Relays Can Provide Alignment for the K-user Interference Channel without Channel State Information at the Transmitters

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***Abstract*—Channel state information at the transmitters (CSIT) is of importance for interference alignment schemes to achieve the optimal degrees of freedom (DoF) for wireless networks. This paper focuses on the K-user interference channel and shows that, employing relays between the sources and the destinations, interference alignment is possible without CSIT. Furthermore, the optimal DoF can be achieved with the help of relays. Specifically, a transmission strategy is designed and it is shown that the DoF $\frac{K}{2}$ can be achieved with a two-slot transmission strategy using either (i) one relay with $K-1$ antennas, or (ii) $K(K-2)$ single antenna relays. It is also shown that when each relay is equipped with L antennas, we need $\left\lceil \frac{K(K-2)}{L^2} \right\rceil$ relays to achieve the optimal DoF.**

I. Introduction

Interference alignment [1], is a powerful technique that can be used to achieve optimal degrees of freedom (DoF) for a variety of wireless networks [1]–[3]. With the help of the channel state information (CSI) at the transmitters (CSIT), the transmitters can steer the signals in such a way that the interfering signals occupy the fewest signal space dimensions at each receiver. Not surprisingly, a key assumption for the interference alignment schemes is that the transmitters and the receivers both have accurate instant global CSI, which we term as *full CSI*. In a practical system, however, obtaining full CSIT is difficult. Consequently, references [4], [5] have studied the DoF of various channel models with the assumption that no CSIT is available, and loss of DoF is observed for many scenarios. Without CSIT, the transmitters cannot steer the signals to the exact desired directions to guarantee that the interference is aligned together at the receivers, which causes the performance degradation in terms of DoF.

To address the loss of DoF caused by the absence of CSIT and in the meantime alleviate the difficulty of obtaining full CSIT, references [6], [7] have shown that as long as the correlation structure of the channel is known at the transmitters, without any knowledge of the exact channel coefficient, interference alignment is still possible for certain wireless networks to achieve the optimal DoF. Another interesting and practical assumption about CSIT is the delayed CSIT model proposed by reference [8]. The delayed CSIT model characterizes the channel variation and the delay in the feedback of CSI from receivers, and thus is important in both theoretical and practical sense. The delayed CSIT assumption is first studied in the context of the K-user broadcast channel [8], i.e., a channel with a transmitter having K antennas and K receivers each having a single antenna, where the transmitter has accurate and global CSIT, but with one slot delay. It is shown that the delayed CSIT can still be useful for interference alignment and the DoF can be improved significantly compared to the case without CSIT. This delayed CSIT assumption is then applied to various of channel models such as the general broadcast channels, interference channels and X channels, and improvement on the DoF compared to the cases without CSIT is found in references [9]–[12] and the references therein. However, with delayed CSIT, the interference alignment schemes cannot achieve the same optimal DoF of the networks as when full CSIT is available.

The addition of a relaying node, although is beneficial in improving the achievable rates, is shown in reference [13] to be unable to improve the DoF of interference and X channels. In this reference, each node has full CSI and the network is fully connected. The impact of relaying on the interference alignment schemes of fully connected wireless networks so far mainly focuses on making the schemes more practical, that is, to reduce the requirement for time extension or to accommodate for static channels [14]–[17]. For networks with relays that are not fully connected, e.g., the multi-hop relay networks, reference [18] has studied the DoF under full CSIT and proposed an interference neutralization to achieve the min-cut DoF of the network. When nodes only have local CSI, it is shown in reference [19] that relaying can help achieve the full DoF of the K-user interference channel, when the relay has more antennas than the total number of transmitters.

In this work, we study the impact of relaying on the DoF from another perspective. We focus on understanding whether, and to what extend, relays can improve the DoF of wireless networks when no CSIT is available. Our previous work [20] has considered using relays to facilitate interference alignment for the 2-user X channel without CSIT, and it is shown that the full DoF $\frac{4}{3}$ can be achieved using one relay with two antennas with delayed CSI, or using one relay with one antenna with full

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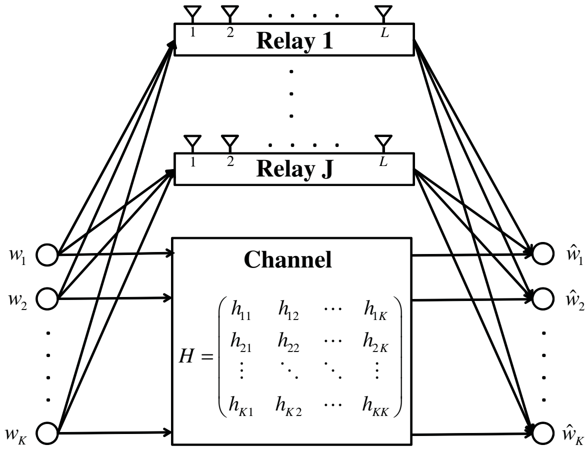

Fig. 1. K-user interference channel with relays.

