Relaying for Multiuser Networks in the Absence of Codebook Information

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Abstract—This paper considers relay assisted transmission for multiuser networks when the relay has no access to the codebooks used by the transmitters. The relay is called oblivious for this reason. Of particular interest is the generalized compress-and-forward (GCF) strategy, where the destinations jointly decode the compression indices and the transmitted messages, and their optimality in this setting. The relay-to-destination links are assumed to be out-of-band with finite capacity. Two models are investigated: 1) the multiple access relay channel (MARC) and 2) the interference relay channel (IFRC). For the MARC with an oblivious relay, a new outerbound is derived and it is shown to be tight by means of achievability of the capacity region using GCF scheme. For the IFRC with an oblivious relay, a new strong interference condition is established, under which the capacity region is found by deriving a new outerbound and showing that it is achievable using GCF scheme. The result is further extended to establish the capacity region of M-user MARC with an oblivious relay, and multicast networks containing *M* sources and *K* destinations with an oblivious relay.

Index Terms—Oblivious relay, generalized compress and forward, capacity, interference relay channel, multiple access relay channel.

I. INTRODUCTION

RELAYING is fundamental to the operation in a wireless network. The simplest model that includes this operation is the classical relay channel with one source, one destination and one relay, in the context of which various relaying strategies have been proposed [2], [3]. The capacity of the relay channel is known for special cases, e.g., degraded/reversely degraded [2], and deterministic [3] channels, where the optimality of decode-and-forward (DF) and hash-and-forward is established respectively. For multi-source relay networks, the multiple access relay channel (MARC) and the interference relay channel (IFRC) have been studied in [4]–[14], where various achievable rates and outerbounds have been developed. The capacity for MARC and IFRC are also known under special conditions using DF [4], [12].

A default assumption in all above is that the codebooks used by the sources are known at the relay. In a real system, the codebook can represent the modulation scheme, channel coding scheme and the interleaving pattern used by the sources. In future wireless networks, the wireless devices for different applications may co-exist in the same area sharing the same resources, each having a different codebook, and the membership in these wireless networks can change often due to mobility and the time varying nature of the medium. When a relay is used to assist communication between different user pairs, it is required to gather all the codebook information from the sources, in order to perform DF relaying. Exchanging this codebook information can lead to excessive overhead, and inefficient use of wireless bandwidth. This concern is especially relevant in scenarios when dynamic ad hoc network of heterogeneous devices need to be deployed for instance in a disaster scenario where existing networks experience service interruptions and connections need to be set up through external relay nodes. It is thus interesting to investigate the fundamental performance limits of building blocks of such wireless networks when the relay(s) does(do) not have access to source codebooks.

To model the uncertainty about the source codebooks at the relay, henceforth called an an oblivious relay, reference [15] has proposed a model which uses randomized encoding at the source, and the codebook becomes common randomness between the source and the destination. The uncertainty about the codebook information at the relay is thus modeled by not informing the relay the common randomness of the codebook. This idea is further investigated in [16], which established the capacity of the *primitive* relay channel with an oblivious relay using compress-and-forward (CF) relaying, where the term *primitive* refers to the relay-destination links being out-of-band and of finite-capacity [3]. The primitive assumption simplifies the model, but retains the ability to characterize the impact of the relay.

For channels with more than one source-destination pairs (e.g. the IFRC), when the uncertainty of codebook information is incorporated into the system model, decoding operation can be classified as *interference-aware decoding* and *interference-oblivious decoding*. When the interferer's codebook is available at the destinations, the decoding is interference-aware and when the interferer's codebook is unavailable at the destinations, the decoding is interference-oblivious. Reference [16] has studied the primitive IFRC (PIFRC) with an oblivious relay under interference-oblivious decoding, and established the capacity region using CF relaying and treating interference
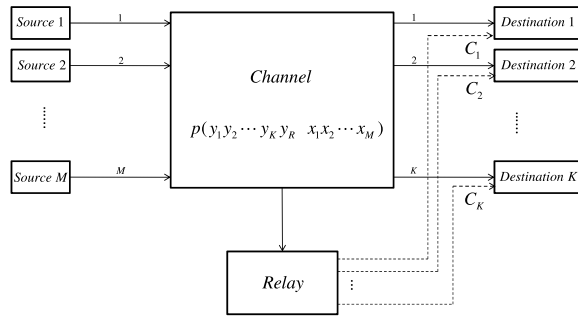
Fig. 1.   Primitive multiuser network with an oblivious relay.

as noise. For the case of interference-aware decoding, the sum capacity is established using CF relaying, for the special case when the destinations have the same statistics of the received signals [16]. This special case reduces the channel to the primitive MARC (PMARC) with an oblivious relay. However, the capacity region of the PMARC and the general PIFRC under interference-aware decoding remains unknown.

In this paper, we consider this oblivious relaying framework for various multiuser networks and establish capacity results that were previously unknown. Our main contribution in this work is summarized as follows:

1) For the PMARC with an oblivious relay, our main contribution is a new outerbound which is shown to be tight using the generalized CF (GCF) scheme. The term GCF refers to the CF strategy with joint decoding of compression index and source messages, as described in [17]. On the other hand, we term CF scheme as the one with sequential decoding of compression index and source messages. Although the GCF scheme achieves the same rate with CF scheme for the relay channel, it can potentially achieve larger capacity region for the PMARC.

2) For the PIFRC with an oblivious relay under interference-aware decoding at the destinations, we propose a new strong interference condition, under which we derive new outerbounds and show that the capacity region can be established using GCF scheme as well.

3) The capacity results are further extended to the M-user PMARC with an oblivious relay and M-source K-destination multicast network with an oblivious relay.

Th remainder of the paper is organized as follows: Section II introduces the system model. Section III establishes the capacity results for MARC with an oblivious relay and multicast networks with an oblivious relay. Section IV establishes the capacity results for IC with an oblivious relay under strong interference conditions. Section V concludes the paper.

II. SYSTEM MODEL

We consider a multiuser network with M sources and K destinations assisted by an oblivious relay, as shown in Figure 1. The relay-to-destination links are digital, i.e., orthogonal with finite capacity C_i, $i = 1, \ldots, K$ for the K destinations as shown in Figure 1. We shall refer to this network as a *primitive* multiuser network. The model is then

specified into MARC and IFRC in the subsequent sections. We define $\mathcal{M} = \{1, 2, \ldots, M\}$ as the index set of the sources and $\mathcal{K} = \{1, 2, \ldots, K\}$ as the index set of the destinations. We define the message set at source i as $\mathcal{W}_i = \{1, 2, \ldots, 2^{nR_i}\}$, and the alphabet for source i as $\mathcal{X}_i$. We also define the set of codebooks at source i as all possible combinations of length n codewords for each message, where the codewords consist of symbols chosen from the alphabet. The number of codebooks is thus $|\mathcal{X}_i|^{n2^{nR_i}}$ for source i. We define an index set of all the codebooks for source i as $\mathcal{F}_i = \{1, 2, \ldots, |\mathcal{X}_i|^{n2^{nR_i}}\}$.

