Sum Capacity of the Deterministic Interference Channel with an Out-of-Band Half-Duplex Relay

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***Abstract*—We consider the linear deterministic model for the two user interference channel (IC) with an out-of-band relay (OBR). In this model, each user has access to two orthogonal bands, where one band forms the IC, and the other band is assisted by a half-duplex relay, i.e., the relay receives and transmits in orthogonal bands. The channel is assumed to be symmetric. We first derive new outerbounds using genie arguments, and then construct optimal relaying strategies. As a result, we characterize the sum capacity of this model for all channel parameters. In particular, it is shown that similar to the case of the IC with output feedback (and without a relay), the "W" curve for the sum capacity of the IC becomes "V" curve as the strength of the links in the OBRC grows. The interference links are classified as extremely strong, very strong, strong, moderate, weak, and very weak. For the IC without the relay, it is known that some signal spaces are left unused for the sum-capacity-optimal transmission strategy. We show that, with an OBR, these spaces can be utilized to achieve the sum capacity of this model improving upon that of the IC without the OBR. We show that for very strong and extremely strong interference, the interference is useful to improve the achievable rates. For weak or very weak interference, the unused signal spaces of the IC can be utilized to transmit new information bits.**

I. Introduction

Broadcast and superposition are two distinct features of the wireless medium. Interference between different wireless devices is an inevitable outcome of these two features, and is a crucial factor that limits the capacity of the wireless networks. Interference channel (IC), which consists of two source-destination pairs, is the simplest model that characterizes the effect of interference in wireless networks, and thus is a basic building block for wireless ad hoc networks. References [1]–[7] studied the IC from an information theoretical perspective to understand the fundamental effect of interference. These works established the capacity for the IC when the interference is either strong or weak. However, for the general case, the capacity is still open.

Relay channel (RC) is another important building block for future wireless networks. It is shown that the relay can cooperate with the source to increase the transmission rate of a point to point channel [8]–[10]. Recent efforts [11]–[21] introduce a relay node in the IC setting, resulting in a new fundamental model termed the interference relay channel (IFRC). In the IFRC, the relay can perform (i) *signal relaying* [14], [18] as the traditional relay channel, (ii) *compute-and-forward* [18], [19], [22] or (iii) *interference forwarding* [11]–[13]. All the schemes can increase the achievable (sum) rate of the IC under different channel conditions.

References [20], [21] derived sum rate upperbounds, which complement each other, for the Gaussian interference relay channel (GIFRC). The capacity of IFRC is only known for special cases [11], [20]. For the general IFRC, the capacity is still open, since it inherits the challenges of both IC and RC, with increased signal interaction. To simplify the channel model and understand the fundamental effect of signal relaying and interference forwarding, references [16], [17] proposed a model where the relay operates in bands orthogonal to the underlying IC, which is called the interference channel with an out-of-band relay (IC-OBR). Reference [16] considered the case when the links associated with the relay are orthogonal to each other, and obtained capacity results for some channel configurations. Reference [17] considered the case when the incoming links of the half-duplex relay are orthogonal to the outgoing links, and provided sum capacity with special channel conditions.

In this work, we consider the IC-OBR as in [17], where the relay operates in a band orthogonal to the IC, and the relay is half-duplex in the sense that the incoming links are orthogonal to the outgoing links. To gain a clear insight into the signal interaction in this model, we study the deterministic model in symmetric case using the approach developed in [23]. The deterministic model allows us to focus on the interaction of the signals by eliminating the noise at the receiver. First proposed in [23], this approach is further utilized in [24]–[27] to obtain approximate capacity results for various channel models.

Our main result is that, for the symmetric deterministic IC-OBR, we characterize the sum capacity for all channel configurations. We observe that the presence of the OBR impacts the capacity in a manner similar to that ovserved for the presence of output feedback for the IC, i.e., the "W" curve for the sum capacity becomes "V" curve as the strength of the links in the OBRC increases. The essence lies in that the signal space resources can be better utilized in the presence of the relay. For the converse, we derive outerbounds via the aid of judiciously designed genie. For achievability, we first observe that for the sum capacity optimal

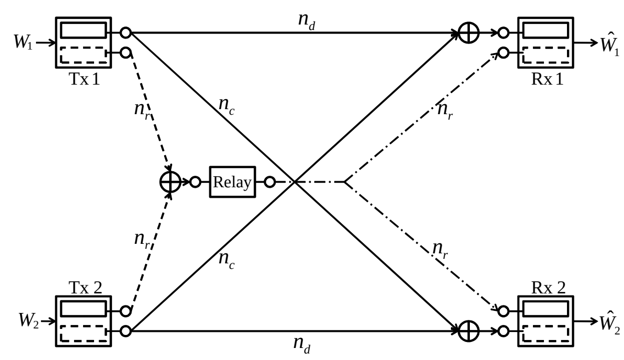

Fig. 1. Deterministic interference channel with an out-of-band relay.

transmission strategies for the deterministic IC, some signal spaces are left unused to avoid interference. Using the out-of-band relay (OBR), we show that these signal spaces can be utilized. We further classify the interference as *extremely strong*, *very strong*, *strong*, *moderate*, *weak* and *very weak*. When the interference is strong and moderate, it is optimal for the sources to transmit independent messages through the IC and the OBRC, that is, separate encoding is optimal. When the interference is very strong or extremely strong, the interference links can carry additional information to facilitate the transmission through the OBRC. When the interference is weak or very weak, the unused signal spaces of the IC can be utilized to transmit new information to the destinations, while the OBRC is used to facilitate interference cancelation. Overall, for all possible cases, we show that the achievable sum rates match the outerbounds. We further show that, in fact, the full capacity region can be characterized when the interference is strong.

