Non-Asymptotic Achievable Rates for Gaussian Energy-Harvesting Channels: Save-and-Transmit and Best-Effort

Silas L. Fong, *Member, IEEE*, Jing Yang, *Member, IEEE*, and Aylin Yener, *Fellow, IEEE*

***Abstract*— An additive white Gaussian noise energy-harvesting channel with an infinite-sized battery is considered. The energy arrival process is modeled as a sequence of independent and identically distributed random variables. The channel capacity $\frac{1}{2}\log(1+P)$ is achievable by the so-called best-effort and save-and-transmit schemes where P denotes the battery recharge rate. This paper analyzes the save-and-transmit scheme whose transmit power is strictly less than P and the best-effort scheme as a special case of save-and-transmit without a saving phase. In the finite blocklength regime, we obtain new non-asymptotic achievable rates for these schemes that approach the capacity with gaps vanishing at rates proportional to $1/\sqrt{n}$ and $((\log n)/n)^{1/2}$ respectively where n denotes the blocklength. The proof technique involves analyzing the escape probability of a Markov process. When P is sufficiently large, we show that allowing the transmit power to back off from P can improve the performance for save-and-transmit. The results are extended to a block energy arrival model where the length of each energy block L grows sublinearly in n. We show that the save-and-transmit and best-effort schemes achieve coding rates that approach the capacity with gaps vanishing at rates proportional to $\sqrt{L/n}$ and $(\max\{\log n, L\}/n)^{1/2}$, respectively.**

***Index Terms*— Best-effort, energy-harvesting channel, escape probability, finite blocklength, save-and-transmit.**

I. Introduction

IN THIS paper, we consider communication over an energy-harvesting (EH) channel which has an input alphabet $\mathcal{X}$, an output alphabet $\mathcal{Y}$ and an infinite-sized battery that stores energy harvested from the environment. The channel law of the EH channel is characterized by a conditional distribution $q_{Y|X}$ where $X \in \mathcal{X}$ and $Y \in \mathcal{Y}$ denote the channel input and output respectively. A source node wants to transmit a message to a destination node through the EH channel. Let $\mathrm{c} : \mathcal{X} \to [0, \infty)$ be a cost function associated with the EH channel, where $\mathrm{c}(x)$ represents the amount of energy used for transmitting $x \in \mathcal{X}$. At each discrete time $k \in \{1, 2, \ldots\}$, a random amount of energy E_k arrives at the battery buffer and the source transmits a symbol $X_k \in \mathcal{X}$ such that

$$\sum_{i=1}^{k} \mathrm{c}(X_i) \le \sum_{i=1}^{k} E_i \qquad \text{almost surely.} \tag{1}$$

This implies that the total harvested energy $\sum_{i=1}^{k} E_i$ must be no smaller than the "energy" of the codeword $\sum_{i=1}^{k} \mathrm{c}(X_i)$ at every discrete time k for transmission to take place successfully. The destination receives Y_k from the channel output in time slot k for each $k \in \{1, 2, \ldots\}$, where (X_k, Y_k) is distributed according to the channel law such that

$$p_{Y_k|X_k}(y_k|x_k) = q_{Y|X}(y_k|x_k)$$

for all $(x_k, y_k) \in \mathcal{X} \times \mathcal{Y}$. We assume that $\{E_i\}_{i=1}^{\infty}$ are independent and identically distributed (i.i.d.), where E_1 is a non-negative random variable. To simplify notation, we write $E \triangleq E_1$ if there is no ambiguity. Throughout the paper, we let

$$P \triangleq \mathbb{E}[E],$$

the expected value of E, denote the battery recharge rate, and we assume that $\mathbb{E}[E^2] < \infty$. All results presented in this paper depend on the random variable E only through its first and second moments rather than its distribution.

This paper focuses on the additive white Gaussian noise (AWGN) model where $\mathcal{X} = \mathcal{Y} = \mathbb{R}$,

$$q_{Y|X}(y|x) \equiv \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{(y-x)^2}{2}}$$

and $\mathrm{c}(x) \equiv x^2$. Under the AWGN model, the received symbol at time k can be expressed as

$$Y_k = X_k + Z_k \tag{2}$$

for each time k where Z_k is a standard normal random variable which is independent of X_k and the random variables $\{Z_k\}_{k=1}^{\infty}$ are independent. Reference [1] has shown that the capacity of this channel is $\frac{1}{2}\log(1+P)$ and proposed two capacity-achieving schemes, namely *save-and-transmit* and *best-effort*.

The save-and-transmit scheme consists of an initial saving phase and a subsequent transmission phase. The transmitter remains silent in the saving phase so that energy accumulates in the battery. In the transmission phase, the transmitter sends the symbols of a random Gaussian codeword with

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S. L. Fong is with the Department of Electrical and Computer Engineering, University of Toronto, Toronto, ON M5S 3G4, Canada (e-mail: silas.fong@utoronto.ca).

J. Yang and A. Yener are with the Department of Electrical Engineering, The Pennsylvania State University, University Park, PA 16802 USA (e-mail: yangjing@psu.edu; yener@engr.psu.edu).

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variance $P - \nu$ as long as the battery has sufficient energy where $\nu \in [0, P)$ denotes some small offset from P.

The best-effort scheme has a simpler design than the save-and-transmit scheme as it does not have an initial saving phase. As long as the transmitter has sufficient energy to output the symbols of a random Gaussian codeword with variance $P - \nu$ for some $\nu \in [0, P)$, information gets transmitted.

Following reference [1], a number of non-asymptotic achievable rates for the save-and-transmit scheme have been presented in references [2]–[4]. By contrast, no non-asymptotic achievable rate exists for the best-effort scheme except for a special discrete memoryless EH channel with infinite battery studied in [5] and a special discrete memoryless EH channel with no battery studied in [6]. A main goal of this paper is to provide a non-asymptotic achievable rate for save-and-transmit with a saving phase of arbitrary length, which will immediately imply a non-asymptotic achievable rate for best-effort. Note that the results in this paper cease to hold if the size of the battery is finite. The channel capacity for the finite battery case is the subject of recent interests, see [7]–[9].

A. Notation

Throughout this paper, we use the following Bachmann-Landau notations with an extra positivity condition to describe finite blocklength results: $O_n(a_n)$, $\Theta_n(b_n)$, $\omega_n(c_n)$ and $o_n(d_n)$ are sequences of *positive* real numbers in n that satisfy

$$\limsup_{n\to\infty} \frac{O_n(a_n)}{a_n} < \infty,$$

$$0 < \liminf_{n\to\infty} \frac{\Theta_n(b_n)}{b_n} \leq \limsup_{n\to\infty} \frac{\Theta_n(b_n)}{b_n} < \infty,$$

$$\limsup_{n\to\infty} \frac{\omega_n(c_n)}{c_n} = \infty,$$

and

$$\lim_{n\to\infty} \frac{o_n(d_n)}{d_n} = 0$$

respectively. The sets of natural numbers, real numbers and non-negative real numbers are denoted by $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{R}_+$ respectively. All logarithms are taken to base e.

We use $\mathbb{P}\{\mathcal{E}\}$ to represent the probability of an event $\mathcal{E}$, and we let $\mathbf{1}\{\mathcal{E}\}$ be the indicator function of $\mathcal{E}$. Random variables are denoted by capital letters (e.g., X), and the realization and the alphabet of a random variable are denoted by the corresponding small letter (e.g., x) and calligraphic font (e.g., $\mathcal{X}$) respectively. We use X^n to denote a random tuple $(X_1, X_2, \ldots, X_n)$, where all of the elements X_k have the same alphabet $\mathcal{X}$. We let p_X and $p_{Y|X}$ denote the probability distribution of X and the conditional probability distribution of Y given X respectively for random variables X and Y. We let $p_X p_{Y|X}$ denote the joint distribution of (X, Y), i.e., $p_X p_{Y|X}(x, y) = p_X(x) p_{Y|X}(y|x)$ for all x and y. For random variable $X \sim p_X$ and any real-valued function g whose domain includes $\mathcal{X}$, we let

$$\mathbb{P}_{p_X}\{g(X) \geq \xi\} \triangleq \int_{\mathcal{X}} p_X(x) \times \mathbf{1}\{g(x) \geq \xi\}\, \mathrm{d}x$$

for any real constant ξ. For any function f whose domain contains $\mathcal{X}$, we use $\mathbb{E}_{p_X}[f(X)]$ to denote the expectation of $f(X)$ where X is distributed according to p_X. For simplicity, we omit the subscript of a notation when there is no ambiguity. The Euclidean norm of a tuple $a^L \in \mathbb{R}^L$ is denoted by

$$\|a^L\| \triangleq \sqrt{\sum_{\ell=1}^{L} a_\ell^2}.$$

The distribution of a Gaussian random variable Z whose mean and variance are μ and σ^2 respectively is denoted by

$$\mathcal{N}(z; \mu, \sigma^2) \triangleq \frac{1}{\sqrt{2\pi\sigma^2}} \mathrm{e}^{-\frac{(z-\mu)^2}{2\sigma^2}}.$$

The cumulative distribution function (cdf) of the standard normal random variable is denoted by Φ where

$$\Phi(t) = \int_{-\infty}^{t} \mathcal{N}(z; 0, 1)\, \mathrm{d}z.$$

The inverse of Φ is denoted by Φ^{-1}.

B. Related Work

The channel capacity of the AWGN EH channel is characterized in [1]. This reference has shown that the capacity of the AWGN channel with an infinite-sized battery subject to EH constraints is equal to the capacity of the same channel with an average power constraint equal to the average recharge rate of the battery. In particular, *save-and-transmit* [1, Sec. IV] and *best-effort* [1, Sec. V] are proposed as capacity-achieving strategies.

For a fixed tolerable error probability ε, reference [2] has performed a finite blocklength analysis of save-and-transmit proposed in [1] and obtained a non-asymptotic achievable rate for the AWGN EH channel. The first-, second- and third-order terms of the non-asymptotic achievable rate presented in [2, Th. 1] are equal to the capacity, $-O_n\left(\sqrt{\frac{\log n}{n}}\right)$ and $-O_n\left(\sqrt{\frac{2+\varepsilon}{n\varepsilon}}\right)$ respectively. Subsequently, reference [3] has refined the analysis in [2] and improved the second-order term to $-O_n(1/\sqrt{n\varepsilon})$. Reference [4] has further improved the second-order term to

$$\sup_{\substack{\varepsilon_1\geq 0,\varepsilon_2\geq 0,\\ \varepsilon_1+\varepsilon_2=\varepsilon}} -O_n\left(\sqrt{\frac{\log(1/\varepsilon_2)}{n}}\right) + \sqrt{\frac{P}{n(P+1)}}\,\Phi^{-1}(\varepsilon_1)$$

for $\varepsilon \in (0, 1/2)$. All the second-order terms obtained by the above studies and the current study are inferior (more negative) to the second-order term $\sqrt{\frac{P(P+2)}{2n(P+1)^2}}\Phi^{-1}(\varepsilon)$ [10, Th. 54] corresponding to the non-EH AWGN channel where all energy is available to the transmitter at the onset and (1) is replaced with the conventional power constraint

$$\mathbb{P}\left\{\frac{1}{n}\sum_{k=1}^{n} X_k^2 \leq nP\right\} = 1.$$

For the block energy arrival model where the length of each energy block L grows sublinearly in n [4], [11], [12], reference [4] has proved that save-and-transmit achieves the second-order term $-O_n\left(\sqrt{\frac{\log(1/\varepsilon)L}{n}}\right)$ for $\varepsilon \in (0, 1/2)$. In addition, a non-asymptotic upper bound $\frac{\sqrt{2P^2+\mathbb{E}[E^2]}}{2(P+1)}\Phi^{-1}(\varepsilon) \times \sqrt{\frac{L}{n}}$ on the second-order term has been proved in [4] for a general coding scheme, implying that save-and-transmit achieves the optimal second-order scaling $-O_n(\sqrt{L/n})$ for $\varepsilon \in (0, 1/2)$.

