# Rank Constrained Temporal-Spatial Filters for CDMA Systems with Base Station Antenna Arrays

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*Abstract* — **In this paper, we consider a minimum mean-squared error (MMSE) based receiver structure which combines multiuser detection (temporal filtering) and receiver beamforming (spatial filtering). Motivated by the high complexity of the optimum receiver, we propose rank constrained temporal-spatial filters which are simpler and near optimum. An iterative algorithm based on alternating minimization is used to find the filter coefficients. The adaptive implementation, built upon least mean squares (LMS), is formulated and its convergence properties are investigated.**

## I. Introduction

CDMA systems are known to suffer from multi-access interference (MAI), which degrades performance. In this work we focus on two receiver signal processing based methods that combat MAI: multiuser detection (temporal filtering) and receiver beamforming (spatial filtering).

Multiuser detection [1] exploits the inherent structure of multiple access interference to suppress it effectively. The high computational complexity of the optimum multiuser detector [2] resulted in the investigation of a number of suboptimum detectors [3, 4]. One of the suboptimum detectors is the MMSE detector, designed to minimize the mean-squared error between the filter output and the information bit. Adaptive implementations of the MMSE detector based on training bits as well as blind versions were studied in [3, 5, 6]. Beamforming using receiver antenna arrays, reduces the interference by separating the desired signal from interfering signals that originate from different locations [7]. It is shown in [8] that for flat-Rayleigh fading channels, interferers can be nulled out with a cost of one degree of freedom per user. The capacity increase that is achieved with base-station antenna arrays in CDMA systems is shown in [9], where perfect instantaneous power control and matched filters are assumed for each user.

Combined temporal-spatial filtering was considered in [10] and the necessary statistics were derived along with several multiuser detectors. Many combined temporal-spatial structures were considered in the past with most of them being in cascade form; beamformers followed by an interference canceller or vice versa [11, 12, 13]. A recent paper [14] examines several two dimensional linear filter structures. It is shown that joint optimum temporal-spatial filter (OTSF) achieves the highest signal-to-interference ratio (SIR). The high computational complexity of OTSF led the authors to propose rank-1 constrained filters.

Rank-1 constrained filter's simplicity is appealing from an implementation point of view but the performance of such filters may be severely suboptimal as compared to the optimal filter due to the constrained solution space, especially under heavy loads. Motivated by the performance gap between the OTSF and the rank-1 constrained filter, in this work, we search for filter structures whose performance lie between that of the OTSF and the rank-1 constrained filter. We propose a general class of rank constrained filters, which are found according to a structural constraint on the receiver filter. It will be shown that the strictness of the constraint determines the resulting filter's performance and complexity. The constraint can be relaxed in order to achieve a near optimum performance, at the expense of additional complexity.

A recent popular approach that reduces the receiver complexity is the reduced-rank method. In reduced-rank methods, the received signal is projected to a lower dimensional subspace [15, 16]. The 'rank' reduction in these methods refer to the rank of the autocorrelation matrix which results in a dimension reduction of the received signal and the corresponding filter. In contrast, in this work, the term 'rank constrained' refers to the rank of the linear matrix filter. More will be said about the differences of these approaches in the remainder of this paper.

In our approach, we express the temporal-spatial filter as a receiver matrix filter. The MSE, with the appropriate rank constraint, is iteratively minimized using an alternating minimization algorithm. We first derive the deterministic iterations, which assume the knowledge of all users' parameters. Motivated by the fact that adaptive implementations may require little information to track the constantly changing parameters in a wireless environment, we consider such implementations in section 5. An adaptive algorithm which is a combination of alternating minimization and least mean squares (LMS) is formulated and the parameters that affect the convergence are explained.

## II. System Model

We consider a single cell synchronous DS-CDMA system with processing gain $N$. An antenna array of $M$ elements is employed. Over one bit period, the received signal at the output of the antenna array is given by:

$$\mathbf{r}(t) = \sum_{k=1}^{K} \sqrt{P_k} b_k s_k(t) \mathbf{a}_k + \mathbf{n}(t) \tag{1}$$

where $P_k$, $b_k$ and $s_k(t)$ represent the transmit power, information bit and the temporal signature of user $k$. $\mathbf{a}_k$ is the

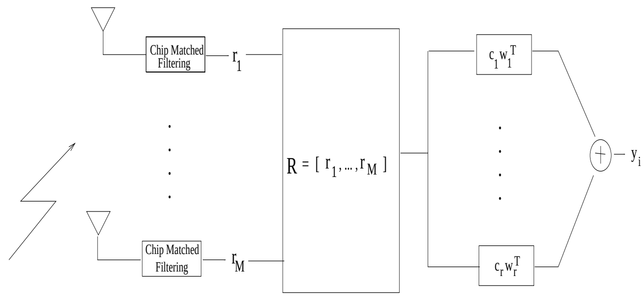


Fig. 1: Receiver structure

combined array response vector and channel coefficient of user $k$. The temporal signatures of the users are of the form:

$$s_k(t) = \sum_{n=1}^{N} s_k^{(n)} \phi(t-(n-1)T_c) \quad \text{with} \quad s_k^{(n)} = \pm 1/\sqrt{N} \quad (2)$$

where $\phi(t)$ is the unit energy chip waveform and $T_c$ is the chip duration. After chip matched filtering and sampling at the chip rate, there are $N$ observations at the output of each of the $M$ antenna elements, which can be arranged in a $N \times M$ dimensional matrix:

$$\mathbf{R} = \sum_{k=1}^{K} \sqrt{P_k} b_k \mathbf{s}_k \mathbf{a}_k^T + \mathbf{N} \quad (3)$$

