Relay-aided Interference Alignment for the X Channel with Limited CSI

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***Abstract*—In this work, we investigate the impact of placing a half-duplex relay in the X channel in the presence of limited channel state information (CSI). Specifically, we consider the scenarios when the sources have no CSI, while the relay either has perfect CSI or delayed CSI. We develop transmission strategies for each scenario to facilitate interference alignment with the help of the relay. We show that for both scenarios, with single antenna at each source and destination, degree of freedom $\frac{4}{3}$ can be achieved. This matches the degree of freedom established in previous work when perfect CSI is available at all nodes. For the case with perfect CSI at the relay, relay only needs to have 1 antenna, and can perform beamforming with the sources to facilitate interference alignment using the proposed scheme. For the case with delayed CSI at the relay, the relay can compensate the staleness of the CSI by adding an antenna. An important observation from this work is that relay can increase the degrees of freedom for the X channel when the sources have no CSI, in contrast with the case when perfect CSI is available at all nodes, where a relay cannot improve the degrees of freedom. The results are also extended to multi-antenna settings.**

I. Introduction

Interference, an inevitable consequence of communicating in the wireless medium, is a major factor that limits the data rate for wireless networks, due to the increasing number of wireless devices sharing the same spectrum. Interference alignment is shown to be a promising technique to combat interference in wireless systems. Utilizing additional signal space dimensions, e.g. those provided by multiple antennas, interference alignment can significantly improve achievable data rates by projecting multiple interfering signals into a limited amount of dimensions. In particular, interference alignment is shown to achieve the maximum degrees of freedom for various wireless network structures [1], [2].

In its original form, interference alignment can be facilitated only with accurate and instantaneous channel state information (CSI) at the source nodes, i.e., *perfect* CSI at transmitters (CSIT). Recent works have considered more practical settings and have demonstrated that imperfect CSIT can also be utilized to facilitate interference alignment [3]–[10]. In particular, references [3], [4] have shown that without the knowledge of the exact channel coefficients at the sources, interference alignment is still possible for certain wireless networks, as long as the channel's correlation structure is known. Reference [5] has studied another interesting scenario, where the sources are assumed to have *delayed CSIT*, i.e., only the CSI from previous time slots is available at the sources in the current time slot. This work established the maximum degree of freedom for K-user broadcast channel with delayed CSIT, where the source is assumed to have K antennas and each receiver has only one antenna. Delayed CSIT is shown to be able to provide significant gain for degree of freedom for many wireless networks such as X channel, interference channel, and general broadcast channel, compared with the cases with no CSIT [6]–[10].

The so called X channel consists of two sources and two destinations where each source has one message for each destination [1]. It is a comprehensive channel model which incorporates broadcast channel, multiple access channel, and interference channel. The degree of freedom of X channel has been extensively studied in various settings [1], [6], [10], [11]. With perfect CSI at all nodes, it is shown that the maximum degree of freedom is $\frac{4}{3}M$ when all nodes have M antennas, achievable using interference alignment [1]. With delayed CSIT, reference [6] has shown that $\frac{8}{7}$ degrees of freedom is achievable when all nodes are equipped with a single antenna. Later, reference [10] has shown that $\frac{6}{5}$ degrees of freedom is actually achievable for the same setting. When there is no CSIT at the sources, reference [11] showed that the maximum degree of freedom reduces to 1 for the single antenna setting, and interference alignment is not feasible.

In this work, we investigate the impact of adding a half-duplex relay for the degree of freedom of the X channel without CSIT at the sources. X channel with a relay with perfect CSI has been studied in reference [12], which considered relay-aided interference alignment for quasi-static $M \times 2$ X channel. Consistent with the result in [13], which shows that the relay cannot provide any degree of freedom gain when perfect CSI is available at all nodes, the work in reference [12] has shown that for the channel model investigated therein, the relay can only facilitate interference alignment to achieve the maximum degrees of freedom for the X channel, but no gain in degrees of freedom is achievable. Other relay-aided interference alignment schemes with CSI at the sources can be found in references [14]–[16].