C. Main Contributions

In this paper, we analyze a save-and-transmit scheme with a saving phase of arbitrary length and derive a non-asymptotic achievable rate. We present the best-effort scheme as a special case of save-and-transmit where the duration of the saving phase is zero. The derivation involves designing the transmit power to be strictly less than the battery recharge rate P[1] so that we can effectively bound the number of mismatched positions between the desired transmitted codeword and the actual transmitted codeword subject to a fixed blocklength. The aforementioned non-asymptotic achievable rate is extended to the block energy arrival model [4], [11], [12] where the length of each energy block L grows sublinearly in n. Our analyzed best-effort and save-and-transmit achieve the second-order scalings $-O_n\left(\sqrt{\frac{\max\{\log n, L\}}{n}}\right)$ and $-O_n(\sqrt{L/n})$ respectively. For $\varepsilon \in (0, 1/2)$, the second-order term for a general coding scheme has been proved to be bounded above by $-O_n(\sqrt{L/n})$ as explained in the previous subsection. This implies that both analyzed schemes achieve the optimal second-order scaling $-O_n(\sqrt{L/n})$ if L grows faster than $\log n$.

In order to compare our results with the existing ones, we focus on the i.i.d. energy arrival case (i.e., $L = 1$) in the remainder of this subsection. We provide the first finite blocklength analysis of the best-effort scheme for the AWGN EH channel and presents a non-asymptotic achievable rate. The first- and second-order terms of the asymptotic achievable rate turn out to be the capacity and $-O_n\left(\sqrt{\frac{\log(1/\varepsilon)\log n}{n}}\right)$ respectively. This second-order scaling $-O_n\left(\sqrt{\frac{\log(1/\varepsilon)\log n}{n}}\right)$ significantly improves the state-of-the-art result in [1] which does not derive a bound on the vanishing rate for the second-order term. In addition, this work obtains a new non-asymptotic achievable rate for save-and-transmit. When P is large, the new non-asymptotic rate outperforms the state-of-the-art result for save-and-transmit [4, Th. 1] that always sets the transmit power equal to P.

D. Paper Outline

The remainder of this paper is organized as follows. Section II presents the model of the AWGN EH channel. Section III describes the save-and-transmit scheme, states the corresponding preliminary results, and presents the main result — a new non-asymptotic achievable rate for save-and-transmit with a saving phase of arbitrary length. A non-asymptotic achievable rate for best-effort is then obtained by setting the length of the saving phase to zero. Section IV generalizes the non-asymptotic results in Section III to the block energy arrival model. Section V presents the proof of the new non-asymptotic achievable rate for save-and-transmit for the block energy arrival model which subsumes the proof for the i.i.d. energy arrival model. Section VI contains numerical results which demonstrate the performance advantage of allowing the transmit power for a save-and-transmit to back off from the battery recharge rate in the high battery recharge rate regime for both i.i.d. and block energy arrivals. Section VII concludes the paper.

[1]This is unlike the design in [2], [4] which sets the transmit power equal to P

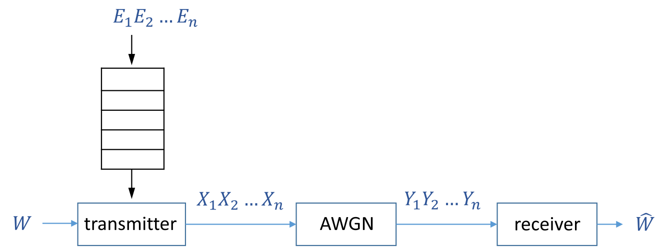


Fig. 1. The AWGN EH channel.

II. The AWGN EH Channel

A. Problem Formulation

The AWGN EH channel, as illustrated in Figure 1, consists of one transmitter and one receiver. Energy harvesting and communication occur in n time slots, i.e., channel uses. In each time slot, a random amount of energy E with alphabet $\mathbb{R}_+$ is harvested where

$$0 < P = \mathbb{E}[E] \quad \text{and} \quad \mathbb{E}[E^2] < \infty. \tag{3}$$

The energy-harvesting process is characterized by n independent copies of E denoted by $E_1, E_2, \ldots, E_n$. Prior to communication, the transmitter chooses a message W. For each $k \in \{1, 2, \ldots, n\}$, the transmitter consumes X_k^2 units of energy to transmit $X_k \in \mathbb{R}$ based on (W, E^k) and the receiver observes $Y_k \in \mathbb{R}$ in time slot k. The energy state information E_k is known by the transmitter at time k before encoding X_k, but the receiver has no access to E_k. For each $k \in \{1, 2, \ldots, n\}$, we have:

(i) E_k and $(W, E^{k-1}, X^{k-1}, Y^{k-1})$ are independent, i.e.,

$$p_{W,E^k,X^{k-1},Y^{k-1}} = p_{E_k} p_{W,E^{k-1},X^{k-1},Y^{k-1}}. \tag{4}$$

(ii) For $w \in \mathcal{W}$ and every $e^n \in \mathbb{R}_+^n$, a transmitted codeword X^n should satisfy

$$\mathbb{P}\left\{ \sum_{i=1}^{k} X_i^2 \le \sum_{i=1}^{k} e_i \,\middle|\, W = w, E^n = e^n \right\} = 1 \tag{5}$$

for each $k \in \{1, 2, \ldots, n\}$.

After the n time slots, the receiver declares $\hat{W}$ to be the transmitted W based on Y^n.

B. Standard Definitions

Formally, we define a code as follows:

Definition 1: An (n, M)*-code* consists of the following:

1) A message set $\mathcal{W} \triangleq \{1, 2, \ldots, M\}$, where W is uniform on $\mathcal{W}$.

2) A sequence of encoding functions $f_k : \mathcal{W} \times \mathbb{R}_+^k \to \mathbb{R}$ for each $k \in \{1, 2, \ldots, n\}$, where f_k is used by the transmitter at time slot k for encoding X_k according to $X_k = f_k(W, E^k)$.
3) A decoding function $\varphi : \mathbb{R}^n \to \mathcal{W}$ for decoding W at the receiver, i.e., $\hat{W} = \varphi(Y^n)$.

If the sequence of encoding functions f_i satisfies (5), the code is also called an *(n, M)-EH code*.

If an (n, M)-code does not satisfy the EH constraints (5) during the encoding process (i.e., X^n is a function of W alone), then the (n, M)-EH code can be viewed as an (n, M)-code for the usual AWGN channel without any cost constraint [13], [14]. The following definition is a formal statement of the channel law (2).

Definition 2: The *AWGN EH channel* is characterized by a conditional probability distribution $q_{Y|X}(y|x) \triangleq \mathcal{N}(y; x, 1)$ such that the following holds for any (n, M)-code: For each $k \in \{1, 2, \ldots, n\}$,

$$p_{W,E^k,X^k,Y^k} = p_{W,E^k,X^k,Y^{k-1}} p_{Y_k|X_k}$$

where

$$p_{Y_k|X_k}(y_k|x_k) = q_{Y|X}(y_k|x_k) = \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{(y_k - x_k)^2}{2}}$$

for all $x_k \in \mathcal{X}$ and $y_k \in \mathcal{Y}$.

For any (n, M)-code defined on the AWGN EH channel, let $p_{W,E^n,X^n,Y^n,\hat{W}}$ be the joint distribution induced by the code. We can factorize $p_{W,E^n,X^n,Y^n,\hat{W}}$ as

$$p_{W,E^n,X^n,Y^n,\hat{W}} = p_W \left(\prod_{k=1}^{n} p_{E_k} p_{X_k|W,E^k} p_{Y_k|X_k} \right) p_{\hat{W}|Y^n}, \quad (6)$$

which follows from the i.i.d. assumption of the EH process E^n in (4), the fact that X_i is a function of (W, E^i) (cf. Definition 1) and the memoryless property of the channel $q_{Y|X}$ described in Definition 2.

Definition 3: For an (n, M)-code defined on the AWGN EH channel, we can calculate according to (6) the *average probability of decoding error* defined as $\mathbb{P}\{\hat{W} \neq W\}$. We call an (n, M)-EH code with average probability of decoding error no larger than ε an *(n, M, ε)-EH code*.

Definition 4: Let $\varepsilon \in (0, 1)$ be a real number. A rate R is said to be *ε-achievable* for the EH channel if there exists a sequence of (n, M_n, ε)-EH codes such that

$$\liminf_{n\to\infty} \frac{1}{n} \log M_n \geq R.$$

Definition 5: The *ε-capacity* of the AWGN EH channel, denoted by C_ε, is defined to be

$$C_\varepsilon \triangleq \sup\{R : R \text{ is } \varepsilon\text{-achievable for the EH channel}\}.$$

The *capacity* of the AWGN EH channel is $C \triangleq \inf_{\varepsilon>0} C_\varepsilon$.

Define the capacity function

$$\mathrm{C}(x) \triangleq \frac{1}{2} \log(1 + x)$$

for all $x \geq 0$. It was shown in [1, Sec. III] (see also [2, Remark 1]) that

$$C_\varepsilon = C = \mathrm{C}(P)$$

for all $\varepsilon \in (0, 1)$ where $P = \mathbb{E}[E]$ can be interpreted as the signal-to-noise ratio (SNR) of the AWGN EH channel.

III. An Achievable Rate for Save-and-Transmit

In this section, we present a non-asymptotic achievable rate for save-and-transmit. To this end, we first formally describe save-and-transmit in the following subsection.

A. Save-and-Transmit Scheme

Fix a blocklength n. Choose a positive $S < P = \mathbb{E}[E]$ that may depend on n and let

$$p_X(x) \equiv \mathcal{N}(x; 0, S) \quad (7)$$

such that $S = \mathbb{E}_{p_X}[X^2]$. The codebook consists of M mutually independent random codewords, which are constructed as follows. For each message $w \in \mathcal{W}$, a length-n codeword $X^n(w) \triangleq (X_1(w), X_2(w), \ldots X_n(w))$ consisting of n i.i.d. symbols is constructed where $X_k(w) \sim p_X$, $k \in 1, 2, \ldots, n$. In other words, the codebook consists of M i.i.d. Gaussian codewords where each codeword consists of n i.i.d. Gaussian random variables with mean S.

