Degrees of Freedom for the MIMO Multi-way Relay Channel

Ye Tian and Aylin Yener
Wireless Communications and Networking Laboratory
Electrical Engineering Department
The Pennsylvania State University, University Park, PA 16802
yetian@psu.edu *yener@ee.psu.edu*

***Abstract*—This paper investigates the degrees of freedom (DoF) of the L-cluster K-user MIMO multi-way relay channel. A DoF upperbound is derived by providing users with carefully designed genie information, and properly enhancing the received signal of one of the users. For the L-cluster K-user MIMO multi-way relay channel in the symmetric setting, conditions under which the DoF upperbound can be achieved using either multiple access transmission or signal space alignment are established, demonstrating that the newly derived upperbound is the first tight DoF upperbound for the general MIMO multi-way relay channel. Additionally, this new upperbound proves the optimality of the achievable DoF for several special cases of the MIMO multi-way relay channel obtained in previous works. The results provide the insight that, with fixed spatial dimension at the relay, increasing the number of users and clusters cannot provide any DoF gain. In addition, it is observed that allowing three or more users to share the spatial dimension of the relay cannot provide any DoF gain.**

I. Introduction

The multi-way relay channel [1] is a fundamental building block for relay networks with multicast transmission. The channel model generalizes a number of relay assisted multi-user models whose performance limits have been studied previously, see for example, [1]–[8] and references therein. For most of these special cases as well as the general model, the exact capacity remains unknown, in turn making it difficult to obtain design insights. The degrees of freedom (DoF) characterization, on the other hand, can provide us with insights on the optimal signal interaction in time/frequency/space dimensions, and can be useful in designing transmission schemes for multicasting scenarios through a relay.

The DoF of a wireless system characterizes its high signal-to-noise ratio (SNR) performance. Signal space alignment has been shown to achieve the optimal DoF for the Y channel in [9], which is a special case of the multi-way relay channel with one cluster and three users. The authors have shown that, by aligning the signals from the users that want to exchange information at the same dimension, network coding can be utilized to maximize the utilization of the spatial dimension available at the relay to achieve the optimal DoF. This technique is further utilized in references [10]–[15] to study the DoF of several special cases of the MIMO multi-way relay channel. The existing *achievable* DoF results for special cases of the MIMO multi-way relay channel are summarized as follows:

- *MIMO three-user Y channel [9]*: This is the MIMO multi-way relay channel with one cluster that contains three users. Each user has M antennas and the relay has N antennas. The optimal DoF, denoted by DoF^* is
$$DoF^* = 3M \quad \text{if } N \geq \left\lceil \frac{3M}{2} \right\rceil. \tag{1}$$
- *MIMO K-user Y channel [10]*: This is the MIMO multi-way relay channel with one cluster that contains K users. User i has M_i antennas and the relay has N antennas. Each user can send $K-1$ independent data streams with DoF d for each stream if
$$M_i \geq d(K-1),\ N \geq \frac{dK(K-1)}{2}$$
$$N < \min\{M_i + M_j - d|\forall i \neq j\}. \tag{2}$$
Note that the optimal DoF is unknown.
- *MIMO two-cluster multi-way relay channel with two users in each cluster [15]*: When each user has M antennas and the relay has N antennas, optimal DoF is
$$DoF^* = 2N, \qquad N \leq \frac{4}{3}M, \tag{3}$$
$$DoF^* = 4M, \qquad N \geq 4M. \tag{4}$$
- *MIMO K-pair two-way relay channel [11]*: This corresponds to the MIMO multi-way relay channel with K clusters each with two users. Each user has M antennas and wants to transmit d data streams with DoF 1. The relay has Kd antennas. To guarantee interference-free transmission, we need
$$K \leq \frac{2M}{d} - 1, \tag{5}$$
and the achievable DoF is $2Kd$. Note that optimal DoF is unknown except for $K = 2$ [15].