In order to fully characterize the impact of relaying with no CSIT at the sources, we consider two scenarios, one with

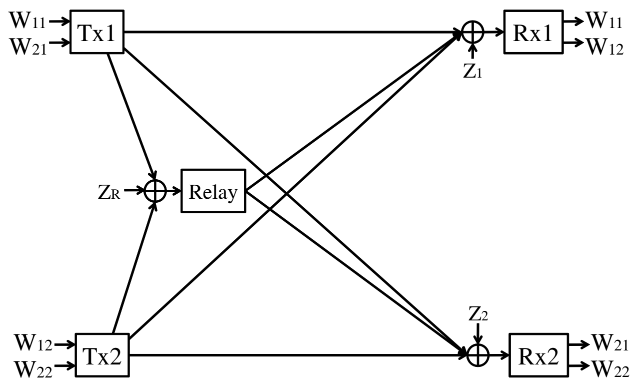

Fig. 1. X channel with a relay.

perfect CSI at the relay, and the other with delayed CSI at the relay. For both cases, we show that the relay can in fact provide a positive degrees of freedom gain. We first study the cases where the sources and the destinations have single antenna. We show that for the case with perfect CSI at the relay, interference alignment is possible. The relay and the sources need to perform *joint beamforming* to steer the interference such that it can be aligned, and the degrees of freedom $\frac{4}{3}$ can be achieved, which is the maximum degrees of freedom for X channel with or without a relay with perfect CSI at all nodes. For the case with delayed CSI at the relay, when the relay is equipped with two antennas, interference alignment is possible and $\frac{4}{3}$ degrees of freedom can be achieved, thus, one additional antenna compensates for staleness of CSI. The results can be extended to the cases with M antennas at the sources and the destinations, to achieve $\frac{4}{3}M$ degrees of freedom.

II. System Model

We consider a Gaussian X channel assisted by a half-duplex relay, as shown in Fig. 1. The X channel consists of two sources and two destinations, where each source has one message for each destination. For the X channel with M_1, M_2 antennas at the sources, M_R antennas at the relay and N_1, N_2 antennas at the destinations, denoting the vectors and matrices with bold letters, the channel outputs are expressed as follows:

When relay listens to the channels, we have:

$$\mathbf{Y}_1(t) = \mathbf{H}_{11}(t)\mathbf{X}_1(t) + \mathbf{H}_{12}(t)\mathbf{X}_2(t) + \mathbf{Z}_1(t), \quad (1)$$

$$\mathbf{Y}_2(t) = \mathbf{H}_{21}(t)\mathbf{X}_1(t) + \mathbf{H}_{22}(t)\mathbf{X}_2(t) + \mathbf{Z}_2(t), \quad (2)$$

$$\mathbf{Y}_R(t) = \mathbf{H}_{R1}(t)\mathbf{X}_1(t) + \mathbf{H}_{R2}(t)\mathbf{X}_2(t) + \mathbf{Z}_R(t). \quad (3)$$

When relay transmits, we have:

$$\mathbf{Y}_1(t) = \mathbf{H}_{11}(t)\mathbf{X}_1(t)+\mathbf{H}_{12}(t)\mathbf{X}_2(t)+\mathbf{H}_{1R}(t)\mathbf{X}_R(t)+\mathbf{Z}_1(t), \quad (4)$$

$$\mathbf{Y}_2(t) = \mathbf{H}_{21}(t)\mathbf{X}_1(t)+\mathbf{H}_{22}(t)\mathbf{X}_2(t)+\mathbf{H}_{2R}(t)\mathbf{X}_R(t)+\mathbf{Z}_2(t), \quad (5)$$

for channel use t.

We consider a fast fading channel changing at each t. Note that each channel use can be a time slot in practical communication system. We also assume that the transmission from the sources and the relay are synchronized in each channel use, i.e., at the slot level. $\mathbf{X}_k(t) \in \mathbb{C}^{M_k}, k = 1, 2, R$ is the transmitted signals from the sources or the relay. The transmitted signals are subject to average power constraint $E(||\mathbf{X}_k||^2) \leq P$. $\mathbf{H}_{ij}(t) \in \mathbb{C}^{N_i \times M_j}, i = 1, 2, j = 1, 2, R$ are the channel matrices between each transmitter-receiver pair. The channel matrices at the receiver side of the relay are $\mathbf{H}_{Rj}(t) \in \mathbb{C}^{M_R \times M_j}, j = 1, 2$. Entries of the channel matrices $\mathbf{H}_{ij}(t)$ are drawn independently from a continuous distribution for each time index. Consequently, the channel matrices have full rank almost surely and are distinct from each other for different time indices [1]. $\mathbf{Z}_k(t) \in \mathbb{C}^{N_k}, k = 1, 2, R$ is the additive white Gaussian noise (AWGN) at the destinations or the relay.