Suppose the transmitter chooses message $w \in \mathcal{W}$ and the realization of E^n is $e^n \in \mathbb{R}_+^n$, i.e., $W = w$ and $E^n = e^n$. Then, the transmitter uses the following *save-and-transmit (n, M)-EH code* with encoding functions $\{f_k\}_{k=1}^n$ and decoding function φ. The save-and-transmit code consists of an initial saving phase and a subsequent transmission phase. Define m to be the number of time slots in the initial saving phase during which energy is harvested but not consumed and no information is conveyed. Define $f_1, f_2, \ldots, f_n$ according to the following recursive formula:

$$f_k(w, e^k) \triangleq \begin{cases} X_k(w) & \text{if } k > m \text{ and} \\ & \big(X_k(w)\big)^2 \leq e_k + \sum_{i=1}^{k-1} \Big(e_i - \big(f_i(w, e^i)\big)^2\Big), \\ 0 & \text{otherwise.} \end{cases} \quad (8)$$

For each $k \in \{1, 2, \ldots, n\}$, let $\tilde{X}_k(W) \triangleq f_k(W, E^k)$ be the symbol transmitted at time k. By construction,

$$\mathbb{P}\left\{ \sum_{i=1}^{k} \big(\tilde{X}_i(w)\big)^2 \leq \sum_{i=1}^{k} e_i \,\middle|\, W = w, E^n = e^n \right\} = 1$$

for each $k \in \{1, 2, \ldots, n\}$. Upon receiving $\tilde{Y}^n(W) \triangleq (\tilde{Y}_1(W), \tilde{Y}_2(W), \ldots, \tilde{Y}_n(W))$ where $\tilde{Y}_k(W)$ is generated according to

$$\mathbb{P}\{\tilde{Y}_k(W) = b \mid \tilde{X}_k(W) = a\} \equiv q_{Y|X}(b|a), \quad (9)$$

the receiver declares that $\varphi(\tilde{Y}^n(W)) = j$ if j is the unique integer in $\mathcal{W}$ that satisfies

$$\sum_{k=m+1}^{n} \log \frac{q_{Y|X}(\tilde{Y}_k(W)|X_k(j))}{p_Y(\tilde{Y}_k(W))} \geq \log \xi,$$

where p_Y is the marginal distribution of $p_X q_{Y|X}$ and $\log \xi$ is an arbitrary threshold to be carefully chosen later (cf. (65)). Otherwise, the receiver chooses $\varphi(\tilde{Y}^n(W)) \in \mathcal{W}$ according to the uniform distribution. The decoding is successful if $j = W$.

B. Preliminaries

An important quantity that determines the performance of the save-and-transmit (n, M)-EH code is

$$\mathcal{Q}^{(n)}(w) \triangleq \left\{ k \in \{m+1, m+2, \ldots, n\} \,\middle|\, \tilde{X}_k(w) \neq X_k(w) \right\}, \tag{10}$$

which is a random set that specifies the mismatched positions between $\tilde{X}^n(w)$ and $X^n(w)$ during the transmission phase when the chosen message W equals w. The following lemma presents an upper bound on the probability of seeing more than $\gamma + 1$ mismatched positions in the transmission phase. The proof, which is based on analyzing the escape probability of a Markov process, is provided in Appendix A.

Lemma 1: Fix any n and any $\rho \in (0, 1)$ such that

$$\frac{\sqrt{42\rho}}{21} < \frac{\sqrt{1-\rho}}{2}, \tag{11}$$

and fix a save-and-transmit (n, M)-EH code with a length-m saving phase where

$$S \triangleq (1-\rho)P. \tag{12}$$

Define

$$\alpha \triangleq \frac{2\rho P}{\mathbb{E}[E^2] + 3S^2} \tag{13}$$

and

$$\beta \triangleq \frac{\alpha}{1 + 63\alpha S}. \tag{14}$$

For any $\gamma \in \mathbb{R}_+$, we have

$$\mathbb{P}\left\{ |\mathcal{Q}^{(n)}(w)| \geq \gamma + 1 \,\middle|\, W = w \right\} \leq \mathrm{e}^{-(m+\gamma)\left(P\beta + \frac{\alpha^2 \mathbb{E}[E^2]}{2}\right)} \tag{15}$$

for each $w \in \mathcal{W}$.

Remark 1: In the proof of Lemma 1 which is readily seen in Appendix A by setting $L = 1$, $\hat{X}_i = X_i$ and $\hat{E}_i = E_i$, an important step is analyzing the escape probability (70) of the Markov process

$$\left\{ \sum_{i=1}^{m} E_i + \sum_{i=m+1}^{m+k} \left(E_i - X_i^2\right) \right\}_{k=1}^{\tau}$$

where τ is the stopping time when the value of the Markov process hits any negative number $a < 0$.

The following lemma [15] is standard for proving achievability results in the finite blocklength regime and its proof can be found in [16, Th. 3.8.1].

Lemma 2 (Implied by Shannon's bound [15, Th. 1]): Let p_{X^n,Y^n} be the probability distribution of a pair of random variables (X^n, Y^n). Suppose $(X^n(1), Y^n(1)) \sim p_{X^n,Y^n}$, and suppose $X^n(2)$ has the same distribution as $X^n(1)$ and is independent of $Y^n(1)$. Then for each $\delta > 0$ and each $M \in \mathbb{N}$, we have

$$\mathbb{P}\left\{ \log \frac{p_{Y^n|X^n}(Y^n(1)|X^n(2))}{p_{Y^n}(Y^n(1))} > \log M + \delta \right\} \leq \frac{\mathrm{e}^{-\delta}}{M}.$$

The following lemma is a modification of the Shannon's bound stated in the previous lemma, and its proof is provided in Appendix B.

Lemma 3: Suppose we are given a save-and-transmit (n, M)-EH code with a length-m saving phase as described in Section III-A. Then for each $\gamma \geq 0$, each $\delta > 0$ and each $M \in \mathbb{N}$, we have

$$\begin{aligned} &\mathbb{P}\left\{ \left. \begin{array}{l} \left\{ \displaystyle\sum_{k=m+1}^{n} \log \frac{p_{Y_k|X_k}(\tilde{Y}_k(1)|X_k(2))}{p_{Y_k}(\tilde{Y}_k(1))} > \log M + \delta \right\} \\ \cap \left\{ |\mathcal{Q}^{(n)}(1)| < \gamma + 1 \right\} \end{array} \right| W = 1 \right\} \\ &\leq \frac{2\mathrm{e}^{-\delta}}{M} \times \left((n-m)\sqrt{S+1}\right)^{\gamma+1}. \end{aligned}$$

C. A Non-Asymptotic Achievable Rate for Save-and-Transmit

The following theorem is the main result of this paper. The proof relies on Lemma 1 and Lemma 3, and will be presented in Section V.

Theorem 1: Fix an $\varepsilon \in (0, 1)$, fix a natural number n, fix a non-negative integer $m < n$, and fix a $\rho \in (0, 1)$ such that (11) holds. Let

$$n_m \triangleq n - m.$$

Define S, α and β as in (12), (13) and (14) respectively. Let $p_X = \mathcal{N}(x; 0, S)$ and let $p_Y = \mathcal{N}(y; 0, S+1)$ be the marginal distribution of $p_X q_{Y|X}$, and let σ^2 and T denote the variance and the third absolute moment of $\log \frac{q_{Y|X}(Y|X)}{p_Y(Y)}$ respectively. For any $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that $\varepsilon_1 + \varepsilon_2 = \varepsilon$, if n and m satisfy

$$\varepsilon_1 - \frac{T}{\sigma^3 \sqrt{n_m}} - \frac{4}{\sqrt{n_m}} > 0,$$

then there exists a save-and-transmit (n, M)-EH code with a length-m saving phase which satisfies

$$\begin{aligned} &\log M \\ &\geq \frac{n_m}{2} \log(1+S) + \sqrt{n_m \sigma^2}\, \Phi^{-1}\left(\varepsilon_1 - \frac{T}{\sigma^3 \sqrt{n_m}} - \frac{4}{\sqrt{n_m}}\right) \\ &\quad - \left(2S\log 2 + \frac{1}{2}\log(1+S) + (8S+1)\log n_m\right)(\gamma(\varepsilon_2) + 1) \\ &\quad - \log\sqrt{n_m} - 1 \end{aligned}$$

and

$$\mathbb{P}\left\{ \varphi\left(\tilde{Y}^n(W)\right) \neq W \right\} \leq \varepsilon$$

where

$$\gamma(\varepsilon_2) \triangleq \max\left\{ \frac{\log \frac{1}{\varepsilon_2}}{P\beta + \frac{\alpha^2 \mathbb{E}[E^2]}{2}} - m, 0 \right\}.$$

In particular, the probability of seeing more than $\gamma(\varepsilon_2) + 1$ mismatch events can be bounded as

$$\mathbb{P}\left\{ |\mathcal{Q}^{(n)}(W)| \geq \gamma(\varepsilon_2) + 1 \right\} \leq \varepsilon_2.$$

The following corollary is a direct consequence of Theorem 1, and it states a non-asymptotic rate for the save-and-transmit scheme whose second-order term scales as $-O_n(1/\sqrt{n})$. The proof of Corollary 4 is provided in Appendix C.

Corollary 4: Fix an $\varepsilon \in (0, 1/2)$, and fix any $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that $\varepsilon_1 + \varepsilon_2 = \varepsilon$. There exists a constant $\kappa > 0$

which does not depend on n such that for all sufficiently large n, we can construct a save-and-transmit (n, M, ε)-EH code which satisfies

$$\frac{1}{n}\log M \geq \frac{1}{2}\log(1+P) - \sqrt{\frac{(\mathbb{E}[E^2]+3P^2)\log(1+P)\log\frac{1}{\varepsilon_2}}{2nP(P+1)}} + \sqrt{\frac{P}{(P+1)n}}\,\Phi^{-1}(\varepsilon_1) - \frac{\kappa}{n^{3/4}}, \tag{16}$$

with ρ being defined as

$$\rho \triangleq \frac{\sqrt{(P+1)(\mathbb{E}[E^2]+3P^2)\log(1+P)\log\frac{1}{\varepsilon_2}}}{P\sqrt{2nP}} = \Theta_n\left(1/\sqrt{n}\right),$$

the average transmit power S being defined as in (12), α and β being defined as in (13) and (14) respectively, and the length of saving phase m being defined as

$$m \triangleq \left\lceil \frac{\log\frac{1}{\varepsilon_2}}{P\beta + \frac{\alpha^2\mathbb{E}[E^2]}{2}} \right\rceil = \Theta_n\left(\sqrt{n}\right)$$

In particular, the probability of seeing a mismatch event in the transmission phase can be bounded as

$$\mathbb{P}\left\{\bigcup_{k=m+1}^{n}\left\{\sum_{i=1}^{k}E_i < \sum_{i=m+1}^{k}X_i^2\right\}\right\} \leq \varepsilon_2$$

where each term in the union characterizes the event that the accumulated energy collected during the first k time slots is insufficient to output the desired codeword symbols from slot $m+1$ to slot k during the transmission phase.

Remark 2: The parameters ρ and m in Corollary 4 have been carefully chosen to achieve the second-order scaling $-O_n(1/\sqrt{n})$ that is optimal [4, Th. 1]. Fix any $\varepsilon \in (0, 1/2)$. The best existing lower bound on the second-order term of $\frac{1}{n}\log M$ was derived in [4, Th. 1], which states that there exists a save-and-transmit (n, M, ε)-EH code that satisfies

$$\liminf_{n\to\infty}\frac{1}{\sqrt{n}}\left(\log M - \frac{n}{2}\log(1+P)\right) \geq -\frac{\log(1+P)}{2P}\sqrt{(\mathbb{E}[E^2]+P^2)\log\frac{1}{\varepsilon_2}} + \sqrt{\frac{P}{P+1}}\,\Phi^{-1}(\varepsilon_1) \tag{17}$$

for any $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that $\varepsilon_1+\varepsilon_2 = \varepsilon$. The save-and-transmit scheme investigated in [4] is similar to that described in Section III-A except that $S = \mathbb{E}[E] = P$ is assumed in [4] while $S < \mathbb{E}[E] = P$ is assumed in this work. Note that the second-order term of the best existing lower bound as stated on the right-hand side (RHS) of (17) decays as

$$-\frac{1}{2}\log(1+P)\sqrt{\left(1+\frac{\mathbb{E}[E^2]}{P^2}\right)\log\frac{1}{\varepsilon_2}} + \Phi^{-1}(\varepsilon_1)$$

as P tends to infinity. On the other hand, it follows from (16) in Corollary 4 that the second-order term of our lower bound decays as

$$-\sqrt{\frac{1}{2}\left(3+\frac{\mathbb{E}[E^2]}{P^2}\right)\log(1+P)\log\frac{1}{\varepsilon_2}} + \Phi^{-1}(\varepsilon_1)$$

as P tends to infinity. Consequently, the second-order term achievable by the save-and-transmit scheme guaranteed by Corollary 4 is strictly larger (less negative) than the best existing bound for all sufficiently large $P > 0$. In other words, letting S be strictly less than instead of equal to P achieves a higher rate in the high SNR regime.