The remainder of the paper is organized as follows: Section II describes the channel model. Section III derives the outerbounds based on genie-aided approaches. Section IV describes the achievable schemes and states the sum capacity results. Section V concludes the paper.

II. System model

A. The deterministic symmetric interference channel with an out-of-band relay (IC-OBR)

The deterministic IC-OBR is shown in Figure 1, which consists of a two-user interference channel (IC) and a relay operating in orthogonal bands, called an out-of-band relay (OBR). The OBR is constrained to be half-duplex and thus it uses part of the bands to receive signals and part of the bands to transmit signals. All the transmitters[1] and receivers share a common band which forms the interference channel. In order to utilize the relay for cooperation, the transmitters have access to the incoming band of the relay and receivers have access to the outgoing band of the relay. For simplicity, we consider the symmetric case, where for the interference channel, the gain of the direct link is n_d and the gain of the interfering link is n_c. The gain of the links associated with the relay is n_r. n_d, n_c, n_r are integers.

[1]We will use transmitter and source, receiver and destination interchangeably throughout the paper.

Let $w_1 \in \{1, 2, \ldots, 2^{nR_1}\}$, $w_2 \in \{1, 2, \ldots, 2^{nR_2}\}$ denote the messages of the two sources. Each transmitter uses an encoding function $\mathbf{x}_i : w_i \to \mathbb{F}_2^q \times \mathbb{F}_2^q$ $(i = 1, 2)$ with $q = \max\{n_d, n_c, n_r\}$, to generate codewords $\mathbf{x}_i^n(w_i) = [\mathbf{x}_{i1}^n, \mathbf{x}_{i2}^{\alpha n}]$, where α is the duplexing factor, $\mathbf{x}_{ik}^n = \{\mathbf{x}_{ik}[1], \cdots, \mathbf{x}_{ik}[n]\}$, $\mathbf{x}_{ik} = [x_{ik,1}, x_{ik,2}, \ldots, x_{ik,q}]^T$ $(k = 1, 2)$, and $x_{ik,m} \in \mathbb{F}_2$ $(m = 1, 2, \ldots, q)$. The OBR sends $\mathbf{x}_r^{\bar{\alpha} n}$ $(\bar{\alpha} = 1 - \alpha)$ to the destinations using the outgoing bands. The signal $\mathbf{x}_r^{\bar{\alpha} n}$ is generated based on the signals $\mathbf{y}_r^{\alpha n}$ received from the incoming bands of the OBR, where $\mathbf{x}_r \in \mathbb{F}_2^q$.

The signal interaction in the deterministic model can be characterized by a series of "add" operations in $\mathbb{F}_2$, and "shift" operations defined by the $q \times q$ shifting matrix $\mathbf{S}$, where

$$\mathbf{S} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{pmatrix} \tag{1}$$

The outputs of the channel can be characterized as the following: For all $t = \{1, 2, \ldots, n\}$

$$\mathbf{y}_{11}[t] = \mathbf{S}^{q-n_d}\mathbf{x}_{11}[t] \oplus \mathbf{S}^{q-n_c}\mathbf{x}_{21}[t] \tag{2}$$

$$\mathbf{y}_{21}[t] = \mathbf{S}^{q-n_c}\mathbf{x}_{11}[t] \oplus \mathbf{S}^{q-n_d}\mathbf{x}_{21}[t] \tag{3}$$

For $t = \{1, 2, \ldots, \alpha n\}$

$$\mathbf{y}_r[t] = \mathbf{S}^{q-n_r}\mathbf{x}_{12}[t] \oplus \mathbf{S}^{q-n_r}\mathbf{x}_{22}[t] \tag{4}$$

For $t = \{\alpha n + 1, \ldots, n\}$

$$\mathbf{y}_{12}[t] = \mathbf{S}^{q-n_r}\mathbf{x}_r[t] \tag{5}$$

$$\mathbf{y}_{22}[t] = \mathbf{S}^{q-n_r}\mathbf{x}_r[t] \tag{6}$$

III. Outerbounds for the Deterministic Symmetric Interference Channel with an Out-of-Band Relay

In this section, we derive outerbounds for the deterministic symmetric IC-OBR using the genie-aided approach. It is easy to show that the optimal duplexing factor is $\frac{1}{2}$ since the channel is symmetric. Due to the orthogonality between IC and OBRC, we assume for simplicity that $\mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{y}_{11}, \mathbf{y}_{21}$ are length $\max\{n_d, n_c\}$ vectors, while $\mathbf{x}_{12}, \mathbf{x}_{22}, \mathbf{y}_r, \mathbf{y}_{12}, \mathbf{y}_{22}$ are length n_r vectors.

Proposition 1: The capacity region of the deterministic symmetric interference channel with an out-of-band half-duplex relay is contained in the region $\mathcal{R} = (R_1, R_2)$ specified by the following rate expressions.

$$R_1 \le n_d + \frac{1}{2}n_r \tag{7}$$

$$R_2 \le n_d + \frac{1}{2}n_r \tag{8}$$

$$R_1 + R_2 \le n_c + \frac{1}{2} n_r, \text{when } n_c \ge n_d \quad (9)$$

$$R_1 + R_2 \le \min\{n_r + 2\max\{n_d - n_c, n_c\}, \\ 2n_d - n_c + \frac{1}{2} n_r\}, \text{when } n_d > n_c \quad (10)$$

Proof: The bounds (7) and (8) can be derived from the cut set bound. We next prove (9)-(10).

