Iterative Transceiver Optimization for Multiuser MIMO Systems

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Abstract

We consider the uplink of a multiuser system where the transmitters as well as the receiver are equipped with multiple antennas. Each user multiplexes its symbols by using a linear precoder through its transmit antennas. We work with the system-wide mean squared error as the performance measure and propose algorithms to find the jointly optimum linear precoders at each transmitter and linear decoders at the receiver. The convergence analysis of the algorithms is given and numerical results are presented.

1 Introduction

Multiple transmit and receive antennas are a means to increase wireless capacity [1, 2]. Recently, there has been considerable research in exploiting the space dimension through transmit diversity, space-time coding and spatial multiplexing [3–5]. In particular, spatial multiplexing can be used to transmit multiple data streams that can be separated using receiver signal processing, e.g., [6, 7].

Spatial multiplexing can significantly benefit from transmit precoding when the channel is known at the transmitter side in addition to the receiver side. In such cases, designing the appropriate precoding strategy has been studied under a variety of system objectives [5, 7–9]. All of these studies, as most of the MIMO system analysis, have been done for a single user system that transmits multiple data streams. Optimum or near optimum transmit strategies that maximize the information theoretic sum capacity of vector multiple access channels have been investigated recently [10, 11]. A recent reference considers optimum transmit strategies relevant for a multicarrier scenario [12].

Transmit and receiver beamforming for multiuser systems when each user is transmitting a single data stream have also been studied extensively up to date. Receiver beamforming has been shown to be effective in interference suppression in multiuser systems [13, 14]. Jointly optimum transmit powers and receiver beamformers were found in [15]. Reference [16] proposed an iterative algorithm for determining the downlink powers and transmit beamformers given a signal-to-interference ratio (SIR) target at the single antenna receiver of each user. The optimality of a similar algorithm was shown in [17]. Algorithms that identify transmit and receiver beamforming strategies and the corresponding transmit power assignments are proposed in [18] with the aim of maximizing the minimum achievable SIR or providing each user with its SIR target. The algorithms suggested were numerically shown to enhance system performance, but were observed to converge to local optima [18].

Our aim in this work is to design algorithms that converge to the optimum transmitters and receivers for all users in a multiuser MIMO system, when users transmit possibly multiple data streams. We assume linear precoders and decoders for all users. We work with a system-wide performance measure for the joint optimization of precoders and decoders, namely the system-wide mean squared error (MSE). In contrast to receiver optimization for fixed transmitters, e.g. in [19], optimization of the individual MSEs is not equivalent to total MSE optimization. However, one can construct iterative algorithms, for the cases where users transmit single or multiple symbols, that monotonically decrease the total MSE under the given system constraints. The proposed algorithms are observed to converge to the best precoder-decoder pairs under the given power constraints, and result in enhanced performance for all users.

2 System Model and Performance Metric

We consider the uplink of a single cell synchronous system with K users. The receiver employs N_R antennas. We assume that the ith user multiplexes M_i data streams through its N_{T_i} transmit antennas employing a linear precoder. Similar to the notation in [7], the received vector is

$$\mathbf{r} = \sum_{j=1}^{K} \mathbf{H}_j \mathbf{F}_j \mathbf{s}_j + \mathbf{n} \tag{1}$$

where, $\mathbf{s}_i$ is the $M_i \times 1$ symbol vector, $\mathbf{H}_i$ is the $N_R \times N_{T_i}$ matrix of channel gains, $\mathbf{F}_i$ is the linear precoder and tr$\{\mathbf{F}_i^\dagger \mathbf{F}_i\} \leq p_i$ is the transmit power constraint for user i; $\mathbf{n}$ is the zero mean Gaussian noise vector with $E[\mathbf{n}\mathbf{n}^\top] = \sigma^2 \mathbf{I}$. The linear receiver (decoder) for user i is denoted by $\mathbf{G}_i$. The decision statistic $\mathbf{y}_i$ is given by

$$\mathbf{y}_i = \mathbf{G}_i \left(\sum_{j=1}^{K} \mathbf{H}_j \mathbf{F}_j \mathbf{s}_j + \mathbf{n} \right) \tag{2}$$