We assume that the sources have no channel state information (CSI), i.e., $\mathbf{H}_{ij}(t)$. The destinations are assumed to have *perfect CSI*, i.e., accurate and instantaneous global CSI. Depending on the scenarios we are interested in, we assume that the relay has either *delayed CSI*, where at time index t the relay knows all channel matrices up to time $t-1$, or *perfect CSI*, where at time index t the relay knows all channel matrices up to time t.

We denote w_{ij} as the message originated from source j to the destination i, where $i = 1, 2, j = 1, 2$, and the rate of message w_{ij} is denoted as $R_{ij}(P)$ with power constraint P. We define $\mathcal{C}(P)$ as the set of all achievable rate tuples $(R_{11}(P), R_{21}(P), R_{12}(P), R_{22}(P))$ with power constraint P. The degree of freedom is defined as

$$DoF = \lim_{P\to\infty} \frac{R_{\sum}(P)}{\log(P)}, \quad (6)$$

where $R_{\sum}(P) = \max_{\mathcal{C}_P}(R_{11}(P) + R_{21}(P) + R_{12}(P) + R_{22}(P))$.

III. Relay-aided Interference Alignment with Perfect CSI at the Relay

In this section, we consider the case where the relay has perfect CSI while the sources have no CSI. The relay has equal number of antennas as the sources and the destinations. We first show that the $\frac{4}{3}$ degrees of freedom is achievable when the sources, the relay and the destinations are all equipped with one antenna, and then extend the result to the multi-antenna setting.

Theorem 1: For the X channel with single antenna at the sources, the relay and the destinations, $\frac{4}{3}$ degrees of freedom is achievable almost surely without CSI at the sources and perfect CSI at the relay.

Proof: We consider sending 4 independent messages $\{w_{11}, w_{12}, w_{21}, w_{22}\}$ over 3 channel uses. Let d_{ij} denote the signals representing the messages w_{ij}.

In the first channel use, source 1 and source 2 send the messages w_{11} and w_{12} to destination 1, i.e., $X_1(1) = d_{11}$, $X_2(1) = d_{12}$. The relay remains slilent in this channel use. Since we are considering the degree of freedom of the channel, which is the behavior of the rates when $P \to \infty$, we need not consider the power constraint and noise in the channel.

The channel outputs for the first channel use are

$$Y_1(1) = h_{11}(1)d_{11} + h_{12}(1)d_{12}, \tag{7}$$

$$Y_2(1) = h_{21}(1)d_{11} + h_{22}(1)d_{12}, \tag{8}$$

$$Y_R(1) = h_{R1}(1)d_{11} + h_{R2}(1)d_{12}. \tag{9}$$

In the second channel use, source 1 and source 2 send the messages w_{21} and w_{22} to destination 2, i.e., $X_1(2) = d_{21}$, $X_2(2) = d_{22}$. The relay also remains silent in this channel use. The channel outputs for the second channel use are

$$Y_1(2) = h_{11}(2)d_{21} + h_{12}(2)d_{22}, \tag{10}$$

$$Y_2(2) = h_{21}(2)d_{21} + h_{22}(2)d_{22}, \tag{11}$$

$$Y_R(2) = h_{R1}(2)d_{21} + h_{R2}(2)d_{22}. \tag{12}$$

There is no decoding performed in the first two channel uses.