When $n_c \ge n_d$, we have

$$\begin{aligned}
&2n(R_1 + R_2) \\
&= H(W_1) + H(W_2) && (11)\\
&= I(W_1; \mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}) + H(W_1|\mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}) \\
&\quad + I(W_2; \mathbf{y}_{21}^{2n}, \mathbf{y}_{22}^{n}) + H(W_2|\mathbf{y}_{21}^{2n}, \mathbf{y}_{22}^{n}) && (12)\\
&\le I(\mathbf{x}_{11}^{2n}, \mathbf{x}_{12}^{n}; \mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}) + I(\mathbf{x}_{21}^{2n}, \mathbf{x}_{22}^{n}, \mathbf{x}_{r}^{n}; \mathbf{y}_{21}^{2n}, \mathbf{y}_{22}^{n}|\mathbf{x}_{11}^{2n}, \mathbf{x}_{12}^{n}) \\
&\quad + 2n\epsilon_1 + 2n\epsilon_2 && (13)\\
&= H(\mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}) - H(\mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}|\mathbf{x}_{11}^{2n}, \mathbf{x}_{12}^{n}) \\
&\quad + H(\mathbf{y}_{21}^{2n}, \mathbf{y}_{22}^{n}|\mathbf{x}_{11}^{2n}, \mathbf{x}_{12}^{n}) && (14)\\
&= H(\mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}) - H(\mathbf{S}^{q-n_c}\mathbf{x}_{21}, \mathbf{y}_{12}^{n}|\mathbf{x}_{11}^{2n}, \mathbf{x}_{12}^{n}) \\
&\quad + H(\mathbf{S}^{q-n_d}\mathbf{x}_{21}, \mathbf{y}_{22}^{n}|\mathbf{x}_{11}^{2n}, \mathbf{x}_{12}^{n}) && (15)\\
&\le H(\mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}) && (16)\\
&\le 2n \cdot n_c + n \cdot n_r && (17)
\end{aligned}$$

where (16) follows because the channel is symmetric and $n_c \ge n_d$. We can write the sum rate upperbound as

$$R_1 + R_2 \le n_c + \frac{1}{2} n_r \quad (18)$$

that is (9).

When $n_d > n_c$,

$$\begin{aligned}
&2n(R_1 + R_2) && (19)\\
&= H(W_1) + H(W_2) && (20)\\
&\le I(\mathbf{x}_{11}^{2n}, \mathbf{x}_{12}^{n}; \mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}, \mathbf{v}_{11}^{2n}) + I(\mathbf{x}_{21}^{2n}, \mathbf{x}_{22}^{n}; \mathbf{y}_{21}^{2n}, \mathbf{y}_{22}^{n}, \mathbf{v}_{21}^{2n}) \\
&= H(\mathbf{v}_{11}^{2n}) + H(\mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}|\mathbf{v}_{11}^{2n}) - H(\mathbf{v}_{21}^{2n}, \mathbf{y}_{12}^{n}|\mathbf{x}_{11}^{2n}, \mathbf{x}_{12}^{n}) \\
&\quad + H(\mathbf{v}_{21}^{2n}) + H(\mathbf{y}_{21}^{2n}, \mathbf{y}_{22}^{n}|\mathbf{v}_{21}^{2n}) - H(\mathbf{v}_{11}^{2n}, \mathbf{y}_{22}^{n}|\mathbf{x}_{21}^{2n}, \mathbf{x}_{22}^{n}) \\
&\le H(\mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}|\mathbf{v}_{11}^{2n}) + H(\mathbf{y}_{21}^{2n}, \mathbf{y}_{22}^{n}|\mathbf{v}_{21}^{2n}) && (21)\\
&\le 2n \cdot n_r + 4n \max\{n_d - n_c, n_c\} && (22)
\end{aligned}$$

where $\mathbf{v}_{11} = \mathbf{S}^{q-n_c}\mathbf{x}_{11}$, $\mathbf{v}_{21} = \mathbf{S}^{q-n_c}\mathbf{x}_{21}$ are the genie information we give to the decoders. The sum rate upperbound can be written as

$$R_1 + R_2 \le n_r + 2\max\{n_d - n_c, n_c\} \quad (23)$$

Alternatively, we can also bound the sum rate as follows [26].