The $m^{th}$ column of $\mathbf{R}$ represents the chip matched filtered observations at the output of the $m^{th}$ antenna element. The $\mathbf{N}$ matrix represents the spatially and temporally white noise, $E[N_{kl}^* N_{mn}] = \sigma^2 \delta_{km} \delta_{ln}$ where $(\cdot)^*$ denotes the conjugate operation. For the rest of this paper, the desired user will be denoted by index $i$. A linear matrix filter $\mathbf{X}_i$ is used to compute the decision statistic $y_i$, and the bit decision is made by taking the sign of the real part of $y_i$:

$$y_i = \sum_{n=1}^{N} \sum_{m=1}^{M} [X_i]_{nm}^* R_{nm} = tr(\mathbf{X}_i^H \mathbf{R}) \quad (4)$$

where $tr(\cdot)$ and $(\cdot)^H$ denote the trace and hermitian operations respectively.

## III. Previous Work

The optimum temporal-spatial filter ($\bar{\mathbf{X}}_i$) minimizes the mean-squared error between the decision statistic and the information bit. Note that $\bar{\mathbf{X}}_i$ also achieves the highest SIR amongst all possible linear matrix filters [17]. We want to find $\bar{\mathbf{X}}_i$ such that:

$$\bar{\mathbf{X}}_i = \arg\min_{\mathbf{X}} E\left[\left| tr\left(\mathbf{X}^H \mathbf{R}\right) - b_i \right|^2\right] \quad (5)$$

After reformulating the optimization problem with vector variables, the solution is given by [17, 14, 3]:

$$\bar{\mathbf{x}}_i = \sqrt{P_k} \left( \sum_{k=1}^{K} P_k \mathbf{q}_k \mathbf{q}_k^H + \sigma^2 \mathbf{I} \right)^{-1} \mathbf{q}_i \quad (6)$$

where $\mathbf{q}_k$ is the long vector obtained by stacking up columns of $\mathbf{s}_k \mathbf{a}_k^T$. To reconstruct $\mathbf{X}_i$, we take every $N$ element of $\mathbf{x}_i$ and place as a column to $\mathbf{X}_i$. From (6), it is seen that finding the optimum filter requires the inversion of an $NM \times NM$ dimensional matrix which may be computationally costly. Motivated by this complexity of OTSF, the authors of [17] proposed a simpler receiver. In this case, the filter space (i.e. the solution space of the optimization problem) is constrained to contain filters of rank 1 only. Note that any $\mathbf{X}_i$ of rank 1 can be decomposed as $\mathbf{X}_i = \mathbf{c}_i \mathbf{w}_i^T$, where $\mathbf{c}_i$ and $\mathbf{w}_i$ are $N$ and $M$ dimensional vectors respectively. The optimization problem and the MSE function are expressed in terms of $\mathbf{c}_i$ and $\mathbf{w}_i$ as:

$$\begin{aligned} [\bar{\mathbf{c}}_i, \bar{\mathbf{w}}_i] &= \arg\min_{\mathbf{c}_i, \mathbf{w}_i} E\left[\left| tr\left(\mathbf{w}_i^* \mathbf{c}_i^H \mathbf{R}\right) - b_i \right|^2\right] \\ &= \arg\min_{\mathbf{c}_i, \mathbf{w}_i} E\left[\left| \mathbf{c}_i^H \mathbf{R} \mathbf{w}_i^* - b_i \right|^2\right] \end{aligned} \quad (7)$$

$$\begin{aligned} \text{MSE} = &\sum_{k=1}^{K} P_k \left| \mathbf{c}_i^H \mathbf{s}_k \mathbf{a}_k^T \mathbf{w}_i^* \right|^2 + \sigma^2 \left(\mathbf{c}_i^H \mathbf{c}_i\right)\left(\mathbf{w}_i^H \mathbf{w}_i\right) \\ &- 2\sqrt{P_k} \Re\left\{ \mathbf{c}_i^H \mathbf{s}_k \mathbf{a}_k^T \mathbf{w}_i^* \right\} + 1 \end{aligned} \quad (8)$$

where $\Re\{\cdot\}$ denotes the real part of a complex number. A closed form expression for the minimizer of MSE does not exist and the MSE is not jointly convex in both vector variables. However, it is convex for a single variable given the other variable is fixed. An alternating minimization based iterative algorithm (Gauss-Seidel iterations) was proposed in [18, 17]. In this iterative method, while one variable (say $\mathbf{w}_i$) is fixed the other variable ($\mathbf{c}_i$) is updated to maximally decrease MSE. At the next step, the previously updated variable ($\mathbf{c}_i$) is kept constant and the previously fixed variable ($\mathbf{w}_i$) is updated. This procedure continues in a round-robbin fashion until convergence. Following the notation in [17], the values of $\mathbf{w}_i$ and $\mathbf{c}_i$ that maximally decrease MSE are given as follows:

