Energy Cooperative Multiple Access Channels with Energy Harvesting Transmitters and Receiver

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***Abstract*—In this work, we study energy cooperation in a two-user Gaussian multiple access channel (MAC), where the transmitters and the receiver are powered by energy harvesters. In particular, the receiver consumes the harvested energy in decoding the received data and bidirectional energy cooperation is allowed between any two nodes in order to facilitate data transmission and decoding. Our objective is to maximize the number of decoded bits at the receiver, assuming offline knowledge of the channel state information (CSI) and the energy harvesting profiles. We first show that the original problem can be decomposed into an inner problem in which we optimize over the energy transfer variables and an outer problem that characterizes the optimal power allocation. For the inner problem, we derive analytical expressions for the energy transfer variables in terms of the fading coefficients and the energy transfer efficiencies between the nodes. On the other hand, the outer problem is solved using a generalized iterative directional water-filling algorithm. The numerical results show that the proposed energy cooperation can significantly enhance the system performance.**

***Index Terms*—Energy harvesting, energy cooperation, decoding cost, optimal energy allocation, multiple access channel.**

I. Introduction

Energy harvesting promises perpetual network operation, as it provides self-sustainability for energy-constrained wireless nodes [1]. In practice, energy can be harvested from several sources such as solar radiation, body heat, and radio frequency (RF) signals. However, a lasting and constant energy harvesting process is rarely found, due to the intermittent nature of the energy sources [2]. Therefore, optimal energy allocation policies have been proposed for various setups in order to prevent energy outages in the system [1], [2]. Additionally, energy cooperation between the network nodes is motivated by the fact that some nodes have abundant energy, while others are energy-deprived. In particular, energy cooperation not only prevents energy outages at energy-deprived nodes, but also enhances the overall system performance. In practice, energy cooperation between the nodes can be realized via multiple techniques, such as resonant inductive coupling and wireless RF energy transfer [3], [4].

References [5]–[10] have studied energy cooperation in different network topologies. In particular, reference [5] has introduced energy cooperation to two-way channels, relay channels, and multiple access channels. Reference [5] has developed a two-dimensional directional water-filling algorithm in order to characterize the optimal energy allocation that maximizes the system throughput. Furthermore, reference [9] has considered bidirectional energy cooperation between the nodes. In particular, a procrastinating policy has been shown to be optimal for the throughput maximization problem.

On the other hand, the energy consumed in decoding the received data at an energy harvesting receiver cannot be neglected when it is comparable to the transmission power, which is the case in short-range communication networks. In reference [11], the decoding cost has been characterized by analyzing the decoder operations in practical systems, while reference [12] has considered an information-theoretic analysis. Additionally, the effect of decoding cost on energy harvesting systems is investigated in [13] for point-to-point links, two-hop channels, multiple access channels, and broadcast channels. Both decoding and processing costs have been considered in [14] for a two-way energy harvesting channel.

In this paper, we study energy cooperation between the nodes in a Gaussian multiple access channel (MAC), which consists of two energy harvesting transmitters and an energy harvesting receiver. In turn, energy cooperation not only facilitates data transmission, but also decoding at the receiver. In particular, we study the throughput maximization problem in a two-user Gaussian MAC channel with block fading, i.e., our objective is to maximize the number of decoded bits at the receiver by jointly optimizing the power allocation and energy transfer variables. We assume an offline setting, where the energy harvesting profiles and the fading coefficients are known at all nodes before the transmission.

For energy cooperation, there exist multiple paths for transferring energy between any two nodes, hence we first show that the optimal energy transfer policy between any two nodes is unidirectional. Furthermore, we show that we can restrict the feasible policies to the set of procrastinating policies without loss of optimality, i.e., we only need to consider policies that delay the energy transfer until it is needed at the receiving node. By considering procrastinating policies, the throughput maximization problem can be decomposed into inner and outer problems. In the inner problem, we find the energy transfer variables at each time slot and show that the energy routing is determined by the fading coefficients and the energy

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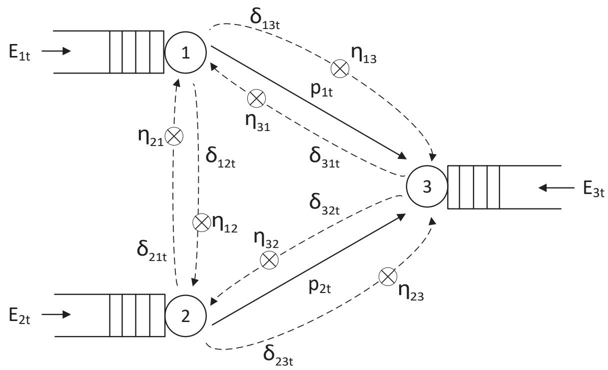

Fig. 1: Two-transmitter multiple access channel with energy cooperation. Dash lines and solid lines indicate energy transfer and information transmission, respectively.

transfer efficiencies between nodes. For the outer problem, a generalized directional water-filling algorithm is applied in order to obtain the optimal power allocation. Finally, we illustrate the structure of the optimal solution via a numerical example.