$$\begin{aligned}
&2n(R_1 + R_2) \\
&= H(W_1) + H(W_2) && (24)\\
&= H(W_1, W_2) && (25)\\
&= I(W_1, W_2; \mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}, \mathbf{u}_{21}^{2n}) + H(W_1, W_2|\mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}, \mathbf{u}_{21}^{2n}) \\
&\le I(\mathbf{x}_{11}^{2n}, \mathbf{x}_{12}^{n}, \mathbf{x}_{21}^{2n}, \mathbf{x}_{22}^{n}, \mathbf{x}_{r}^{n}; \mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}, \mathbf{u}_{21}^{2n}) \\
&\quad + H(W_1|\mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}) + H(W_2|\mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}, \mathbf{u}_{21}^{2n}, W_1) && (26)\\
&\le I(\mathbf{x}_{11}^{2n}, \mathbf{x}_{12}^{n}, \mathbf{x}_{21}^{2n}, \mathbf{x}_{22}^{n}, \mathbf{x}_{r}^{n}; \mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}, \mathbf{u}_{21}^{2n}) + 2n\epsilon_1 \\
&\quad + H(W_2|\mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}, \mathbf{u}_{21}^{2n}, W_1, \mathbf{x}_{11}^{2n}, \mathbf{x}_{12}^{n}) && (27)\\
&\le I(\mathbf{x}_{11}^{2n}, \mathbf{x}_{12}^{n}, \mathbf{x}_{21}^{2n}, \mathbf{x}_{22}^{n}, \mathbf{x}_{r}^{n}; \mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}, \mathbf{u}_{21}^{2n}) + 2n\epsilon_1 \\
&\quad + H(W_2|\mathbf{y}_{21}^{2n}, \mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}, \mathbf{u}_{21}^{2n}, W_1, \mathbf{x}_{11}^{2n}, \mathbf{x}_{12}^{n}) && (28)\\
&\le I(\mathbf{x}_{11}^{2n}, \mathbf{x}_{12}^{n}, \mathbf{x}_{21}^{2n}, \mathbf{x}_{22}^{n}, \mathbf{x}_{r}^{n}; \mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}, \mathbf{u}_{21}^{2n}) \\
&\quad + 2n\epsilon_1 + H(W_2|\mathbf{y}_{21}^{2n}, \mathbf{y}_{22}^{n}) && (29)\\
&\le H(\mathbf{y}_{11}^{2n}, \mathbf{y}_{12}^{n}, \mathbf{u}_{21}^{2n}) + 2n\epsilon_1 + 2n\epsilon_2 && (30)\\
&\le 2n \cdot n_d + n \cdot n_r + 2n \cdot (n_d - n_c) && (31)
\end{aligned}$$

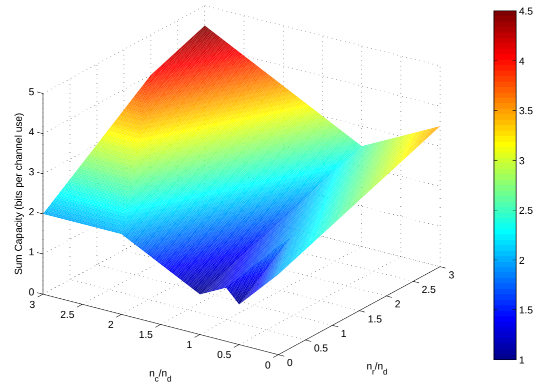

Fig. 2. Sum capacity for the linear deterministic model.

where $\mathbf{u}_{21} = [\mathbf{S}^{q-n_d}\mathbf{x}_{21}]^{\uparrow n_c}$ is the genie information we give to the decoder 1, and $\mathbf{x}^{\uparrow n_c}$ denotes the operation of removing the first n_c elements of the vector $\mathbf{x}$. The step (27) is because $\mathbf{x}_{11}^{2n}$ and $\mathbf{x}_{12}^{n}$ are functions of W_1, and step (28) is because given $\mathbf{y}_{11}^{2n}, \mathbf{u}_{21}^{2n}, \mathbf{x}_{11}^{2n}$, we can recover $\mathbf{y}_{21}^{2n}$, and $\mathbf{y}_{22}^{n} = \mathbf{y}_{12}^{n}$. The sum rate upperbound is

$$R_1 + R_2 \le 2n_d - n_c + \frac{1}{2} n_r \quad (32)$$

(23) and (32) yield (10). ■

IV. Sum Rate Optimal Transmission Strategies

In this section, we demonstrate how to construct the sum rate optimal transmission strategies, and show that the achievable rates match the outerbounds derived in section III.

Theorem 1: For the deterministic symmetric interference channel with an out-of-band half-duplex relay, the sum capacity is

when $n_c \ge 2n_d + \frac{1}{2} n_r$

$$R_1 + R_2 \le 2n_d + n_r \quad (33)$$

when $2n_d + \frac{1}{2} n_r > n_c \ge n_d$

$$R_1 + R_2 \le n_c + \frac{1}{2} n_r \quad (34)$$

when $n_d > n_c \ge \frac{2}{3} n_d$

$$R_1 + R_2 \le 2n_d - n_c + \frac{1}{2} n_r \quad (35)$$

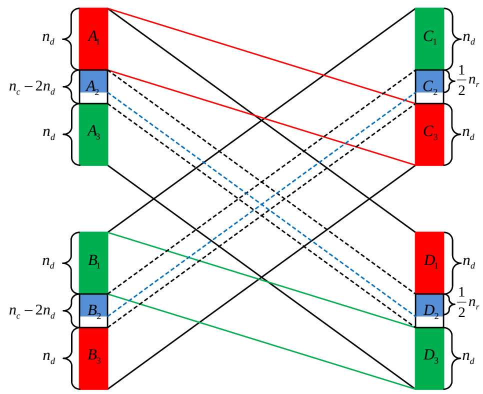

Fig. 3. Signal interaction for the underlying IC when $n_c \geq 2n_d + \frac{1}{2}n_r$.

when $\frac{2}{3}n_d > n_c \geq \frac{1}{2}n_d$

$$R_1 + R_2 \leq \min\{2n_c + n_r, 2n_d - n_c + \frac{1}{2}n_r\} \tag{36}$$

when $\frac{1}{2}n_d > n_c$

$$R_1 + R_2 \leq \min\{2(n_d - n_c) + n_r, 2n_d - n_c + \frac{1}{2}n_r\} \tag{37}$$

Figure 2 shows how the sum capacity scales with the ratio $\frac{n_c}{n_d}$ and the ratio $\frac{n_r}{n_d}$. We can see that when $\frac{n_r}{n_d}$ is small, the sum capacity has a "W" shape as the IC [5]. However, as $\frac{n_r}{n_d}$ grows, the "W" curve gradually turns into a "V" curve. This effect is similar to the IC with output feedback, observed in [24]. We note that, the sum capacity is unbounded as $\frac{n_c}{n_d} \to \infty$ and $\frac{n_r}{n_d} \to \infty$. This thanks to the OBRC utilizing the available signal resources in an efficient manner, as we explain later in the achievable strategies.