In the third channel use, we need to design a scheme that can provide each destination one additional equation to decode the intended messages, without producing extra interference to the destinations. For this end, we let source 1 transmit $X_1(3) = d_{11} + d_{21}$, and the relay transmit $X_R(3) = \alpha Y_R(1) + \beta Y_R(2)$. Source 2 remains silent in this channel use. We need to choose the parameters α and β such that the interference can be aligned. Note that we can also let source 1 transmit $X_1(3) = d_{11}$ and source 2 transmit $X_2(3) = d_{22}$, which will yield the same results from the degrees of freedom perspective. The received signals at the destinations for this channel use are as follows:

$$\begin{aligned}
&Y_1(3) && (13)\\
&= h_{11}(3)X_1(3) + h_{1R}(3)X_R(3) && (14)\\
&= h_{11}(3)d_{11} + h_{11}(3)d_{21} && (15)\\
&\quad + \alpha h_{1R}(3)h_{R1}(1)d_{11} + \alpha h_{1R}(3)h_{R2}(1)d_{12} && (16)\\
&\quad + \beta h_{1R}(3)h_{R1}(2)d_{21} + \beta h_{1R}(3)h_{R2}(2)d_{22} && (17)\\
&= (h_{11}(3) + \alpha h_{1R}(3)h_{R1}(1))\, d_{11} + \alpha h_{1R}(3)h_{R2}(1)d_{12}\\
&\quad + (h_{11}(3) + \beta h_{1R}(3)h_{R1}(2))\, d_{21} + \beta h_{1R}(3)h_{R2}(2)d_{22} && (18)
\end{aligned}$$

Similarly, we have

$$\begin{aligned}
&Y_2(3) && (19)\\
&= (h_{21}(3) + \alpha h_{2R}(3)h_{R1}(1))\, d_{11} + \alpha h_{2R}(3)h_{R2}(1)d_{12}\\
&\quad + (h_{21}(3) + \beta h_{2R}(3)h_{R1}(2))\, d_{21} + \beta h_{2R}(3)h_{R2}(2)d_{22} && (20)
\end{aligned}$$

If we let

$$\frac{h_{11}(3) + \beta h_{1R}(3)h_{R1}(2)}{\beta h_{1R}(3)h_{R2}(2)} = \frac{h_{11}(2)}{h_{12}(2)} \tag{21}$$

and

$$\frac{h_{21}(3) + \alpha h_{2R}(3)h_{R1}(1)}{\alpha h_{2R}(3)h_{R2}(1)} = \frac{h_{21}(1)}{h_{22}(1)}, \tag{22}$$

then destination 1 can subtract

$$Y_1(2) \cdot \frac{\beta h_{1R}(3)h_{R2}(2)}{h_{12}(2)} \tag{23}$$

from $Y_1(3)$ and destination 2 can subtract

$$Y_2(1) \cdot \frac{\alpha h_{2R}(3)h_{R2}(1)}{h_{22}(1)} \tag{24}$$

from $Y_2(3)$ to produce desired equation for decoding.

This can be done by choosing

$$\alpha = \frac{h_{21}(3)h_{22}(1)}{h_{21}(1)h_{2R}(3)h_{R2}(1) - h_{22}(1)h_{2R}(3)h_{R1}(1)} \tag{25}$$

$$\beta = \frac{h_{11}(3)h_{12}(2)}{h_{11}(2)h_{1R}(3)h_{R2}(2) - h_{12}(2)h_{1R}(3)h_{R1}(2)}. \tag{26}$$

Since relay is assumed to have perfect CSI, α and β can be readily computed at the relay. Also, the destinations are assumed to have perfect CSI, and thus they can recover α and β as well. The destinations can subtract the interference and obtain

$$\begin{aligned}
Y_1'(3) &= Y_1(3) - Y_1(2) \cdot \frac{\beta h_{1R}(3)h_{R2}(2)}{h_{12}(2)}\\
&= (h_{11}(3) + \alpha h_{1R}(3)h_{R1}(1))\, d_{11} + \alpha h_{1R}(3)h_{R2}(1)d_{12}
\end{aligned}$$

$$\begin{aligned}
Y_2'(3) &= Y_2(3) - Y_2(1) \cdot \frac{\beta h_{2R}(3)h_{R2}(1)}{h_{22}(1)}\\
&= (h_{21}(3) + \beta h_{2R}(3)h_{R1}(2))\, d_{21} + \beta h_{2R}(3)h_{R2}(2)d_{22}.
\end{aligned}$$

It is then easy to see that $Y_1'(3)$ and $Y_1(1)$, $Y_2'(3)$ and $Y_2(2)$ are pairwise linearly independent almost surely. With two linearly independent equations, each destination can recover these intended messages. We thus conclude that $\frac{4}{3}$ degrees of freedom is achievable. ■

We can also interpret the choice of parameters α and β in the following way:

The received signal at destination 1 can be written as

$$\begin{aligned}
\begin{pmatrix} Y_1(1)\\ Y_1(2)\\ Y_1(3) \end{pmatrix} &= \begin{pmatrix} h_{11}(1)\\ 0\\ h_{11}(3) + \alpha h_{1R}(3)h_{R1}(1) \end{pmatrix} d_{11}\\
&+ \begin{pmatrix} h_{12}(1)\\ 0\\ \alpha h_{1R}(3)h_{R2}(1) \end{pmatrix} d_{12}\\
&+ \begin{pmatrix} 0\\ h_{11}(2)\\ h_{11}(3) + \beta h_{1R}(3)h_{R1}(2) \end{pmatrix} d_{21}\\
&+ \begin{pmatrix} 0\\ h_{12}(2)\\ \beta h_{1R}(3)h_{R2}(2) \end{pmatrix} d_{22}.
\end{aligned} \tag{27}$$

We can see that the way we choose the parameters α and β is to align the interference signals d_{21} and d_{22} such that they only occupy a one-dimensional space, and in the mean time the messages d_{11} and d_{12} span a two-dimensional space. In fact, we use the relay to perform *joint beamforming* with the sources to facilitate interference alignment, which is otherwise infeasible without the relay.

Remark 1: Assume the channel matrices are drawn from the Rayleigh distribution, i.e., each entry in the matrices is a complex Gaussian random variable. In reference [11], the authors have shown that under Rayleigh fading environment, without CSIT, the degree of freedom of the X channel with single antenna at all nodes has only 1 degree of freedom. Our scheme showed that for this case, relay can provide additional degree of freedom gain. Note that this is different from the conclusion made in reference [13], where it is shown that for X channel with perfect CSI at all nodes, relaying does not provide any degree of freedom gain. With limited CSIT, relay becomes a useful entity and increases the degrees of freedom.

Remark 2: It is shown in reference [1] that the X channel with perfect CSI at all nodes has $\frac{4}{3}$ degrees of freedom. Since relaying does not provide any degree of freedom gain with perfect CSI [13], the maximum number of degrees of freedom for X channel with a relay is also $\frac{4}{3}$. Therefore our scheme is *optimal* in terms of degree of freedom.

The scheme can be extended to the case where the sources, the relay and the destinations all have M antennas. It can be shown that $\frac{4}{3}M$ degrees of freedom is achievable using similar strategy as the single antenna case. To do so, we send $4M$ independent messages using 3 channel uses. In the first two channel uses, each source sends a vector of M signal streams representing M independent messages.

The channel outputs for the first channel use are

$$\mathbf{Y}_1(1) = \mathbf{H}_{11}(1)\mathbf{d}_{11} + \mathbf{H}_{12}(1)\mathbf{d}_{12}, \tag{28}$$

$$\mathbf{Y}_2(1) = \mathbf{H}_{21}(1)\mathbf{d}_{11} + \mathbf{H}_{22}(1)\mathbf{d}_{12}, \tag{29}$$

$$\mathbf{Y}_R(1) = \mathbf{H}_{R1}(1)\mathbf{d}_{11} + \mathbf{H}_{R2}(1)\mathbf{d}_{12}, \tag{30}$$

where $\mathbf{d}_{ij} = [d_{ij,1}, \cdots, d_{ij,M}]^T$ is the vector of signal streams from source j to destination i.

The channel outputs for the second channel use are

$$\mathbf{Y}_1(2) = \mathbf{H}_{11}(2)\mathbf{d}_{21} + \mathbf{H}_{12}(2)\mathbf{d}_{22}, \tag{31}$$

$$\mathbf{Y}_2(2) = \mathbf{H}_{21}(2)\mathbf{d}_{21} + \mathbf{H}_{22}(2)\mathbf{d}_{22}, \tag{32}$$

$$\mathbf{Y}_R(2) = \mathbf{H}_{R1}(2)\mathbf{d}_{21} + \mathbf{H}_{R2}(2)\mathbf{d}_{22}. \tag{33}$$

In the third channel use, instead of choosing the parameters α and β, we need to choose the matrices $\mathbf{A}$ and $\mathbf{B}$ such that all the interference signals are aligned in a space of dimension M, and the intended messages span the rest $2M$ dimensional space. We let source 1 transmit $\mathbf{X}_1(3) = \mathbf{d}_{11} + \mathbf{d}_{21}$, and let the relay transmit $\mathbf{X}_R(3) = \mathbf{A}\mathbf{Y}_R(1) + \mathbf{B}\mathbf{Y}_R(2)$.