A. Encoders

We follow the definition in [16] to allow time sharing. We define a $(n, R_1, R_2, \ldots, R_M)$ code for the M-source K-destination channel assisted by an oblivious relay with time-sharing as

$$(P_{F_i|Q^n}, \phi_i^n) \quad i \in \mathcal{M}, \tag{1}$$

where $P_{F_i|Q^n}(f_i|q^n)$ is the probability of choosing the codebook $f_i \in \mathcal{F}_i$ conditioned on the time sharing sequence $q^n \in \mathcal{Q}^n$. ϕ_i^n is the encoding function such that $x_i^n = \phi_i^n(w_i, f_i)$, where $w_i \in \mathcal{W}_i$. The probability of selecting the codebook f_i for source i conditioned on q^n is

$$P_{F_i|Q^n}(f_i|q^n) = \prod_{w_i=1}^{2^{nR_i}} P_{X_i^n|Q^n}(\phi_i^n(w_i, f_i)|q^n) \tag{2}$$

where

$$P_{X_i^n|Q^n}(x_i^n|q^n) = \prod_{t=1}^{n} P_{X_i|Q}(x_{i,t}|q_t). \tag{3}$$

Note that the codebook and the message are selected independently, i.e.,

$$P_{F_i W_i|Q^n}(f_i, w_i|q^n) = P_{F_i|Q^n}(f_i|q^n) \cdot 2^{-nR_i}. \tag{4}$$

Based on this formulation, we have the following lemma:

Lemma 1:

$$P_{X_i^n|Q^n}(x_i^n|q^n) = \prod_{j=1}^{n} P_{X_i|Q}(x_{i,j}|q_j), \tag{5}$$

$$P_{Y_i^n|Q^n}(y_i^n|q^n) = \prod_{j=1}^{n} P_{Y_i|Q}(y_{i,j}|q_j) \tag{6}$$

Proof: This relation can be derived following the steps of in [15, Lemma 1], and the details of the derivation are provided in Appendix V for completeness. ∎

The property in *Lemma 1* is a result of the defined codebook selection probability in (2) and (3). Intuitively, when sampling from the whole set of codebooks containing all combinations of i.i.d. generated codewords, any codeword alone will appear to be i.i.d. generated. In the context of oblivious nodes, without the codebook information, the oblivious node sees the transmitted sequence from the source and its received sequence as generated independently from each symbol, given the time sharing sequence q^n. Therefore the received sequence and the transmitted sequence have no structure when no

codebook information is available. However, in the presence of codebook information, the probability $P\left\{X_i^n = x_i^n|q^n, f_i\right\}$ does not have this property. Given the codebook information, the transmitted and received sequences indeed are structured, which is inherited from the known codebook information.

As explained in reference [16], the above construction of the codebook, i.e., equation (2) and (3), limits the available coding schemes that can be utilized. Specifically, it rules out the use of multi-letter coding schemes. On the other hand, all the single-letter coding schemes are still available under this setting. This is because in single-letter coding schemes, the probability of generating a specific codebook equals the probability of selecting the same codebook from the set of all codebooks, which is defined in equation (2) and (3). With this construction of the codebook specified by equation (2) and (3), we have the property that the transmitted and received sequences X_i^n and Y_i^n have no structure without the knowledge of the codebook information, which is a good way to model the uncertainty of codebook information at the relay. A detailed elaboration on the proceeding discussion can be found in [15] and [16, Remarks 2–4].

B. Channel

The channel is discrete memoryless, and consists of M input alphabets and $K + 1$ output alphabets, a channel transition probability, and K out-of-band finite capacity links from the relay to the destinations, i.e.,

$$\mathcal{X}_{\mathcal{M}}, p(y_1 y_2 \ldots y_K y_R | x_1 x_2 \ldots x_M), \mathcal{Y}_{\mathcal{K}}, \mathcal{Y}_R, C_{\mathcal{K}}. \quad (7)$$

C. Relay Encoder

The relay does not know the codebooks used by the sources. It communicates to each destination with an out-of-band finite capacity link. The messages are generated according to an encoding function

$$\phi_R^n : \mathcal{Y}_R^n \times \mathcal{Q}^n \to \mathcal{S}_1 \times \mathcal{S}_2 \ldots \times \mathcal{S}_K \quad (8)$$

with $\mathcal{S}_k = \{1, 2, \ldots, 2^{nC_k}\}, k \in \mathcal{K}$. We denote $(S_1, S_2, \ldots, S_K) = \phi_R^n(Y_R^n|q^n)$ as the messages generated by the relay.

D. Decoders

We assume that the destinations know all the codebooks used by the sources. We consider both *multicast* (MC) and *unicast* (UC) transmissions. For *multicast transmission*, i.e., each source wishes to transmit a message to all destinations, we define the decoding function at destination $j \in \mathcal{K}$ as

$$g_j^{MC} : \mathcal{Q}^n \times \mathcal{F}_1 \times \mathcal{F}_2 \ldots \times \mathcal{F}_M \times \mathcal{S}_j \times \mathcal{Y}_j^n$$
$$\to \mathcal{W}_1 \times \mathcal{W}_2 \ldots \times \mathcal{W}_M. \quad (9)$$

A set of rates $(R_1, R_2, \ldots, R_M)$ is achievable if there exists $(P_{F_i|Q^n}, \phi_i^n)$ for all $i \in \mathcal{K}$ such that $(\hat{W}_1^i, \ldots, \hat{W}_M^i) = g_i^{MC}(Q^n, F_1, \ldots, F_M, S_i, Y_i^n)$, and $\Pr\{\cup_{i=1}^M \cup_{j=1}^K \hat{W}_i^j \neq W_i\} \to 0$ as $n \to \infty$.

For *unicast transmission*, each source only wishes to transmit a message to its intended destination. We assume that
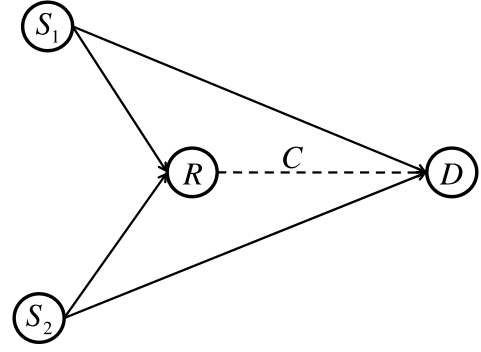

Fig. 2. Primitive multiple access relay channel with an oblivious relay.

$\mathcal{M} = \mathcal{K}$. We define the decoding function at destination $j \in \mathcal{M}$ as

$$g_j^{UC} : \mathcal{Q}^n \times \mathcal{F}_1 \times \mathcal{F}_2 \ldots \times \mathcal{F}_M \times \mathcal{S}_j \times \mathcal{Y}_j^n \to \mathcal{W}_j. \quad (10)$$

A set of rates $(R_1, R_2, \ldots, R_M)$ is achievable if there exists $(P_{F_i|Q^n}, \phi_i^n)$ for all $i \in \mathcal{K}$ such that $\hat{W}_i = g_i^{UC}(Q^n, F_1, \ldots, F_M, S_i, Y_i^n)$, and $\Pr\{\cup_{i=1}^M \hat{W}_i \neq W_i\} \to 0$ as $n \to \infty$.