CSI. The achievable scheme is based on *joint beamforming*. Here, we use the idea developed in [20] to study the DoF of the K-user interference channel without CSIT. The relays are assumed to have full CSI. We show that interference alignment is still possible utilizing the relays. Specifically, we design a 2-slot transmission strategy and show that the optimal DoF $\frac{K}{2}$ can be achieved using one relay with $K-1$ antennas, or $K(K-2)$ single antenna relays. When we have one relay with $K-1$ antennas, joint beamforming is not necessary for interference alignment and the channel does not need to be time varying. However, when we have $K(K-2)$ relays each with one antenna, joint beamforming is required to achieve interference alignment and the channel does need to be time varying. For the general case, we require $\left\lceil \frac{K(K-2)}{L^2} \right\rceil$ relays with L antennas to achieve the optimal DoF.

In the remainder of the paper, we use bold capital letters, e.g. $\mathbf{H}$, to denote matrices. We use bold letters, e.g., $\mathbf{h}$ to denote vectors. We use ordinary capital letters, e.g. H, to denote scalars. $\{A_i\}$ denotes a set of variables $\{A_1, A_2, \cdots\}$.

II. System Model

Fig. 1 shows the K-user interference channel with intermediate relays. In the model we have K transmitters and K receivers, and each transmitter has a message to be communicated with its unique intended receiver. It is assumed that the transmitters and receivers are equipped with one antenna each. There are J half-duplex relays available to help the transmission. Each relay is assumed to have L antennas. We denote w_k as the message from transmitter k to its intended receiver k, $k = 1, \cdots, K$. The transmitted signal from transmitter k is denoted as $X_k(t) \in \mathbb{C}$ and the signal from relay $j = 1, \cdots, J$ is denoted as $\mathbf{X}_{R_j}(t) \in \mathbb{C}^L$, where t is the time index denoting the slot in which the signal is transmitted.

When the relays listen to the channel, the received signals at the receivers are

$$Y_n(t) = \sum_{k=1}^{K} h_{nk}(t) X_k(t) + Z_n(t), \ n = 1, \cdots, K \quad (1)$$

and the received signals at the relays are

$$Y_{R_j}(t) = \sum_{k=1}^{K} \mathbf{h}_{R_j k}(t) X_k(t) + \mathbf{Z}_{R_j}(t), \ j = 1, \cdots, J. \quad (2)$$

When the relays transmit, the received signals at the receivers are

$$Y_n(t) = \sum_{k=1}^{K} h_{nk}(t) X_k(t) + \sum_{j=1}^{J} \mathbf{h}_{nR_j}(t)^T \mathbf{X}_{R_j}(t) + Z_n(t). \quad (3)$$

In the above expressions, the transmitted signals are subject to average power constraint $E(||\mathbf{X}_{R_j}(t)||^2) \leq P$, $E(|X_k(t)|^2) \leq P$. $h_{nm} \in \mathbb{C}$ is the channel coefficient from transmitter m to the receiver n. $\mathbf{h}_{R_j m}(t) \in \mathbb{C}^L$ is the channel vector between transmitter m and relay R_j, and $\mathbf{h}_{nR_j}(t) \in \mathbb{C}^L$ is the channel vector between relay R_j and receiver n. It is assumed that the channel coefficients are independently drawn from a continuous distribution for each time index. The channel can be time varying or static. $Z_n(t)$ and $\mathbf{Z}_{R_j}(t)$ are Gaussian random variable with zero mean and unit variance, or Gaussian random vector with identity covariance matrix, respectively.

We assume the rate of message w_k is $R_k(P)$ for power constraint P. We define $\mathcal{C}(P)$ as the set of all achievable rate tuples $\{R_k(P)\}$ for power constraint P. The degree of freedom is defined as

$$DoF = \lim_{P \to \infty} \frac{R_{\sum}(P)}{\log(P)}, \quad (4)$$

where $R_{\sum}(P) = \max_{\mathcal{C}(P)} \left(\sum_{k=1}^{K} R_k(P) \right)$.