D. A Non-Asymptotic Achievable Rate for Best-Effort

We call a save-and-transmit scheme a *best-effort scheme* if the duration of the saving phase equals zero, i.e., $m = 0$. By setting $m = 0$, Theorem 1 reduces to the following corollary, which states that the best-effort scheme achieves a non-asymptotic rate whose second-order term scales as $-O_n\big(\sqrt{(\log n)/n}\big)$. The proof of Corollary 5 is provided in Appendix D.

Corollary 5: Fix an $\varepsilon \in (0, 1/2)$, and fix any $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that $\varepsilon_1 + \varepsilon_2 = \varepsilon$. Define

$$\lambda_1 \triangleq 2P\log 2 + \frac{1}{2}\log(1+P) \tag{23}$$

and

$$\lambda_2 \triangleq 8P + 1. \tag{24}$$

There exists a constant $\kappa > 0$ which does not depend on n such that for all sufficiently large n, we can construct a best-effort (n, M, ε)-EH code with

$$\rho \triangleq \frac{\sqrt{(\lambda_1+\lambda_2\log n)(P+1)(\mathbb{E}[E^2]+3P^2)\log\frac{1}{\varepsilon_2}}}{P^{3/2}\sqrt{n}} = \Theta_n\big(\sqrt{(\log n)/n}\big)$$

and

$$S = P(1-\rho) = P - \Theta_n\big(\sqrt{(\log n)/n}\big)$$

which satisfies

$$\frac{1}{n}\log M \geq \frac{1}{2}\log(1+P) - \sqrt{\frac{(\lambda_1+\lambda_2\log n)(\mathbb{E}[E^2]+3P^2)\log\frac{1}{\varepsilon_2}}{P(P+1)n}} - \sqrt{\frac{P}{(P+1)n}}\,\Phi^{-1}(\varepsilon_1) - \frac{\kappa\log n}{n}. \tag{25}$$

In particular, the probability of seeing more than

$$\gamma(\varepsilon_2) \triangleq \frac{\log\frac{1}{\varepsilon_2}}{P\beta + \frac{\alpha^2\mathbb{E}[E^2]}{2}} = \Theta_n\left(\sqrt{\frac{n}{\log n}}\right)$$

mismatch events can be bounded as

$$\mathbb{P}\big\{|\mathcal{Q}^{(n)}(W)| \geq \gamma(\varepsilon_2)+1\big\} \leq \varepsilon_2.$$

Remark 3: Although the achievable second-order scaling for best-effort in Corollary 5 is not optimal (the optimal scaling is $-O_n(1/\sqrt{n})$ [4, Th. 1]), it is a significant improvement compared to the state of the art [1, Sec. V] where the achievable second-order scaling therein for best-effort is $-o_n(1)$.

IV. The Block Energy Arrival Model

In this section, we generalize our achievable rates for save-and-transmit and best-effort to the block energy arrival model [4], [11], [12], which is useful for modeling practical scenarios when the energy-arrival process (e.g., solar energy, wind energy, ambient radio-frequency (RF) energy, etc.) evolves at a slower timescale compared to the transmission process.

A. Block Energy Arrivals

We follow the formulation in [4], which assumes that $\{E_i\}_{i=1}^{\infty}$ arrive at the buffer in a block-by-block manner as follows: For each $\ell \in \mathbb{N}$, let

$$b_\ell \triangleq (\ell-1)L \tag{21}$$

such that $b_\ell + 1$ is the index of the first channel use within the ℓ^{th} block of energy arrivals, where L denotes the length of each block. The EH random variables that mark the starting positions of the blocks (i.e., $\{E_{b_\ell+1}\}_{\ell=1}^{\infty}$) are assumed to be i.i.d. random variables where $E_1 = E$ satisfies (3). In addition, we assume

$$E_{b_\ell+1} = E_{b_\ell+2} = \ldots = E_{b_\ell+L}$$

for all $\ell \in \mathbb{N}$. In other words, the harvested energy in each channel use within a block remains constant while the harvested energy across different blocks is characterized by a sequence of i.i.d. random variables with mean equal to P. By construction, we have the following for each $k \in \{1, 2, \ldots, n\}$ and all $e^k \in \mathbb{R}_+^k$:

$$p_{E_k|E^{k-1}}(e_k|e^{k-1}) = \begin{cases} p_{E_1}(e_k) & \text{if } k = b_\ell + 1 \text{ for some } \ell \in \mathbb{N}, \\ \mathbf{1}\{e_k = e_{k-1}\} & \text{otherwise.} \end{cases}$$

The length of each energy-arrival block L is assumed to remain constant or grow sublinearly in n.

B. Blockwise Save-and-Transmit

Fix a blocklength n and choose an $L = o_n(n)$. Choose a positive real number $S < P = \mathbb{E}[E]$ and let p_X be as defined in (7) such that $S = \mathbb{E}_{p_X}[X^2]$. The codebook consists of M mutually independent random codewords denoted by $\{X^n(w) \mid w \in \mathcal{W}\}$, which are constructed as described in Section III-A. Suppose $W = w$ and $E^n = e^n$. Then, the transmitter uses the following *blockwise save-and-transmit* (n, M)*-EH code* with encoding functions $\{f_k\}_{k=1}^{\bar{n}}$ and decoding function φ where $\bar{n} \triangleq \lceil n/L \rceil$. The saving phase consists of m blocks of L consecutive time slots. Define $f_1, f_2, \ldots, f_{\bar{n}}$ in a recursive manner according to (22) as shown at the bottom of this page. In other words, the transmitter outputs the block of L symbols $\big(X_{b_\ell+1}(w), X_{b_\ell+2}(w), \ldots, X_{b_\ell+L}(w)\big)$ in the transmission phase during time $b_\ell+1$ to $b_\ell+L$ if the energy in the battery at time $b_\ell+1$ (i.e., $\sum_{k=1}^{b_\ell+1} e_k - \sum_{i=1}^{\ell-1} \|f_i(w, e^{b_i+1})\|^2$) can support the transmission of the whole block of symbols starting at time $b_\ell + 1$. If $L = 1$, the blockwise save-and-transmit scheme defined by (22) reduces to the save-and-transmit scheme presented in Section III-A defined by (8). Let $\tilde{X}_k(W)$ be the symbol transmitted at time k for each $k \in \{1, 2, \ldots, n\}$ such that

$$(\tilde{X}_{b_\ell+1}(W), \tilde{X}_{b_\ell+2}(W), \ldots, \tilde{X}_{\min\{b_\ell+L,n\}}(W)) \triangleq f_\ell(W, E^{b_\ell+1}) \tag{23}$$

for each $\ell \in \{1, 2, \ldots, \bar{n}\}$. Upon receiving

$$\tilde{Y}^n(W) \triangleq (\tilde{Y}_1(W), \tilde{Y}_2(W), \ldots, \tilde{Y}_n(W))$$

where $\tilde{Y}_k(W)$ is generated according to (9), the receiver declares that $\varphi(\tilde{Y}^n(W)) = j$ if j is the unique integer in $\mathcal{W}$ that satisfies

$$\sum_{k=mL+1}^{n} \log \frac{q_{Y|X}(\tilde{Y}_k(W)|X_k(j))}{p_Y(\tilde{Y}_k(W))} \geq \log \xi, \tag{24}$$

where p_Y is the marginal distribution of $p_X q_{Y|X}$ and $\log \xi$ is an arbitrary threshold to be carefully chosen later (cf. (65)). Otherwise, the receiver chooses $\varphi(\tilde{Y}^n(W)) \in \mathcal{W}$ according to the uniform distribution. The decoding is successful if $j = W$.

The following lemma is an extension of Lemma 1 which states an upper bound on the probability of seeing more than $L\gamma + 1$ mismatched positions in the transmission phase. The proof of Lemma 6 is contained in Appendix A.

$$f_\ell(w, e^{b_\ell+1}) \triangleq \begin{cases} \big(X_{b_\ell+1}(w), X_{b_\ell+2}(w), \ldots, X_{b_\ell+L}(w)\big) & \text{if } m < \ell < \bar{n} \text{ and } \sum_{j=1}^{L} \big(X_{b_\ell+j}(w)\big)^2 \leq \sum_{k=1}^{b_\ell+1} e_k - \sum_{i=1}^{\ell-1} \big\| f_i(w, e^{b_i+1}) \big\|^2, \\ \big(X_{b_{\bar{n}}+1}(w), X_{b_{\bar{n}}+2}(w), \ldots, X_n(w)\big) & \text{if } \ell = \bar{n} \text{ and } \sum_{k=b_{\bar{n}}+1}^{n} \big(X_k(w)\big)^2 \leq \sum_{k=1}^{b_{\bar{n}}+1} e_k - \sum_{i=1}^{\bar{n}-1} \big\| f_i(w, e^{b_i+1}) \big\|^2, \\ \underbrace{(0, 0, \ldots, 0)}_{\min\{L, n-b_\ell\} \text{ times}} & \text{otherwise.} \end{cases} \tag{22}$$

Lemma 6: Fix any n, any $L \leq n$ and any $\rho \in (0, 1)$ such that (11) holds, and fix a blockwise save-and-transmit (n, M)-EH code with S being defined as in (12). Define

$$\alpha \triangleq \frac{2\rho P}{L\mathbb{E}[E^2] + 3S^2} \tag{25}$$

and

$$\beta \triangleq \frac{\alpha}{1 + 63\alpha S}. \tag{26}$$

For any $\gamma \in \mathbb{R}_+$, we have

$$\mathbb{P}\left\{|\mathcal{Q}^{(n)}(W)| \geq L\gamma + 1\right\} \leq \mathrm{e}^{-L(m+\gamma)\left(P\beta + \frac{L\alpha^2\mathbb{E}[E^2]}{2}\right)}. \tag{27}$$

Remark 4: In the proof of Lemma 6 in Appendix A, an important step is analyzing the escape probability (70) of the Markov process

$$\left\{\sum_{i=1}^{m} LE_{b_i+1} + \sum_{i=m+1}^{m+k} \left(LE_{b_i+1} - \sum_{\ell=1}^{L} X_{b_i+\ell}^2\right)\right\}_{k=1}^{\tau}$$

where τ is the stopping time when the value of the Markov process hits any negative number $a < 0$.

The following lemma is a generalization of Lemma 3. The proof of Lemma 7 is contained in Appendix B.