II. System Model and Problem Formulation

We consider a two-user Gaussian multiple access channel (MAC) with energy harvesting transmitters and an energy harvesting receiver, as shown in Fig. 1. In particular, each node can harvest energy and participate in energy cooperation simultaneously. Additionally, we assume bidirectional energy transfer links with different transfer efficiencies for each direction. The system is assumed to be time-slotted, and each time slot is of length 1 second. The energy harvesting profiles of all nodes are known apriori for a transmission period of T slots, i.e., we assume offline knowledge of the amount of harvested energy, and the energy arrival times. The harvested energy in slot t at node k is denoted by $E_{k,t} \geq 0$, where $k = 1, 2, 3$, and $t = 1, 2, \ldots, T$. In particular, $k = 1, 2$, refers to transmitters 1 and 2, while $k = 3$ refers to the receiver. We assume that each node is equipped with a sufficiently large battery in order to avoid any overflow as in [5], [7]. Furthermore, $\delta_{kj,t}$ denotes the amount of energy transferred from node k to node j in slot t with transfer efficiency η_{kj}, $k \neq j$, thus, the received energy at node j is $\eta_{kj}\delta_{kj,t}$. We assume that $0 < \eta_{kj} < 1$, and it is constant throughout the transmission period.

We consider a Gaussian multiple access channel (MAC) with block fading. In particular, the number of received bits in slot t is given as

$$R_t = \frac{1}{2}\log_2(1 + h_{1,t}p_{1,t} + h_{2,t}p_{2,t}),\ t = 1, \ldots, T, \quad (1)$$

where $p_{k,t}$ denotes the transmission power of transmitter k at slot t, and $h_{k,t}$ is the channel gain between transmitter k and the receiver at slot t, which is normalized by the noise power.

Additionally, we assume that the receiver consumes energy in decoding the received data. The decoding cost function is assumed to be convex, and nondecreasing in the decoding rate as in [11], [12], [15]. To avoid data buffer overflow at the receiver and to avoid decoding delays, the decoding rate is assumed to be equal to the received data rate in (1). Specifically, we consider the exponential decoding cost function $\phi(x) = 2^{2x} - 1$ for simplicity and in order to obtain a tractable analytical solution [13]. Therefore, the power consumed in decoding at the receiver is given as

$$p_{3,t} = \phi(R_t) = h_{1,t}p_{1,t} + h_{2,t}p_{2,t},\ t = 1, \ldots, T. \quad (2)$$

Our objective is to maximize the number of decoded bits at the receiver in T slots. The problem is formulated as follows.

$$\max_{p_{k,t},\delta_{kj,t}} \sum_{t=1}^{T} \frac{1}{2}\log_2(1 + h_{1,t}p_{1,t} + h_{2,t}p_{2,t}) \quad (3a)$$

$$\text{s.t.} \sum_{i=1}^{t}\Big(p_{k,i} + \sum_{\substack{j=1\\ j\neq k}}^{3}(\delta_{kj,i} - \eta_{jk}\delta_{jk,i})\Big) \leq \sum_{i=1}^{t} E_{k,i}, \forall k, t, \quad (3b)$$

$$p_{3,t} = h_{1,t}p_{1,t} + h_{2,t}p_{2,t},\ \forall t, \quad (3c)$$

$$p_{k,t},\ \delta_{kj,t} \geq 0,\ \forall k, j \neq k, t. \quad (3d)$$

In particular, (3b) represents the energy causality constraint at node k. One can easily verify that the problem is convex due the concavity of the objective function and the linearity of the constraints.

III. Problem Decomposition

In this section, we decompose the original problem in (3) into outer and inner problems, which characterize the optimal power allocation and energy transfer, respectively. In particular, without loss of optimality, we restrict the feasible solutions to procrastinating policies as in [9], i.e., we show that one of the optimal policies is a procrastinating policy.

First, we prove that the optimal energy transfer between any two nodes is unidirectional.

Lemma 1. *The optimal energy transfer satisfies* $\delta_{kj,t}\delta_{jk,t} = 0$, $k, j = 1, 2, 3,\ k \neq j,\ \forall t$.

Proof: The proof follows from Lemma 2 in [9]. In Appendix A, we provide an alternative proof based on the Karush-Kuhn-Tucker (KKT) conditions. ∎

Furthermore, we denote the consumed power at node k in slot t by

$$\bar{p}_{k,t} \triangleq p_{k,t} + \sum_{j=1, j\neq k}^{3}(\delta_{kj,t} - \eta_{jk}\delta_{jk,t}), \quad (4)$$

which is called a procrastinating policy if it satisfies $\bar{p}_{k,t} \geq 0$, $\forall k, t$. In the next lemma, we show that one of the optimal solutions to (3) is a procrastinating policy, which guarantees that energy transfer is deferred until it is needed.

Lemma 2. *There exists an optimal solution to problem (3) that satisfies the condition* $\bar{p}_{k,t} \geq 0$, $\forall k, t$.