The received signal at destination 1 is

$$\begin{aligned}&\mathbf{Y}_1(3)\\ &= (\mathbf{H}_{11}(3) + \mathbf{H}_{1R}(3)\mathbf{A}\mathbf{H}_{R1}(1))\mathbf{d}_{11} + \mathbf{H}_{1R}(3)\mathbf{A}\mathbf{H}_{R2}(1)\mathbf{d}_{12}\\ &+ (\mathbf{H}_{11}(3) + \mathbf{H}_{1R}(3)\mathbf{B}\mathbf{H}_{R1}(2))\mathbf{d}_{21} + \mathbf{H}_{1R}(3)\mathbf{B}\mathbf{H}_{R2}(2)\mathbf{d}_{22}.\end{aligned}$$

To align $\mathbf{d}_{21}$ and $\mathbf{d}_{22}$ in a M dimensional space while keep $\mathbf{d}_{11}$ and $\mathbf{d}_{12}$ in the other $2M$ dimensional space, we can choose the following matrix $\mathbf{B}$

$$\begin{aligned}\mathbf{B} = \mathbf{H}_{1R}(3)^{-1}\mathbf{H}_{11}(3)\Big(&\mathbf{H}_{R2}(2)\mathbf{H}_{12}(2)^{-1}\mathbf{H}_{11}(2)\\ &- \mathbf{H}_{R1}(2)\Big)^{-1}.\end{aligned} \tag{34}$$

Similarly, we can choose the matrix $\mathbf{A}$ as

$$\begin{aligned}\mathbf{A} = \mathbf{H}_{2R}(3)^{-1}\mathbf{H}_{21}(3)\Big(&\mathbf{H}_{R2}(1)\mathbf{H}_{22}(1)^{-1}\mathbf{H}_{21}(1)\\ &- \mathbf{H}_{R1}(1)\Big)^{-1}.\end{aligned} \tag{35}$$

We can then decode the intended message using zero-forcing. Therefore $\frac{4}{3}M$ degrees of freedom is achievable.

Remark 3: Based on results from [1] [13], $\frac{4}{3}M$ is the maximum degree of freedom for the X channel with a relay with perfect CSI at all nodes. This shows the *optimality* of our scheme for the multi-antenna setting as well.

Remark 4: Reference [11] has shown that when the channel matrices are drawn from the Rayleigh distribution, the degree of freedom of the X channel with M antennas at the sources and the destinations without CSIT is M. Again, our scheme shows that a relay can provide significant gain for degrees of freedom when sources have no CSI available.

IV. Relay-aided Interference Alignment with Delayed CSI at the Relay

In this section, we consider the case when the relay does not have perfect CSI. Instead, it is assumed to have delayed CSI only. The sources are assumed to have no CSI available. For this case, the scheme in the previous section is not applicable. We show that for this case, if the relay has more antennas than the sources and destinations, interference alignment is still feasible.

Theorem 2: The X channel with single antenna at the sources and the destinations, when assisted by a relay with *two* antennas, can achieve $\frac{4}{3}$ degrees of freedom almost surely without CSI at the sources and delayed CSI at the relay.

Proof: We consider sending 4 independent messages using 3 channel uses, similar with the case discussed in Section III. For the first two channel uses, the scheme is the same as the one we used in proving *Theorem 1*. The channel outputs for the first channel use can be described as follows:

$$Y_1(1) = h_{11}(1)d_{11} + h_{12}(1)d_{12}, \tag{36}$$

$$Y_2(1) = h_{21}(1)d_{11} + h_{22}(1)d_{12}, \tag{37}$$

$$\mathbf{Y}_R(1) = \mathbf{H}_{R1}(1)d_{11} + \mathbf{H}_{R2}(1)d_{12}, \tag{38}$$

where $\mathbf{H}_{R1}(1)$ and $\mathbf{H}_{R1}(1)$ are 2×1 vectors. By assumption, the matrix $[\mathbf{H}_{R1}(1)\ \mathbf{H}_{R2}(1)]$ has full rank almost surely, and hence the relay is able to decode d_{11} and d_{12} via zero-forcing after it obtains the delayed CSI after the first channel use.