III. CAPACITY REGION FOR THE PRIMITIVE MULTIPLE ACCESS CHANNEL WITH AN OBLIVIOUS RELAY

In this section, we study the PMARC with an oblivious relay. To begin with, we investigate the channel with two sources, one destination, and one oblivious relay, i.e., $M = 2$, $K = 1$ as in the definition of multicast transmission in Section II. The channel model is shown in Figure 2. The relay-destination link is out-of-band with finite capacity C, and the message transmitted from relay to the destination is denoted as S. The sum capacity of this channel is obtained in [16], where the CF strategy from [2] with sequential decoding of the compression index and source messages is shown to be *sum-rate optimal*. By deriving new outerbounds, we show that, for the entire capacity region, the GCF scheme is optimal.

A. Main Result

Theorem 1: The following rate region is the capacity region of PMARC with oblivious relaying:

$$R_1 < I(X_1; \hat{Y}_R Y | X_2 Q) \quad (11)$$
$$R_1 < I(X_1; Y | X_2 Q) + C - I(Y_R; \hat{Y}_R | X_1 X_2 Y Q) \quad (12)$$
$$R_2 < I(X_2; \hat{Y}_R Y | X_1 Q) \quad (13)$$
$$R_2 < I(X_2; Y | X_1 Q) + C - I(Y_R; \hat{Y}_R | X_1 X_2 Y Q) \quad (14)$$
$$R_1 + R_2 < I(X_1 X_2; \hat{Y}_R Y | Q) \quad (15)$$
$$R_1 + R_2 < I(X_1 X_2; Y | Q) + C - I(Y_R; \hat{Y}_R | X_1 X_2 Y Q) \quad (16)$$

for all distributions

$$p(q)p(x_1|q)p(x_2|q)p(\hat{y}_R|y_R q)p(yy_R|x_1 x_2), \quad (17)$$

where $|\hat{\mathcal{Y}}_R| \le |\mathcal{Y}_R| + 3$, $|\mathcal{Q}| \le 6$.

Proof: The achievability is using GCF relaying, where the relay compresses the received signal and uses
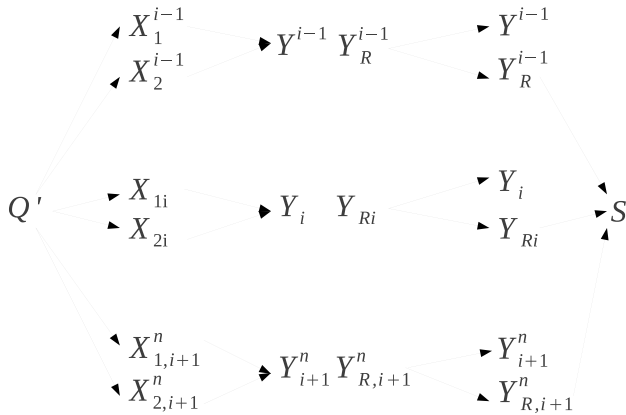
Fig. 3. Markov chain between random variables.

Wyner-Ziv binning to convey the bin index to the destination. The destination uses the bin index to jointly decode the source messages and the compression index. The scheme follows from [17, Ch. 16.7] with generalization to the multi-user set up and can be obtained from a more general achievable rate in [18, Th. 1].

For outerbounds, we need to utilize the property (5) and appropriately define the random variable $\hat{Y}_{Ri}$. To illustrate the approach and keep the clarity of the proof, we only present the proof for the individual rates. The proof for the sum rate bound follows from a similar approach and is deferred to Appendix B. For the individual rate R_1, we have

$$nR_1 = H(W_1) \tag{18}$$

$$= H(W_1|Q') \tag{19}$$

$$\leq I(W_1; Y^n S F_1 F_2|Q' W_2) + n\epsilon_n \tag{20}$$

$$= I(W_1; F_1 F_2|Q' W_2)$$
$$\quad + I(W_1; Y^n S|Q' W_2 F_2 F_1) + n\epsilon_n \tag{21}$$

$$\leq I(F_1 W_1; Y^n S|Q' W_2 F_2) + n\epsilon_n \tag{22}$$

$$= I(F_1 W_1 X_1^n; Y^n S|Q' W_2 F_2 X_2^n) + n\epsilon_n \tag{23}$$

$$\leq H(Y^n S|Q' X_2^n) - H(Y^n S|Q' X_2^n X_1^n) + n\epsilon_n \tag{24}$$

$$\leq I(X_1^n; Y^n S|Q' X_2^n) + n\epsilon_n \tag{25}$$

where $Q' = Q^n$, $\epsilon_n \to 0$ as $n \to \infty$ and (21) follows from the independence between W_1 and $F_1 F_2$. Due to *Lemma 1*, we have that the symbols from source sequences are independent for each instance i conditioned on Q' without conditioning on the codebook information. When we combine this property and the memoryless property of the channel, together with the definition of relay encoder, we have the Markov chain between the random variables illustrated in Figure 3, based on the graphical method described in [18, pp. 166–168]. Note that we do not consider messages W_1, W_2 in the figure since the channel outputs only depend on the channel inputs, and we can upperbound (22) by (25). Without conditioning on the codebooks, we have *Lemma 1* and it is easy to see that the message and source sequence are independent. It thus suffices to consider the random variables shown in Figure 3.

We can further bound (25) with two different methods.

$$I(X_1^n; Y^n S|Q' X_2^n) \tag{26}$$

$$= H(X_1^n|Q' X_2^n) - H(X_1^n|Y^n S Q' X_2^n) \tag{27}$$

$$= \sum_{i=1}^{n} H(X_{1i}|X_{2i} Q') - \sum_{i=1}^{n} H(X_{1i}|Y^n S Q' X_2^n X_1^{i-1}) \tag{28}$$

$$\leq \sum_{i=1}^{n} H(X_{1i}|X_{2i} Q') - \sum_{i=1}^{n} H(X_{1i}|S X_1^{i-1} X_{1,i+1}^n$$
$$\times X_2^{i-1} X_{2,i+1}^n Y^{i-1} Y_{i+1}^n Y_R^{i-1} X_{2i} Y_i Q') \tag{29}$$

$$= \sum_{i=1}^{n} H(X_{1i}|X_{2i} Q') - \sum_{i=1}^{n} H(X_{1i}|\hat{Y}_{Ri} X_{2i} Y_i Q') \tag{30}$$

$$= \sum_{i=1}^{n} I(X_{1i}; \hat{Y}_{Ri} Y_i|X_{2i} Q') \tag{31}$$

where (28) follows from *Lemma 1* and the fact that there is no cooperation between the sources. In (30), since conditioning reduces entropy, we introduce some additional random variables in the condition of the second term to form $\hat{Y}_{Ri}$, which is defined as

$$\hat{Y}_{Ri} = S X_1^{i-1} X_{1,i+1}^n X_2^{i-1} X_{2,i+1}^n Y^{i-1} Y_{i+1}^n Y_R^{i-1}. \tag{32}$$