Proof: Suppose that $\{p^*_{k,t}, \delta^*_{kj,t}\}$ is an optimal solution to (3). We can construct a procrastinating policy that achieves the optimal objective value by repeating the following process for all t and k. If for some t and k, we have $p^*_{k,t} < \sum_{j=1,j\neq k}^{3}(\eta_{jk}\delta^*_{jk,t} - \delta^*_{kj,t})$, then define

$$\theta_{k,t} \triangleq \sum_{j=1,j\neq k}^{3}(\eta_{jk}\delta^*_{jk,t} - \delta^*_{kj,t}) - p^*_{k,t}. \tag{5}$$

In order to decrease the actual received energy at node k by $\theta_{k,t}$, we obtain $\tilde{\delta}_{jk,t} \geq 0$ and $\tilde{\delta}_{kj,t} \geq 0$ by decreasing $\delta^*_{jk,t}$ and/or increasing $\delta^*_{kj,t}$ such that

$$p^*_{k,t} = \sum_{j=1,j\neq k}^{3}(\eta_{jk}\tilde{\delta}_{jk,t} - \tilde{\delta}_{kj,t}), \tag{6}$$

and $\tilde{\delta}_{kj,t}\tilde{\delta}_{jk,t} = 0$ are satisfied. To maintain the feasibility, energy equal to $\theta_{k,t}$ is transferred to node k in later slots by increasing $\delta^*_{jk,l}$ and/or decreasing $\delta^*_{kj,l}$ for some $l > t$. Therefore, one can construct a procrastinating policy that achieves the optimal objective value, since the transmission power $p^*_{k,t}$ is unchanged. ∎

Next, by considering only procrastinating policies, the throughput maximization problem can be expressed in terms of the optimization variables $\{\bar{p}_{k,t}, \delta_{kj,t}\}$ as follows.

$$\max_{\bar{p}_{k,t},\delta_{kj,t}} \quad \sum_{t=1}^{T}\frac{1}{2}\log_2(1+\varphi_1(\bar{p}_{1,t},\bar{p}_{2,t})) \tag{7a}$$

$$\text{s.t.} \quad \sum_{i=1}^{t}\bar{p}_{k,i} \leq \sum_{i=1}^{t}E_{k,i}, \ \forall k,t, \tag{7b}$$

$$\bar{p}_{k,t} \geq \sum_{j=1,j\neq k}^{3}(\delta_{kj,t} - \eta_{jk}\delta_{jk,t}), \ \forall k,t, \tag{7c}$$

$$\varphi_1(\bar{p}_{1,t},\bar{p}_{2,t}) = \varphi_2(\bar{p}_{3,t}), \ \forall t, \tag{7d}$$

$$\bar{p}_{k,t}, \ \delta_{kj,t} \geq 0, \ \forall k, j\neq k, t, \tag{7e}$$

where

$$\begin{aligned}\varphi_1(\bar{p}_{1,t},\bar{p}_{2,t}) =& (h_{2,t}\eta_{12} - h_{1,t})\delta_{12,t} + (h_{1,t}\eta_{21} - h_{2,t})\delta_{21,t} \\ &+ \sum_{k=1}^{2} h_{k,t}(\bar{p}_{k,t} + \eta_{3k}\delta_{3k,t} - \delta_{k3,t}),\end{aligned} \tag{8}$$

$$\varphi_2(\bar{p}_{3,t}) = \bar{p}_{3,t} - \delta_{31,t} - \delta_{32,t} + \eta_{13}\delta_{13,t} + \eta_{23}\delta_{23,t}. \tag{9}$$

In particular, (7c) is due to the nonnegativity of $p_{k,t}$.

In the transformed problem (7), the energy transfer variables at slot t are constrained by (7c) and (7d), while the consumed powers are constrained by the energy causality constraints in (7b). Based on these observations, the problem in (7) can be decomposed into outer and inner problems as follows.

$$\max_{\bar{p}_{k,t}} \quad \sum_{t=1}^{T}\frac{1}{2}\log_2(1+\bar{\varphi}(\bar{p}_{1,t},\bar{p}_{2,t},\bar{p}_{3,t})) \tag{10a}$$

$$\text{s.t.} \quad \sum_{i=1}^{t}\bar{p}_{k,i} \leq \sum_{i=1}^{t}E_{k,i}, \ \forall k,t, \tag{10b}$$

$$\bar{p}_{k,t} \geq 0, \ \forall k,t, \tag{10c}$$

where $\bar{\varphi}(\bar{p}_{1,t},\bar{p}_{2,t},\bar{p}_{3,t})$, $t = 1,2,\ldots,T$ is given as

$$\bar{\varphi}(\bar{p}_{1,t},\bar{p}_{2,t},\bar{p}_{3,t}) = \max_{\delta_{kj,t}} \ \varphi_1(\bar{p}_{1,t},\bar{p}_{2,t}) \tag{11a}$$

$$\text{s.t.} \ \bar{p}_{k,t} \geq \sum_{\substack{j=1\\ j\neq k}}^{3}(\delta_{kj,t} - \eta_{jk}\delta_{jk,t}), \ \forall k, \tag{11b}$$

$$\varphi_1(\bar{p}_{1,t},\bar{p}_{2,t}) = \varphi_2(\bar{p}_{3,t}), \tag{11c}$$

$$\delta_{kj,t} \geq 0, \ \forall k, j \neq k. \tag{11d}$$

In the outer problem (10), the energy allocation is obtained by optimizing the consumed powers, and the energy cooperation is characterized in the inner problem (11) by optimizing the energy transfer variables for given consumed powers in each slot t.

IV. Optimal Solution

In this section, we first show that an analytical solution is possible to be obtained for the inner energy transfer problem. Then, we solve the outer energy allocation problem using a generalized water-filling algorithm.