For the DoF characterization of MIMO multi-way relay channels, the DoF upperbound obtained to date is a cut-set bound, which can provide a tight upperbound for the three-user Y channel [9] and two-cluster multiway relay channel with two users in each cluster [15], but can be arbitrarily loose

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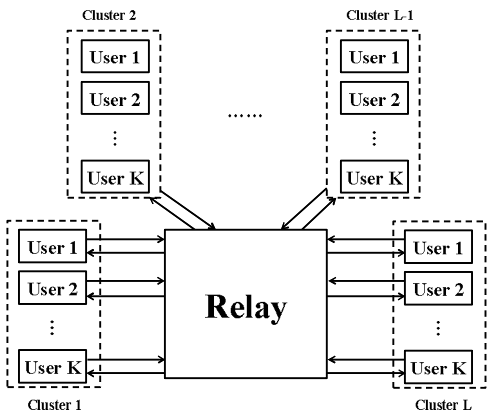

Fig. 1. L-cluster K-user MIMO multi-way relay channel.

for other instances of the model.

In this work, we derive a new DoF upperbound for the L-cluster K-user MIMO multi-way relay channel using a genie-aided approach and enhancing the received signal at one of the users following the idea from [16]. In the sequel, we will refer to this as the *general* MIMO multi-way relay channel. We show that the DoF for the general MIMO multi-way relay channel is always upperbounded by $2N$ with N being the number of antennas at the relay. This DoF upperbound, combined with the cut-set bound, provides us a comprehensive set of DoF upperbounds for the general MIMO multi-way relay channel. With the new upperbound, we then study the achievable DoF for the general MIMO multi-way relay channel in the symmetric setting, where all users have the same number of antennas. It is shown that the newly derived DoF upperbound can be achieved for several scenarios of interests using signal space alignment or multiple-access transmission. The result shows that the DoF for the MIMO multi-way relay channel is always limited by the spatial dimension available at the relay, and that we cannot get any DoF gain by letting three or more users to share the same spatial dimension of the relay.

II. Channel Model

The L-cluster K-user MIMO multi-way relay channel is shown in Fig. 1, which has L clusters, and each cluster has K users. User k $(k = 1, 2, \cdots, K)$ in cluster l $(l = 1, 2, \cdots, L)$ is assumed to have M_k^l antennas, and the relay is assumed to have N antennas. Without loss of generality, we assume that $M_1^l \geq M_2^l \geq \cdots \geq M_K^l$.

In cluster l, user k has a message W_{ik}^l $(i = 1, 2, \cdots, K, i \neq k)$, for all the other users in cluster l. We denote $\mathcal{W}_k^l$ as the message set originated from user k in cluster l for all the other users in the same cluster, i.e.,

$$\mathcal{W}_k^l = \{W_{1k}^l, W_{2k}^l, \cdots, W_{k-1,k}^l, W_{k+1,k}^l, \cdots, W_{Kk}^l\}. \quad (6)$$

It is assumed that the users can communicate only through the relay and no direct links exist between any pairs of users [1]. All the nodes in the network are assumed to be full duplex. The transmitted signal from user k in cluster l for channel use t is denoted as $\mathbf{X}_{k,l}(t) \in \mathbb{C}^{M_k^l}$. The received signal at the relay for channel use t is denoted as $\mathbf{Y}_R(t) \in \mathbb{C}^N$. The received signal at user k in cluster l for channel use t is defined as $\mathbf{Y}_{k,l}(t) \in \mathbb{C}^{M_k^l}$. The channel matrix from user k in cluster l to the relay is denoted as $\mathbf{H}_{R(k,l)}(t) \in \mathbb{C}^{N \times M_k^l}$. The channel matrix from the relay to user k in cluster l is denoted as $\mathbf{H}_{(k,l)R}(t) \in \mathbb{C}^{M_k^l \times N}$. It is assumed that the entries of the channel matrices are drawn independently from a continuous distribution, which guarantees that the channel matrices have full rank almost surely.

The encoding function at user k in cluster l is defined as

$$\mathbf{X}_{k,l}(t) = f_{k,l}(\mathcal{W}_k^l, \mathbf{Y}_{k,l}^{t-1}), \quad (7)$$

where $\mathbf{Y}_{k,l}^{t-1} = [\mathbf{Y}_{k,l}(1), \cdots, \mathbf{Y}_{k,l}(t-1)]$.