In this work, we aim to design precoder-decoder pairs that minimize the system-wide MSE. The MSE incurred by user i, MSE$_i$ is:

$$\begin{aligned} \text{MSE}_i &= E\left[\|\mathbf{y}_i - \mathbf{s}_i\|^2 \right] \\ &= \text{tr}\left\{ \sum_{j=1}^{K} \mathbf{F}_j^\dagger \mathbf{H}_j^\dagger \mathbf{G}_i^\dagger \mathbf{G}_i \mathbf{H}_j \mathbf{F}_j - \mathbf{F}_i^\dagger \mathbf{H}_i^\dagger \mathbf{G}_i^\dagger - \mathbf{G}_i \mathbf{H}_i \mathbf{F}_i + \mathbf{I} + \sigma^2 \mathbf{G}_i \mathbf{G}_i^\dagger \right\} \end{aligned} \tag{3}$$

where tr$(\mathbf{A})$ denotes the trace of matrix $\mathbf{A}$. The total MSE of all users in the system is

$$\text{MSE} = \text{tr}\left\{ \sum_{i=1}^{K} \sum_{j=1}^{K} \mathbf{F}_j^\dagger \mathbf{H}_j^\dagger \mathbf{G}_i^\dagger \mathbf{G}_i \mathbf{H}_j \mathbf{F}_j - \mathbf{F}_i^\dagger \mathbf{H}_i^\dagger \mathbf{G}_i^\dagger - \mathbf{G}_i \mathbf{H}_i \mathbf{F}_i + \mathbf{I} + \sigma^2 \mathbf{G}_i \mathbf{G}_i^\dagger \right\} \tag{4}$$

Total MSE minimization by choosing the transmitters and receivers has recently been studied for synchronous CDMA systems with single antennas in the context of CDMA signature optimization [20]. This performance measure is desirable to work with in transmitter (precoder) optimization, in contrast with each user minimizing its own MSE as is adapted in receiver optimization [19]. This is because the choice of the transmitter of a user affects the MSE of each user in the system. In the following sections, we pose the problem of minimizing the total MSE in the presence of power constraints, and devise iterative algorithms that reach the solution of the corresponding problems. This problem is a generalized version of what is posed in [12] without any constraints on the number of users or channel structure.

3 MSE Minimization

We now pose the problem of minimizing the MSE subject to a transmit power constraint for each user. As is explained in Section 2, transmit power constraint for user i can be expressed as tr$(\mathbf{F}_i^\dagger \mathbf{F}_i) \leq p_i$. The Lagrangian dual objective of this optimization problem is:

$$\text{tr}\left\{\sum_{i=1}^{K}\sum_{j=1}^{K}\mathbf{F}_j^\dagger\mathbf{H}_j^\dagger\mathbf{G}_i^\dagger\mathbf{G}_i\mathbf{H}_j\mathbf{F}_j - \mathbf{F}_i^\dagger\mathbf{H}_i^\dagger\mathbf{G}_i^\dagger - \mathbf{G}_i\mathbf{H}_i\mathbf{F}_i + \mathbf{I} + \sigma^2\mathbf{G}_i\mathbf{G}_i^\dagger\right\} + \sum_{i=1}^{K}\mu_i\left[\text{tr}\left\{\mathbf{F}_i^\dagger\mathbf{F}_i\right\} - p_i\right] \tag{5}$$

where $\mu_i \geq 0$ is the Lagrange multiplier associated with the transmit power constraint of user i. Optimum precoder and decoder structures should satisfy the first order optimality conditions for each user in the same manner as in the single user case [7]. Simply taking the derivative with respect to the transmitter and the receiver of user k and equating it to zero, we arrive at:

$$\mathbf{G}_k = \mathbf{F}_k^\dagger\mathbf{H}_k^\dagger\left(\sigma^2\mathbf{I} + \sum_{i=1}^{K}\mathbf{H}_i\mathbf{F}_i\mathbf{F}_i^\dagger\mathbf{H}_i^\dagger\right)^{-1} \tag{6}$$

$$\mathbf{F}_k = \left(\mu_k\mathbf{I} + \sum_{i=1}^{K}\mathbf{H}_k^\dagger\mathbf{G}_i^\dagger\mathbf{G}_i\mathbf{H}_k\right)^{-1}\mathbf{H}_k^\dagger\mathbf{G}_k^\dagger \tag{7}$$

Note that, as expected, the optimum decoders (receivers) for a given set of precoders are in the form of the well-known *MMSE receivers* [19]. Note also that the precoders are functions of decoders of all users, while the decoders are functions of precoders of all users. To find the joint optimum set of precoders and decoders, one can devise iterative algorithms that monotonically decrease the total MSE. In particular, alternating minimization where variables are optimized one at a time, keeping all others fixed, proves attractive in the design of iterative algorithms [21]. Equations (6) and (7) describe the precoder-decoder updates we can perform. The algorithm starts with a given set of precoders-decoders and we can update the decoders $\{\mathbf{G}_k\}$ and precoders $\{\mathbf{F}_k\}$ independently in a parallel fashion using (6) and (7). Note that at each iteration, the Lagrange multiplier, μ_k in (7), should be calculated such that the transmit power constraint is satisfied.