1) Inner Problem: The inner problem is a linear program, which can be solved analytically. In particular, for fixed consumed powers, the energy transfer variables are determined by the fading coefficients and the energy transfer efficiencies. More specifically, the solution to the linear program can be evaluated in two steps: 1) we determine the energy routing, 2) we calculate the amount of transferred energy.

First, note that for systems with more than two nodes, there exist multiple possible routes for transfer of energy from one node to another. For example, in order to transfer energy from node 1 to node 3 in our system, we either transfer energy from node 1 to node 3 directly or via node 2. To minimize the energy loss during transfer, we always choose the route with the highest energy transfer efficiency, i.e., $\max\{\eta_{kj}, \eta_{kl_1}\eta_{l_1 j}, \eta_{kl_1}\eta_{l_1 l_2}\eta_{l_2 j}, \eta_{kl_2}\eta_{l_2 j}, \ldots\}$ for transferring energy from node k to node j. In the sequel, we use $\tilde{\eta}_{kj}$ to denote the transfer efficiency of the best route from node k to node j.

The next step is to calculate the energy transfer variables. In our system, there are at most two nonzero energy transfer variables, which follows from Lemma 1. In turn, the inner problem can be solved analytically. In particular, the energy transfer variables are functions of the consumed powers, the fading coefficients, and the energy transfer efficiencies. We provide the analytical solution to the inner linear program in Appendix B.

In order to maximize the objective function, the following observations must hold [6], [7]. (i) If $h_{k,t} < \tilde{\eta}_{kj}h_{j,t}, \forall j$, then transmitter k transfers all of its harvested and received energy to other nodes, i.e., it works as an energy relaying node. (ii) If $\varphi_1(\bar{p}_{1,t},\bar{p}_{2,t}) > \varphi_2(\bar{p}_{3,t})$, then energy has to be transferred from the transmitters to the receiver in order to achieve the equality in (11c). In particular, if $\frac{\tilde{\eta}_{k3}}{h_k} > \frac{\tilde{\eta}_{j3}}{h_j}$, then transmitter k has a higher energy transfer priority than transmitter j. (iii) If $\varphi_1(\bar{p}_{1,t},\bar{p}_{2,t}) < \varphi_2(\bar{p}_{3,t})$, then energy has to be transferred from the receiver to the transmitters. In particular, if $h_k\tilde{\eta}_{3k} > h_j\tilde{\eta}_{3j}$, then transmitter k has a higher priority in receiving energy than transmitter j.

2) Outer Problem: The outer problem is a convex problem. Thus, the KKT conditions are sufficient and necessary conditions for optimality. The Lagrangian function is given as

$$\begin{aligned}\mathcal{L}(\bar{\mathbf{p}},\boldsymbol{\lambda},\boldsymbol{\beta}) = \\ -\sum_{t=1}^{T}\frac{1}{2}\log_2(1+\bar{\varphi}(\bar{p}_{1,t},\bar{p}_{2,t},\bar{p}_{3,t})) - \sum_{k=1}^{3}\sum_{t=1}^{T}\beta_{k,t}\bar{p}_{k,t} \\ +\sum_{k=1}^{3}\sum_{t=1}^{T}\lambda_{k,t}\Big(\sum_{i=1}^{t}\bar{p}_{k,i}-\sum_{i=1}^{t}E_{k,i}\Big),\end{aligned} \tag{12}$$

where $\lambda_{k,t}, \beta_{k,t}$ are the Lagrangian multipliers associated with the energy causality constraints in (10b) and the nonnegativity constraints in (10c). From the KKT conditions, for all k,t, we have

$$-\frac{\frac{\partial\bar{\varphi}(\bar{p}_{1,t},\bar{p}_{2,t},\bar{p}_{3,t})}{\partial\bar{p}_{k,i}}}{2\ln(2)(1+\bar{\varphi}(\bar{p}_{1,t},\bar{p}_{2,t},\bar{p}_{3,t}))}+\sum_{n=t}^{T}\lambda_{k,n}-\beta_{k,t}=0, \tag{13a}$$

$$\lambda_{k,t}\Big(\sum_{i=1}^{t}\bar{p}_{k,i}-\sum_{i=1}^{t}E_{k,i}\Big)=0, \tag{13b}$$

$$\beta_{k,t}\bar{p}_{k,t}=0, \tag{13c}$$

$$\lambda_{k,t},\beta_{k,t}\geq 0. \tag{13d}$$

From (13a), we define the generalized water level $w_{k,t}$ as

$$w_{k,t} \triangleq \frac{(1+\bar{\varphi}(\bar{p}_{1,t},\bar{p}_{2,t},\bar{p}_{3,t}))}{\frac{\partial\bar{\varphi}(\bar{p}_{1,t},\bar{p}_{2,t},\bar{p}_{3,t})}{\partial\bar{p}_{k,i}}} \tag{14}$$

$$= \frac{1}{2\ln(2)\big(\sum_{n=t}^{T}\lambda_{k,n}-\beta_{k,t}\big)}. \tag{15}$$