III. Relay-aided Interference Alignment for K-user Interference Channel

In this section, we investigate impact of relays on the DoF of the K-user interference channel without CSIT. Relays are assumed to have full CSI. The DoF of this channel is upper bounded by $\frac{K}{2}$, which is the maximum DoF for K-user interference channel with full CSIT [2], since relaying does not provide any DoF gain for interference channels with full CSI at all nodes [13]. It is easy to see that when a relay has K antennas, the DoF $\frac{K}{2}$ can be achieved using a 2-slot transmission scheme: In the first slot the transmitters send messages to the relay, and relay decodes all messages. In the second slot the relay broadcasts all the messages to the receivers. In the following theorem, we show that we can actually achieve the maximum DoF $\frac{K}{2}$ using a relay with only $K-1$ antennas.

Theorem 1: For the K-user interference channel without CSIT, when there is a relay with $K-1$ antennas and full CSI, the DoF $\frac{K}{2}$ is achievable.

Proof: To show the achievability of $\frac{K}{2}$ degrees of freedom, we construct a 2-slot transmission scheme.

In the first slot, each transmitter sends a message to the intended receiver, i.e.,

$$X_k(1) = d_k, \tag{5}$$

where d_k denotes the data stream carrying the message w_k, and $k = 1, \cdots, K$.

The signals received at the receivers and the relay are

$$Y_k(1) = \sum_{i=1}^{K} h_{ki}(1) d_i, \tag{6}$$

$$\mathbf{Y}_R = \sum_{i=1}^{K} \mathbf{h}_{Ri}(1) d_i, \tag{7}$$

where $\mathbf{Y}_R, \mathbf{h}_{Ri}(1) \in \mathbb{C}^{K-1}$. Note that we also omitted the noise in the expressions since we are considering DoF of the channel.

Since we use a 2-slot transmission scheme, the signal space at the receivers has 2 dimensions in time. To decode the intended message, the receivers need to keep all the other $K-1$ interference signals aligned into a one dimensional space. For this end, the relay applies a precoding matrix to the received signal vector, and transmits the following signal vector in the second slot:

$$\mathbf{X}_R = \mathbf{U}\mathbf{Y}_R \tag{8}$$

where $\mathbf{U} \in \mathbb{C}^{(K-1)\times(K-1)}$, which is to be determined later.

The received signals at the receivers for slot 2 is

$$Y_k(2) = \sum_{i=1}^{K} \mathbf{h}_{kR}^T(2)\mathbf{U}\mathbf{h}_{Ri}(1) d_i. \tag{9}$$

If we combine the received signals at receiver k from 2 slots into one vector, we have

$$\mathbf{Y}_k = \begin{pmatrix} h_{kk}(1) \\ \mathbf{h}_{kR}^T(2)\mathbf{U}\mathbf{h}_{Rk}(1) \end{pmatrix} d_k + \tag{10}$$

$$\sum_{i \neq k} \begin{pmatrix} h_{ki}(1) \\ \mathbf{h}_{kR}^T(2)\mathbf{U}\mathbf{h}_{Ri}(1) \end{pmatrix} d_i \tag{11}$$

In order to align all the interference signals into a one dimensional space, we need

$$\frac{\mathbf{h}_{kR}^T(2)\mathbf{U}\mathbf{h}_{Ri}(1)}{h_{ki}(1)} = \frac{\mathbf{h}_{kR}^T(2)\mathbf{U}\mathbf{h}_{Rj}(1)}{h_{kj}(1)} \tag{12}$$

where $i = 2$ if $k = 1$, and $i = 1$ if $k \neq 1$. In addition, we need $j \neq k, j \neq i$.