We can also bound the term (25) in the following way:

$$I(X_1^n; Y^n S|Q' X_2^n) \tag{33}$$

$$= I(X_1^n; Y^n|Q' X_2^n) + I(X_1^n; S|Q' X_2^n Y^n) \tag{34}$$

$$= H(Y^n|Q' X_2^n) - H(Y^n|Q' X_2^n X_1^n)$$
$$\quad + H(S|Q' X_2^n Y^n) - H(S|Q' X_1^n X_2^n Y^n) \tag{35}$$

$$\leq \sum_{i=1}^{n} H(Y_i|X_{2i} Q') - \sum_{i=1}^{n} H(Y_i|X_{1i} X_{2i} Q') + H(S)$$
$$\quad - \big(H(S|Q' X_1^n X_2^n Y^n) - H(S|Q' X_1^n X_2^n Y^n Y_R^n)\big) \tag{36}$$

$$\leq \sum_{i=1}^{n} I(X_{1i}; Y_i|X_{2i} Q') + nC - I(S; Y_R^n|Q' X_1^n X_2^n Y^n) \tag{37}$$

$$= \sum_{i=1}^{n} I(X_{1i}; Y_i|X_{2i} Q') + nC$$
$$\quad - \sum_{i=1}^{n} I(S; Y_{Ri}|Q' X_1^n X_2^n Y^n Y_R^{i-1}) \tag{38}$$

$$= \sum_{i=1}^{n} I(X_{1i}; Y_i|X_{2i} Q') + nC$$
$$\quad - \sum_{i=1}^{n} \Big(H(Y_{Ri}|Q' X_1^n X_2^n Y^n Y_R^{i-1})$$
$$\quad\quad - H(Y_{Ri}|S Q' X_1^n X_2^n Y^n Y_R^{i-1})\Big) \tag{39}$$

$$= \sum_{i=1}^{n} I(X_{1i}; Y_i|X_{2i} Q') + nC$$
$$\quad - \sum_{i=1}^{n} \Big(H(Y_{Ri}|Q' X_{1i} X_{2i} Y_i) - H(Y_{Ri}|S X_1^{i-1} X_{1,i+1}^n$$
$$\quad\quad X_2^{i-1} X_{2,i+1}^n Y^{i-1} Y_{i+1}^n Y_R^{i-1} X_{1i} X_{2i} Y_i Q')\Big) \tag{40}$$

$$= \sum_{i=1}^{n} I(X_{1i}; Y_i|X_{2i} Q') + nC$$
$$\quad - \sum_{i=1}^{n} I(\hat{Y}_{Ri}; Y_{Ri}|X_{1i} X_{2i} Y_i Q') \tag{41}$$

where (40) follows from the Markov chain in Figure 3, i.e., conditioned on Q', there is no path between the random variables with different time instances. Note that the random variable S and the related edges should not be considered here since S is not present in the third term in (39) and (40) (see [19], pages 166-168 for detailed explanation). The end result can be obtained by introducing another time sharing random variable $Q'' \sim \mathcal{U}(\{1, 2, \ldots, n\})$ and setting $Q = (Q'', Q')$. The way we define the random variable $\hat{Y}_{Ri}$ implies the Markov chain $X_{1i} X_{2i} \rightarrow Y_{Ri} Q' \rightarrow \hat{Y}_{Ri}$ and thus the distribution (17). The individual rate R_2 can be obtained in a similar fashion.

The cardinality bounds on random variables $\hat{Y}_R$ and Q follow from the *convex cover method* introduced in [17, Appendix C] in a straightforward manner. ∎

Remark 1: It can be shown that the achievable rate region obtained from GCF includes the one obtained by CF. Specifically, these two schemes have the same maximum sum rate and individual rates, but the rate region due to GCF is potentially larger than that of CF. In fact, using GCF scheme, the random variable $\hat{Y}_R$ can be chosen from a larger set, which leads to a potential improvement in terms of rate region.

B. Capacity Without Codebook Information vs Capacity With Codebook Information

In this section, we compare the capacity result obtained for the PMARC without codebook information in Section III-A and the capacity results obtained for the PMARC in the general setting in reference [7].

1) Physically Degraded PMARC: When the channel is physically degraded, i.e.,

$$p(Y_R, Y | X_1, X_2) = p(Y_R | X_1, X_2) p(Y | Y_R), \quad (42)$$

it is shown in [7] that the capacity region for PMARC with codebook information is

$$R_1 \leq I(X_1; Y | X_2 Q) + \min\{C, I(X_1; Y_R | Y X_2 Q)\} \quad (43)$$

$$R_2 \leq I(X_2; Y | X_1 Q) + \min\{C, I(X_2; Y_R | Y X_1 Q)\} \quad (44)$$

$$R_1 + R_2 \leq I(X_1 X_2; Y | Q) + \min\{C, I(X_1 X_2; Y_R | Y Q)\}. \quad (45)$$

We can rewrite the capacity region for PMARC without codebook information obtained in *Theorem 1* as

$$R_1 \leq I(X_1; Y | X_2 Q) + \min\{C - I(Y_R; \hat{Y}_R | X_1 X_2 Y Q),$$
$$I(X_1; \hat{Y}_R | Y X_2 Q)\} \quad (46)$$

$$R_2 \leq I(X_2; Y | X_1 Q) + \min\{C - I(Y_R; \hat{Y}_R | X_1 X_2 Y Q),$$
$$I(X_2; \hat{Y}_R | Y X_1 Q)\} \quad (47)$$

$$R_1 + R_2 \leq I(X_1 X_2; Y | Q) + \min$$
$$\{C - I(Y_R; \hat{Y}_R | X_1 X_2 Y Q), I(X_1 X_2; \hat{Y}_R | Y Q)\}. \quad (48)$$

Considering the fact that given X_2, Y and Q, the random variables $X_1 \rightarrow Y_R \rightarrow \hat{Y}_R$ form a Markov chain, it is implied that the capacity region with codebook information at the relay is strictly larger than the capacity region for PMARC without codebook information. Therefore, for the physically degraded PMARC, there *is* a rate loss for the capacity without codebook

information due to the fact that the relay is unable to decode the source messages.

2) Semi-Deterministic PMARC: When the channel inputs and outputs for the PMARC satisfy that $H(Y_R | Y, X_1) = H(Y_R | Y, X_2) = 0$, it is shown in [7] that the capacity region without codebook information is

$$R_1 \leq I(X_1; Y | X_2 Q) \quad (49)$$

$$R_2 \leq I(X_2; Y | X_1 Q) \quad (50)$$

$$R_1 + R_2 \leq I(X_1 X_2; Y | Q) + \min\{C, H(Y_R | Y Q)\}. \quad (51)$$

It is easy to see that under the semi-deterministic condition $H(Y_R | Y, X_1) = H(Y_R | Y, X_2) = 0$, the capacity region we obtained in *Theorem 1* reduces to the same region. Therefore there is *no* capacity loss for the semi-deterministic PMARC, since for this case, GCF scheme is optimal regardless the existence of codebook information at the relay.

Remark 2: Under the semi-deterministic condition, CF scheme and GCF scheme achieve the same rate region. However, in general, GCF scheme can achieve a larger rate region, as stated in *Remark 1*.

