_{l(\boldsymbol{a})}, \tag{2}$$

where $\boldsymbol{R} = \boldsymbol{\Lambda\Gamma\Lambda}$. This problem is NP hard. One solution is to relax [4] the constraint set in (2) and use a convex superset of $\{-1,1\}^K$, denoted with $\boldsymbol{\Omega}_c$, instead. The relaxed solution, denoted with $\boldsymbol{a}^*$, is typically quantized to the closest solution in the feasible set $\{-1,1\}^K$.

III. Constrained Slowest Descent

The slowest descent search [3] suggests studying the slope of the likelihood function in a neighborhood of a relaxed maximizer $\boldsymbol{a}^*$ and generating an *expanded* set of candidate feasible points closest (in Euclidean distance) to P lines of the least decrease in the likelihood function. The slowest descent estimate is the best (most likely) feasible point from this subset. Here, we define the lines of the constrained slowest descent as defined by $\boldsymbol{a}^*$ and vectors

$$\hat{\boldsymbol{\Delta}}^i_{min} \leftarrow \hat{\boldsymbol{\Delta}}^i_{min}(\epsilon) \quad as \quad \epsilon \to 0+ \tag{3}$$

where $\hat{\boldsymbol{\Delta}}^i_{min}(\epsilon) = \arg\min_{\boldsymbol{\Delta}\in\boldsymbol{\Omega}^{i-1}_{\boldsymbol{\Delta}}(\epsilon)} \{l(\boldsymbol{a}^*) - l(\boldsymbol{a}^* + \epsilon\boldsymbol{\Delta})\}$ and

$$\boldsymbol{\Omega}^i_{\boldsymbol{\Delta}}(\epsilon) = \left\{\boldsymbol{\Delta} : \|\boldsymbol{\Delta}\| = 1,\ \boldsymbol{a}^* + \epsilon\boldsymbol{\Delta} \in \boldsymbol{\Omega}_c,\ \boldsymbol{\Delta}^T\hat{\boldsymbol{\Delta}}^k_{min} = 0,\ k \le i\right\} \tag{4}$$

The constrained slowest descent directions (3) are the Hessian eigenvectors at $\boldsymbol{a}^*$ of the restriction of $l(\boldsymbol{a})$ onto the intersection of the hypertangents of $\boldsymbol{\Omega}_c$ at $\boldsymbol{a}^*$ and the hypertangent of the equi-value contours of $l(\boldsymbol{a})$ at $\boldsymbol{a}^*$.

For a hypercube $\boldsymbol{\Omega}_c = \{\boldsymbol{a} : |a_i| \le 1\}$, $\boldsymbol{a}^*$ is a soft interference canceller [4] and $\hat{\boldsymbol{\Delta}}^i_{min}$ are eigenvectors of $\tilde{\mathbf{P}}\mathbf{R}\tilde{\mathbf{P}}$. Here, $\tilde{\mathbf{P}}$ is the projection matrix $\tilde{\mathbf{P}} = \mathrm{diag}\{\delta(|a^*_i| < 1) \text{ for } i = 1,\ldots,K\}$, where $\delta(x) = 1$ for $x = 1$ and 0 otherwise.

For a hyperball $\boldsymbol{\Omega}_c = \{\boldsymbol{a} : \|\boldsymbol{a}\| \le 1\}$, $\boldsymbol{a}^*$ is a generalized MMSE estimate [4] and $\hat{\boldsymbol{\Delta}}^i_{min} = \mathbf{U}_r\mathbf{v}^i$, where $\mathbf{v}^i$ are eigenvectors of $\mathbf{U}_r^T\mathbf{R}\mathbf{U}_r$ and the columns of $\mathbf{U}_r$ form an orthonormal basis of the space orthogonal to $\boldsymbol{a}^*$.

References

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