Lemma 7: Suppose we are given a blockwise save-and-transmit (n, M)-EH code with a saving phase of length mL as described in Section IV-B. Then for each natural number $L < n/m$, each $\gamma \geq 0$, each $\delta > 0$ and each $M \in \mathbb{N}$, we have

$$\mathbb{P}\left\{\left.\begin{array}{l}\left\{\sum_{k=mL+1}^{n} \log \frac{p_{Y_k|X_k}(\tilde{Y}_k(1)|X_k(2))}{p_{Y_k}(\tilde{Y}_k(1))} > \log M + \delta\right\} \\ \cap \left\{|\mathcal{Q}^{(n)}(1)| < L\gamma + 1\right\}\end{array}\right| W = 1\right\}$$
$$\leq \frac{2\mathrm{e}^{-\delta}}{M} \times \left((n - mL)(S+1)^{L/2}\right)^{\gamma+1}. \tag{28}$$

C. A Non-Asymptotic Achievable Rate for Blockwise Save-and-Transmit

The following theorem is the main result under the block energy arrival model. The proof relies on Lemma 6 and Lemma 2, and will be provided in Section V.

Theorem 2: Fix an $\varepsilon \in (0, 1)$, fix a natural number $n \geq 2$, fix a natural number $L \leq n$, fix a non-negative integer $m < n$, and fix a $\rho \in (0, 1)$ such that (11) holds. Let

$$n_m \triangleq n - mL.$$

Define S, α and β as in (12), (25) and (26) respectively. Let $p_X = \mathcal{N}(x; 0, S)$ and let $p_Y = \mathcal{N}(y; 0, S+1)$ be the marginal distribution of $p_X q_{Y|X}$, and let σ^2 and T denote the variance and the third absolute moment of $\log \frac{q_{Y|X}(Y|X)}{p_Y(Y)}$ respectively. For any $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that $\varepsilon_1 + \varepsilon_2 = \varepsilon$, if n and m satisfy

$$\varepsilon_1 - \frac{T}{\sigma^3\sqrt{n_m}} - \frac{4}{\sqrt{n_m}} > 0, \tag{29}$$

then there exists a blockwise save-and-transmit (n, M)-EH code with a saving phase of length mL such that

$$\begin{aligned}&\log M \\ &\geq \frac{n_m}{2}\log(1+S) + \sqrt{n_m\sigma^2}\,\Phi^{-1}\left(\varepsilon_1 - \frac{T}{\sigma^3\sqrt{n_m}} - \frac{4}{\sqrt{n_m}}\right) \\ &- \left(L\left(2S\log 2 + \frac{\log(1+S)}{2}\right) + (8S+1)\log n_m\right)(\gamma(\varepsilon_2)+1) \\ &- \log\sqrt{n_m} - 1\end{aligned} \tag{30}$$

and

$$\mathbb{P}\left\{\varphi\left(\tilde{Y}^n(W)\right) \neq W\right\} \leq \varepsilon \tag{31}$$

where

$$\gamma(\varepsilon_2) \triangleq \max\left\{\frac{\log\frac{1}{\varepsilon_2}}{LP\beta + \frac{L^2\alpha^2\mathbb{E}[E^2]}{2}} - m, 0\right\}. \tag{32}$$

In particular, the probability of seeing more than $L\gamma(\varepsilon_2) + 1$ mismatch events can be bounded as

$$\mathbb{P}\left\{|\mathcal{Q}^{(n)}(W)| \geq L\gamma(\varepsilon_2) + 1\right\} \leq \varepsilon_2. \tag{33}$$

The following corollary is a direct consequence of Theorem 2, and it states a non-asymptotic rate for the blockwise save-and-transmit scheme whose second-order term scales as $-O_n(\sqrt{L/n})$. The proof of Corollary 8 is provided in Appendix C.

Corollary 8: Fix an $\varepsilon \in (0, 1/2)$, and fix any $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that $\varepsilon_1 + \varepsilon_2 = \varepsilon$. Suppose $L = o_n(n)$. There exists a constant $\kappa > 0$ which does not depend on n such that for all sufficiently large n, we can construct a blockwise save-and-transmit (n, M, ε)-EH code that satisfies

$$\begin{aligned}&\frac{1}{n}\log M \\ &\geq \frac{1}{2}\log(1+P) - \sqrt{\frac{(L\mathbb{E}[E^2] + 3P^2)\log(1+P)\log\frac{1}{\varepsilon_2}}{2nP(P+1)}} \\ &+ \sqrt{\frac{P}{(P+1)n}}\,\Phi^{-1}(\varepsilon_1) - \kappa\max\left\{\frac{L^{1/4}}{n^{3/4}}, \frac{L}{n}\right\},\end{aligned} \tag{34}$$

with ρ being defined as

$$\begin{aligned}\rho &\triangleq \frac{\sqrt{(P+1)(L\mathbb{E}[E^2] + 3P^2)\log(1+P)\log\frac{1}{\varepsilon_2}}}{P\sqrt{2nP}} \\ &= \Theta_n\left(\sqrt{L/n}\right),\end{aligned} \tag{35}$$

the average transmit power S being defined as in (12), α and β being defined as in (25) and (26) respectively, and the length of saving phase mL being defined as

$$mL \triangleq L\left\lceil\frac{\log\frac{1}{\varepsilon_2}}{LP\beta + \frac{L^2\alpha^2\mathbb{E}[E^2]}{2}}\right\rceil = \Theta(\sqrt{nL}). \tag{36}$$

In particular, the probability of seeing a mismatch event in the transmission phase can be bounded as

$$\mathbb{P}\left\{\bigcup_{k=mL+1}^{n}\left\{\sum_{i=1}^{k} E_i < \sum_{i=mL+1}^{k} X_i^2\right\}\right\} \leq \varepsilon_2. \tag{37}$$

The following result is a direct consequence of Corollary 8.

Theorem 3: Fix any $\varepsilon \in (0, 1/2)$. Suppose

$$L = \omega_n(1) \cap o_n(n),$$

i.e.,

$$\lim_{n\to\infty} \frac{1}{L} = \lim_{n\to\infty} \frac{L}{n} = 0.$$

Then for all sufficiently large n, there exists a blockwise save-and-transmit (n, M, ε)-EH code such that

$$\frac{1}{n}\log M \geq \frac{1}{2}\log(1+P) - \sqrt{\frac{\mathbb{E}[E^2]\log(1+P)\log\frac{1}{\varepsilon}}{2P(P+1)}} \cdot \sqrt{\frac{L}{n}} - o_n\left(\sqrt{\frac{L}{n}}\right). \tag{38}$$

Proof: It follows from Corollary 8 that for all sufficiently large n, there exists a blockwise save-and-transmit (n, M, ε)-EH code that satisfies (34), which together with the hypothesis regarding L implies (38). ■

Remark 5: Fix any $\varepsilon \in (0, 1/2)$ and fix any $L = \omega_n(1) \cap o_n(n)$. The best existing lower bound on the second-order term of $\frac{1}{n}\log M$ was derived in [4, Th. 1], which states that there exists a save-and-transmit (n, M, ε)-EH code that satisfies

$$\liminf_{n\to\infty} \frac{1}{\sqrt{Ln}}\left(\log M - \frac{n}{2}\log(1+P)\right) \geq -\frac{\log(1+P)}{2P}\sqrt{(\mathbb{E}[E^2]+P^2)\log\frac{1}{\varepsilon}}. \tag{39}$$

The blockwise save-and-transmit scheme investigated in [4] is similar to that described in Section IV-B except that $S = \mathbb{E}[E] = P$ is assumed in [4] while $S < \mathbb{E}[E] = P$ is assumed in this work. Note that the second-order term of the best existing lower bound as stated on the RHS of (39) decays as

$$-\frac{1}{2}\log(1+P)\sqrt{\left(1+\frac{\mathbb{E}[E^2]}{P^2}\right)\log\frac{1}{\varepsilon}}$$

as P tends to infinity. On the other hand, it follows from (38) in Theorem 3 that the second-order term of our lower bound decays as

$$-\sqrt{\frac{\mathbb{E}[E^2]}{2P^2}\log(1+P)\log\frac{1}{\varepsilon}}$$

as P tends to infinity. Consequently, the second-order term achievable by the save-and-transmit scheme guaranteed by Theorem 3 is strictly larger (less negative) than the best existing bound for all sufficiently large $P > 0$.

D. A Non-Asymptotic Achievable Rate for Blockwise Best-Effort

We call a blockwise save-and-transmit scheme a *blockwise best-effort scheme* if the length of saving phase equals zero, i.e., $m = 0$. By setting $m = 0$, Theorem 2 reduces to the following corollary, which states that blockwise best-effort achieves a non-asymptotic rate whose second-order term scales as $-O_n\left(\sqrt{\max\{\log n, L\}/n}\right)$. The proof of Corollary 9 is provided in Appendix D.

Corollary 9: Fix an $\varepsilon \in (0, 1/2)$, and fix any $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that $\varepsilon_1 + \varepsilon_2 = \varepsilon$. Define λ_1 and λ_2 as in (23) and (24) respectively. There exists a constant $\kappa > 0$ which does not depend on n such that for all sufficiently large n and any $L \leq n$, we can construct a blockwise best-effort (n, M, ε)-EH code with

$$\rho \triangleq \frac{\sqrt{(\lambda_1 L + \lambda_2\log n)(P+1)(L\mathbb{E}[E^2]+3P^2)\log\frac{1}{\varepsilon_2}}}{P\sqrt{PLn}} = \Theta_n\left(\sqrt{\max\{\log n, L\}/n}\right) \tag{40}$$

and

$$S = P(1-\rho) = P - \Theta_n\left(\sqrt{\max\{\log n, L\}/n}\right)$$

which satisfies

$$\frac{1}{n}\log M \geq \frac{1}{2}\log(1+P) - \sqrt{\frac{(\lambda_1 L+\lambda_2\log n)(L\mathbb{E}[E^2]+3P^2)\log\frac{1}{\varepsilon_2}}{LP(P+1)n}} - \sqrt{\frac{P}{(P+1)n}}\,\Phi^{-1}(\varepsilon_1) - \frac{\kappa\max\{\log n, L\}}{n}. \tag{41}$$

In particular, the probability of seeing more than $L\gamma(\varepsilon_2)+1$ mismatch events with

$$\gamma(\varepsilon_2) \triangleq \frac{\log\frac{1}{\varepsilon_2}}{LP\beta + \frac{L^2\alpha^2\mathbb{E}[E^2]}{2}} = \Theta_n\left(\sqrt{\frac{n}{\max\{\log n, L\}}}\right) \tag{42}$$

can be bounded as

$$\mathbb{P}\left\{|\mathcal{Q}^{(n)}(W)| \geq L\gamma(\varepsilon_2)+1\right\} \leq \varepsilon_2. \tag{43}$$

Remark 6: The parameters ρ and $\gamma(\varepsilon_2)$ in Corollary 9 have been optimized to achieve the second-order scaling $-O_n\left(\sqrt{\max\{\log n, L\}/n}\right)$.

The following result is a direct consequence of Corollary 9.