Alternatively, if we assume that each receiver (decoder) is updated instantaneously when the precoder is updated, we can reduce the two step iteration given by (6), (7) to a single iteration. This is accomplished by inserting the resulting decoders of (6) in (7). Let us define

$$\mathbf{T} = \sigma^2\mathbf{I} + \sum_{i=1}^{K}\mathbf{H}_i\mathbf{F}_i\mathbf{F}_i^\dagger\mathbf{H}_i^\dagger \tag{8}$$

Then, following some straightforward algebra, we arrive at the following iteration:

$$\mathbf{F}_k^\star = \left(\mu_k\mathbf{I} + \mathbf{H}_k^\dagger(\mathbf{T}^{-1} - \sigma^2\mathbf{T}^{-2})\mathbf{H}_k\right)^{-1}\mathbf{H}_k^\dagger\mathbf{T}^{-1}\mathbf{H}_k\mathbf{F}_k \tag{9}$$

Note that the iterative algorithm defined by (9) may choose to iterate over each user's transmitter by updating (8) with the newest transmitter found before the next user's iteration for faster convergence. Both the two-step algorithm (6), (7), and the algorithm given by (9) decrease the MSE at each step.

4 Convergence Issues

The iterative algorithms defined in the previous section are obtained by minimizing the system-wide MSE function with respect to the precoder (decoder) matrix of each user keeping precoder and decoder matrices of all other users fixed. We iterate over the users each time decreasing the total MSE monotonically. The fact that the MSE function is bounded below implies that our algorithm, which produces a decreasing sequence which is lower bounded, is convergent. Unfortunately, while the MSE function is convex over each of the precoder and decoder matrices, it is not jointly convex on all the variables. Therefore, while each step of our algorithm finds the minimum of the MSE function over the variable we optimize over, the fixed point of the algorithm is not guaranteed to converge to the global minimum due to the possible multimodality of the MSE function.

At the fixed point of our algorithm, the set of precoders remains unchanged when the iteration is performed over all the users:

$$\left(\mu_k \mathbf{I} + \mathbf{H}_k^\dagger \left(\mathbf{T}^{-1} - \sigma^2 \mathbf{T}^{-2}\right) \mathbf{H}_k\right)^{-1} \mathbf{H}_k^\dagger \mathbf{T}^{-1} \mathbf{H}_k \mathbf{F}_k = \mathbf{F}_k \tag{10}$$

which leads to

$$\mathbf{H}_k^\dagger \mathbf{T}^{-2} \mathbf{H}_k \mathbf{F}_k = \mu_k / \sigma^2 \mathbf{F}_k \tag{11}$$

Thus, at the fixed point, the columns of precoder matrix of each user are eigenvectors of the matrix $\mathbf{H}_k^\dagger \mathbf{T}^{-2} \mathbf{H}_k$ with the same eigenvalue of μ_k/σ^2.

An observation that can readily be made from (11) is that the set of precoder matrices at the fixed point is not unique. For example, if $\mathbf{F}_k = [\mathbf{f}_{k1}\mathbf{f}_{k2} \cdots \mathbf{f}_{ki}]$ is the precoder matrix of user k transmitting i symbols at the fixed point, then phase addition to each column and/or permutation of the column vectors, $\mathbf{F}_k^\star = \left[\mathbf{f}_{k2}e^{j\theta_1}\mathbf{f}_{k1}e^{j\theta_2} \cdots \mathbf{f}_{ki}e^{j\theta_i}\right]$, will also satisfy the fixed point equation.

It is also easily seen that if all columns of the initial precoder matrix of any user are in the null space of the channel matrix of that user, then the algorithm will yield the undesirable fixed point of the all zero precoder matrix. Such undesirable starting points should be avoided while implementing this algorithm, for example, by choosing random starting points.

In general, the nonconvexity of the problem may prevent the convergence of the algorithm to the global optimum. As is common with nonconvex problems, our hope then lies in finding the optimum by choosing multiple random starting points and adopting to the best of the fixed points of multiple runs. A moment's thought reveals however, that, in certain cases, we may be able to construct a mechanism to check if the fixed point we arrived is indeed the global optimum.