The received signal at the relay is

$$\mathbf{Y}_R(t) = \sum_{l=1}^{L}\sum_{k=1}^{K} \mathbf{H}_{R(k,l)}(t)\mathbf{X}_{k,l}(t) + \mathbf{Z}_R(t). \quad (8)$$

For channel use t, the transmitted signal $\mathbf{X}_R(t) \in \mathbb{C}^N$ from the relay is a function of its received signals from channel use 1 to $t-1$, i.e.,

$$\mathbf{X}_R(t) = f_R(\mathbf{Y}_R^{t-1}). \quad (9)$$

The received signal at user k in cluster l for channel use t is

$$\mathbf{Y}_{k,l}(t) = \mathbf{H}_{(k,l)R}(t)\mathbf{X}_R(t) + \mathbf{Z}_{k,l}(t). \quad (10)$$

In the above expressions, $\mathbf{Z}_{k,l}(t) \in \mathbb{C}^{M_k^l}, \mathbf{Z}_R(t) \in \mathbb{C}^N$ are additive white Gaussian noise vectors with zero mean and identity covariance matrices. The transmitted signals from the users and the relay satisfy the following power constraints:

$$E\left[\mathrm{tr}\left(\mathbf{X}_{k,l}(t)\mathbf{X}_{k,l}(t)^\dagger\right)\right] \leq P, \quad (11)$$

$$E\left[\mathrm{tr}\left(\mathbf{X}_R(t)\mathbf{X}_R(t)^\dagger\right)\right] \leq P. \quad (12)$$

Based on the received signals and the message set $\mathcal{W}_k^l$, user k in cluster l tries to decode all the messages intended for it, which is denoted as

$$\hat{\mathcal{W}}_k^l = \{\hat{W}_{k,1}^l, \hat{W}_{k,2}^l, \cdots, \hat{W}_{k,k-1}^l, \hat{W}_{k,k+1}^l, \cdots, \hat{W}_{k,K}^l\}.$$

We also have

$$\hat{\mathcal{W}}_k^l = g_{k,l}(\mathbf{Y}_{k,l}^n, \mathcal{W}_k^l) \quad (13)$$

where $g_{k,l}$ is the decoding function for user k in cluster l.

We assume the rate of message W_{ik}^l is $R_{ik}^l(P)$ under power constraint P. A rate tuple $\{R_{ik}^l(P)\}$ with $l = 1, \cdots, L$, $k = 1, \cdots, K$ and $i = 1, \cdots, K$, $i \neq k$ is achievable if the error probability $P_e^n = \Pr(\cup_{l,k,i}\hat{W}_{i,k}^l \neq W_{i,k}^l) \to 0$ as $n \to \infty$.

We define $\mathcal{C}(P)$ as the set of all achievable rate tuples $\{R_{ik}^l(P)\}$, under power constraint P. The degrees of freedom is defined as

$$DoF = \lim_{P \to \infty} \frac{R_{\sum}(P)}{\log(P)}, \quad (14)$$

where

$$R_{\sum}(P) = \sup_{\{R_{ik}^l(P)\}\in\mathcal{C}(P)} \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{\substack{i=1\\i\neq k}}^{K} R_{ik}^l(P) \quad (15)$$

is the sum capacity under power constraint P.

III. DoF Upperbound for the General MIMO multi-way Relay Channel

Theorem 1: For the L-cluster K-user MIMO multi-way relay channel, the DoF upperbound is

$$DoF \leq \min\left\{\sum_{l=1}^{L}\sum_{k=1}^{K} M_k^l, 2\sum_{l=1}^{L}\sum_{k=2}^{K} M_k^l, 2N\right\}. \quad (16)$$