Theorem 4: Fix any $\varepsilon \in (0, 1/2)$. Suppose $L = \omega_n(\log n) \cap o_n(n)$, i.e., $\lim_{n\to\infty}\frac{\log n}{L} = \lim_{n\to\infty}\frac{L}{n} = 0$. Then for all sufficiently large n, there exists a blockwise best-effort (n, M, ε)-EH code such that

$$\frac{1}{n}\log M \geq \frac{1}{2}\log(1+P) - \sqrt{\frac{\left(2P\log 2+\frac{1}{2}\log(1+P)\right)\mathbb{E}[E^2]\log\frac{1}{\varepsilon}}{P(P+1)}} \cdot \sqrt{\frac{L}{n}} - o_n\left(\sqrt{L/n}\right). \tag{44}$$

Proof: It follows from Corollary 9 that for all sufficiently large n, there exists a blockwise best-effort (n, M, ε)-EH code that satisfies (41) where ε_1 and ε_2 are chosen to be ε/n and $\varepsilon(1-1/n)$ respectively, which together with the definitions of λ_1 and λ_2 in (23) and (24) and the hypothesis regarding L implies (44). ■

Remark 7: If $L = \omega_n(\log n) \cap o_n(n)$, the achievable second-order scaling for blockwise best-effort in Theorem 4 is $O_n(\sqrt{L/n})$ which is optimal [4, Th. 1]. However, we can see from Theorem 3 and Theorem 4 that blockwise best-effort always achieves a smaller (more negative) coefficient for the second-order term than save-and-transmit.

V. Proofs of Theorem 1 and Theorem 2

Since save-and-transmit defined in Section III-A is a special case of blockwise save-and-transmit defined in Section IV-B with $L = 1$ and Theorem 1 is a special case of Theorem 2 with $L = 1$, it suffices to prove Theorem 2.

Fix an $\varepsilon \in (0, 1)$ and any $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that $\varepsilon_1 + \varepsilon_2 = \varepsilon$. Fix an $n \in \mathbb{N}$, an $L < n$, a non-negative integer $m \le n$ and a $\rho \in (0, 1)$ that satisfies (11). Consider a blockwise save-and-transmit (n, M)-code described in Section IV-B where the corresponding S and p_X are defined according to (12) and (7) respectively. In addition, let $p_Y(y) = \mathcal{N}(y; 0, S+1)$ be the marginal distribution of $p_X q_{Y|X}$, and define α, β and $\gamma(\varepsilon_2)$ as in (25), (26) and (32) respectively. Consider the probability of decoding error

$$\begin{aligned}&\mathbb{P}\left\{\left\{\varphi\left(\tilde{Y}^n(W)\right) \neq W\right\}\right\}\\&\le \mathbb{P}\left\{\left\{\varphi\left(\tilde{Y}^n(W)\right) \neq W\right\} \cap \left\{|\mathcal{Q}^{(n)}(W)| < L\gamma(\varepsilon_2)+1\right\}\right\} + \varepsilon_2\end{aligned} \tag{45}$$

which is due to the union bound and the following fact by Lemma 6 (Lemma 1 suffices for the case $L = 1$) and the definition of $\gamma(\varepsilon_2)$ in (32):

$$\mathbb{P}\left\{|\mathcal{Q}^{(n)}(W)| \ge L\gamma(\varepsilon_2)+1\right\} \le \mathrm{e}^{-\log\frac{1}{\varepsilon_2}} = \varepsilon_2. \tag{46}$$

Recall that $n_m = n - mL$ and $b_\ell + 1$ (which was defined in (21)) denotes the first channel use within the ℓ^{th} block of energy arrivals. Using the convention that $X_k(1) = 0$ deterministically for all $k > n$, it follows from the code construction that

$$\begin{aligned}&\mathbb{P}\left\{\begin{array}{l}\max\limits_{m+1\le\ell\le\lceil n/L\rceil}\left\|(X_{b_\ell+1}(1), X_{b_\ell+2}(1), \ldots, X_{b_\ell+L}(1))\right\|^2\\ \quad \ge 2S(L\log 2 + 3\log n_m)\end{array}\right\}\\
&\le \frac{n_m}{L}\,\mathbb{P}_{p_{X^n}}\left\{\sum_{k=1}^{L} X_k^2 \ge 2S(L\log 2 + 3\log n_m)\right\}\\
&= \frac{n_m}{L}\,\mathbb{P}_{p_{X^n}}\left\{\mathrm{e}^{\sum_{k=1}^{L}\frac{X_k^2}{4S}} \ge n_m^{3/2} 2^{L/2}\right\}\\
&\le \frac{n_m}{L} \times \frac{1}{n_m^{3/2}2^{L/2}}\left(\mathbb{E}_{p_X}\left[\mathrm{e}^{\frac{X^2}{4S}}\right]\right)^L \qquad (47)\\
&\le \frac{1}{\sqrt{n_m}}, \qquad (48)\end{aligned}$$

where (47) follows from Markov's inequality and (48) is due to the fact that $X \sim \mathcal{N}(x; 0, S)$. To simplify notation, define

$$\Delta \triangleq L\log 2 + 3\log n_m \tag{49}$$

and

$$\tilde{\Delta} \triangleq L\log 2 + 4\log n_m, \tag{50}$$

and define the events

$$\mathcal{E}_1 \triangleq \left\{|\mathcal{Q}^{(n)}(1)| < L\gamma(\varepsilon_2)+1\right\} \tag{51}$$

and

$$\mathcal{E}_2 \triangleq \left\{\max_{\ell\in\{m+1,m+2,\ldots,\lceil n/L\rceil\}}\sum_{j=1}^{L}(X_{b_\ell+j}(1))^2 < 2S\Delta\right\}.$$

In addition, define

$$\begin{aligned}\imath(a;b) &\triangleq \log\frac{q_{Y|X}(b|a)}{p_Y(b)}\\&= \frac{1}{2}\log(1+S) + \frac{-S^2(b-a)^2 + 2a(b-a) + a^2}{2(S+1)}\end{aligned} \tag{52}$$

for all $(a, b) \in \mathbb{R}_+^2$ where $\imath(a; b)$ is used in the decoding rule specified by (24). Following (45) and letting $\xi > 0$ be an arbitrary positive number to be determined later in (65), we obtain from the symmetry of the codebook, the encoding rule (22), the decoding rule (24), the union bound and (48) that

$$\begin{aligned}&\mathbb{P}\left\{\left\{\varphi\left(\tilde{Y}^n(W)\right) \neq W\right\} \cap \{|\mathcal{Q}^{(n)}(W)| < L\gamma(\varepsilon_2)+1\}\right\}\\
&= \mathbb{P}\left\{\begin{array}{l}\left\{\varphi\left(\tilde{Y}^n(1)\right) \neq 1\right\}\\ \cap\{|\mathcal{Q}^{(n)}(1)| < L\gamma(\varepsilon_2)+1\}\end{array}\middle| W=1\right\}\\
&\le \mathbb{P}\left\{\left(\begin{array}{l}\left\{\sum\limits_{k=mL+1}^{n}\imath(X_k(1);\tilde{Y}_k(1)) < \log\xi\right\}\cup\\ \bigcup\limits_{i=2}^{M}\left\{\sum\limits_{k=mL+1}^{n}\imath(X_k(i);\tilde{Y}_k(1)) \ge \log\xi\right\}\end{array}\right)\cap\mathcal{E}_1\middle| W=1\right\}\\
&\le \mathbb{P}\left\{\left\{\sum_{k=mL+1}^{n}\imath(X_k(1);\tilde{Y}_k(1)) < \log\xi\right\}\cap\mathcal{E}_1\cap\mathcal{E}_2\middle| W=1\right\}\\
&\quad + \mathbb{P}\left\{\begin{array}{l}\bigcup\limits_{i=2}^{M}\left\{\sum\limits_{k=mL+1}^{n}\imath(X_k(i);\tilde{Y}_k(1)) \ge \log\xi\right\}\\ \cap\,\mathcal{E}_1\cap\mathcal{E}_2\end{array}\middle| W=1\right\}\\
&\quad + \frac{1}{\sqrt{n_m}}.\end{aligned} \tag{53}$$

In order to bound the first term in (53), we construct

$$Y_k(1) \triangleq X_k(1) + \tilde{Y}_k(1) - \tilde{X}_k(1) \tag{54}$$

for each $k \in \{1, 2, \ldots, n\}$ and consider

$$\begin{aligned}&\mathbb{P}\left\{\left\{\sum_{k=mL+1}^{n}\imath(X_k(1);\tilde{Y}_k(1)) < \log\xi\right\}\cap\mathcal{E}_1\cap\mathcal{E}_2\middle| W=1\right\}\\
&= \mathbb{P}\left\{\begin{array}{l}\left\{\sum\limits_{k=mL+1}^{n}\imath(X_k(1);Y_k(1)) + \sum\limits_{k\in\mathcal{Q}^{(n)}(1)}\imath(X_k(1);\tilde{Y}_k(1))\right.\\ \left.\quad - \sum\limits_{k\in\mathcal{Q}^{(n)}(1)}\imath(X_k(1);Y_k(1)) < \log\xi\right\}\cap\mathcal{E}_1\cap\mathcal{E}_2\end{array}\middle| W=1\right\}.\end{aligned} \tag{55}$$

Combining (54), (23), (22), (2) and (9), we have

$$\tilde{Y}_k(1) - \tilde{X}_k(1) = Y_k(1) - X_k(1) = Z_k \tag{56}$$

for each $k \in \{1, 2, \ldots, n\}$ where

$$p_{Z^n}(z^n) = \prod_{k=1}^{n}\mathcal{N}(z_k; 0, 1).$$

$$
\begin{aligned}
&\mathbb{P}\left\{\left\{\sum_{k=b_\ell+1}^{b_\ell+L}\imath(X_k(1);\tilde{Y}_k(1))-\imath(X_k(1);Y_k(1))\le -2S\tilde{\Delta}\right\}\cap\mathcal{E}_2\middle| W=1,\mathcal{Q}^{(n)}(1)=\mathcal{A}\right\}\\
&=\mathbb{P}_{p_{Z^n}p_{X^n|W=1,\mathcal{Q}^{(n)}(1)=\mathcal{A}}}\left\{\left\{\sum_{k=b_\ell+1}^{b_\ell+L}\frac{2SX_kZ_k-(S+2)X_k^2}{2(S+1)}\le -2S\tilde{\Delta}\right\}\cap\left\{\sum_{j=1}^{L}X_{b_\ell+j}^2<2S\Delta\right\}\right\} \qquad (58)\\
&\le \sup_{x^L:\|x^L\|^2<2S\Delta}\mathbb{P}_{p_{Z^n}}\left\{\sum_{k=1}^{L}\frac{2Sx_kZ_k-(S+2)x_k^2}{2(S+1)}\le -2S\tilde{\Delta}\right\}\\
&=\sup_{x^L:\|x^L\|^2<2S\Delta}\mathbb{P}_{p_Z^n}\left\{\mathrm{e}^{-\sum_{k=1}^{L}x_kZ_k}\ge \mathrm{e}^{2(S+1)\tilde{\Delta}}\mathrm{e}^{-\sum_{k=1}^{L}\frac{(S+2)x_k^2}{2S}}\right\}\\
&\le \sup_{x^L:\|x^L\|^2<2S\Delta}\mathbb{E}_{p_Z}\left[\mathrm{e}^{-\sum_{k=1}^{L}x_kZ_k}\right]\mathrm{e}^{-2(S+1)\tilde{\Delta}}\mathrm{e}^{\sum_{k=1}^{L}\frac{(S+2)x_k^2}{2S}} \qquad (59)\\
&=\sup_{x^L:\|x^L\|^2<2S\Delta}\mathrm{e}^{-2(S+1)\tilde{\Delta}}\mathrm{e}^{\sum_{k=1}^{L}\frac{(S+2)x_k^2}{2S}+\frac{x_k^2}{2}}\\
&<\mathrm{e}^{-2(S+1)\tilde{\Delta}}\mathrm{e}^{2(S+1)\Delta}\\
&<\frac{1}{n_m^2} \qquad (60)
\end{aligned}
$$

For each $k\in\mathcal{Q}^{(n)}(1)$, since $\tilde{X}_k(1)=0$ holds almost surely (cf. the definition of $\mathcal{Q}^{(n)}(1)$ in (10)), it follows from (56) that

$$\tilde{Y}_k(1)=Y_k(1)-X_k(1)=Z_k \quad \text{almost surely.} \qquad (57)$$

Conditioned on the event $\{\mathcal{Q}^{(n)}(1)=\mathcal{A}\}$, we consider the chain of inequalities leading to (60) as shown at the top of this page for each block of L consecutive mismatched positions in $\mathcal{A}$ denoted by $b_\ell+1,b_\ell+2,\ldots,b_\ell+L$ where

- (58), as shown at the top of this page, follows from the following fact due to the definition of $\imath(\,\cdot\,;\,\cdot\,)$ in (52) and (57): For each $k\in\mathcal{A}$,
$$\begin{aligned}&\imath(X_k(1);\tilde{Y}_k(1))-\imath(X_k(1);Y_k(1))\\&\quad=\frac{2SX_k(1)(Y_k(1)-X_k(1))-(S+2)(X_k(1))^2}{2(S+1)}\end{aligned}$$
holds almost surely.
- (59), as shown at the top of this page, is due to Markov's inequality.
- (60) is due to the definitions of Δ and $\tilde{\Delta}$ in (49) and (50) respectively.