$$\begin{aligned} \hat{\mathbf{c}}_i &= \text{MMSE}(\mathbf{w}_i) \\ &= \sqrt{P_k}(\mathbf{w}_i^H \mathbf{a}_i) \left( \sum_{k=1}^{K} P_k |\mathbf{w}_i^H \mathbf{a}_k|^2 \mathbf{s}_k \mathbf{s}_k^H + \sigma^2 \mathbf{I} \right)^{-1} \mathbf{s}_i \end{aligned} \quad (9)$$

$$\begin{aligned} \hat{\mathbf{w}}_i &= \text{MMSE}(\mathbf{c}_i) \\ &= \sqrt{P_k}(\mathbf{c}_i^H \mathbf{s}_i) \left( \sum_{k=1}^{K} P_k |\mathbf{c}_i^H \mathbf{s}_k|^2 \mathbf{a}_k \mathbf{a}_k^H + \sigma^2 \mathbf{I} \right)^{-1} \mathbf{a}_i \end{aligned} \quad (10)$$

Note that each step of this iterative algorithm consists of 2 sub-steps: equations (9) and (10). The $n+1^{th}$ step is shown below:

$$\mathbf{c}_i(n+1) = \text{MMSE}\left(\mathbf{w}_i(n)\right) \quad (11)$$

$$\mathbf{w}_i(n+1) = \text{MMSE}\left(\mathbf{c}_i(n+1)\right) \quad (12)$$

It was observed that with power control the performance of rank-1 constrained filters were near optimal [17].

## IV. Rank Constrained Temporal-Spatial Filters

Rank-1 constrained filter's simplicity is an advantage, but because of the tight constraint on the solution space, its performance is suboptimal as compared to the OTSF's. This performance difference could be quite pronounced in heavily loaded systems: as the number of interfering users increase with respect to the dimensions provided by the temporal and spatial domains, the solutions found in the constrained space of rank-1 filters become more inadequate. Under such conditions, filters whose performance lie between that of OTSF and the rank-1 constrained filter may be desired. We propose to achieve this performance increase by replacing the rank constraint with a looser version. By relaxing the constraint, the solution space will expand (including the matrices of rank 1) and the filters found in this new larger space will possibly perform better. Here we will investigate the general class of rank-$r$ constrained filters, where $1 \leq r \leq \min\{N, M\}$. In other words, the solution space of the optimization problem will be the space of up to rank $r$ matrices in $C^{N\times M}$. Note that any matrix filter whose rank is less than or equal to $r$ can be expressed in terms of at most $r$ temporal-spatial filter pairs:

$$\mathbf{X}_i = \sum_{j=1}^{r} \mathbf{c}_{ij}\mathbf{w}_{ij}^T \tag{13}$$

where $\mathbf{c}_{ij}$ and $\mathbf{w}_{ij}$ are $N$ and $M$ dimensional vectors respectively. With this new representation, the decision statistic and the MSE can be expressed as:

$$\begin{aligned} y_i &= tr\left(\sum_{j=1}^{r} \mathbf{w}_{ij}^*\mathbf{c}_{ij}^H\mathbf{R}\right) \\ &= \sum_{j=1}^{r} \mathbf{c}_{ij}^H\mathbf{R}\mathbf{w}_{ij}^* \end{aligned} \tag{14}$$

$$\begin{aligned} \text{MSE} =& \sum_{l=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{K} P_k\mathbf{c}_{il}^H\mathbf{Q}_k\mathbf{w}_{il}^*\mathbf{w}_{ij}^T\mathbf{Q}_k^H\mathbf{c}_{ij} \\ &+ \sigma^2\sum_{l=1}^{r}\sum_{j=1}^{r}\left(\mathbf{c}_{il}^H\mathbf{c}_{ij}\right)\left(\mathbf{w}_{il}^H\mathbf{w}_{ij}\right) \\ &- 2\sum_{j=1}^{r}\sqrt{P_i}\Re\left(\mathbf{c}_{ij}^H\mathbf{Q}_i\mathbf{w}_{ij}^*\right) + 1 \end{aligned} \tag{15}$$