To see this let us rewrite the MSE function as

$$\text{MSE} = \sum_{i=1}^{K} M_i - \text{tr}\left\{\sum_{j=1}^{K} \mathbf{F}_j^\dagger \mathbf{H}_j^\dagger \mathbf{T}^{-1} \mathbf{H}_j \mathbf{F}_j\right\} = \sum_{i=1}^{K} M_i - N_R + \sigma^2 \text{tr}\left\{\mathbf{T}^{-1}\right\} \tag{12}$$

where we have used MMSE receivers for all users. Now let us define $\mathbf{R}_k = \mathbf{F}_k \mathbf{F}_k^\dagger$. The total MSE optimization problem can be restated as

$$\min \quad \text{tr}\left\{\mathbf{T}^{-1}\right\} \tag{13}$$

$$\text{s.t.} \quad \mathbf{T} \leq \sigma^2 \mathbf{I} + \sum_{i=1}^{K} \mathbf{H}_i \mathbf{R}_i \mathbf{H}_i^\dagger \tag{14}$$

$$\text{tr}\{\mathbf{R}_i\} \leq p_i; \quad \mathbf{R}_i \geq 0 \qquad i = 1, \cdots, K \tag{15}$$

$$\text{rank}(\mathbf{R}_i) \leq \min(N_{T_i}, M_i) \qquad i = 1, \cdots, K \tag{16}$$

where $\mathbf{A} \geq 0$ refers to the positive semidefiniteness constraint on $\mathbf{A}$.

It can be shown that the MSE function is jointly convex in $\{\mathbf{R}_k\}$. In addition, the constraints defined by (14) and (15) are convex constraints. Thus, short of the rank constraint defined by (16), which is a consequence of the fact that each $\mathbf{F}_k$ is $N_{T_k} \times M_k$, we have a convex optimization problem at hand. If we are able to discard this constraint, then the convex optimization problem defined by (13)-(15) may prove useful in our quest to test the optimality of the algorithm described in Section 3.

Specifically, the KKT conditions are necessary and sufficient to test the optimality of the $\{\mathbf{R}_k\}$ which can be readily constructed from the precoders at the fixed point of our algorithm. Constructing the Lagrangian dual problem, one can come up with the following KKT conditions for this problem [22]:

$$\lambda_k \mathbf{I} = \mathbf{H}_k^\dagger \mathbf{T}^{-2} \mathbf{H}_k + \Psi_k \tag{17}$$
$$tr\{\mathbf{R}_k\} = p_k \tag{18}$$
$$tr\{\Psi_k \mathbf{R}_k\} = 0 \tag{19}$$
$$\Psi_k, \mathbf{R}_k, \lambda_k \geq 0 \tag{20}$$

where $\{\Psi_k, \lambda_k\}$ are the dual variables. Note that, it can be shown that the optimum $\{\mathbf{R}_k\}$ set is a fixed point of our algorithm. Thus, if for given precoders, which lead to a set of $\{\mathbf{R}_k\}$, the above conditions are satisfied, i.e. the corresponding dual variables are found, then we are at the global minimizer of the MSE function.

It is important to note that the above serves as an optimality check only when the rank constraint is redundant for the problem. Observe that the rank constraint for the $N_{T_k} \times N_{T_k}$ matrix $\mathbf{R}_k$ is necessary only for the case when $M_k < N_{T_k}$.Thus, for the cases where all users have $M_k \geq N_{T_k}$, the MSE minimization problem formulated as a function of the precoders with transmit power constraints, and the optimization problem defined by (13)-(15) in terms of $\{\mathbf{R}_k\}$ are equivalent and the optimality check above is exact. For systems, where $M_k < N_{T_k}$ for at least one user, the KKT conditions correspond to the *relaxed* problem where the rank constraint is relaxed. Thus, in this case, the KKT conditions above are sufficient but not necessary. Specifically, we may be at the optimum point of our rank constrained problem described by (13)-(16) and not at the optimum point of the convex problem described by (13)-(15). This may lead to cases where we misjudge the fixed point of our algorithm as a local minimizer. Our numerical observations so far have not revealed such points.

5 Single Symbol Transmission

A special case of the multiuser MIMO system is when each user transmits a single data stream. In this case, the communication model is the same as the system studied in [18]. The precoder and decoder matrices become vectors, and the algorithm developed in the previous section is fully applicable. In addition, an alternative iterative algorithm can be devised as follows. We will denote the precoder and decoder (MMSE receiver) vectors for user i by $\mathbf{f}_i$ and $\mathbf{g}_i$ respectively. The decoder (MMSE receiver) for user i is given by