From the complementary slackness condition in (13c), we have $\beta_{k,t}=0$ for positive consumed powers. In turn, the generalized water level $w_{k,t}$ is monotonically increasing. In particular, the water level increases, i.e., $w_{k,t} < w_{k,t+1}$, only when the energy at node k is depleted at slot t. Furthermore, the following relation holds

$$\alpha_{1,t}w_{1,t}=\alpha_{2,t}w_{2,t}=\alpha_{3,t}w_{3,t}=1+\bar{\varphi}(\bar{p}_{1,t},\bar{p}_{2,t},\bar{p}_{3,t}), \tag{16}$$

where $\alpha_{k,t} = \frac{\partial\bar{\varphi}(\bar{p}_{1,t},\bar{p}_{2,t},\bar{p}_{3,t})}{\partial\bar{p}_{k,i}}$. As a result, the generalized water levels satisfy $\frac{w_{k,t}}{w_{k,t+1}} = \frac{w_{j,t}}{w_{j,t+1}}$, $\forall k, j \neq k$ when $\alpha_{k,t} = \alpha_{k,t+1}$, $\forall k$. That is, the generalized water level at any node is equalized between consecutive slots if all nodes have nonempty batteries and increases whenever at least one node has depleted its energy.

The optimal values for the consumed powers can be obtained by applying a generalized iterative water-filling algorithm [9]. In particular, we initialize the consumed powers as $\bar{p}_{k,t}=E_{k,t}$. Next, we fix $\bar{p}_{1,t}$ and $\bar{p}_{2,t}$ and iterate on $\bar{p}_{3,t}$. If $w_{3,t} > w_{3,t+1}$, then we decrease $\bar{p}_{3,t}$ to make energy flow to future slots until the water level is monotonically increasing. Then, we fix $\bar{p}_{3,t}$ and iterate on $\bar{p}_{1,t}$ and $\bar{p}_{2,t}$ in the same way. We terminate the process when the objective value converges.

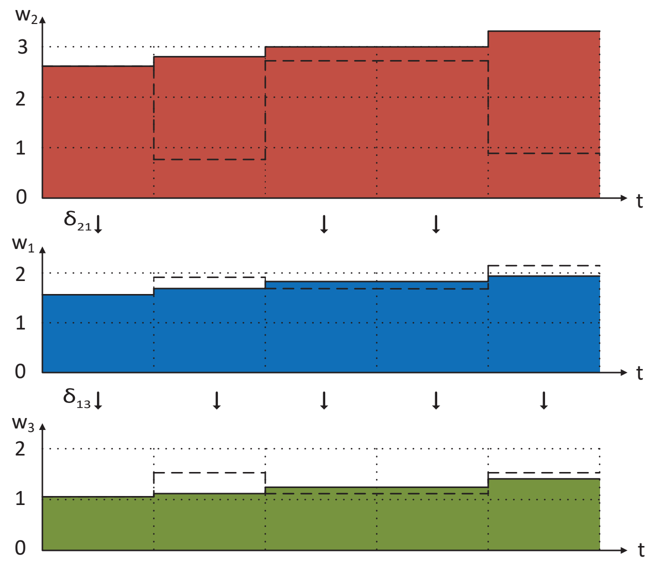

Fig. 2: The optimal water levels obtained from the generalized directional water-filling algorithm. The dash lines represent initial water levels for node 2, node 1, and node 3, respectively, while the solid lines represent the optimal water levels.

Due to the convexity of the outer problem, the convergence is always guaranteed.

V. Numerical Results

Next, we explain the solution obtained from the generalized iterative water-filling algorithm. Fig. 2 illustrates the optimal water levels of the three nodes, for $t = 1, 2, \ldots, 5$. In particular, we consider the energy transfer efficiencies $\eta_{12} = 0.4$, $\eta_{13} = 0.7$, $\eta_{21} = 0.6$, $\eta_{23} = 0.3$, $\eta_{31} = 0.5$, and $\eta_{32} = 0.5$. The energy harvesting profiles are $\mathbf{E}_1 = (23, 12, 38, 38, 37)$ mJ, $\mathbf{E}_2 = (37, 5, 34, 23, 11)$ mJ, and $\mathbf{E}_3 = (49, 412, 88, 82, 333)$ mJ. The fading coefficients are fixed throughout the transmission as $h_1 = 6, h_2 = 20$. The optimal consumed powers are $\bar{\mathbf{p}}_1 = (23, 7.2, 24.7, 56.1, 37)$ mJ, $\bar{\mathbf{p}}_2 = (31.2, 10.8, 20.5, 29.6, 17.9)$ mJ, and $\bar{\mathbf{p}}_3 = (49, 211.2, 198.3, 172.5, 333)$ mJ. The energy is transferred via the route $2 \rightarrow 1 \rightarrow 3$ with $\boldsymbol{\delta}_{21} = (27.4, 0, 9.5, 18.6, 0)$ mJ, and $\boldsymbol{\delta}_{13} = (39.4, 7.2, 30.4, 67.2, 37)$ mJ. Node 1 works as an energy relay, which transfers all of its harvested and received energy to node 3 in order to facilitate decoding, and all the data is transmitted by node 2. The energy is depleted at $t = 1$ and $t = 4$ at node 1 and node 3 and at $t = 2$ at node 2, in turn the optimal water levels increase, as shown in Fig. 2.