where $\mathbf{Q}_k = \mathbf{s}_k\mathbf{a}_k^T$. As in the case with the rank-1 constrained filter, there is no closed form expression for the minimizer of (15) and MSE is not jointly convex in all vector variables. Note that in this case we have $2r$ vector variables $\{\mathbf{c}_{i1}, \ldots, \mathbf{c}_{ir}, \mathbf{w}_{i1}, \ldots, \mathbf{w}_{ir}\}$. Fortunately, since MSE is convex for each variable given that the remaining $2r-1$ variables are fixed, alternating minimization approach can be used here as well to iteratively minimize the MSE. The expanded solution space causes the increase in the number of variables, and consequently, each step of the iterative algorithm will consist of $2r$ sub-steps. At each sub-step of the algorithm, a single variable will be updated and the remaining $2r-1$ variables will be fixed. With some abuse of notation let $\text{MMSE}(\{\tilde{\mathbf{c}}_{ij}\}_{j\neq x}, \{\tilde{\mathbf{w}}_{ij}\}_{j=1}^r)$ and $\text{MMSE}(\{\tilde{\mathbf{c}}_{ij}\}_{j=1}^r, \{\tilde{\mathbf{w}}_{ij}\}_{j\neq x})$ denote values of $\mathbf{c}_{ix}$ and $\mathbf{w}_{ix}$ that minimize the MSE given that the remaining $2r-1$ variables are fixed. After setting the gradient equal to zero and solving for vector variables, the following equations are found:

$$\begin{aligned} \mathbf{c}_{ix} =& \text{MMSE}(\{\mathbf{c}_{ij}\}_{j\neq x}, \{\mathbf{w}_{ij}\}_{j=1}^r) \\ =& \left(\sum_{k=1}^{K} P_k\mathbf{Q}_k\mathbf{w}_{ix}^*\mathbf{w}_{ix}^T\mathbf{Q}_k^H + \sigma^2|\mathbf{w}_{ix}|^2\mathbf{I}\right)^{-1} \\ \times&\left(\sqrt{P_i}\mathbf{Q}_i\mathbf{w}_{ix}^* - \sum_{j\neq x}^{r}\left(\sum_{k=1}^{K} P_k\mathbf{Q}_k\mathbf{w}_{ix}^*\mathbf{w}_{ij}^T\mathbf{Q}_k^H + \sigma^2\mathbf{w}_{ix}^H\mathbf{w}_{ij}\mathbf{I}\right)\mathbf{c}_{ij}\right) \end{aligned} \tag{16}$$

| |
|---|
| FOR $s = 1:S$ DO |
|     FOR $x = 1:r$ DO |
|         $\hat{\mathbf{c}}_{ix} = \text{MMSE}(\{\tilde{\mathbf{c}}_{ij}\}_{j\neq x}, \{\tilde{\mathbf{w}}_{ij}\}_{j=1}^r)$ |
|         $\hat{\mathbf{w}}_{ix} = \text{MMSE}(\{\tilde{\mathbf{c}}_{ij}\}_{j=1}^r, \{\tilde{\mathbf{w}}_{ij}\}_{j\neq x})$ |
|     END |
| END |

Table 1: Summary of the alternating minimization algorithm for the rank $r$ case

$$\begin{aligned} \mathbf{w}_{ix} =& \text{MMSE}(\{\mathbf{c}_{ij}\}_{j=1}^r, \{\mathbf{w}_{ij}\}_{j\neq x}) \\ =& \left(\sum_{k=1}^{K} P_k\mathbf{Q}_k^T\mathbf{c}_{ix}^*\mathbf{c}_{ix}^T\mathbf{Q}_k^* + \sigma^2|\mathbf{c}_{ix}|^2\mathbf{I}\right)^{-1} \\ &\left(\sqrt{P_i}\mathbf{Q}_i^T\mathbf{c}_{ix}^* - \sum_{j\neq x}^{r}\left(\sum_{k=1}^{K} P_k\mathbf{Q}_k^T\mathbf{c}_{ix}^*\mathbf{c}_{ij}^T\mathbf{Q}_k^* + \sigma^2\mathbf{c}_{ix}^H\mathbf{c}_{ij}\mathbf{I}\right)\mathbf{w}_{ij}\right) \end{aligned} \tag{17}$$

A summary of this algorithm is shown in Table 1. Although Table 1 shows that the variables are updated in the order $\{\mathbf{c}_{i1}, \mathbf{w}_{i1}, \ldots, \mathbf{c}_{ir}, \mathbf{w}_{ir}\}$, in general we may exchange the order in which the variables are updated. The convergence of the overall algorithm is guaranteed by the fact that each sub-step guarantees to decrease the MSE function which is bounded below. Due to the nonconvexity of MSE (8), the global minimum is not attained by a unique filter pair. This can be observed by noting that for any nonzero value of $\beta$, all filter pairs $[\beta\mathbf{c}_{ix}, \mathbf{w}_{ix}/\beta]$ will produce the same MSE. As a final remark, note that since MSE is possibly multimodal, the algorithm may get stuck in a local minima. However experimentally, our full-rank filter ($r = min\{N, M\}$) always converged to the MMSE value that OTSF achieved. For the case of $1 \leq r < min\{N, M\}$, the randomly initialized filter coefficients always converged to the same MSE value.

## V. Adaptive Implementations

In this section, adaptive implementations of the rank constrained filters will be formulated. The deterministic case assumes the knowledge of all users' temporal signatures, array response vectors, transmit powers and channel information. Not all these parameters may be available to the system, especially in a cellular scenario. Adaptive implementations are preferred because the only side information required are the desired user's training bits and signature sequence.