Remark 3: The above comparison is for the PMARC. For the relay channel, the capacity is also known for degraded case [2] and semi-deterministic case [3], and the comparison between the capacity results with codebook information and without codebook information follows the same argument, hence is omitted here to avoid repetition.

C. Extension to $M > 2$ Nodes

Using GCF relaying, and the outerbounding techniques we used in *Theorem 1*, we can derive the capacity region of the M-user MAC, i.e., a channel with M sources and $K = 1$ destination as in the definition of multicast transmission in Section II:

Corollary 1: Let $\mathcal{S} \subseteq \mathcal{M}$, $R_{\mathcal{S}} \triangleq \sum_{i \in \mathcal{S}} R_i$. The capacity region of the M-user MAC can be specified by the rate vector $R_{\mathcal{S}}$ satisfying

$$R_{\mathcal{S}} \leq I(X_{\mathcal{S}}; \hat{Y}_R Y | X_{\mathcal{S}^c} Q) \quad (52)$$

$$R_{\mathcal{S}} \leq I(X_{\mathcal{S}}; Y | X_{\mathcal{S}^c} Q) + C - I(\hat{Y}_R; Y_R | X_{\mathcal{M}} Y Q), \quad (53)$$

for $\forall \mathcal{S} \in \mathcal{M}$ and all distributions

$$p(q) \prod_{i=1}^{M} p(x_i | q) p(\hat{y}_R | y_R q), \quad (54)$$

where $|\hat{\mathcal{Y}}_R| \leq |\mathcal{Y}_R| + 2^{|\mathcal{S}|} - 1$ and $|\mathcal{Q}| \leq 2(2^{|\mathcal{S}|} - 1)$.

The proof is an extension of *Theorem 1*, and the cardinality bounds on random variables $\hat{Y}_R$ and Q follow from the *convex cover method* introduced in [17, Appendix C]. Note that for the M-user MAC with one relay, the increase of the achievable rate is at most the capacity of the out-of-band link between the relay and the destination, i.e., when the negative term of (53) is 0, and (53) is the dominating term. For this ideal setting, the rate improvement of C bits is shared by all the M-users. Note that as the number of users increases, the average rate improvement for all the M sources, which is $\frac{C}{M}$, becomes negligible. This issue can be resolved by introducing multiple relays using GCF relaying.
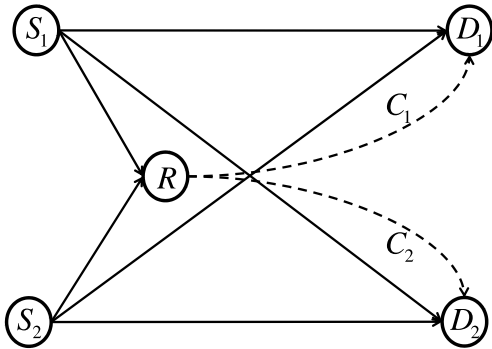
Fig. 4.   Primitive interference relay channel with an oblivious relay.

IV. CAPACITY REGION FOR MULTI-DESTINATION NETWORKS WITH AN OBLIVIOUS RELAY

A. Capacity Region for the Primitive Interference Relay Channel With an Oblivious Relay

In this section, we present a result for the primitive interference relay channel (PIFRC) with an oblivious relay, where the relay connects to destination 1 (2) with an out-of-band link with capacity C_1 (C_2) and the destinations are assumed to be interference-aware. The signals sent from the relay to the destinations are denoted as S_1 and S_2. The channel model is shown in Figure 4. We first establish a new strong interference condition, and then derive outerbounds to show that GCF achieves the capacity region.

Theorem 2: When the channel transition probability leads to the following strong interference conditions for all input distributions $p(x_1)p(x_2)$

$$I(X_1; Y_1 Y_R | X_2) \le I(X_1; Y_2 | X_2), \tag{55}$$

$$I(X_2; Y_2 Y_R | X_1) \le I(X_2; Y_1 | X_1), \tag{56}$$

then, the following rate region is the capacity region for the PIFRC with an oblivious relay:

$$R_1 < I(X_1; \hat{Y}_{R1} Y_1 | X_2 Q) \tag{57}$$

$$R_1 < I(X_1; Y_1 | X_2 Q) + C_1 - I(Y_R; \hat{Y}_{R1} | X_1 X_2 Y_1 Q) \tag{58}$$

$$R_2 < I(X_2; \hat{Y}_{R2} Y_2 | X_1 Q) \tag{59}$$

$$R_2 < I(X_2; Y_2 | X_1 Q) + C_2 - I(Y_R; \hat{Y}_{R2} | X_1 X_2 Y_2 Q) \tag{60}$$

$$R_1 + R_2 < I(X_1 X_2; \hat{Y}_{R1} Y_1 | Q) \tag{61}$$

$$R_1 + R_2 < I(X_1 X_2; Y_1 | Q) + C_1 - I(Y_R; \hat{Y}_{R1} | X_1 X_2 Y_1 Q) \tag{62}$$

$$R_1 + R_2 < I(X_1 X_2; \hat{Y}_{R2} Y_2 | Q) \tag{63}$$

$$R_1 + R_2 < I(X_1 X_2; Y_2 | Q) + C_2 - I(Y_R; \hat{Y}_{R2} | X_1 X_2 Y_2 Q) \tag{64}$$

for all input distributions

$$p(q)p(x_1|q)p(x_2|q)p(\hat{y}_{R1}|y_R q)p(\hat{y}_{R2}|y_R q), \tag{65}$$

where $|\hat{\mathcal{Y}}_{R1}| \le |\mathcal{Y}_{R1}| + 2$, $|\hat{\mathcal{Y}}_{R2}| \le |\mathcal{Y}_{R2}| + 2$ and $|\mathcal{Q}| \le 8$.

Proof: The achievability follows from the GCF relaying, which is similar to the one we used for *Theorem 1*. The only difference here is that the relay needs to generate two compression codebooks for each destination, and use different Wyner-Ziv binning for each destination. Using the bin index obtained from the out-of-band relay link, each

destination jointly decodes the messages from both sources and the compression index. Note that the rate constraints on interfering messages at each destination are removed from the rate region since the destination is only interested in the intended message. The detailed scheme is omitted and the reader is referred to [17, Ch. 16.7] and [18, Th. 2].