Combining (55) and (60) and using the union bound, we have

$$
\begin{aligned}
&\mathbb{P}\left\{\left\{\sum_{k=mL+1}^{n}\imath(X_k(1);\tilde{Y}_k(1))<\log\xi\right\}\cap\mathcal{E}_1\cap\mathcal{E}_2\middle| W=1\right\}\\
&\le\mathbb{P}\left\{\begin{matrix}\left\{\sum_{k=mL+1}^{n}\imath(X_k(1);Y_k(1))-2S\tilde{\Delta}\left\lceil\frac{|\mathcal{Q}^{(n)}(1)|}{L}\right\rceil<\log\xi\right\}\\ \cap\mathcal{E}_1\cap\mathcal{E}_2\end{matrix}\middle|\begin{matrix}W\\=1\end{matrix}\right\}\\
&\quad+\frac{1}{n_m} \qquad (61)\\
&\le\mathbb{P}\left\{\sum_{k=mL+1}^{n}\imath(X_k(1);Y_k(1))<\log\xi+2S\tilde{\Delta}(\gamma(\varepsilon_2)+1)\right\}\\
&\quad+\frac{1}{n_m} \qquad (62)
\end{aligned}
$$

where (61) is due to the union bound, the fact that $\mathcal{Q}^{(n)}(1)$ has at most $\left\lceil\frac{|\mathcal{Q}^{(n)}(1)|}{L}\right\rceil$ blocks of consecutive mismatched positions (only the last block may have length other than L), and the fact that (60) holds if L is replaced with any natural number $L^*\le L$; and (62) follows from the definition of $\mathcal{E}_1$ in (51). The first term in (62) can be bounded by standard procedures which will be elaborated later. In order to bound the second term in (53), we use Lemma 7 (Lemma 3 suffices for the case $L=1$) to obtain

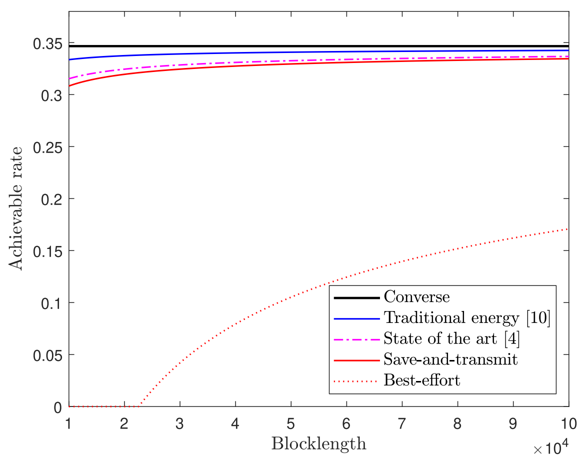
$$
\begin{aligned}
&\mathbb{P}\left\{\left\{\sum_{k=mL+1}^{n}\imath(X_k(2);\tilde{Y}_k(1))\ge\log\xi\right\}\cap\mathcal{E}_1\cap\mathcal{E}_2\middle| W=1\right\}\\
&\le\frac{2}{M}\mathrm{e}^{-(\log\xi-\log M)}\times\left(n_m(S+1)^{L/2}\right)^{\gamma(\varepsilon_2)+1}. \qquad (63)
\end{aligned}
$$

Consequently, it follows from (45), (53), (62) and (63) that

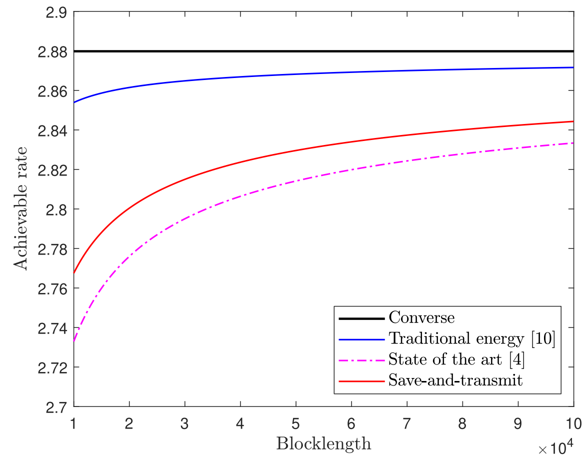
$$
\begin{aligned}
&\mathbb{P}\left\{\varphi\left(\tilde{Y}^n(W)\right)\ne W\right\}\\
&\le\mathbb{P}\left\{\sum_{k=mL+1}^{n}\imath(X_k(1);Y_k(1))<\log\xi+2S\tilde{\Delta}(\gamma(\varepsilon_2)+1)\right\}\\
&\quad+2\mathrm{e}^{-\left(\log\xi-\log M-(\gamma(\varepsilon_2)+1)\log(n_m(S+1)^{L/2})\right)}+\varepsilon_2\\
&\quad+\frac{1}{\sqrt{n_m}}+\frac{1}{n_m}. \qquad (64)
\end{aligned}
$$

The remainder of the proof follows from standard steps, outlined below for the sake of completeness. Let $\mu=\frac{1}{2}\log(1+S)$, $\sigma^2=\frac{S}{S+1}>0$ and $T<\infty$ denote the mean, the variance and the third absolute moment of $\imath(X;Y)$ respectively, where the finiteness of T is due to (52) and the fact that $|S|\le P$. Choose

$$
\begin{aligned}
\log\xi&\triangleq n_m\mu+\sqrt{n_m\sigma^2}\,\Phi^{-1}\left(\varepsilon_1-\frac{T}{\sigma^3\sqrt{n_m}}-\frac{4}{\sqrt{n_m}}\right)\\
&\quad-2S\tilde{\Delta}(\gamma(\varepsilon_2)+1). \qquad (65)
\end{aligned}
$$



(a) SNR = 0 dB



(b) SNR = 25 dB

Fig. 2. Achievable rates for save-and-transmit, best-effort and the state of the art [4] for $L = 1$ where $\varepsilon_1 = \varepsilon_2 = 0.01$.

It then follows from Berry-Esséen theorem [17], i.e.,

$$\left|\mathbb{P}\left\{\frac{\sum_{k=1}^{n}\imath(X_k;Y_k)-n\mu}{\sqrt{n\sigma^2}}\le a\right\}-\Phi(a)\right|\le\frac{T}{\sigma^3\sqrt{n}}$$

for all $a\in\mathbb{R}$, that

$$\mathbb{P}\left\{\sum_{k=mL+1}^{n}\imath(X_k(1);Y_k(1))<\log\xi+2S\tilde{\Delta}(\gamma(\varepsilon_2)+1)\right\}\le\varepsilon_1-\frac{4}{\sqrt{n_m}}. \tag{66}$$

In order to bound the second term in (64), we choose

$$\log M \triangleq \left\lfloor\log\xi-(\gamma(\varepsilon_2)+1)\left(\frac{L}{2}\log(S+1)+\log n_m\right)-\log\sqrt{n_m}\right\rfloor \tag{67}$$

$$\ge\log\xi-(\gamma(\varepsilon_2)+1)\left(\frac{L}{2}\log(S+1)+\log n_m\right)-\log\sqrt{n_m}-1. \tag{68}$$

Consequently, (31) follows from (64), (66) and (67), and (30) follows from (65) and (68). In addition, (33) follows from (46).

VI. Numerical Results

In this section, we numerically compare the performance of our analyzed save-and-transmit with the state-of-the-art save-and-transmit in [4] under the following two cases: The i.i.d. energy arrival case with $L = 1$, and the block energy arrival case with $L = \lceil\sqrt{n}\rceil$. In both cases, we assume that $\mathbb{E}[E^2] = 3(\mathbb{E}[E])^2$. An example for E is $E = U^2$ where $U \sim \mathcal{N}(u;0,P)$. The major difference between the two save-and-transmit strategies is that the former uses a transmit power S strictly less than the battery recharge rate P while the latter one always assumes $S = P$. The difference in transmitting power results in different achievable rates as shown in the rest of the section.

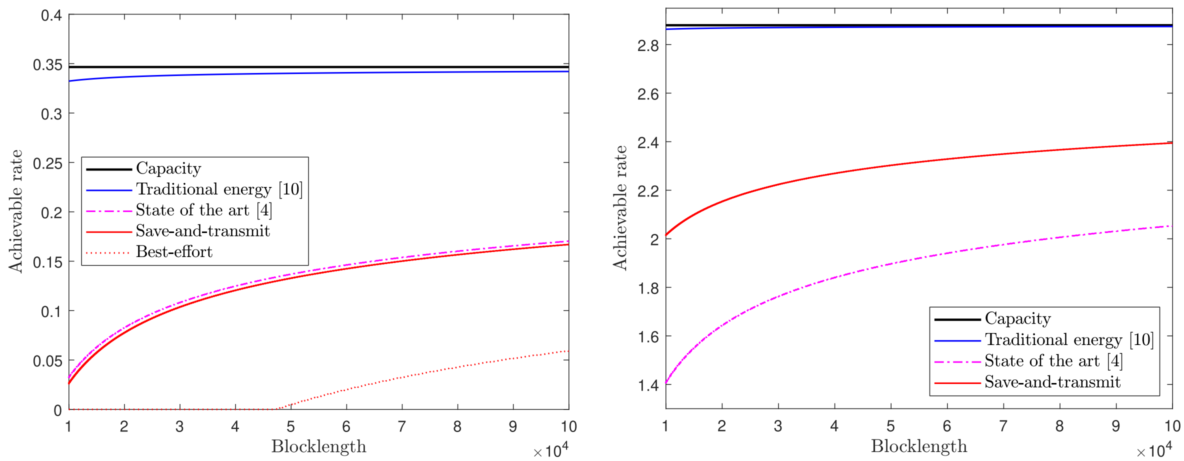
A. Case $L = 1$

Figure 2(a) plots the achievable rate up to the $\Theta(1/\sqrt{n})$ term of our analyzed save-and-transmit scheme, our analyzed best-effort scheme and the state-of-the-art save-and-transmit [4, Th. 1] according to (16), (25) and (17) respectively for the low SNR (i.e., low battery recharge rate) regime where $P = 0$ dB, $\mathbb{E}[X^2] = 3P^2$, and $\varepsilon_1 = \varepsilon_2 = 0.01$. In addition, we compare in Figure 2(b) our analyzed save-and-transmit and the state-of-the-art for the high SNR regime $P = 25$ dB, where best-effort is omitted because it does not achieve a positive rate in this regime (the magnitude of the backoff term in (25) is larger than the capacity $\mathrm{C}(P)$ for large P). For the high SNR regime, Figure 2(b) shows that save-and-transmit outperforms the state of the art at reasonable values of the blocklength. On the other hand, the state of the art outperforms save-and-transmit for the low SNR regime as shown in Figure 2(a). The two plots in Figure 2 agree with Remark 2 and Remark 3. To demonstrate the effect of EH constraints (5) on the AWGN channel, we also plot the following maximum achievable rate up to the $\Theta(1/\sqrt{n})$ term [10, Th. 54, Eq. (294)] when the EH constraints are replaced with the conventional power constraint $\mathbb{P}\{\frac{1}{n}\sum_{k=1}^{n}X_k^2\le nP\}=1$:

$$\frac{1}{2}\log(1+P)+\sqrt{\frac{P(P+2)}{2n(P+1)^2}}\Phi^{-1}(\varepsilon_1+\varepsilon_2). \tag{69}$$