Based on the strength of n_d, n_r and n_c, we show the achievability of the above rates for the following cases[2]. In particular, for the out-of-band half-duplex relay, we consider a two stage transmission scheme with duplexing factor $\alpha = \frac{1}{2}$.

A. Case 1: $n_c \geq 2n_d + \frac{1}{2}n_r$

For this case, the interference links are extremely strong. The signal interaction is shown in Figure 3. Without the OBR, each source can only send information bits using the signal spaces which are visible to its intended receiver, e.g., spaces A_1 and B_1. The other signal spaces, e.g., A_2, A_3 and B_2, B_3, are unused, since the signals sent from these spaces are only visible to the other destination. With the OBR, part of the unused signal spaces can be utilized to facilitate interference cancelation. Specifically, the sources transmit n_r new information bits using the signal spaces of the OBRC *in common*. The n_r signal bits received at the OBR are the sum of the signal bits sent from the sources in $\mathbb{F}_2$, which means the signals sent from two sources interfere with each

[2]In each case we describe the achievability scheme in accordance with space limitations of the paper. Further details of the transmission strategies can be found in our upcoming journal submission.

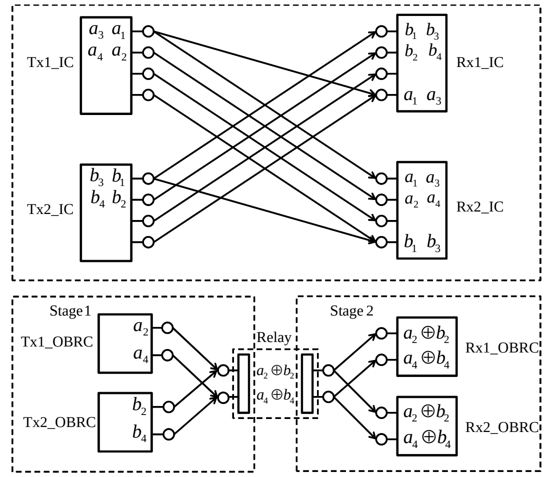

Fig. 4. Transmission scheme when $n_c = 4$, $n_d = 1$, $n_r = 2$.

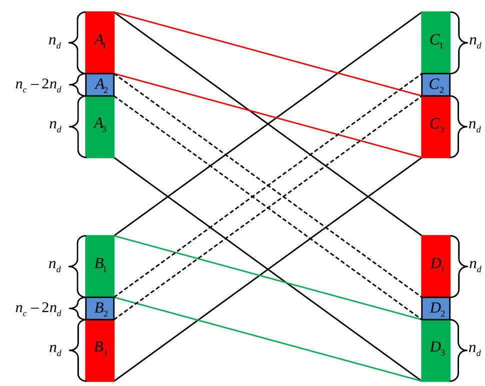

Fig. 5. Signal interaction for the underlying IC when $2n_d + \frac{1}{2}n_r > n_c \geq 2n_d$.

other in the OBRC. For the IC, since $n_c \geq 2n_d + \frac{1}{2}n_r$, $\frac{1}{2}n_r$ bits of the spaces A_2 and B_2 are visible to the other destination without corrupting other signal bits for each stage. Using these signal spaces, the sources can send the information bits transmitted into the OBRC to the destinations. Thus for the two stage transmission, each destination receives n_r clean interfering signal bits. These signal bits can be used to cancel the interference in the signal received from the OBRC. The detailed transmission scheme is omitted.

This scheme achieves the rate pair $(R_1, R_2) = (n_d + \frac{1}{2}n_r, n_d + \frac{1}{2}n_r)$, which is exactly the cut set bound for the individual rates, and thus the capacity region for this scenario is characterized. For illustration, an example of the transmission scheme is shown in Figure 4.

B. Case 2: $2n_d + \frac{1}{2}n_r > n_c \geq 2n_d$

For this case, the interference links are very strong. The signal interaction for this case is shown in Figure 5. Similar to the case in section IV-A, without the OBR, each source only transmits information bits using spaces A_1 and B_1. With the OBR, however, since $2n_d + \frac{1}{2}n_r > n_c \geq 2n_d$, the sources can use $2(n_c - 2n_d)$ bits of the OBR in common to transmit new information. The rest $n_r - 2(n_c - 2n_d)$ bits of the OBR can only be used by one source, or divided between two sources to transmit new information. Those commonly used signal bits of the OBR are corrupted by interference, and the $n_c - 2n_d$ signal bits in spaces A_2 or B_2 are used to cancel the interference. The detailed transmission strategy is omitted. The sum rate achieved is $n_c + \frac{1}{2}n_r$ bits per channel use, which is exactly the sum capacity of this channel according to the upperbound (9). From the cut set bound for individual rates, we can see that this scheme also achieves the corner points of the capacity region, and thus we can fully characterize the capacity region for this case.