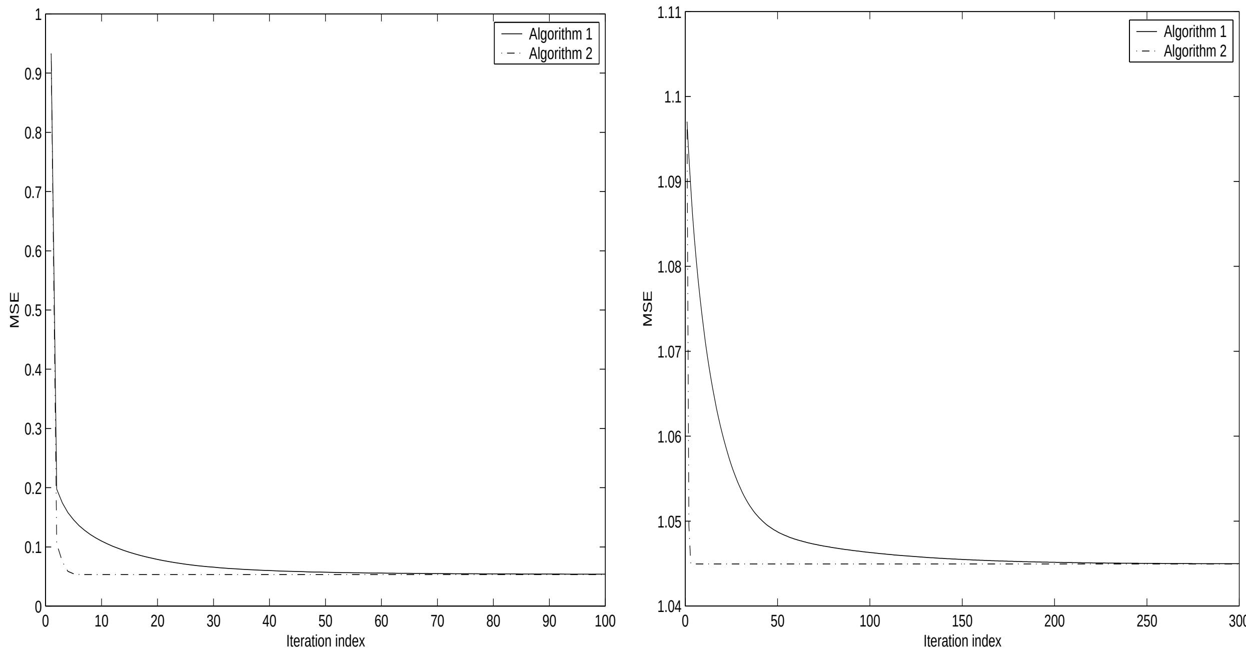
$$\mathbf{g}_i = \mathbf{f}_i^\dagger \mathbf{H}_i^\dagger \mathbf{T}^{-1}$$

where $\mathbf{T}$ is given by (8), i.e., $\mathbf{T} = \sigma^2 \mathbf{I} + \sum_{i=1}^{K} \mathbf{H}_i \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}_i^\dagger$. Substituting the above for the receivers, MSE of user i becomes

$$\text{MSE}_i = 1 - \mathbf{f}_i^\dagger \mathbf{H}_i^\dagger \mathbf{T}^{-1} \mathbf{H}_i \mathbf{f}_i \tag{21}$$

Total MSE of all users in the system is

$$\text{MSE} = K - \sum_{i=1}^{K} \mathbf{f}_i^\dagger \mathbf{H}_i^\dagger \mathbf{T}^{-1} \mathbf{H}_i \mathbf{f}_i = K - \text{tr}\{\mathbf{I}\} + \sigma^2 \text{tr}\{\mathbf{T}^{-1}\} \tag{22}$$



(a) (b)

Figure 1: MIMO system with single data stream, each user has $N_{T_i} = 4$ antennas, receiver has $N_R = 4$ antennas. (a) $K = 4$ users; (b) $K = 5$ users

Next we note that, using the matrix inversion lemma, $\mathbf{T}^{-1}$ can be expressed as

$$\mathbf{T}^{-1} = \mathbf{E}_k^{-1} - \frac{\mathbf{E}_k^{-1}\mathbf{H}_k\mathbf{f}_k\mathbf{f}_k^\dagger\mathbf{H}_k^\dagger\mathbf{E}_k^{-1}}{1 + \mathbf{f}_k^\dagger\mathbf{H}_k^\dagger\mathbf{E}_k^{-1}\mathbf{H}_k\mathbf{f}_k} \tag{23}$$

with $\mathbf{E}_k = \sum_{i \neq k} \mathbf{H}_i\mathbf{f}_i\mathbf{f}_i^\dagger\mathbf{H}_i^\dagger + \sigma^2\mathbf{I}$. Since $\mathbf{E}_k$ does not depend on $\mathbf{f}_k$, we can easily express the total MSE as

$$\text{MSE} = C_k - \sigma^2 \left(\frac{\mathbf{f}_k^\dagger\mathbf{H}_k^\dagger\mathbf{E}_k^{-2}\mathbf{H}_k\mathbf{f}_k}{1 + \mathbf{f}_k^\dagger\mathbf{H}_k^\dagger\mathbf{E}_k^{-1}\mathbf{H}_k\mathbf{f}_k} \right) \tag{24}$$