Proof: The first two terms of the upperbound can be derived using cut set bound. Note that by assumption we have $M_1^l \geq M_2^l \geq \cdots \geq M_K^l$. For the messages in cluster l, we remove the messages from other clusters. This operation does not affect the rate of the messages in cluster l. Now the channel is effectively a MIMO multi-way relay channel with a single cluster. For the messages intended for user i, we can combine all the other users except for user i, which yields a two-way relay channel with user i as a node, and all the other users as a node. We can then bound the DoF for the messages in cluster l in the following fashion:

$$\sum_{k=2}^{K} d_{1k}^l \leq \min\{M_1^l, N, \sum_{k=2}^{K} M_k^l\} \quad (17)$$

$$\sum_{\substack{k=1\\k\neq i}}^{K} d_{ik}^l \leq \min\{M_i^l, N\},\ i\neq 1, \quad (18)$$

where d_{ik}^l denotes the DoF for message W_{ik}^l. This yields the desired DoF upperbound for the first two terms in (16).

To prove the term $DoF \leq 2N$, we first assume that $M_k^l \geq N$. If $M_k^l < N$, we can always add more antennas to user k in cluster l, which does not reduce the capacity. The received signal at user k in cluster l is

$$\mathbf{Y}_{k,l} = \mathbf{H}_{(k,l)R}\mathbf{X}_R + \mathbf{Z}_{k,l}. \quad (19)$$

We consider users in cluster l. We wish to enhance the received signal at user 1 in cluster l such that it has a stronger signal than all the other users in cluster l. To this end, we perform singular value decomposition for $\mathbf{H}_{(k,l)R}$, $k\neq 1$:

$$\mathbf{H}_{(k,l)R} = \mathbf{U}_{k,l}\mathbf{\Sigma}_{(k,l)R}\mathbf{V}_{k,l}^{\dagger}. \quad (20)$$

We have

$$\mathbf{U}_{k,l}^{\dagger}\mathbf{Y}_{k,l} = \mathbf{\Sigma}_{(k,l)R}\mathbf{V}_{k,l}^{\dagger}\mathbf{X}_R + \mathbf{U}_{k,l}^{\dagger}\mathbf{Z}_{k,l}. \quad (21)$$

With some change of notation, we have

$$\mathbf{Y}'_{k,l} = \mathbf{\Sigma}_{(k,l)R}\mathbf{X}'_{R,(k,l)} + \mathbf{Z}'_{k,l}, \quad (22)$$

where $\mathbf{Y}'_{k,l} = \mathbf{U}_{k,l}^{\dagger}\mathbf{Y}_{k,l}$, $\mathbf{X}'_{R,(k,l)} = \mathbf{V}_{k,l}^{\dagger}\mathbf{X}_R$, $\mathbf{Z}'_{k,l} = \mathbf{U}_{k,l}^{\dagger}\mathbf{Z}_{k,l}$, and $\mathbf{Z}'_{k,l}$ has the same distribution as $\mathbf{Z}_{k,l}$. Note that the above operation is invertible and does not change the capacity region.

We further consider an enhanced version of the received signal at user 1:

$$\mathbf{Y}_{1,l}^{*} = \mathbf{H}_{(1,l)R}\mathbf{X}_R + \mathbf{N}_{1,l} \quad (23)$$

where $\mathbf{N}_{1,l} \sim \mathcal{CN}(0,\mathbf{K})$ and

$$\mathbf{K} = \mathbf{I}_M - \mathbf{H}_{(1,l)R}\left(\mathbf{H}_{(1,l)R}^{\dagger}\mathbf{H}_{(1,l)R}\right)^{-1}\mathbf{H}_{(1,l)R}^{\dagger} + \frac{1}{\alpha}\mathbf{H}_{(1,l)R}\mathbf{H}_{(1,l)R}^{\dagger} \quad (24)$$

and α is the maximum of all the singular values of matrices $\mathbf{H}_{(k,l)R}$. Note that using the enhanced version of $\mathbf{Y}_{1,l}$ will not reduce the capacity region since we can add artificial noise to the signal to obtain a statistically equivalent signal as the original received signal. Since we have $M_1^l \geq N$, we can let user 1 in cluster l to perform zero-forcing to obtain a stronger signal than the received signal at user k in cluster l

$$\mathbf{Y}'_{1,(k,l)} = \mathbf{X}'_{R,(k,l)} + \mathbf{N}'_{1,(k,l)} \quad (25)$$

where

$$\mathbf{N}'_{1,(k,l)} = \mathbf{V}_{k,l}^{\dagger}\left(\mathbf{H}_{(1,l)R}^{\dagger}\mathbf{H}_{(1,l)R}\right)^{-1}\mathbf{H}_{(1,l)R}^{\dagger}\mathbf{N}_{1,l}. \quad (26)$$