For the outerbounds, we need to utilize the strong interference conditions (55) and (56). In fact, the conditions (55) and (56) imply that (see [20], [21] for details)

$$I(X_1^n; Y_1^n Y_R^n | X_2^n U) \le I(X_1^n; Y_2^n | X_2^n U) \tag{66}$$

$$I(X_2^n; Y_2^n Y_R^n | X_1^n U) \le I(X_2^n; Y_1^n | X_1^n U) \tag{67}$$

Since S_1 and S_2 are functions of Y_R^n, we have the following lemma:

Lemma 2:

$$I(X_1^n; Y_1^n S_1 | X_2^n U) \le I(X_1^n; Y_1^n Y_R^n | X_2^n U)$$
$$\le I(X_1^n; Y_2^n | X_2^n U) \le I(X_1^n; Y_2^n S_2 | X_2^n U) \tag{68}$$

$$I(X_2^n; Y_2^n S_2 | X_1^n U) \le I(X_2^n; Y_2^n Y_R^n | X_1^n U)$$
$$\le I(X_2^n; Y_1^n | X_1^n U) \le I(X_2^n; Y_1^n S_1 | X_1^n U) \tag{69}$$

Proof: To prove this relation, we proceed as follows:

$$I(X_1^n; Y_1^n Y_R^n S_1 | X_2^n U) \tag{70}$$

$$= I(X_1^n; Y_1^n S_1 | X_2^n U) + I(X_1^n; Y_R^n | X_2^n Y_1^n S_1 U) \tag{71}$$

$$= I(X_1^n; Y_1^n Y_R^n | X_2^n U) + I(X_1^n; S_1 | X_2^n Y_1^n Y_R^n U) \tag{72}$$

Since $(S_1, S_2) = f(Y_R^n)$, we have

$$I(X_1^n; S_1 | X_2^n Y_1^n Y_R^n U) = 0 \tag{73}$$

Consequently, we can obtain the following inequality

$$I(X_1^n; Y_1^n S_1 | X_2^n U) \le I(X_1^n; Y_1^n Y_R^n | X_2^n U), \tag{74}$$

and the relation (68) can be obtained accordingly. The relation (69) can be obtained in the same fashion. ∎

We are now ready to prove the capacity results. The outerbounds for individual rates can be obtained by setting

$$\hat{Y}_{R1,i} = S_1 X_1^{i-1} X_{1,i+1}^n X_2^{i-1} X_{2,i+1}^n Y_1^{i-1} Y_{1,i+1}^n Y_R^{i-1}, \tag{75}$$

and

$$\hat{Y}_{R2,i} = S_2 X_1^{i-1} X_{1,i+1}^n X_2^{i-1} X_{2,i+1}^n Y_2^{i-1} Y_{2,i+1}^n Y_R^{i-1}. \tag{76}$$

using similar steps as in *Theorem 1*.

For the sum rate outerbounds, we have

$$n(R_1 + R_2)$$

$$= H(W_1) + H(W_2) \tag{77}$$

$$= H(W_1 | Q) + H(W_2 | Q) \tag{78}$$

$$\le I(W_1; Y_1^n S_1 F_1 F_2 | W_2 Q) + I(W_2; Y_2^n S_2 F_1 F_2 | Q) + n\epsilon_n \tag{79}$$

$$= I(W_1; F_1 F_2 | Q W_2) + I(W_1; Y_1^n S_1 | Q W_2 F_1 F_2)$$
$$+ I(W_2; F_1 F_2 | Q) + I(W_2; Y_2^n S_2 | Q F_1 F_2) + n\epsilon_n \tag{80}$$

$$\le I(W_1; Y_1^n S_1 | Q F_2 W_2 F_1) + I(W_2 F_2; Y_2^n S_2 | Q F_1) + n\epsilon_n \tag{81}$$

$$\le I(X_1^n; Y_1^n S_1 | Q X_2^n F_1) + I(X_2^n; Y_2^n S_2 | Q F_1) + n\epsilon_n \tag{82}$$

$$\le I(X_1^n; Y_2^n S_2 | Q X_2^n F_1) + I(X_2^n; Y_2^n S_2 | Q F_1) + n\epsilon_n \tag{83}$$

$$\le I(X_1^n X_2^n; Y_2^n S_2 | Q) \tag{84}$$

where in (83) we used condition (68) with with $U = QF_1$. From this we can derive the bounds (63) and (64) following similar steps as in *Theorem 1*, and the bounds (61) and (62) can be obtained using condition (69).

With the auxiliary random variables specified by (75) and (76), the probability distribution is factorized as

$$p(q)p(x_1|q)p(x_2|q)p(\hat{y}_{R1}\hat{y}_{R2}|y_Rq)p(y_1y_2y_R|x_1x_2) \quad (85)$$

Note that the input distribution

$$p(q)p(x_1|q)p(x_2|q)p(\hat{y}_{R1}|y_Rq)p(\hat{y}_{R2}|y_Rq)p(y_1y_2y_R|x_1x_2) \quad (86)$$

yields the same rate region as the one specified by (85). This is because all the rate expressions only depend on the marginal distribution

$$p(q)p(x_1|q)p(x_2|q)p(\hat{y}_{R1}|y_Rq)p(y_1y_R|x_1x_2), \quad (87)$$

and

$$p(q)p(x_1|q)p(x_2|q)p(\hat{y}_{R2}|y_Rq)p(y_2y_R|x_1x_2). \quad (88)$$

For each distribution factorized as (85), we can always find a distribution factorized as (86) that yields the same marginal distribution (87) and (88).

It then suffices to constrain the probability distribution to the set specified by (65).

The cardinality bounds on the random variables $\hat{Y}_{R1}$, $\hat{Y}_{R2}$ and Q can be obtained using the *convex cover method* introduced in [17, Appendix C]. ∎

Remark 4: The above Theorem shows that it is optimal for the relay to generate the compression codebooks for different destinations independently. The probability distribution used for generating the codebooks can be optimized independently for each destination.

Remark 5: Similar as PMARC, noisy network coding yields the same rate region as GCF for PIFRC. For the general IFRC, noisy network coding can have better performance, since it does not use Wyner-Ziv binning and can overcome the limitation placed on GCF by the receiver with the worst relay-destination channel. In PIFRC, however, relay has two digital links to the destinations with different capacity, and thus the rate of the bin indices in the GCF scheme is no longer limited by the receiver with the worst relay-destination channel, leading to identical rates for GCF and noisy network coding.

B. Discussion on the Strong Interference Condition

The strong interference condition here differs from that of the interference channel, which is of the form $I(X_1; Y_1|X_2) \leq I(X_1; Y_2|X_2)$. In the strong interference condition (68) and (69), the relay serves as a virtual antenna shared by both destinations.

To gain a more intuitive grasp of these conditions, consider the Gaussian IFRC

$$Y_1 = h_{11}X_1 + h_{21}X_2 + Z_1 \quad (89)$$
$$Y_2 = h_{12}X_1 + h_{22}X_2 + Z_2 \quad (90)$$
$$Y_R = h_{1R}X_1 + h_{2R}X_2 + Z_R, \quad (91)$$

where Z_1, Z_2, Z_R are zero-mean Gaussian random variables with unit variance.

For this model, an equivalent condition for the strong interference conditions (55) and (56) is

$$h_{12}^2 \geq h_{11}^2 + h_{1R}^2 \quad (92)$$
$$h_{21}^2 \geq h_{22}^2 + h_{2R}^2, \quad (93)$$

i.e., the strength of the interference link is greater than the sum of the direct link and the corresponding source-relay link. To see this, we recognize that the left-hand-side of (55) under the Gaussian model becomes

$$I(X_1; \mathbf{h}X_1 + \mathbf{Z}) \quad (94)$$

$$= I\left(X_1; \frac{1}{\sqrt{h_{11}^2 + h_{1R}^2}}\mathbf{H}(\mathbf{h}X_1 + \mathbf{Z})\right) \quad (95)$$

$$= I\left(X_1; \sqrt{h_{11}^2 + h_{1R}^2}X_1 + Z\right) \quad (96)$$

where

$$\mathbf{h} = \begin{pmatrix} h_{11} \\ h_{1R} \end{pmatrix} \quad (97)$$

$$\mathbf{H} = \begin{pmatrix} h_{11} & h_{1R} \\ h_{1R} & -h_{11} \end{pmatrix} \quad (98)$$

$$\mathbf{Z} = \begin{pmatrix} Z_1 \\ Z_R \end{pmatrix} \quad (99)$$

and Z is a zero-mean Gaussian random variable with unit variance.