The adaptive implementation that we propose here will be a combination of the alternating minimization approach of the previous section and the least mean squares (LMS) algorithm. While keeping the main structure of the alternating minimization algorithm, each sub-step (Equations (16) and (17)) will be treated as an independent LMS problem. In the deterministic case at each sub-step, one variable is updated to maximally decrease MSE. LMS on the other hand is a recursive method that uses noisy estimates of the gradient to update the filter estimate along the direction of the steepest descent. Because of the stochastic nature of LMS, in principle infinite iterations are required to reach the optimal point, whereas in the deterministic case, the same is accomplished with a single update (equations (16) and (17)). Since it is not feasible to wait for such long periods, for each sub-step, we will only use $B$ training bits. When the algorithm moves on to the next sub-step, the MSE function (error surface) will be changed and a new

LMS algorithm will begin. Therefore, the same set of training bits could be reused, for a more efficient use of the resources, bandwidth and time.

The classic update rule of LMS is given by [19]:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \left[b(n) - \mathbf{w}^H(n)\mathbf{u}(n)\right]^* \mathbf{u}(n) \qquad (18)$$

where $\mathbf{w}(n)$, $\mu$, $\mathbf{u}(n)$ and $b(n)$ represent the filter estimate, step size, received signal and the desired response respectively. Applying this rule to our case results in the following equations:

$$\begin{aligned}\mathbf{c}_{ix}(n+1) =& \mathbf{c}_{ix}(n) \\ &+\mu \left[b(n) - tr\left(\mathbf{c}_i^H(n)\mathbf{R}(n)\mathbf{w}_i^*\right)\right]^* \mathbf{R}(n)\mathbf{w}_{ix}^*\end{aligned} \qquad (19)$$

$$\begin{aligned}\mathbf{w}_{ix}(n+1) =& \mathbf{w}_{ix}(n) \\ &+\mu \left[b(n) - tr\left(\mathbf{c}_i^H(n)\mathbf{R}(n)\mathbf{w}_i(n)^*\right)\right]^* \mathbf{R}^T(n)\mathbf{c}_{ix}^*\end{aligned} \qquad (20)$$

Step size is an important parameter in LMS algorithms. When a small step size is used, the algorithm converges slower but eventually it achieves better performance (higher SIR in this case) with respect to a larger step size. We expect our algorithm to exhibit the same characteristics. Although in principle an infinite number of iterations are required to reach the optimal point in the LMS algorithm, in our implementation we limit the number of recursive iterations to $B$. The value of $B$ should be chosen large enough to avoid premature jumping to the next step, but small enough to avoid unacceptably slow convergence of the overall adaptive algorithm. Both the step size ($\mu$) and the number of recursive iterations ($B$) are important parameters that affect the convergence of the adaptive algorithm as investigated in the next section.

## VI. Numerical Results and Discussions

We consider a single cell CDMA system that employs a base station antenna array. We assume a linear array of antennas equispaced at half a wavelength [20]. The results are time averages of 100 runs, both the temporal signatures and the users' positions are randomly generated for each run. The directions of arrival of users' signals are uniformly distributed over $(-\pi/3, \pi/3]$. To simulate a *near-far* situation, interferers' powers are set 10 dB stronger than the desired user and the SNR level of the desired user is kept at 10 dB. For different experiments, the signal-to-interference ratio (SIR) for the desired user versus the iteration index is plotted to investigate the behaviors of the algorithms.

We first consider a system with processing gain $N = 16$, $M = 8$ array elements and $K = 40$ users. Figures 2 and 3 show the output MSE and SIR respectively. Note that with appropriate scaling, MSE and SIR produced by every linear matrix filter can be related [17], thus for the remaining experiments we will only plot the SIR graphs. It is seen that each iteration decreases the MSE and increases the SIR monotonically, which indicates that the alternating minimization algorithm is working as expected. Increasing the maximum allowable rank of $\mathbf{X}_i$ (relaxing the constraint to expand the solution space) also increases the realized SIR level. At the convergence point, rank-1 constrained filter achieves 4.23 dB SIR whereas rank-2 and rank-4 filters converge to values of 6.35 and 7.08 dB respectively. Note that the maximum SIR that OTSF performs is very close to rank-4's SIR level. This indicates that near full-rank ($r = 8$ in this case) performance can be achieved with a mild increase in complexity.

In Figure 3, we see that there is approximately 3dB gap between the OTSF and the rank-1 constrained filter. It was mentioned earlier that this difference is smaller in lightly loaded systems. In Figure 4, a system with processing gain $N = 16$, $M = 8$ array elements and $K = 10$ users is considered. It is seen that the difference between possible filter structures is rather indistinguishable with only 0.5 dB difference between OTSF and rank-1 constrained filter. For such systems, the performance improvement that is gained from extra complexity may become insignificant, however we observe that the rank-4 constrained filter converges faster than its rank-1 counterpart. This is due to the fact that the number of coefficients updated at each step (iteration index) of rank-4 is four times the number of coefficients updated in rank-1 case.