It is easy to see that

$$\mathbf{N}'_{1,(k,l)} \sim \mathcal{N}(0, \frac{1}{\alpha}\mathbf{I}_M) \quad (27)$$

and thus $\mathbf{Y}'_{1,(k,l)}$ is less noisy than $\mathbf{Y}'_{k,l}$, and $\mathbf{Y}_{1,l}^{*}$ is a stronger signal than all the other signals received at user $k\neq 1$ in cluster l[1].

By assumption, user k in cluster l can decode messages $\{W_{ki}^l\}$ for $i\in\{1,\cdots,K\}/\{k\}$ given the received signal $Y_{k,l}^n$ and the side information $\mathcal{W}_k^l$. We now use the following steps to derive the DoF upperbound.

- Step 1:
  - User 1 in cluster l has side information $\mathcal{W}_{j1}^l$, $j = 2,\cdots,K$.
  - User 1 in cluster l can decode messages $\{W_{1i}^l\}$ for $i = 2,\cdots,K$.
- Step 2:
  - Let a genie provide user 1 in cluster l with messages $\{W_{i2}\}$, $i = 3,\cdots,K$.
  - User 1 now has the messages $\{W_{i2}\}$, $i = 1,3,\cdots,K$, which is exactly all the side information available at user 2 in cluster l.
  - With the enhanced signal $Y_{1,l}^{*}$, user 1 in cluster l can decode all the messages intended for user 2 in cluster l, i.e., the messages $\{W_{2i}^l\}$ for $i = 3,\cdots,K$, since we can obtain a stronger signal than the received signal at user 2 in cluster l from $Y_{1,l}^{*}$.
- Step 3:

[1] Note that the above channel enhancement arguments can be simplified by providing user 1 in cluster l with all the received signals from the other users in the same cluster as suggested by the reviewer. See [17] for details.

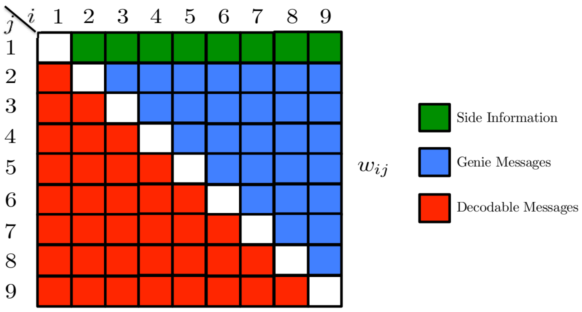

Fig. 2. Illustration for side information, genie information and decodable messages for the DoF upperbound at user 1.

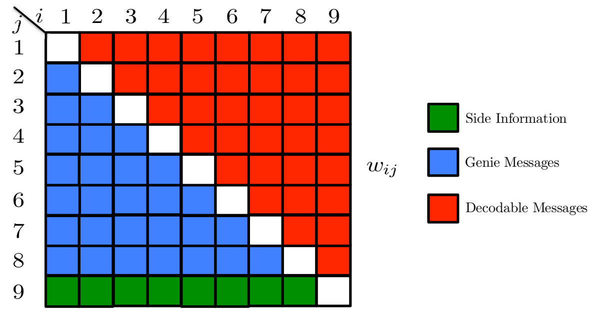

Fig. 3. Illustration for side information, genie information and decodable messages for the DoF upperbound at user K.

- Let a genie provide user 1 in cluster l with messages $\{W_{i3}\}$, $i = 4, \cdots, K$.
- User 1 now has the messages $\{W_{i3}\}$, $i = 1, 2, 4, \cdots, K$, which is exactly all the side information available at user 3 in cluster l.
- With the enhanced signal $Y_{1,l}^*$, user 1 in cluster l can decode the messages $\left\{W_{3i}^l\right\}$ for $i = 4, \cdots, K$.