In Fig. 3, we simulate a wireless AWGN channel between each transmitter and the receiver over a distance of 1 meter. In particular, we adopt the indoor free space path loss model with path loss exponent 1.6 [16]. We assume that the carrier frequency is 2.4 GHz and the bandwidth is 1 MHz. The noise power spectral density is 10^{-19} W/Hz. The antenna gain at the receiver is 6 dB. The average channel gain is set to -85 dB. The amount of harvested energy per slot is uniformly dis-

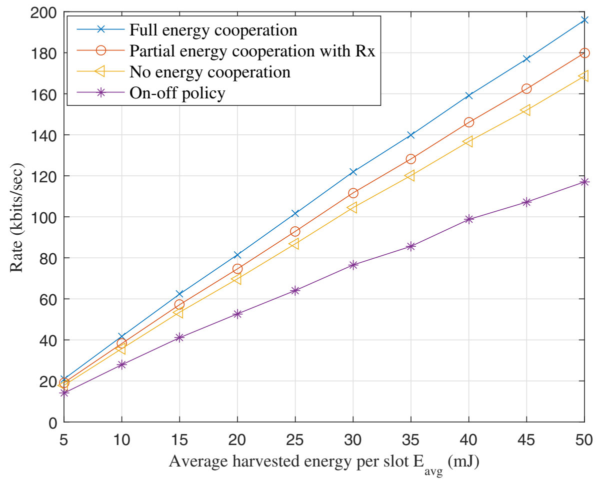

Fig. 3: Decoded data rate by varying average harvested energy in one slot.

tributed with average value E_{avg}, i.e., $E_{k,t} \sim \text{Unif}\ (0, 2E_{\text{avg}})$. The following results are averaged over 500 realizations for the channel gains and energy harvesting profiles. In Fig. 3, we compare the proposed energy cooperation to partial energy cooperation, no energy cooperation, and an on-off policy. In particular, in a partial energy cooperation scheme energy transfer is only allowed between transmitters and between one transmitter and the receiver. While, in the on-off policy, the nodes transmit with the average harvested energy and the receiver decodes the received signal as long as it has enough energy. Fig. 3 shows the decoded data rate versus the average harvested energy per slot. Intuitively, the decoded data rate is increasing with the average harvested energy for all schemes. Also, the proposed energy cooperation achieves the best performance.

VI. Conclusion

In this paper, we have studied energy cooperation in a two-user Gaussian multiple access channel, where all nodes are powered with energy harvesters and participate in energy cooperation. We have solved the problem of maximizing the number of decoded bits at the receiver, by jointly optimizing the power allocation and the energy transfer variables. In particular, we decompose the original problem into outer and inner problems by restricting the set of feasible policies to procrastinating policies. In the inner problem, the energy transfer variables are optimized for given consumed powers. While, in the outer problem, the consumed powers are optimized for given energy transfer variables using a generalized iterative directional water-filling algorithm. Finally, we have demonstrated numerically that energy cooperation with the receiver significantly improves the system performance.

Appendix A
Proof of Lemma 1

The unidirectional energy transfer property is proved using the KKT condition of problem (3). By taking the derivatives of the Lagrangian with respect to $\delta_{12,t}$ and $\delta_{21,t}$ and equating them to zero, we obtain

$$\xi_{12,t} = -\sum_{n=t}^{T} \nu_{1,n} + \eta_{12} \sum_{n=t}^{T} \nu_{2,n}, \tag{17}$$

$$\xi_{21,t} = \eta_{21} \sum_{n=t}^{T} \nu_{1,n} - \sum_{n=t}^{T} \nu_{2,n}, \tag{18}$$

where $\xi_{12,t}$ and $\xi_{21,t}$ are the Lagrangian multipliers associated with the nonnegativity of $\delta_{12,t}$ and $\delta_{21,t}$, respectively. While, $\nu_{1,t}$ and $\nu_{2,t}$ are the Lagrangian multipliers associated with the energy causality constraint at node 1 and 2, respectively. From (17) and (18), we have

$$\xi_{21,t} = -\eta_{21}\Big(-\sum_{n=t}^{T} \nu_{1,n} + \frac{1}{\eta_{21}} \sum_{n=t}^{T} \nu_{2,n}\Big) \tag{19}$$

$$\overset{(a)}{<} -\eta_{21}\Big(-\sum_{n=t}^{T} \nu_{1,n} + \eta_{12} \sum_{n=t}^{T} \nu_{2,n}\Big) = -\eta_{21}\xi_{12,t}, \tag{20}$$

where (a) is due to $0 < \eta_{kj} < 1$. Hence, $\xi_{12,t}$ and $\xi_{21,t}$ cannot be both zero. From the complementary slackness condition, we observe that $\delta_{12,t}$ and $\delta_{21,t}$ cannot be both positive, i.e., $\delta_{12,t}\delta_{21,t} = 0$. Similarly, the unidirectional property can be proved for other energy transfer variables.

Appendix B
Analytical Solution of the Inner Problem

For ease of exposition, we omit the slot index t and use the notation $\max\{x, 0\} \triangleq [x]^+$.