Remark 1: For the cases discussed above, we have considered very strong, or extremely strong interference. The key idea is to let the sources transmit new information bits using the signal spaces of OBRC in common, while the strong interference links can provide some side information to the destinations to facilitate interference cancelation. To transmit new information bits through the OBRC, the sources can split the signal spaces of the OBRC, or use the signal spaces of the OBRC in common. The first approach does not incur interference at the destinations, while the second approach causes corruption of the transmitted signal bits. However, the second approach is more beneficial since one bit of the OBRC can help each source transmit one bit, that is, we can trade one bit of the OBRC for the transmission of two information bits. For the first approach, we can only trade one bit of the OBRC for the transmission of one information bit. The interference links are critical for the destinations to recover the intended information bits from the OBRC when the sources use the second approach. For the case of extremely strong interference, the interference links carry a large amount of side information. Therefore the sources use the second approach to transmit all the new information bits through the OBRC, such that the side information transmitted through the interference link can be utilized to the fullest extent. For the case of very strong interference, the interference links carry less side information, and thus only parts of the signal spaces of the OBRC are used with the second approach to fully utilize the interference links. The rest signal spaces of the OBRC are utilized with the first approach. The optimality of our achievable strategy shows that when using the resources of the OBRC under extremely strong and very strong interference, we should first consider making use of the side information transmitted through the interference links, since this provides the largest payoff. For the following cases when interference is strong or moderate, we will adopt a different approach to construct the optimal transmission strategies.

C. *Case 3:* $2n_d > n_c \geq n_d$

For this case, the interference links are strong. The signal interaction is shown in Figure 6. Without the OBR, to achieve the sum capacity of the IC, source 1 transmits n_d information bits using the signal space A_1, A_2, while source 2 transmits $n_c - n_d$ information bits using all the $2n_d - n_c$ bits in signal

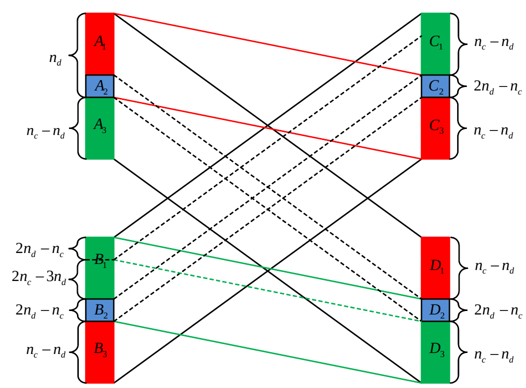

Fig. 6. Signal interaction for the underlying IC when $2n_d > n_c \geq n_d$.

space B_2 and the lower $2n_c - 3n_d$ bits in signal space B_1, as shown in figure 6. The $2n_d - n_c$ bits in signal space B_2 cause interference at the signal space C_2 at destination 1. Source 2 use the higher level $2n_d - n_c$ bits in signal space B_1 to transmit another copy of the signal bits in space B_2. The higher level $2n_d - n_c$ bits in signal space B_1 are visible to destination 1 without corrupting other signals. Destination 1 can thus remove the interference and obtain a clean signal. This way, the sum capacity n_c bits can be achieved. Different from the previous cases in section IV-A,IV-B, with the scheme that achieves sum capacity of the IC, there is no additional signal space available at the sources that does not cause interference at the destinations. Therefore the sources cannot use the signal spaces of the OBRC in common to transmit new information bits. The signal spaces of the OBRC can only be used by one source, or divided between two sources. Since there are n_r bits available at the OBRC, the sum rate achieved in two stages is $n_c + \frac{1}{2}n_r$ bits per channel use. Based on the outerbound (9), when $n_c \geq n_d$, this is exactly the sum capacity. According to the cut set bound for individual rate, the corner points $(n_d + \frac{1}{2}n_r, n_c - n_d)$ and $(n_c - n_d, n_d + \frac{1}{2}n_r,)$ can also be achieved. Thus, full capacity region, for this case, is characterized.

D. *Case 4:* $n_d > n_c \geq \frac{2}{3}n_d$

For this case, the interference is moderate. The signal interaction for the IC is shown in Figure 7. Without the OBR, it is known that the sum capacity for this case is $R_1 + R_2 = 2n_d - n_c$ [25]. Similar to the case in the previous section (section IV-C), for the sum capacity optimal strategy for the IC, there is no additional signal space available at the sources that does not cause interference at the destinations. The signal spaces of the OBR can be used by only one source or divided between two sources to transmit n_r new information bits in two stages. The sum rate achieved by this scheme is $R_1 + R_2 = 2n_d - n_c + \frac{1}{2}n_r$ bits per channel use, which matches the outerbound in (10). Thus the sum capacity is characterized.

Remark 2: It is easy to verify that the individual rate $n_d + \frac{1}{2}n_r$ of the cut set bound can be achieved by allowing only

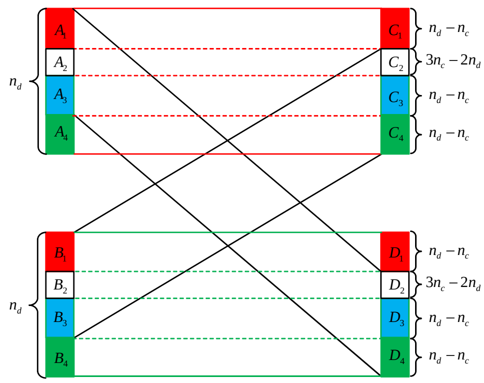

Fig. 7. Signal interaction for the underlying IC for the case $n_d > n_c \geq \frac{2}{3}n_d$.