If we denote the entries of $\mathbf{U}$ as u_{mn}, where $m, n = 1, \cdots, K-1$, equation (12) can be written as

$$\sum_{m=1}^{K-1} \sum_{n=1}^{K-1} h_{kR,m}(2) \left(\frac{h_{Ri,n}(1)}{h_{ki}(1)} - \frac{h_{Rj,n}(1)}{h_{kj}(1)} \right) u_{mn} = 0. \tag{13}$$

If we let

$$\mathbf{h}_{ki} = [h_{kR,1}(2)h_{Ri,1}(1), \cdots, h_{kR,1}(2)h_{Ri,K-1}(1), \cdots, h_{kR,K-1}(2)h_{Ri,1}(1), \cdots, h_{kR,K-1}(2)h_{Ri,K-1}(1)]^T,$$

and reorganize u_{mn} to form a vector

$$\mathbf{u} = [u_{11}, \cdots, u_{1,K-1}, \cdots, u_{K-1,1}, \cdots, u_{K-1,K-1}]^T, \tag{14}$$

then all the linear equations can be written as

$$\mathbf{H}\mathbf{u} = \mathbf{0}, \tag{15}$$

where

$$\mathbf{H} = \begin{pmatrix} \frac{\mathbf{h}_{12}^T}{h_{12}(1)} - \frac{\mathbf{h}_{13}^T}{h_{13}(1)} \\ \frac{\mathbf{h}_{12}^T}{h_{12}(1)} - \frac{\mathbf{h}_{14}^T}{h_{14}(1)} \\ \vdots \\ \frac{\mathbf{h}_{12}^T}{h_{12}(1)} - \frac{\mathbf{h}_{1K}^T}{h_{1K}(1)} \\ \frac{\mathbf{h}_{21}^T}{h_{21}(1)} - \frac{\mathbf{h}_{23}^T}{h_{23}(1)} \\ \frac{\mathbf{h}_{21}^T}{h_{21}(1)} - \frac{\mathbf{h}_{24}^T}{h_{24}(1)} \\ \vdots \\ \frac{\mathbf{h}_{21}^T}{h_{21}(1)} - \frac{\mathbf{h}_{2K}^T}{h_{2K}(1)} \\ \vdots \\ \frac{\mathbf{h}_{K1}^T}{h_{K1}(1)} - \frac{\mathbf{h}_{K2}^T}{h_{K2}(1)} \\ \frac{\mathbf{h}_{K1}^T}{h_{K1}(1)} - \frac{\mathbf{h}_{K3}^T}{h_{K3}(1)} \\ \vdots \\ \frac{\mathbf{h}_{K1}^T}{h_{K1}(1)} - \frac{\mathbf{h}_{K,K-1}^T}{h_{K,K-1}(1)} \end{pmatrix} \tag{16}$$

The dimension of the matrix $\mathbf{H}$ is $K(K-2) \times (K-1)^2$. Therefore we can choose a non-zero vector $\mathbf{u}$ from the null space of matrix $\mathbf{H}$. Since the relay has full CSI, the precoding matrix can be found before the transmission in the second time slot. With the resulting precoding matrix $\mathbf{U}$ at the relay, all the interference streams can be aligned into a one dimensional space at each receiver. Since the channel coefficients are drawn from a continuous distribution, the intended signal occupies the other dimension of the signal space almost surely. We can then use a zero-forcing decoder to recover the intended message, and the full DoF $\frac{K}{2}$ can be achieved. ■

Remark 1: From equation (11), we can see that the channel needs not to be time varying for the scheme to work. The relay can provide sufficient channel variation to separate the intended signal and the interfering signals.

The essence of the above scheme we constructed is to utilize the precoding matrix at the relay to provide enough number of variables to satisfy the number of equations needed to align all the interfering signals.

Using similar idea, we now focus on the case when relays only have a single antenna, and investigate how many relays are needed to achieve the maximum DoF $\frac{K}{2}$.

Theorem 2: For the K-user interference channel without CSIT, when there are $K(K-2)$ relays with single antenna and full CSI, the DoF $\frac{K}{2}$ is achievable.

Proof: We construct a two slot communication scheme to show that $\frac{K}{2}$ degrees of freedom is achievable.

In slot 1, each transmitter sends a message to the intended receiver, i.e., transmitter k sends $X_k(1) = d_k$. The signal received at receiver k is

$$Y_k(1) = \sum_{i=1}^{K} h_{ki}(1) d_i \tag{17}$$

where $k = 1, 2, \cdots, K$.

The received signals at the relays can be expressed as

$$Y_{R_t}(1) = \sum_{i=1}^{K} h_{R_t i}(1) d_i \tag{18}$$

where $t = 1, 2, \cdots, K(K-2)$.