1) $\delta_{31} \geq 0$, $\delta_{32} \geq 0$, $h_1 \geq h_2\eta_{12}$, $h_2 \geq h_1\eta_{21}$,

$$\delta_{31} = \begin{cases} 0, & \text{if } h_2\eta_{32} > h_1\eta_{31} \\ \frac{[\bar{p}_3 - h_1\bar{p}_1 + h_2\bar{p}_2]^+}{1 + h_1\eta_{31}}, & \text{o.w.} \end{cases} \tag{21}$$

$$\delta_{32} = \begin{cases} \frac{[\bar{p}_3 - h_1\bar{p}_1 + h_2\bar{p}_2]^+}{1 + h_2\eta_{32}}, & \text{if } h_2\eta_{32} > h_1\eta_{31} \\ 0, & \text{o.w.} \end{cases} \tag{22}$$

2) $\delta_{21} \geq 0$, $\delta_{31} \geq 0$, $h_2 < h_1\eta_{21}$,

$$\delta_{21} = \bar{p}_2, \quad \delta_{31} = \frac{[-h_1(\eta_{21}\bar{p}_2 + h_1\bar{p}_1) + \bar{p}_3]^+}{1 + h_1\eta_{31}} \tag{23}$$

3) $\delta_{12} \geq 0$, $\delta_{32} \geq 0$, $h_1 < h_2\eta_{12}$

$$\delta_{12} = \bar{p}_1, \quad \delta_{32} = \frac{[-h_2(\eta_{12}\bar{p}_1 + h_2\bar{p}_2) + \bar{p}_3]^+}{1 + h_2\eta_{32}} \tag{24}$$

4) $\delta_{12} \geq 0, \delta_{23} \geq 0$

$$\delta_{12} = \begin{cases} \bar{p}_1, & \text{if } h_2\eta_{12} > h_1 \\ \frac{[-\eta_{23}\bar{p}_2 + h_1\bar{p}_1 - \bar{p}_3]^+}{h_1 + \eta_{12}\eta_{23}}, & \text{o.w.} \end{cases} \tag{25}$$

$$\delta_{23} = \begin{cases} \frac{[h_1(\bar{p}_2 + \eta_{12}\bar{p}_1) - \bar{p}_3]^+}{h_2 + \eta_{23}}, & \text{if } h_2\eta_{12} > h_1 \\ \frac{[h_1\bar{p}_1 + h_2\bar{p}_2 - \bar{p}_3]^+}{h_2 + \eta_{23}}, & \text{if } \frac{h_2\eta_{12}}{h_1} \leq 1, \frac{h_1\eta_{21}}{h_2} \leq 1, \\ & \text{and } h_1\bar{p}_1 - \eta_{23}\bar{p}_2 \leq \bar{p}_3 \\ \frac{[h_1(\bar{p}_2 + \eta_{12}\bar{p}_1) - \eta_{12}\bar{p}_3]^+}{h_1 + \eta_{12}\eta_{23}}, & \text{o.w.} \end{cases} \tag{26}$$

5) $\delta_{21} \geq 0, \delta_{13} \geq 0$

$$\delta_{21}=\begin{cases} \bar{p}_2, & \text{if } h_1\eta_{21} > h_2 \\ \frac{[-\eta_{13}\bar{p}_1+h_2\bar{p}_2-\bar{p}_3]^+}{h_2+\eta_{21}\eta_{13}}, & \text{o.w.} \end{cases} \tag{27}$$

$$\delta_{13}=\begin{cases} \frac{[h_1(\bar{p}_1+\eta_{21}\bar{p}_2)-\bar{p}_3]^+}{h_1+\eta_{13}}, & \text{if } h_1\eta_{21} > h_2 \\ \frac{[h_1\bar{p}_1+h_2\bar{p}_2-\bar{p}_3]^+}{h_1+\eta_{13}}, & \text{if } \frac{h_2\eta_{12}}{h_1}\leq 1, \frac{h_1\eta_{21}}{h_2}\leq 1, \\ & \text{and } h_2\bar{p}_2-\eta_{13}\bar{p}_1 \leq \bar{p}_3 \\ \frac{[h_2(\bar{p}_1+\eta_{21}\bar{p}_2)-\eta_{21}\bar{p}_3]^+}{h_2+\eta_{21}\eta_{13}}, & \text{o.w.} \end{cases} \tag{28}$$

6) $\delta_{13} \geq 0, \delta_{23} \geq 0$

$$\delta_{13}=\begin{cases} \frac{[h_1\bar{p}_1-\eta_{23}\bar{p}_2-\bar{p}_3]^+}{h_1+\eta_{13}}, & \text{if } \frac{h_1\eta_{21}}{h_2}>1, \text{or} \frac{h_1\bar{p}_1-\eta_{23}\bar{p}_2}{\bar{p}_3}\leq 1, \\ & \text{and } \frac{h_2\eta_{12}}{h_1}\leq 1, \frac{h_1\eta_{21}}{h_2}\leq 1 \\ \frac{[h_1\bar{p}_1+h_2\bar{p}_2-\bar{p}_3]^+}{h_1+\eta_{13}}, & \text{if } \frac{h_2\eta_{12}}{h_1}\leq 1, \frac{h_1\eta_{21}}{h_2}\leq 1, \\ & \text{and } h_2\bar{p}_2-\eta_{13}\bar{p}_1 \leq \bar{p}_3 \\ \bar{p}_1, & \text{o.w.} \end{cases} \tag{29}$$

$$\delta_{23}=\begin{cases} \bar{p}_2, & \text{if } \frac{h_1\eta_{21}}{h_2}>1 \\ \frac{[h_1\bar{p}_1+h_2\bar{p}_2-\bar{p}_3]^+}{h_2+\eta_{23}}, & \text{if } \frac{h_2\eta_{12}}{h_1}\leq 1, \frac{h_1\eta_{21}}{h_2}\leq 1, \\ & \text{and } h_1\bar{p}_1-\eta_{23}\bar{p}_2 \leq \bar{p}_3 \\ \frac{[h_2\bar{p}_2-\eta_{13}\bar{p}_1-\bar{p}_3]^+}{h_2+\eta_{23}}, & \text{o.w.} \end{cases} \tag{30}$$