• Proceed in the same fashion, for Step k:
  - Let a genie provide user 1 in cluster l with messages $\{W_{ik}\}$, $i = k+1, \cdots, K$.
  - User 1 now has the messages $\{W_{ik}\}$, $i \in \{1, \cdots, K\}/\{k\}$, which is exactly all the side information available at user k in cluster l.
  - With the enhanced signal $Y_{1,l}^*$, user 1 in cluster l can decode the messages $\left\{W_{ki}^l\right\}$ for $i = k+1, \cdots, K$.

• Step $K-1$:
  - Let a genie provide user 1 in cluster l with message $W_{K,K-1}^l$.
  - User 1 now has the messages $\{W_{i,K-1}\}$, $i = 1, \cdots, K-2, K$, which is exactly all the side information available at user $K-1$ in cluster l.
  - User 1 in cluster l can decode the message $W_{K-1,K}$.

Based on the above arguments, user 1 in cluster l can decode the messages $\left\{W_{ki}^l\right\}$ for $k = 1, \cdots, K-1$, $i = k+1, \cdots, K$, which is illustrated in Fig. 2 for $K = 9$. We can see that half of all the messages can be decoded at user 1 in cluster l based on the received signal $Y_{1,1}^n$, the side information $\mathcal{W}_1^l$, and the genie information $\{W_{ik}^l\}$, for $k = 2, \cdots, K-1$, $i = k+1, \cdots, K$.

We define $\mathcal{W}_d^l$ as the set of messages $\left\{W_{ki}^l\right\}$ for $k = 1, \cdots, K-1$, $i = k+1, \cdots, K$ for cluster l, which are messages that can be decoded by user 1 in cluster l, $\mathcal{W}^l$ as all the messages from cluster l and $\mathcal{W}_d^{lc}$ to denote the set of messages $\mathcal{W}^l/\mathcal{W}_d^l$. We can then bound the rate of these decodable messages as follow:

$$n \sum_{l=1}^{L} \sum_{k=1}^{K-1} \sum_{i=k+1}^{K} R_{ki}^l \tag{28}$$

$$= H(\mathcal{W}_d^1, \cdots, \mathcal{W}_d^L | \mathcal{W}_d^{1c}, \cdots, \mathcal{W}_d^{Lc}) \tag{29}$$

$$= I(\mathcal{W}_d^1, \cdots, \mathcal{W}_d^L; Y_{1,1}^{*n}, \cdots, Y_{1,L}^{*n} | \mathcal{W}_d^{1c}, \cdots, \mathcal{W}_d^{Lc}) + n\epsilon_n$$

$$\leq H(Y_{1,1}^{*n}, \cdots, Y_{1,L}^{*n}) - H(Y_{1,1}^{*n}, \cdots, Y_{1,L}^{*n} | X_R^n) + n\epsilon_n \tag{30}$$

$$= I(X_R^n; Y_{1,1}^{*n}, \cdots, Y_{1,L}^{*n}) \tag{31}$$

where equation (30) is because conditioning reduces entropy and the received signals at the users only depend on the transmitted signal from the relay.

From equation (31), we can see that

$$\lim_{SNR \to \infty} \frac{\sum_{l=1}^{L} \sum_{k=1}^{K-1} \sum_{i=k+1}^{K} R_{ki}^l}{\log SNR} \leq N \tag{32}$$

We now have an upperbound for half of the messages from all users. We can bound the DoF for the rest of the messages by enhancing the received signal of user K and provide genie information to user K in a similar fashion, as illustrated in Fig. 3, which yields

$$\lim_{SNR \to \infty} \frac{\sum_{l=1}^{L} \sum_{k=2}^{K} \sum_{i=1}^{k-1} R_{ki}^l}{\log SNR} \leq N \tag{33}$$

Based on equations (32) and (33), we have $DoF \leq 2N$. ■

A. Optimality of Achievable DoF for Special Cases

We can now evaluate the optimality of the achievable DoF for the special cases of the MIMO multi-way relay channel we have reviewed in Section I using the newly derived DoF upperbound.