where C_k represents the terms independent of user k. Thus, from the perspective of user k, MSE can be minimized by choosing $\mathbf{f}_k$ to minimize the second term in equation (24). It is easily shown that we need $\mathbf{f}_k^\dagger\mathbf{f}_k = p_k$ to maximize the second term. Thus

$$\begin{aligned} \text{MSE} &= C_k - \sigma^2 \left(\frac{\mathbf{f}_k^\dagger\mathbf{H}_k^\dagger\mathbf{E}_k^{-2}\mathbf{H}_k\mathbf{f}_k}{1/p_k\mathbf{f}_k^\dagger\mathbf{f}_k + \mathbf{f}_k^\dagger\mathbf{H}_k^\dagger\mathbf{E}_k^{-1}\mathbf{H}_k\mathbf{f}_k} \right) && (25) \\ &= C_k - \sigma^2 \left(\frac{\mathbf{f}_k^\dagger\mathbf{H}_k^\dagger\mathbf{E}_k^{-2}\mathbf{H}_k\mathbf{f}_k}{\mathbf{f}_k^\dagger \left(1/p_k\mathbf{I} + \mathbf{H}_k^\dagger\mathbf{E}_k^{-1}\mathbf{H}_k \right) \mathbf{f}_k} \right) && (26) \end{aligned}$$

The minimization of the second term is accomplished simply by choosing $\mathbf{f}_k$ to be the maximum generalized eigenvalued eigenvector of $\mathbf{H}_k^\dagger\mathbf{E}_k^{-2}\mathbf{H}_k$ and $1/p_k\mathbf{I} + \mathbf{H}_k^\dagger\mathbf{E}_k^{-1}\mathbf{H}_k$.

An iterative algorithm that minimizes the MSE can be devised as follows: each user takes turns in optimizing the MSE function from its perspective as explained above, monotonically decreasing the MSE function at each iteration. Performance examples for this algorithm is given in the next section.

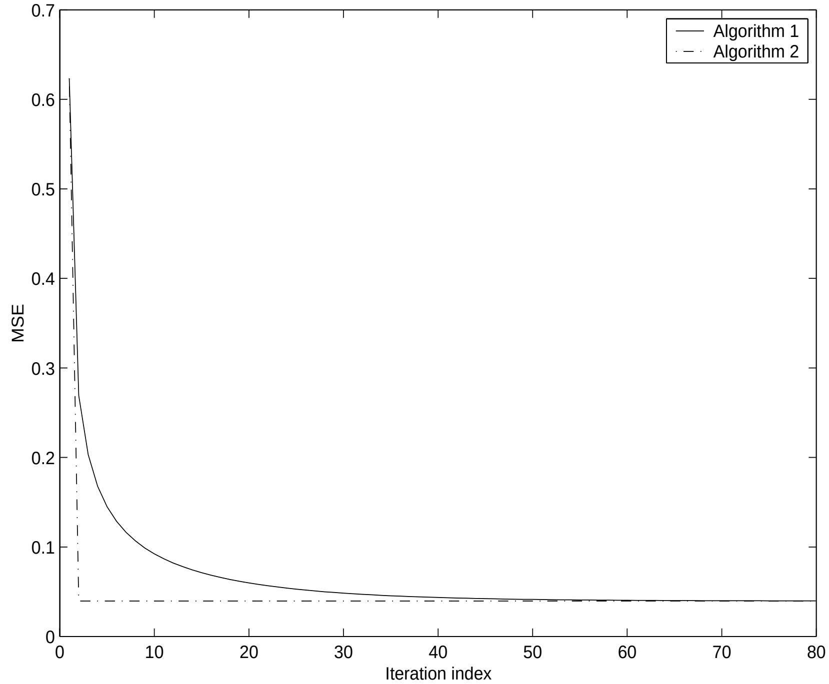


Figure 2: $K = 4$ user MIMO system with single data stream; $\mathbf{H}_i = \mathbf{I}$. Resulting precoders are orthogonal to each other.

6 Numerical Results and Conclusion

In this section, we offer numerical results to support our analysis. The simulations are performed for a multiuser system where each user is equipped with $N_{T_i} = N_T = 4$, for all i, transmit antennas and the receiver has $N_R = 4$ antennas. Transmit power constraint for each user is unity. Each plot shows the MSE value versus the algorithm iteration index, where an iteration signifies updating all users' precoders, i.e., K updates.