In slot 2, the transmitters and the relays perform joint beamforming to align the interference. The transmitters send

$$X_k(2) = d_k. \tag{19}$$

The relays transmit a scaled version of their received signals in previous slot, i.e.,

$$X_{R_t}(2) = \alpha_t Y_{R_t}(1) = \alpha_t \sum_{i=1}^{K} h_{R_t i}(1) d_i. \tag{20}$$

The received signal at receiver k is

$$Y_k(2) = \sum_{i=1}^{K} h_{ki}(2) d_i + \sum_{t=1}^{K(K-2)} h_{kR_t}(2) X_{R_t}(2) \tag{21}$$

$$= \left(h_{kk}(2) + \sum_{t=1}^{K(K-2)} h_{kR_t}(2) \alpha_t h_{R_t k}(1) \right) d_k$$

$$+ \sum_{i \neq k} \left(h_{ki}(2) + \sum_{t=1}^{K(K-2)} h_{kR_t}(2) \alpha_t h_{R_t i}(1) \right) d_i \tag{22}$$

Combining the received signals from slot 1 and slot 2, we have

$$\mathbf{Y}_k = \begin{pmatrix} h_{kk}(1) \\ h_{kk}(2) + \sum_{t=1}^{K(K-2)} h_{kR_t}(2) \alpha_t h_{R_t k}(1) \end{pmatrix} d_k + \sum_{i \neq k} \begin{pmatrix} h_{ki}(1) \\ h_{ki}(2) + \sum_{t=1}^{K(K-2)} h_{kR_t}(2) \alpha_t h_{R_t i}(1) \end{pmatrix} d_i \tag{23}$$

Now we set

$$\mathbf{h}_{ki} = [h_{kR_1}(2) h_{R_1 i}(1),\ h_{kR_2}(2) h_{R_2 i}(1),\ \cdots \ h_{kR_{K(K-2)}}(2) h_{R_{K(K-2)} i}(1)]^T \tag{24}$$

and

$$\alpha = \left[\alpha_1,\ \alpha_2,\ \cdots\ \alpha_{K(K-2)}\right]^T. \tag{25}$$

From (23), we can see that if we can align all the interference vectors in a one dimensional space, then we can zero force all the interference to decode the intended signal. This requires

$$\frac{h_{ki}(2) + \mathbf{h}_{ki}^T \alpha}{h_{ki}(1)} = \frac{h_{kj}(2) + \mathbf{h}_{kj}^T \alpha}{h_{kj}(1)} \tag{26}$$

where $i = 2$ if $k = 1$, and $i = 1$ if $k \neq 1$. We also need $j \neq k, j \neq i$.

Note that (26) can be written as

$$\left(\frac{\mathbf{h}_{ki}}{h_{ki}(1)} - \frac{\mathbf{h}_{kj}}{h_{kj}(1)} \right)^T \alpha = \left(\frac{h_{kj}(2)}{h_{kj}(1)} - \frac{h_{ki}(2)}{h_{ki}(1)} \right) \tag{27}$$

It is easy to see that we have $K(K-2)$ variables and $K(K-2)$ equations. Now we can write the equations in matrix form as follows

$$\mathbf{H}\alpha = \mathbf{b} \tag{28}$$

where the matrix $\mathbf{H}$ is of similar form as equation (16) except that the vector $\mathbf{h}_{ki}$ is defined by equation (24). The vector $\mathbf{b}$ is obtained from the right hand side of equation (27).

Since the channel coefficients are drawn from a continuous distribution, the matrix $\mathbf{H}$ is full rank with probability one, and thus the scaling factors at the relays can be solved by $\alpha = \mathbf{H}^{-1}\mathbf{b}$. Since each relay has full CSI, the ith relay can obtain the scaling factor by finding the inner product between the ith row of matrix $\mathbf{H}^{-1}$ and the vector $\mathbf{b}$. Therefore using the relays, all the interference from the $K-1$ users can be aligned into a one dimensional space. Since the channel coefficients are drawn from a continuous distribution, we can verify that the intended signal vector and the interfering signals occupy different signal dimensions, and thus all the $K-1$ interfering signals can be zero-forced to decode the intended signal, to achieve the DoF $\frac{K}{2}$. ■

Different from *Theorem 1*, for the above strategy, if we let the transmitters remain silent during the second time slot, or allow the channel to be static, the right hand side of equation (27) will be zero. Combining all the equations required to align the interference yields an equation in the form of $\mathbf{H}\alpha = \mathbf{0}$, with $\mathbf{H}$ invertible almost surely. This gives us an all zero vector α, which reduces the total number of dimensions of the signal spaces at the receivers to one. The reason here is that after the transmission in the first time slot, all the signals occupy a single dimension at the receivers, i.e., all the interfering signals and the intended signal are aligned together. If we do not let the transmitters send anything in the second time slot, or the channel is static, relays can still keep the interfering signals aligned at the receivers by not transmitting anything. However, we need another signal dimension at the receivers to separate the intended message. For this reason, we need the channel to be time varying, and so that we can let the transmitters and the relays perform joint beamforming, in order to keep the interference aligned and at the same time do not reduce the total number of dimensions at the receivers.