The purpose of Figure 5 is to emphasize the difference between rank-constrained filters and reduced-rank methods. For the system described in Figure 3, the maximum SIR of rank constrained filters are compared with the reduced-rank multistage wiener filter (MSWF). The MSWF curve is generated by formulating equation (5) with long vector variables (matrices $\mathbf{X}$ and $\mathbf{R}$ in equation (5) become vectors $\mathbf{x}$ and $\mathbf{r}$ ) and applying the MSWF techniques described in [15, 16]. The reduced-rank algorithms project the received signal $\mathbf{r}$ to a lower dimensional subspace and the 'rank' term refers to the rank of the autocorrelation matrix $E[\mathbf{r}\mathbf{r}^H]$. In these methods, the dimension of the $\mathbf{r}$ vector and correspondingly the dimension of the receiver filter $\mathbf{x}$ is decreased. Full-rank indicates an $\mathbf{x}$ vector of length $NM$. In rank constrained filters on the other hand, 'rank' refers to the rank of $\mathbf{X}$, with full-rank being equal to $min\{N, M\}$. Unlike the reduced-rank methods, the dimensions of the 'constrained' and 'optimum' filters are the same. Figure 5 shows that the near optimum performance of our rank-4 filter is achieved by a reduced-rank filter of dimension 7.

For the rest of this section, the system considered has $M = 8$ antenna array elements, $N = 16$ processing gain and $K = 40$ users. Figure 6 compares the adaptive implementations of different rank constrained filters. Blocks of $B = 100$ training bits are used at each sub-step of the alternating minimization algorithm such that each block of 100 bits corresponds to one step (iteration index) of the deterministic case. Similar to Figure 3, filters with higher maximum allowable rank perform better. Although the performance difference between different filters is preserved, adaptive implementations typically perform approximately 2dB worse than their deterministic counterparts. For the same system, the effects of the step size and $B$ on convergence are shown in Figures 7 and 8. In classic LMS applications, small step size indicates slower convergence but a higher SIR. The same phenomenon can be observed in Figure 7; the curve with $\mu = 0.001$ displays slower convergence than the curve with $\mu = 0.01$, but as the algorithm evolves, it achieves a higher SIR. The effect of $B$ on the convergence of the adaptive algorithm is seen in Figure 8. Note that for a duration of 1000 bits, the curve with $B = 100$ corresponds to an alternating minimization algorithm with 10 steps where each step uses 100 bits, $B = 1$ case, on the other hand, is an alternating minimization algorithm of 1000 steps with only 1 bit per step. Even though the latter case has more steps of alternating minimization, because each step performed poorly as compared to the former, it converges to

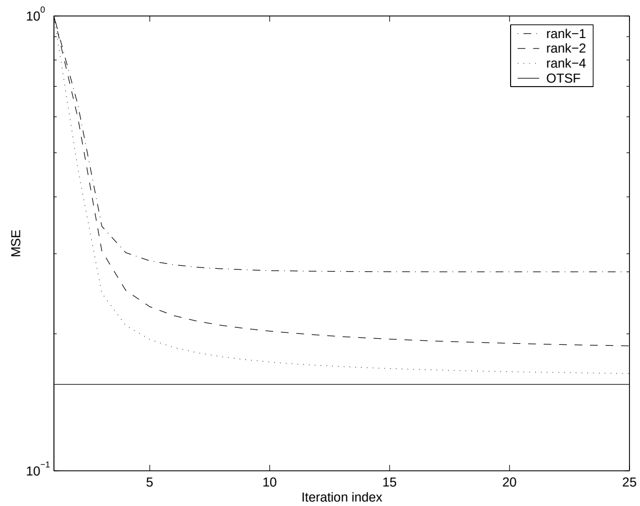


Fig. 2: $K = 40,\ N = 16,\ M = 8$

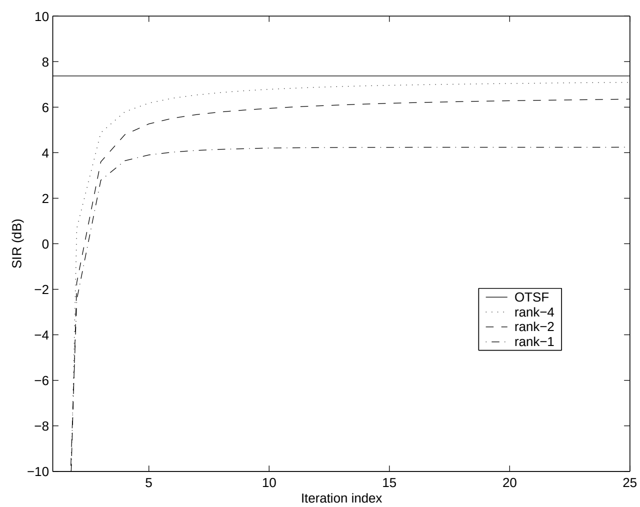


Fig. 3: $K = 40,\ N = 16,\ M = 8$

a lower SIR value.