First, we consider a $K = 4$ user MIMO system with each user sending a single data stream. Both the algorithm described by (9), dubbed *Algorithm 1*, and the algorithm described in Section 5, dubbed *Algorithm 2*, are simulated. Figure 1(a) shows the evolution of both algorithms as they converge to the optimum MSE value. Although both algorithms converge to the optimum value, we observe that the convergence of Algorithm 2 which chooses the maximum generalized eigenvalued eigenvector of $\mathbf{H}_k^\dagger \mathbf{E}_k^{-2} \mathbf{H}_k$ and $\mathbf{I} + \mathbf{H}_k^\dagger \mathbf{E}_k^{-1} \mathbf{H}_k$, is faster. This is expected since Algorithm 2 assumes instantaneous receiver updates with each user's precoder update, and jointly optimizes the transmitter and the receiver for each user, while Algorithm 1 performs transmitter and receiver optimization for each user in tandem.

Figure 1(b) shows the MSE evolution when a fifth user becomes active in the previous system. The minimum achievable MSE of the system increases as expected due to higher number of users and channel constraints.

For the special case of $\mathbf{H}_i = \mathbf{I}$, precoder vectors of each user are unit energy spatial (transmit) signatures. Such a system is mathematically equivalent to a CDMA system with a processing gain N_T and equal received powers. When MSE minimization algorithms are applied to such systems, optimum precoders are found to be orthogonal column vectors when $N_T \geq K$ [20]. Performance of the MSE minimization algorithms for $N_T = K = 4$ users is given in Figure 2.

Figure 3 shows the performance of Algorithm 1 for the case of multiple symbol transmission by $K = 2$ users, each equipped with $N_T = 4$ transmitter antennas and transmitting

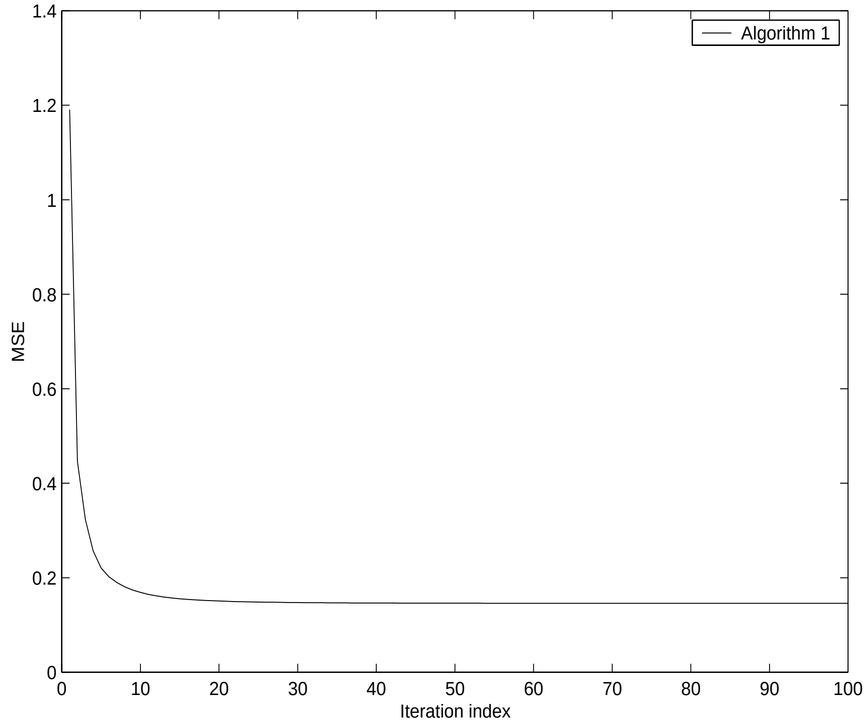


Figure 3: $K = 2$ user MIMO system with $M_i = 2$ data streams per user. $N_{T_i} = N_R = 4$

$M_1 = M_2 = 2$ data streams. Total MSE monotonically decreases and converges to its minimum value. When number of transmitter antennas of each user is decreased to $N_T = 2$, the performance degradation of the system at the optimum point due to the reduction in spatial dimensions is shown in Figure 4.

Finally, Figure 5 shows the evolution of the iterative algorithm described by (9) for a $K = 3$ user system where each user transmits 2 data streams with five random starting points. Total MSE monotonically decreases and converges to its minimum value for each starting precoder set. Although the resulting precoder sets are different, the total MSE of the system and $\mathbf{R}_k$ of each user at the fixed point are the same for each starting point. When precoder matrices are started randomly, we observed they consistently converged to the same total MSE and $\{\mathbf{R}_k\}$ set for a given set of channel matrices $\{\mathbf{H}_k\}$. We also were able to verify the optimality in each case.

In this paper, we proposed transmitter (and receiver) update algorithms that are geared towards enhancing the system performance by minimizing the total MSE in a multiuser MIMO system. We have investigated the fixed point properties and identified an optimality check mechanism for the algorithm proposed for the system where each user transmits multiple symbols. For the special case where each user sends one symbol, we proposed an alternative iterative algorithm that is observed to have faster convergence.