Remark 2: If we assume that the channel coefficients are drawn from the Rayleigh distribution, then it is shown in [5] that the DoF for the K-user interference channel without CSIT and relays is upper bounded by 1. It is thus clear that relays can provide DoF gain for the K-user interference channel without CSIT.

Remark 3: When the number of single-antenna relays is of the order $\mathcal{O}(K)$, it is easy to see that the DoF $\frac{\lfloor \sqrt{K} \rfloor}{2}$ can be achieved. This is still a significant improvement compared to the DoF with no relays for the Rayleigh fading channel [5].

Remark 4: In reference [2], the $\frac{K}{2}$ DoF for the K-user interference channel is achieved via time extension of the channel, which requires infinite channel uses to achieve exactly $\frac{K}{2}$ DoF. In our scheme, however, the optimal DoF $\frac{K}{2}$ is achieved via a two-slot transmission scheme.

Based on the transmission schemes developed above, we can now generalize the result to the general K-user interference channel with J relays, each having L antennas.

Theorem 3: For the K-user interference channel without CSIT, when there is full CSI at the relays, the optimal DoF $\frac{K}{2}$ can be achieved using $\left\lceil \frac{K(K-2)}{L^2} \right\rceil$ relays with L antennas.

Proof: This result can be shown using similar techniques as in *Theorem 1* and *Theorem 2* and thus we only provide an outline to the proof due to limited space.

We use a two-slot transmission scheme, where in the first slot the transmitters send messages to the receivers, and the relays remain silent.

In the second slot, each relay applies a precoding matrix of dimension $L \times L$ to the signal it received in the first slot, and transmits the resulting signal to the destinations. Since the number of users is fixed, the number of equations needed to align the interference is also fixed, which is $K(K-2)$. However, the number of variables the relays can provide depends on the product of the number of relays and the number of antennas at the relays. In order to provide a proof for the general case, we use joint beamforming to guarantee that the scheme yields a set of linear equations of the form $\mathbf{Hu} = \mathbf{b}$. Therefore, we let the transmitters also send the same messages to the receivers as in slot 1. With the choice $J = \left\lceil \frac{K(K-2)}{L^2} \right\rceil$, the number of equations is always less than or equal to the number of variables, which guarantees the existence of a set of non-zero precoding matrices for all the cases.

When the number of equations is equal to the number of variables, each relay can find the precoding matrix by calculating $\mathbf{H}^{-1}\mathbf{b}$, since it has full CSI. When the number of equations is less than the number of variables, the matrix $\mathbf{H}$ has full rank almost surely. The vector $\mathbf{u}$ can be calculated using $\mathbf{H}^{\dagger}(\mathbf{H}\mathbf{H}^{\dagger})^{-1}\mathbf{b}$. With the resulting precoding matrices, at each receiver, interference can be aligned into a one dimensional space and the intended message occupies the remaining dimension. A zero-forcing decoder can be used to recover the intended message, and the DoF $\frac{K}{2}$ can be achieved. ■

IV. Conclusion

In this paper, we have investigated relay-aided interference alignment schemes for the K-user interference channel, when no channel state information (CSI) at the transmitters (CSIT) is available. It is assumed that relays and the destinations have instant global CSI. We have shown that without CSIT, relays can provide interference alignment to achieve the optimal DoF $\frac{K}{2}$ using a 2-slot transmission scheme. When relay has $K-1$ antennas, one relay is sufficient to achieve the optimal DoF. When relays are equipped with a single antenna, a sufficient condition to achieve the optimal DoF is to have $K(K-2)$ relays. In general, we have shown that the optimal DoF $\frac{K}{2}$ can be achieved using $\left\lceil \frac{K(K-2)}{L^2} \right\rceil$ relays with L antennas.

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