## VII. Conclusion

In this paper, we have proposed the rank constrained temporal-spatial filters and derived their deterministic and adaptive implementations. It is shown that with a looser rank constraint, better performance can be achieved at the expense of additional complexity. Even in heavily loaded systems, where there is a significant performance gap between OTSF and the rank-1 filter, near full-rank (optimal) performance can be achieved with a mild increase in complexity with respect to the rank-1 constrained filter. Adaptive implementations based on LMS are formulated and their convergence properties are investigated. It is seen that when a combination of LMS and alternating minimization is employed, the number of recursive iterations ($B$), becomes an important parameter that affect convergence, together with step size.

We conclude by noting that interference management capability of the system can be further enhanced by combining the rank constrained filters with power control algorithms [17].

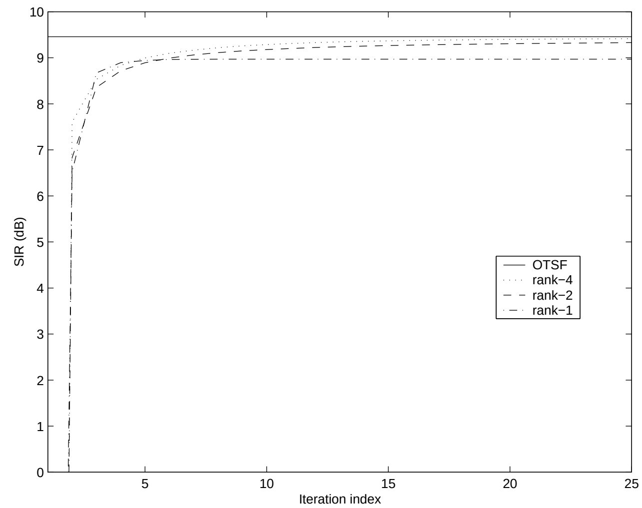


Fig. 4: A lightly loaded system $K = 10,\ N = 16,\ M = 8$

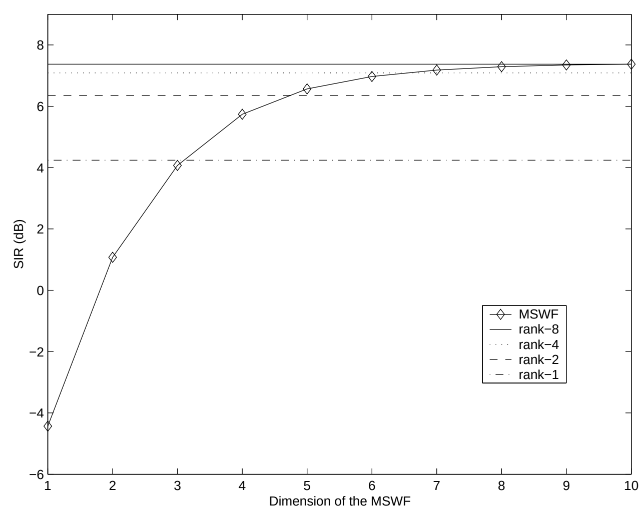


Fig. 5: MSWF and rank constrained filters

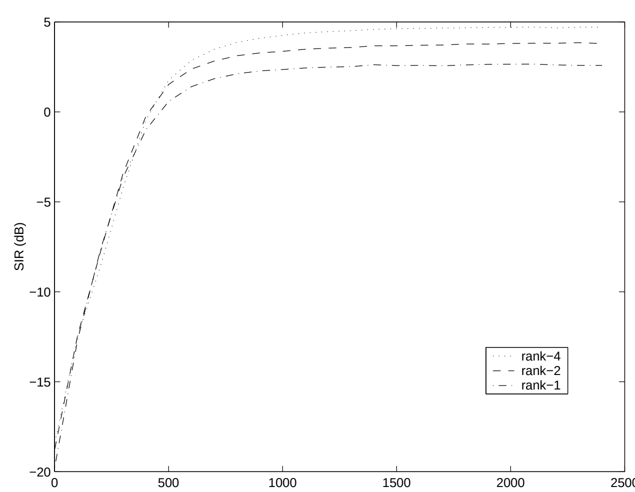


Fig. 6: $K = 40,\ N = 16,\ M = 8$

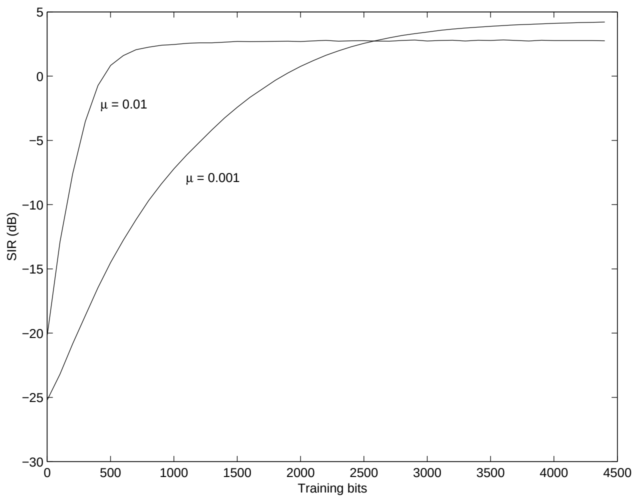


Fig. 7: $K = 40,\ N = 16,\ M = 8$

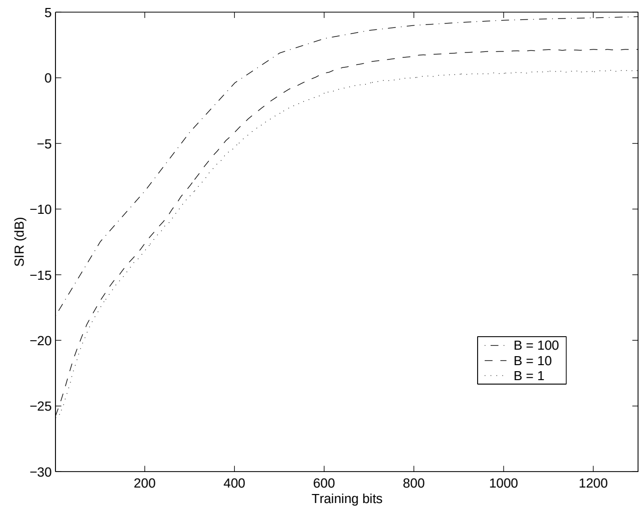


Fig. 8: $K = 40,\ N = 16,\ M = 8$

# References


[1] S. Verdú. *Multiuser Detection*. Cambridge, 1998.

[2] S. Verdú. Computational complexity of multiuser detection. *Algorithmica*, 4(4):303–312, 1989.

[3] U. Madhow and M. L. Honig. MMSE interference suppression for direct-sequence spread-spectrum cdma. *IEEE Transactions on Communications*, 42(12):3178–3188, December 1994.

[4] A. Duel-Hallen. Decorrelating decision-feedback multiuser detector for synchronous code-division multiple access channels. *IEEE Transactions on Communications*, 41(2):285–290, February 1993.

[5] M. L. Honig, U. Madhow, and S. Verdú. Blind adaptive multiuser detection. *IEEE Transactions on Information Theory*, 41(4):944–960, July 1995.

[6] X. Wang and H. V. Poor. Blind multiuser detection: A subspace approach. *IEEE Transactions on Information Theory*, 44(2):677–690, March 1998.

[7] B. D. Van Veen and K. M. Buckley. Beamforming: A versatile approach to spatial filtering. *IEEE ASSP Magazine*, 5(2):4–24, April 1988.