7) $\delta_{23} \geq 0,\ \delta_{31} \geq 0$

$$\delta_{23} = \begin{cases} \frac{[h_1\bar{p}_1+h_2\bar{p}_2-\bar{p}_3]^+}{h_2+\eta_{23}}, & \text{if } \frac{h_2\eta_{12}}{h_1}\leq 1, \frac{h_1\eta_{21}}{h_2}\leq 1, \\ & \text{and } \frac{h_2+\eta_{23}}{\eta_{23}(1+h_1\eta_{31})} < 1 \\ \bar{p}_2, & \text{o.w.} \end{cases} \tag{31}$$

$$\delta_{31} = \begin{cases} \frac{[-h_1\bar{p}_1-h_2\bar{p}_2+\bar{p}_3]^+}{1+h_1\eta_{31}}, & \text{if } \frac{h_2\eta_{12}}{h_1}\leq 1, \frac{h_1\eta_{21}}{h_2}\leq 1, \\ & \text{and } \frac{h_2+\eta_{23}}{\eta_{23}(1+h_1\eta_{31})} < 1 \\ \frac{[\eta_{23}\bar{p}_2-h_1\bar{p}_1+\bar{p}_3]^+}{1+h_1\eta_{31}}, & \text{o.w.} \end{cases} \tag{32}$$

8) $\delta_{13} \geq 0,\ \delta_{32} \geq 0$

$$\delta_{13} = \begin{cases} \frac{[h_1\bar{p}_1+h_2\bar{p}_2-\bar{p}_3]^+}{h_1+\eta_{13}}, & \text{if } \frac{h_2\eta_{12}}{h_1}\leq 1, \frac{h_1\eta_{21}}{h_2}\leq 1, \\ & \text{and } \frac{h_1+\eta_{13}}{\eta_{13}(1+h_2\eta_{32})} < 1 \\ \bar{p}_1, & \text{o.w.} \end{cases} \tag{33}$$

$$\delta_{32} = \begin{cases} \frac{[-h_1\bar{p}_1-h_2\bar{p}_2+\bar{p}_3]^+}{1+h_2\eta_{32}}, & \text{if } \frac{h_2\eta_{12}}{h_1}\leq 1, \frac{h_1\eta_{21}}{h_2}\leq 1, \\ & \text{and } \frac{h_1+\eta_{13}}{\eta_{13}(1+h_2\eta_{32})} < 1 \\ \frac{[\eta_{13}\bar{p}_1-h_2\bar{p}_2+\bar{p}_3]^+}{1+h_2\eta_{32}}, & \text{o.w.} \end{cases} \tag{34}$$

9) $\delta_{21} \geq 0,\ \delta_{23} \geq 0,\ h_2 < h_1\eta_{21}$,

$$\delta_{21} = \frac{[\eta_{23}\bar{p}_2 - h_1\bar{p}_1 + \bar{p}_3]^+}{\eta_{23} + h_1\eta_{21}} \tag{35}$$

$$\delta_{23} = \frac{[h_1(\eta_{21}\bar{p}_2 + \bar{p}_1) - \bar{p}_3]^+}{\eta_{13} + h_1\eta_{21}} \tag{36}$$

10) $\delta_{21} \geq 0,\ \delta_{32} \geq 0,\ h_2 < h_1\eta_{21}$,

$$\delta_{21} = \frac{[\bar{p}_2 - \eta_{32}(h_1\bar{p}_1 + \bar{p}_3)]^+}{1 + h_2\eta_{32}\eta_{21}} \tag{37}$$

$$\delta_{32} = \frac{[-h_1(\eta_{21}\bar{p}_2 + \bar{p}_1) + \bar{p}_3]^+}{1 + h_1\eta_{32}\eta_{21}} \tag{38}$$

11) $\delta_{12} \geq 0,\ \delta_{13} \geq 0,\ h_1 < h_2\eta_{12}$

$$\delta_{12} = \frac{[\eta_{13}\bar{p}_1 - h_2\bar{p}_2 + \bar{p}_3]^+}{\eta_{13} + h_2\eta_{12}} \tag{39}$$

$$\delta_{13} = \frac{[h_2(\eta_{12}\bar{p}_1 + \bar{p}_2) - \bar{p}_3]^+}{\eta_{13} + h_2\eta_{12}} \tag{40}$$

12) $\delta_{12} \geq 0,\ \delta_{31} \geq 0,\ h_1 < h_2\eta_{12}$

$$\delta_{12} = \frac{[\bar{p}_1 - \eta_{31}(h_2\bar{p}_2 + \bar{p}_3)]^+}{1 + h_2\eta_{31}\eta_{12}} \tag{41}$$

$$\delta_{31} = \frac{[-h_2(\eta_{12}\bar{p}_1 + \bar{p}_2) + \bar{p}_3]^+}{1 + h_2\eta_{31}\eta_{12}} \tag{42}$$

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