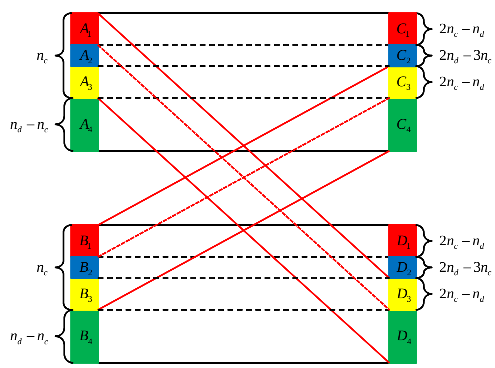

Fig. 8. Signal interaction for the underlying IC when $\frac{2}{3}n_d > n_c \geq \frac{1}{2}n_d$

one user to use the channel. However, the maximum rate of the other user is 0. The sum rate for this case is less than the sum capacity derived above. The reason is that there may exist another bound of the form $2R_1 + R_2$ which is active in this case.

For the cases described in section IV-C, IV-D, the sources cannot use the signal spaces of the OBRC in common to transmit new information, since no signal space of the IC can be used to cancel the interference. The signal spaces of the OBRC can be used by only one source, or divided between two sources. For the following cases when the interference links are weaker, we will adopt another approach to construct the transmission strategy.

E. *Case 5:* $\frac{2}{3}n_d > n_c \geq \frac{1}{2}n_d$

For this case, the interference is weak. The signal interaction in the IC is shown in Figure 8. The signal bits from $A_1, A_2, A_3, B_1, B_2, B_3$ are all common information bits, and the signal bits from A_4, B_4 are private information bits. Without the relay, the sum rate optimal transmission strategy for the IC is to use the signal spaces A_1 and B_1 to transmit common information, which is to be decoded at both destinations, and use the signal spaces A_4 and B_4 to transmit private information, which is only to be decoded at the intended destinations. The condition $\frac{2}{3}n_d > n_c \geq \frac{1}{2}n_d$ guarantees that the signal bits from signal spaces A_1, A_4, B_1, B_4 are aligned at the receivers such that they do not interfere with each other. The rest signal spaces A_2, A_3, B_2, B_3 are left unused, since the information bits transmitted using these signal spaces cause interference at the receivers. We will show that, with the OBR, the interference caused by using spaces A_2, B_2 can be removed, and the sum capacity can be achieved. However, the extent to which we can use the signal spaces A_2, B_2 depends on the strength of the links in the OBRC. We consider the following subcases:

1) $n_r \geq 4n_d - 6n_c$*:* For this case, the sources can use all the signal bits in spaces A_2, B_2, in addition to spaces A_1, B_1, A_4, B_4 to transmit new information in both stages through the IC. Note that the signal bits transmitted from spaces A_1, A_2 and B_1, B_2 can be decoded directly at the intended destinations since they are not corrupted by interference. However, $2n_d - 3n_c$ signal bits received at spaces C_4 and D_4 are corrupted by interference signal bits from spaces B_2 and A_2 for each stage. The sources use $4n_d - 6n_c$ bits of the OBRC in common to transmit the signal bits from A_2 and B_2, and the rest $n_r - 4n_d + 6n_c$ signal bits of the OBRC can be used by one source or divided between two sources to transmit additional new information. The relay simply forwards all the information bits to the destinations. At the destinations, the $4n_d - 6n_c$ bits received from the OBRC carry the modulo sum of information bits from spaces A_2 and B_2. Since each destination knows the signal bits from one of the spaces, it can recover the signal bits from the other space. Therefore the interference bits in spaces C_4 and D_4 can be removed. The sum rate can be achieved is $2n_d - n_c + \frac{1}{2}n_r$, which matches the outerbound (10).

For illustration, we provide an example in Figure 9. Receiver 1 first decodes the bits a_2, a_7 from the signals received from the IC. It then decodes b_2, b_7 from the signals received from the OBRC. Based on these signal bits, it can decode all the information bits.

2) $4n_d - 6n_c > n_r$*:* For this case, the sources can only use $\frac{1}{2}n_r$ bits of the spaces A_2 and B_2 to transmit information through the IC for each stage, in addition to the spaces A_1, A_4 and B_1, B_4. All the signal bits of the OBRC are used in common by two sources to transmit the information bits from spaces A_2 and B_2. At destination 1, the decoder first decodes the signal bits transmitted from space A_1 and part of the space A_2. It can then recover the signal bits transmitted from space B_2 utilizing the OBRC, and thus the interference signal bits in space C_4 can be removed. Destination 2 uses similar decoding method. The sum rate achieved is $n_r + 2n_c$, which matches the outerbound (10)

Remark 3: Note that for weak interference, we only utilize the common information bits from A_2 and B_2 to transmit new information bits, but the signal spaces A_3 and B_3 are left

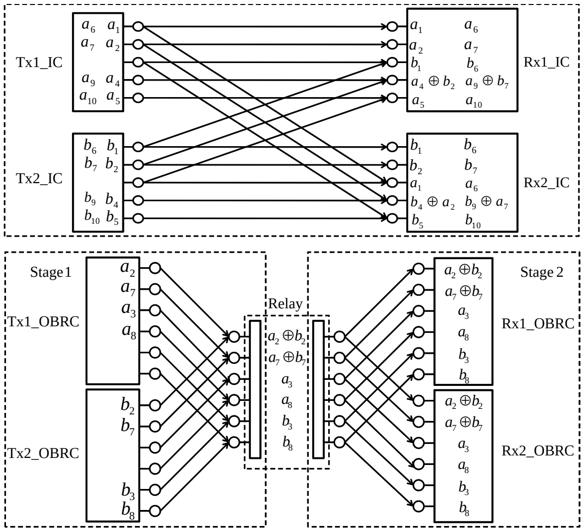

Fig. 9. Transmission scheme when $n_c = 3$, $n_d = 5$, $n_r = 6$.