[8] J. H. Winters, J. Salz, and R. D. Gitlin. The impact of antenna diversity on the capacity of wireless communication systems. *IEEE Transactions on Communications*, 42(2):1740–1751, Feb/March/April 1994.

[9] A. F. Naguib, A. Paulraj, and T. Kailath. Capacity improvement with base-station antenna arrays in cellular CDMA. *IEEE Transactions on Vehicular Technology*, 43(3):691–698, August 1994.

[10] S. Y. Miller and S. C. Schwartz. Integrated spatial-temporal detectors for asynchronous gaussian multiple-access channels. *IEEE Transactions on Communications*, 43(2/3/4):396–411, Feb/March/April 1995.

[11] S. Kapoor, S. Gollamudi, S. Nagaraj, and Y. F. Huang. Adaptive multiuser detection and beamforming for interference suppression in CDMA mobile radio systems. *IEEE Transactions on Vehicular Technology*, 48(5):1341–1355, September 1999.

[12] N. A. Mohamed and J. G. Dunham. A low-complexity combined antenna array and interference cancellation DS-CDMA receiver in multipath fading channels. *IEEE Journal on Selected Areas in Communications*, 20(2):248–256, February 2002.

[13] T. S. Lee and T. C. Tsai. A beamspace-time interference cancelling CDMA receiver for sectored communications in a multipath environment. *IEEE Journal on Selected Areas in Communications*, 19(7):1374–1384, July 2001.

[14] A. Yener, R. D. Yates, and S. Ulukus. Combined multiuser detection and beamforming for CDMA systems: filter structures. *IEEE Transactions on Vehicular Technology*, 51(5):1087–1095, September 2002.

[15] J. S. Goldstein, I. S. Reed, and L L. Scharf. A multistage representation of the wiener filter based on orthogonal projections. *IEEE Transactions on Information Theory*, 44(7):2943–2959, November 1998.

[16] M. L. Honig and W. Xiao. Performance of reduced-rank linear interference suppression. *IEEE Transactions on Information Theory*, 47(5):1928–1946, July 2001.

[17] A. Yener, R. D. Yates, and S. Ulukus. Interference management for CDMA systems through power control, multiuser detection, and beamforming. *IEEE Transactions on Communications*, 49(7):1227–1239, July 2001.

[18] D. Bertsekas. *Nonlinear Programming*. Belmont, MA: Athena Scientific, 1995.

[19] S. Haykin. *Adaptive Filter Theory*. Prentice Hall, 2001.

[20] X. Wang and H.V. Poor. Space-time multiuser detection in multipath CDMA channels. *IEEE Transactions on Signal Processing*, 47(9):2356–2374, September 1999.