Moreover, the right-hand-side of (55) becomes

$$I(X_1; h_{12}X_1 + Z_2). \quad (100)$$

The equivalence between the conditions (55), (56) and the conditions (92), (93) are then immediate.

We can see that the strong interference condition implies that the interference link from S_1 to D_2 can provide the destination D_2 with more information regarding X_1 than the amount of information of X_1 contained in the combination of links S_1 to D_1 and S_1 to relay. Therefore the destinations are able to decode both messages.

Remark 6: Note that different from the interference channel, the capacity result cannot be extended to the Gaussian case directly, since the optimal input distribution is not necessarily Gaussian.

C. Extension to M-Source K-Destination Multicast Networks

The result for the PIFRC with an oblivious relay is obtained by establishing the strong interference condition under which the channel is equivalent to a compound PMARC with an oblivious relay. This result can be extended to M-source K-destination multicast network with an oblivious relay, as defined in Section II.

Corollary 2: Let $\mathcal{S} \subseteq \mathcal{M}$, $d \in \mathcal{K}$, $R_{\mathcal{S}} \triangleq \sum_{i \in \mathcal{S}} R_i$. The capacity region of the M-source K-destination multicast network with an oblivious relay can be specified by the rate

vector $R_{\mathcal{M}}$ satisfying

$$R_{\mathcal{S}} \leq \min_{d \in \mathcal{K}} \left\{ I(X_{\mathcal{S}}; \hat{Y}_{R,d} Y_d | X_{\mathcal{S}^c} Q) \right\} \tag{101}$$

$$R_{\mathcal{S}} \leq \min_{d \in \mathcal{K}} \left\{ I(X_{\mathcal{S}}; Y_d | X_{\mathcal{S}^c} Q) \right.$$
$$\left. + C_d - I(\hat{Y}_{R,d}; Y_R | X_{\mathcal{M}} Y_d Q) \right\} \tag{102}$$

for $\forall \mathcal{S} \in \mathcal{M}$ and all distributions

$$p(q) \left(\prod_{i=1}^{M} p(x_i | q) \right) \left(\prod_{j=1}^{K} p(\hat{y}_{R,j} | y_R q) \right) p(y_{\mathcal{K}} y_R | x_{\mathcal{M}}), \tag{103}$$

where $|\hat{\mathcal{Y}}_{Rd}| \leq |\mathcal{Y}_{Rd}| + 2^{|\mathcal{S}|} - 1$ and $|\mathcal{Q}| \leq 2d(2^{|\mathcal{S}|} - 1)$.

The proof of this proposition follows the same lines of reasoning as in *Theorem 1*, where the auxiliary random variable $\hat{Y}_{Ri,d}$ is defined as

$$\hat{Y}_{Ri,d} = S_d X_{\mathcal{M}}^{i-1} X_{\mathcal{M},i+1}^{n} Y_d^{i-1} Y_{d,i+1}^{n} Y_R^{i-1}. \tag{104}$$

Note that under this definition, the auxiliary random variables $\hat{Y}_{Ri,d}$ are not independent conditioned on Y_R and Q. However, the distribution $\prod_{j=1}^{K} p(\hat{y}_{R,j} | y_R q)$ and $p(\hat{y}_{R,\mathcal{K}} | y_R q)$ yields the same rate region as discussed in the proof of *Theorem 2*. Thus it is equivalent to assume that $\hat{Y}_{Ri,d}$ are independent conditioned on Y_R and Q. The cardinality bounds on the random variables $\hat{Y}_{Rd}$ and Q once again follow from the *convex cover method* in [17, Appendix C].

Remark 7: For the multicast model, the maximum rate improvement is limited by the out-of-band link from the relay to the destinations with the smallest capacity. The average rate improvement for each source also becomes negligible as the number of sources increases, which can be resolved by introducing multiple relays, as discussed in Section III-C.

V. CONCLUSION

In this paper, we have studied network information theoretic models that enjoy the cooperation of a relay node despite the relay having no information about the codebooks used by the sources. We have focused on a class of primitive multiuser networks with an oblivious relay where the relay-destination links are out-of-band with finite capacity. Different than previous work considering special cases of such networks, we have employed the generalized compress-and-forward (GCF) relaying scheme, where the compression index and source messages are decoded jointly at the destination and found it to be capacity achieving under certain conditions. Specifically, we have established the capacity region of the primitive multiple access relay channel (PMARC) with an oblivious relay by deriving new outerbounds and showing they are tight by means of GCF. We have next extended the result to establish the capacity region for the M-user PMARC with an oblivious relay. For the primitive interference relay channel (PIFRC) with an oblivious relay, we have established a new strong interference condition, and derived the capacity region under this condition, which again can be achieved by GCF relaying. This result is further extended to M-source K-destination multicast network with an oblivious relay.

The results obtained in this paper are intended to provide design insights towards optimal relaying strategies in wireless ad hoc networks when the codebook information is absent at the relay nodes. The capacity region of an ad hoc network with arbitrary size and topology remains open.

APPENDIX A
PROOF OF LEMMA 1

Proof: We omit the subscripts i in x_i^n and y_i^n for clarity.

$$P_{X^n | Q^n}(x^n | q^n) \tag{105}$$

$$= \sum_{f, W : \phi^n(W, f) = x^n} P(f, W | q^n) \tag{106}$$

$$= \sum_{W=1}^{2^{nR}} \sum_{f : \phi^n(W, f) = x^n} P(f | q^n) P(W) \tag{107}$$

$$= \sum_{f : \phi^n(1, f) = x^n} P(f | q^n) \tag{108}$$

$$= \prod_{i=1}^{n} P(x_i | q_i) \sum_{x^n(2) \ldots x^n(2^{nR})} \prod_{i=1}^{n} \prod_{l=2}^{2^{nR}} P(x_i(l) | q_i) \tag{109}$$

$$= \prod_{i=1}^{n} P(x_i | q_i) \sum_{x_2^n(2) \ldots x_2^n(2^{nR})} \sum_{x_1(2) \ldots x_1(2^{nR})}$$
$$\times \left(\prod_{l=2}^{2^{nR}} P(x_1(l) | q_1) \prod_{i=2}^{n} \prod_{l=2}^{2^{nR}} P(x_i(l) | q_i) \right) \tag{110}$$

$$= \prod_{i=1}^{n} P(x_i | q_i) \sum_{x_2^n(2) \ldots x_2^n(2^{nR})}$$
$$\times \left(\sum_{x_1(2) \ldots x_1(2^{nR})} \prod_{l=2}^{2^{nR}} P(x_1(l) | q_1) \right) \prod_{i=2}^{n} \prod_{l=2}^{2^{nR}} P(x_i(l) | q_i) \tag{111}$$