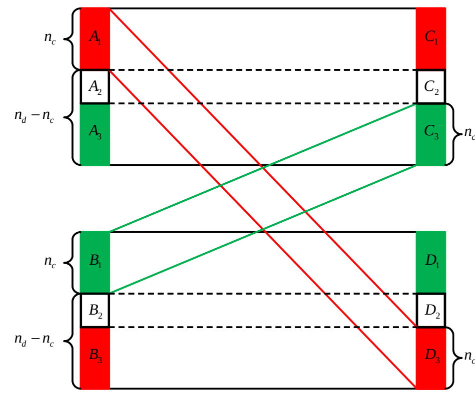

Fig. 10. Signal interaction for the underlying IC when $\frac{1}{2}n_d > n_c$

unused. The reason is that the signal bits from A_2 and B_2 only cause interference at D_4 and C_4, respectively, but they are not interfered by other signal bits. However, the signal bits from A_3 and B_3 not only cause interference at the other destination, but they are also interfered by the other source. To recover one bit from A_2 and one bit at the corresponding level from B_2, we only need one bit from the OBRC, that is, we trade one bit of the OBRC for the transmission of two information bits. However, to cancel the interference caused by using one bit from A_3 and one bit from the corresponding level of B_3, we need two bits from the OBRC, i.e., we only trade one bit of the OBRC for the transmission of one information bit, which is the same as the case when the signal spaces of the OBRC are used by only one source, or divided between two sources, to transmit new information. In addition, using the spaces A_3 and B_3 makes the signal interaction more complicated, and requires a more involved achievable strategy.

F. Case 6: $\frac{1}{2}n_d > n_c$

For this case, the interference is very weak. The signal interaction for the IC is shown in Figure 10. Without the OBR, the optimal transmission scheme is to transmit "private" information, i.e., to transmit information using signal spaces A_2, A_3 and B_2, B_3, since these signal bits are invisible to the other destination. The signal spaces A_1 and B_1 are left unused, since the signal bits from these spaces are common information bits, and they cause interference at the destinations. With the OBR, however, the signal spaces A_1 and B_1 can be utilized to transmit additional information bits, and the resulting interference can be removed. Similar to the strategy described in section IV-E, the extent to which we can use the signal spaces A_1 and B_1 depends on the strength of the links in the OBRC. We thus consider the following subcases.

1) $n_r \geq 2n_c$: For this case, the sources use all the signal spaces to transmit information through the IC. In the OBRC, the sources use $2n_c$ signal bits of the OBRC in common to transmit all the signal bits from spaces A_1 and B_1. The rest $n_r - 2n_c$ bits of the OBR can be used by one source, or divided between two sources to transmit new information bits. At destination 1, the decoder first decodes the signal bits sent from space A_1. It can then recover the interfering signal bits from B_1 using the signal obtained from the OBRC. With all the interference signal bits, it can decode all the intended information bits. The sum rate achieved is $2n_d - n_c + \frac{1}{2}n_r$ bits per channel use, which coincides with the upperbound (10).

2) $2n_c > n_r$: For this case, since the resources at the OBRC are limited, the sources can transmit their information bits using all signal spaces A_2, A_3 and B_2, B_3, and $\frac{1}{2}n_r$ bits of spaces A_1 and B_1 for each stage. All the signal bits in the OBRC are used by two sources in common to transmit the information bits from spaces A_1 and B_1. The decoding process is similar to the one we used in section IV-F1. The sum rate achieved for this case is $n_r + 2n_d - 2n_c$ bits per channel use, which matches the upperbound (10).

Remark 4: For the channel settings discussed in section IV-E2 and IV-F2, the OBRC cannot help the sources to transmit new information bits. It can only facilitate interference cancelation. However, for the channel settings discussed in section IV-E1 and IV-F1, the OBRC can help the sources to transmit new information bits in addition to facilitate interference cancelation, since the OBRC has more resources to be utilized. When the OBRC is used for interference cancelation, one bit of the OBRC can help each source to transmit one information bit, which means we trade one bit of the OBRC for two information bits. The optimality of our achievable strategy shows that when using the resources of the OBRC under weak and very weak interference, we should first consider using the OBRC to facilitate canceling the interference caused by transmitting additional common information bits, since this provides the largest payoff.

V. Conclusion

In this paper, we have considered the interference channel with an out-of-band relay. To explore and facilitate the proper interaction between the signals, we have focused on the deterministic model with symmetric channel gains. We have established the sum capacity for all channel gain values. Based on the sum rate optimal transmission strategy for the IC, we have shown that when the interference is strong or moderate, separate encoding is optimal. When the interference is extremely strong or very strong, the interference links are useful to make the transmission through the OBRC more efficient. When the interference is weak or very weak, the unused signal spaces in the IC are used to transmit new information, while the OBRC is used to facilitate interference cancelation. Depending on the strength of the links in the OBRC, the OBR can also help the sources to transmit new information. The sum rate optimal strategy for the deterministic model is a first step towards insights for the design of transmission strategies for the Gaussian model. Future work includes extending the result to Gaussian case and asymmetric channel settings.

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