• *MIMO K-user Y channel [10]*: The DoF upper bound for this case is

$$DoF \leq \min\{\sum_{i=1}^{K} M_i, 2N\}. \tag{34}$$

If we have $M_i \geq d(K-1)$ and fix $N = \frac{dK(K-1)}{2}$, the DoF upperbound becomes

$$DoF \leq dK(K-1). \tag{35}$$

If we further have $M_i > \frac{K^2-K+2}{4}$, the condition $N < \min\{M_i + M_j - d | \forall i \neq j\}$ is also satisfied, and the DoF upper bound implies that $dK(K-1)$ is indeed the optimal DoF.

• *MIMO K-pair two-way relay channel [11]*: For this case, the DoF upperbound becomes

$$DoF \leq \min\{2KM, 2Kd\}. \tag{36}$$

When $K \leq \frac{2M}{d} - 1$, we have $2Kd \leq 2KM$ for $K \geq 2$. The DoF upperbound $2Kd$ is thus achievable and is the optimal DoF.

IV. Optimal DoF for L-Cluster K-User MIMO multi-way Relay Channel

In this section, we study the L-cluster K-user MIMO multi-way relay channel in the symmetric setting, i.e., all the users have the same number of antennas.

Theorem 2: For the symmetric L-cluster K-user MIMO multi-way relay channel, where all users have M antennas and the relay has N antennas, the optimal DoF is

$$DoF^* = KLM \quad \text{if } N \geq KLM, \tag{37}$$

$$DoF^* = 2N \quad \text{if } L\binom{K}{2}(2M - N) \geq N. \tag{38}$$

Proof: For this case, the DoF upperbound in equation (16) becomes

$$DoF \leq \min\{KLM, 2N\}. \tag{39}$$

When $2N > KLM$, the DoF upperbound becomes KLM. The DoF upper bound can be achieved when $N \geq KLM$. Under this condition, the relay can decode all the messages from all the users and can broadcast the messages to the intended users using zero-forcing.

When $2N \leq KLM$, the DoF upperbound becomes $2N$. To achieve this upperbound, we let each signal dimension at the relay to be shared by a pair of users. From the result in [9], any pair of users can share $2M - N$ dimensional signal space at the relay, if $2M \geq N$. Therefore, we need

$$L\binom{K}{2}(2M - N) \geq N, \tag{40}$$

such that all the signal dimension at the relay can be shared by a pair of users. We can choose any pair of users to exchange data streams without exceeding their maximum allowed dimension of shared signal space $2M - N$. We let the users exchange N pairs of data streams, and the relay can decode the sum of each pair of the data streams and broadcast to the users with proper receiver-side processing. The detailed scheme is omitted due to space limits. ■

Remark 1: This is the first DoF result for the general MIMO multi-way relay channel. We can see that the DoF is always limited by the available spatial dimensions at the relay. With fixed number of antennas at the relay, increasing the number of users and the number of clusters cannot provide any DoF gain. In the meantime, the DoF upperbound $2N$ also provides us insights regarding how the resources of the relay can be utilized: the optimal way to utilize the resources of the relay is to share the relay between two users. We cannot obtain DoF gain by letting three or more users to share the resources of the relay.

V. Conclusion

In this paper, we have investigated the DoF for the L-cluster K-user MIMO multi-way relay channel and established the optimal DoF for several scenarios of interests. We have derived a new DoF upperbound using genie-aided approach and channel enhancement, which is shown to be tight for several scenarios of interests. We have also studied the L-cluster K-user MIMO multi-way relay channel with equal number of antennas at the users, and established the optimal DoF. The DoF results imply that the DoF of the MIMO multi-way relay channel is always limited by the spatial dimensions available at the relay. With fixed number of antennas at the relay, increasing the number of users and clusters cannot provide any DoF gain. The results also imply that allowing three or more users to share the resources of the relay cannot provide any DoF gain.

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