For the middle term in the above expression, we have

$$\sum_{x_1(2) \ldots x_1(2^{nR})} \prod_{l=2}^{2^{nR}} P(x_1(l) | q_1) \tag{112}$$

$$= \sum_{x_1(3) \ldots x_1(2^{nR})} \sum_{x_1(2)} P(x_1(2) | q_1) \prod_{l=3}^{2^{nR}} P(x_1(l) | q_1) \tag{113}$$

$$= \sum_{x_1(3) \ldots x_1(2^{nR})} \left(\sum_{x_1(2)} P(x_1(2) | q_1) \right) \prod_{l=3}^{2^{nR}} P(x_1(l) | q_1). \tag{114}$$

Now it is easy to see that $\sum_{x_1(2)} P(x_1(2) | q_1) = 1$. Continue with this approach, we have

$$\sum_{x_1(2) \ldots x_1(2^{nR})} \prod_{l=2}^{2^{nR}} P(x_1(l) | q_1) = 1. \tag{115}$$

Substitute (115) into (111), and proceed with the rest of the terms, we have

$$P_{X^n | Q^n}(x^n | q^n) = \prod_{i=1}^{n} P_{X|Q}(x_i | q_i) \tag{116}$$

Using this result, we have

$$P_{Y^n|Q^n}(y^n|q^n) \tag{117}$$

$$= \sum_{x^n} P_{X^nY^n|Q^n}(x^ny^n|q^n) \tag{118}$$

$$= \sum_{x^n} P_{X^n|Q^n}(x^n|q^n) \prod_{i=1}^{n} P_{Y|X}(y_i|x_i) \tag{119}$$

$$= \sum_{x^n} \prod_{i=1}^{n} P_{X|Q}(x_i|q_i) P_{Y|X}(y_i|x_i) \tag{120}$$

$$= \sum_{x^n} \prod_{i=1}^{n} P_{XY|Q}(x_iy_i|q_i) \tag{121}$$

$$= \prod_{i=1}^{n} \sum_{x_i} P_{XY|Q}(x_iy_i|q_i) \tag{122}$$

$$= \prod_{i=1}^{n} P_{Y|Q}(y_i|q_i) \tag{123}$$

∎

APPENDIX B
PROOF OF THEOREM 1

Proof: **Sum rate upperbound**: For the sum rate, we have

$$n(R_1 + R_2) = H(W_1W_2) \tag{124}$$

$$= H(W_1W_2|Q') \tag{125}$$

$$\leq I(W_1W_2; Y^nSF_1F_2|Q') + n\epsilon_n \tag{126}$$

$$= I(W_1W_2; F_1F_2|Q'W_2)$$
$$\quad + I(W_1W_2; Y^nS|Q'F_1F_2) + n\epsilon_n \tag{127}$$

$$\leq I(F_1W_1F_2W_2; Y^nS|Q') + n\epsilon_n \tag{128}$$

$$\leq I(X_1^nX_2^n; Y^nS|Q') + n\epsilon_n \tag{129}$$

We can further bound (129) in two different ways.

$$I(X_1^nX_2^n; Y^nS|Q') \tag{130}$$

$$= H(X_1^nX_2^n|Q') - H(X_1^nX_2^n|Y^nSQ') \tag{131}$$

$$= \sum_{i=1}^{n} H(X_{1i}X_{2i}|Q')$$
$$\quad - \sum_{i=1}^{n} H(X_{1i}X_{2i}|Y^nSQ'X_1^{i-1}X_2^{i-1}) \tag{132}$$

$$\leq \sum_{i=1}^{n} H(X_{1i}X_{2i}|Q') - \sum_{i=1}^{n} H(X_{1i}X_{2i}|SX_1^{i-1}X_{1,i+1}^n$$
$$X_2^{i-1}X_{2,i+1}^n Y^{i-1}Y_{i+1}^n Y_R^n Y_i Q') \tag{133}$$

$$= \sum_{i=1}^{n} H(X_{1i}X_{2i}|Q') - \sum_{i=1}^{n} H(X_{1i}X_{2i}|\hat{Y}_{Ri}Y_iQ') \tag{134}$$

$$= \sum_{i=1}^{n} I(X_{1i}X_{2i}; \hat{Y}_{Ri}Y_i|Q'), \tag{135}$$

and

$$I(X_1^nX_2^n; Y^nS|Q') \tag{136}$$

$$= I(X_1^nX_2^n; Y^n|Q') + I(X_1^nX_2^n; S|Q'Y^n) \tag{137}$$

$$= H(Y^n|Q') - H(Y^n|Q'X_2^nX_1^n) + H(S|Q'Y^n)$$
$$\quad - H(S|Q'X_1^nX_2^nY^n) \tag{138}$$

$$\leq \sum_{i=1}^{n} H(Y_i|Q') - \sum_{i=1}^{n} H(Y_i|X_{1i}X_{2i}Q') + H(S)$$
$$\quad - \left(H(S|Q'X_1^nX_2^nY^n) - H(S|Q'X_1^nX_2^nY^nY_R^n) \right) \tag{139}$$

$$\leq \sum_{i=1}^{n} I(X_{1i}X_{2i}; Y_i|Q') + nC - I(S; Y_R^n|Q'X_1^nX_2^nY^n) \tag{140}$$

$$= \sum_{i=1}^{n} I(X_{1i}X_{2i}; Y_i|Q') + nC$$
$$\quad - \sum_{i=1}^{n} I(S; Y_{Ri}|Q'X_1^nX_2^nY^nY_R^{i-1}) \tag{141}$$

$$= \sum_{i=1}^{n} I(X_{1i}X_{2i}; Y_i|Q') + nC$$
$$\quad - \sum_{i=1}^{n} \left(H(Y_{Ri}|Q'X_1^nX_2^nY^nY_R^{i-1}) \right.$$
$$\quad \left. - H(Y_{Ri}|SQ'X_1^nX_2^nY^nY_R^{i-1}) \right) \tag{142}$$

$$= \sum_{i=1}^{n} I(X_{1i}X_{2i}; Y_i|Q') + nC$$
$$\quad - \sum_{i=1}^{n} \left(H(Y_{Ri}|Q'X_{1i}X_{2i}Y_i) \right.$$
$$\quad - H(Y_{Ri}|SX_1^{i-1}X_{1,i+1}^nX_2^{i-1}X_{2,i+1}^n$$
$$\quad \left. \times Y^{i-1}Y_{i+1}^nY_R^{i-1}X_{1i}X_{2i}Y_iQ') \right) \tag{143}$$

$$= \sum_{i=1}^{n} I(X_{1i}X_{2i}; Y_i|Q') + nC$$
$$\quad - \sum_{i=1}^{n} I(\hat{Y}_{Ri}; Y_{Ri}|X_{1i}X_{2i}Y_iQ') \tag{144}$$

The result can be obtained by introducing another time sharing random variable $Q'' \sim \mathcal{U}(\{1, 2, \ldots, n\})$ and setting $Q = (Q'', Q')$. The way we define the random variable $\hat{Y}_{Ri}$ implies the distribution (17). ∎

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