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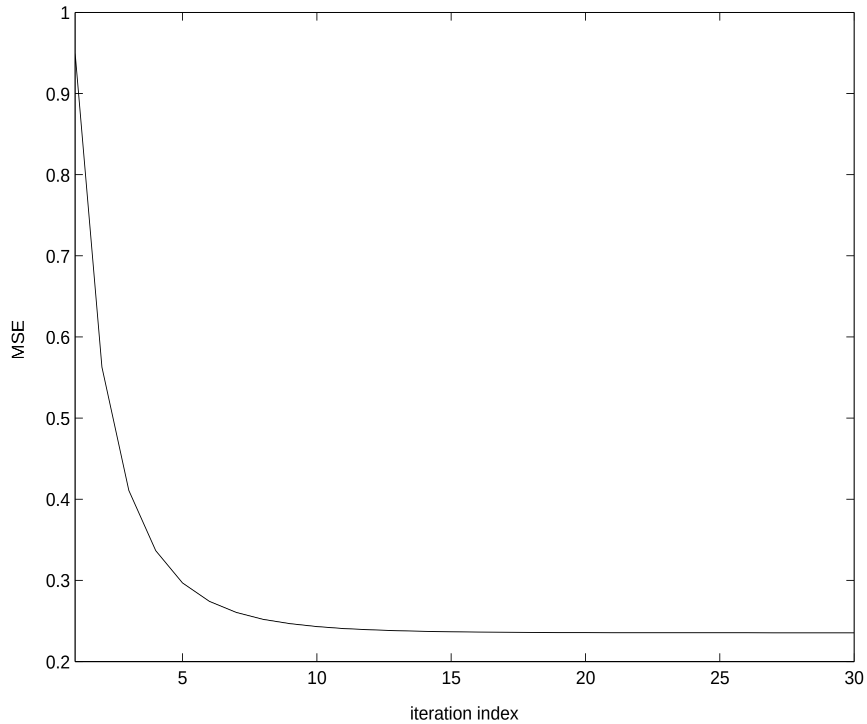

Figure 4: $K = 2$ user MIMO system with $M_i = 2$ data streams per user. $N_{T_i} = 2, N_R = 4$

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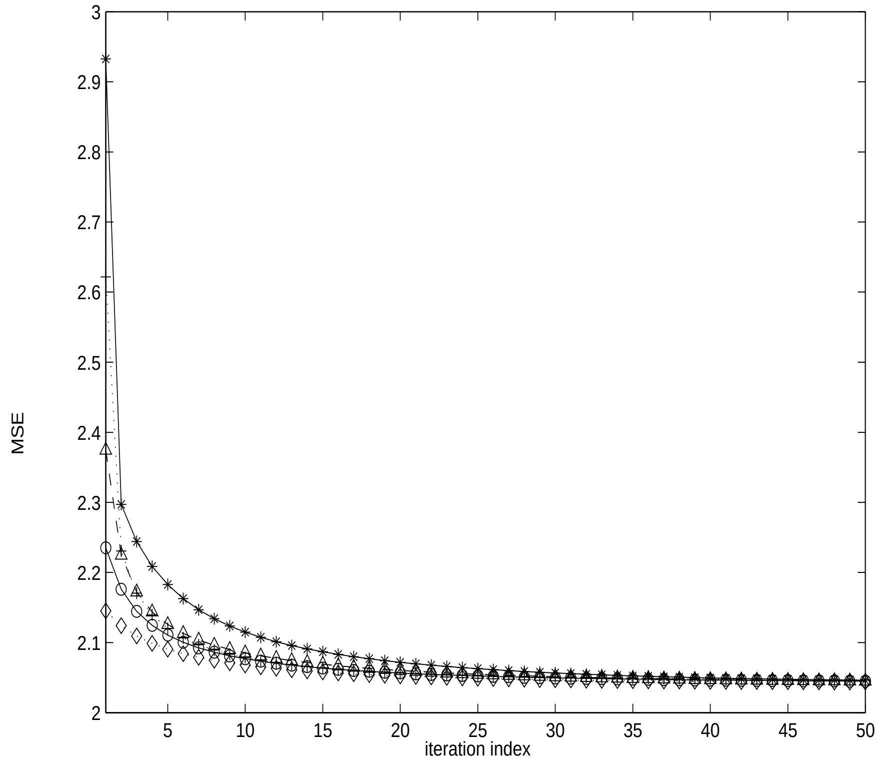

Figure 5: $K = 3$ user MIMO system with $M_i = 2$ data streams per user. $N_{T_i} = N_R = 4$ Performance of Algorithm 1 with 5 different starting precoder sets

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