The channel outputs for the second channel use can be described as follows:

$$Y_1(2) = h_{11}(2)d_{21} + h_{12}(2)d_{22}, \tag{39}$$

$$Y_2(2) = h_{21}(2)d_{21} + h_{22}(2)d_{22}, \tag{40}$$

$$\mathbf{Y}_R(2) = \mathbf{H}_{R1}(2)d_{21} + \mathbf{H}_{R2}(2)d_{22}. \quad (41)$$

Similar to the first channel use, the relay is able to decode d_{21} and d_{22} via zero-forcing after the second channel use.

In the third channel use, since the relay has delayed CSI from the previous two channel uses, it can decode all the messages. In addition, it can reconstruct $Y_2(1)$ and $Y_1(2)$. The relay transmits $[Y_2(1)\ Y_1(2)]^T$ using two antennas. The sources remain silent in this channel use. The channel outputs can be described as follows:

$$Y_1(3) = h_{1R,1}(3)Y_2(1) + h_{1R,2}(3)Y_1(2), \quad (42)$$

$$Y_2(3) = h_{2R,1}(3)Y_2(1) + h_{2R,2}(3)Y_1(2). \quad (43)$$

Destination 1 has $Y_1(2)$ and thus can recover $Y_2(1)$. Now it has two equations, which are linearly independent of each other almost surely. Therefore, it can decode the two unknown variables d_{11} and d_{12}. Similarly, destination 2 can decode d_{21} and d_{22} almost surely.

Based on the above scheme, it is easy to see that $\frac{4}{3}$ degrees of freedom is achievable. ■

The scheme can be explained in the following way to show how interference is aligned via delayed CSI:

Based on the transmission scheme, the received signals at destination 1 in the 3 channel uses can be written as

$$\begin{pmatrix} Y_1(1) \\ Y_1(2) \\ Y_1(3) \end{pmatrix} = \begin{pmatrix} h_{11}(1) \\ 0 \\ h_{1R,1}(3)h_{21}(1) \end{pmatrix} d_{11} + \begin{pmatrix} h_{12}(1) \\ 0 \\ h_{1R,1}(3)h_{22}(1) \end{pmatrix} d_{12} + \begin{pmatrix} 0 \\ h_{11}(2) \\ h_{1R,2}(3)h_{11}(2) \end{pmatrix} d_{21} + \begin{pmatrix} 0 \\ h_{12}(2) \\ h_{1R,2}(3)h_{12}(2) \end{pmatrix} d_{22}. \quad (44)$$

We can see that the interference signals d_{21} and d_{22} are aligned at a one-dimensional space, while the other two messages span the rest two-dimensional space.

Remark 5: The result showed that using an additional antenna, the relay can compensate the staleness of the CSI to facilitate interference alignment. The scheme is optimal in terms of degree of freedom, i.e., the achieved degrees of freedom matches the one for X channel with a relay with full CSI at all nodes [1], [13]. In addition, with only delayed CSI, the relay can still provide degree of freedom gain compared with the X channel without a relay, when the sources have no CSI available.

Remark 6: The above strategy can be easily extended to the case when the sources and the destinations have M antennas and the relay has $2M$ antennas, to achieve $\frac{4}{3}M$ degrees of freedom. Details are omitted due to limited space.

V. Conclusion

In this work, we have studied the impact of relay in the X channel with no CSI at the sources. We have shown that the relay can increase the degrees of freedom with both perfect and delayed CSI. Specifically, for the case when sources and destinations have single antenna, we have shown that the degrees of freedom $\frac{4}{3}$ can be achieved with the help of the relay, which matches the degrees of freedom established in previous work when perfect CSI is available at all nodes. For the case with perfect CSI at the relay, relay only needs to have 1 antenna, and it can perform joint beamforming with the sources to facilitate interference alignment. On the other hand, for the case with delayed CSI, the relay can *compensate* the staleness of the CSI by adding one antenna, in order to facilitate interference alignment. The results are also extended to multi-antenna settings. Future work includes extending the results to X networks, and investigating other channel models to study the impact of relay in terms of the degrees of freedom.

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