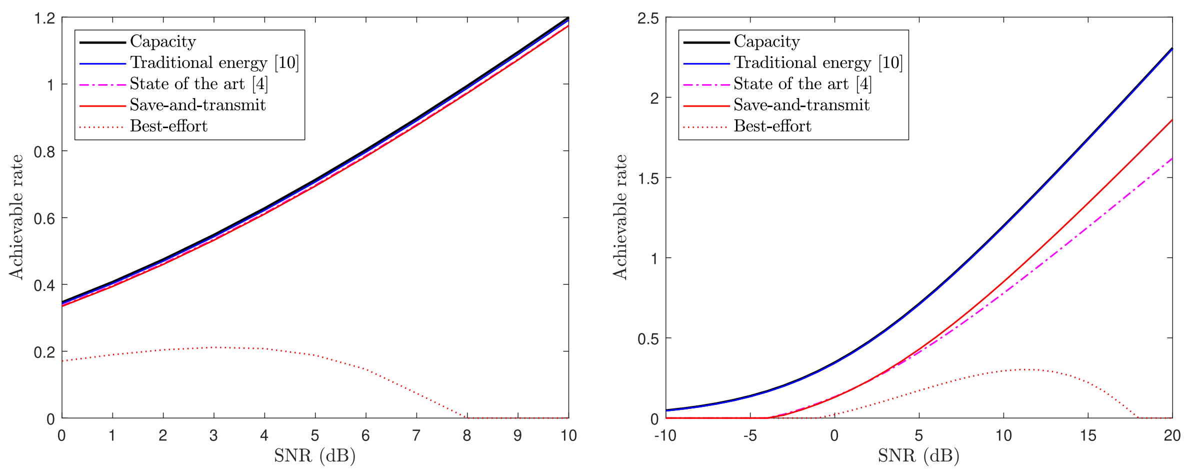
B. Case $L = \lceil\sqrt{n}\rceil$

Figure 3(a) plots the achievable rate up to the $\Theta(\sqrt{L/n})$ term of our analyzed save-and-transmit scheme, our analyzed best-effort scheme and the state-of-the-art save-and-transmit [4, Th. 1] according to (38), (44) and (39) respectively for the low SNR regime $P = 0$ dB and $\mathbb{E}[X^2] = 3P^2$ and for $\varepsilon = 0.01$. In addition, we compare in Figure 3(b) the three schemes for the high SNR regime $P = 25$ dB, where best-effort is omitted because it does not achieve a positive rate in this regime (the magnitude of the backoff term in (44) is larger than the capacity $\mathrm{C}(P)$ for large P). For the high SNR



(a) SNR = 0 dB (b) SNR = 25 dB

Fig. 3. Achievable rates for save-and-transmit, best-effort and the state of the art [4] for $L = \lceil\sqrt{n}\,\rceil$ and $\varepsilon = 0.01$.



(a) $L = 1$ and $\varepsilon_1 = \varepsilon_2 = 0.01$ (b) $L = \lceil\sqrt{n}\,\rceil$ and $\varepsilon = 0.01$

Fig. 4. Achievable rates for save-and-transmit, best-effort and the state of the art [4] for $n = 10^5$.

regime, Figure 3(b) shows that save-and-transmit outperforms the state of the art at reasonable values of the blocklength. On the other hand, the state of the art outperforms save-and-transmit for the low SNR regime as shown in Figure 3(a). The two plots in Figure 3 agree with Remark 5 and Remark 7. To demonstrate the effect of EH constraints (5) on the AWGN channel, we also plot the maximum achievable rate (69) up to the $\Theta(1/\sqrt{n})$ term with $\varepsilon_1 + \varepsilon_2 = 0.01$ when the EH constraints are replaced by $\mathbb{P}\{\sum_{k=1}^{n} X_k^2 \le nP\} = 1$.

C. Impact of SNR

In order to illustrate how the SNR impacts the performance of the save-and-transmit, best-effort and the state of the art, we plot in Figure 4(a) their achievable rates at a fixed blocklength up to the $\Theta(1/\sqrt{n})$ term against SNR for $L = 1$, $n = 10^5$ and $\varepsilon_1 = \varepsilon_2 = 0.01$. Similarly, we plot in Figure 4(b) their achievable rates up to the $\Theta(\sqrt{L/n})$ term against SNR for $L = \lceil\sqrt{n}\,\rceil$, $n = 10^5$ and $\varepsilon = 0.01$. For $L = 1$, save-and-transmit and the state of the art have similar performance. In contrast, for $L = \lceil\sqrt{n}\,\rceil$, save-and-transmit outperforms the state of the art when the SNR is larger than 5 dB. For both cases $L = 1$ and $L = \lceil\sqrt{n}\,\rceil$, best-effort achieves a positive rate only within a range of SNRs. Therefore, recalling the major difference between save-and-transmit and the state of the art explained at the beginning of this section, we conclude that allowing the transmit power to be strictly less than the SNR (i.e., battery recharge rate) can be beneficial for the block energy arrival case.

VII. Concluding Remarks and Future Work

In this paper, we have studied in the finite blocklength regime the save-and-transmit scheme with a saving phase of arbitrary length m over the AWGN EH channel, and also the best-effort scheme through setting $m = 0$. A new non-asymptotic achievable rate is obtained for save-and-transmit, which directly implies a new non-asymptotic achievable rate for best-effort. The non-asymptotic result implies that the save-and-transmit scheme achieves the optimal second-order scaling $-O_n(1/\sqrt{n})$, and that the best-effort scheme achieves the second-order scaling $-O_n\big(\sqrt{(\log n)/n}\big)$. The achievable rates for the schemes are extended to the block energy arrival model where $L = o_n(n)$, and are shown to attain the second-order scalings $-O_n\big(\sqrt{\max\{\log n, L\}/n}\big)$ and $-O_n(\sqrt{L/n})$ respectively. Compared to the state-of-the-art save-and-transmit scheme [4], our save-and-transmit has a better finite-blocklength performance for sufficiently large P. In addition, our theoretical and simulation results reveal that save-and-transmit significantly outperforms best-effort for all blocklengths, which prompts us to conjecture that it is always beneficial to first accumulate some energy before the actual transmission.

For the simplest case $L = 1$, the best-effort scheme does not achieve the optimal scaling $-O_n(1/\sqrt{n})$. A straightforward verification by MATLAB reveals that under the assumption $E = U^2$ where $U \sim \mathcal{N}(u; 0, 1)$, the average number of mismatched positions for the best-effort scheme is of the order $o_n(\sqrt{n})$. A future direction may improve the second-order scaling $-O_n\big(\sqrt{(\log n)/n}\big)$ for $L = 1$ for best-effort schemes by possibly developing a sharper probability bound than (15) in Lemma 1. On the other hand, although save-and-transmit achieves the optimal second-order scaling, its achievable rate is obtained based on a two-step separation approach, which naturally results in two error events – one for energy shortage and another for random coding error (cf. (16)). For the traditional AWGN channel in the finite blocklength regime [10, Th. 54], the former event was made to have zero measure to obtain the optimal backoff from capacity. In contrast, the energy shortage event for save-and-transmit has non-vanishing measure. Therefore, any future attempts of designing joint energy-harvesting and channel coding schemes that unify the two error events may yield possible coding gain for save-and-transmit. Another interesting direction is to tighten the existing non-asymptotic upper bound for a general coding scheme presented in [4, Th. 1], which states that the second-order term is bounded above by $\frac{\sqrt{2P^2+\mathbb{E}[E^2]}}{2(P+1)}\Phi^{-1}(\varepsilon) \times \sqrt{\frac{L}{n}}$. The upper bound is potentially loose because it considers only the last EH constraint $\sum_{i=1}^n X_k^2 \le \sum_{i=1}^n E_k$ rather than the n EH constraints in (5). Last but not least, a natural extension of this work is to explore non-asymptotic achievable rates for EH channels with finite battery [8], [9].

Appendix A
Proofs of Lemma 1 and Lemma 6

Since save-and-transmit defined in Section III-A is a special case of blockwise save-and-transmit defined in Section IV-B with $L = 1$ and Lemma 1 is a special case of Lemma 6 with $L = 1$, it suffices to prove Lemma 6.

Fix an $n \in \mathbb{N}$, a natural number $L < n$ and a $\rho \in (0, 1)$ that satisfies (11), and fix a blockwise save-and-transmit (n, M)-EH code as described in Section IV-B. Let p_X be as defined in (7) where S is as defined in (12). Define $p_{\hat{E}}$ to be the distribution of

$$\hat{E} \triangleq \sum_{j=1}^{L} E_j = LE$$

where E satisfies (3), and define $p_{\hat{X}}$ to be the distribution of

$$\hat{X} \triangleq \sqrt{\sum_{j=1}^{L} X_j^2}.$$

In this proof, all the probability, expectation and variance terms are evaluated according to $p_{\hat{X}^\infty} p_{\hat{E}^\infty}$ where $p_{\hat{X}^\infty} = \prod_{k=1}^\infty p_{\hat{X}_k}$ and $p_{\hat{E}^\infty} = \prod_{k=1}^\infty p_{\hat{E}_k}$ denote the infinite product distributions of $p_{\hat{X}}$ and $p_{\hat{E}}$ respectively. Consider the Markov process

$$\left\{\sum_{i=1}^{m} \hat{E}_i + \sum_{i=m+1}^{m+k} \left(\hat{E}_i - \hat{X}_i^2\right)\right\}_{k=1}^{\tau(m)}$$

where m is an arbitrary non-negative integer and $\tau(m)$ is the stopping time when the value of the Markov process hits any $a < 0$. By definition of $\tau(m)$, we have

$$\mathbb{P}\left\{\sum_{i=1}^{m} \hat{E}_i + \sum_{i=m+1}^{m+\tau(m)} \left(\hat{E}_i - \hat{X}_i^2\right) < 0 \,\middle|\, \tau(m) < \infty\right\} = 1$$

and

$$\begin{aligned}&\mathbb{P}\{\tau(m) = \infty\}\\ &\quad = \mathbb{P}\left\{\bigcap_{k=1}^{\infty}\left\{\sum_{i=1}^{m} \hat{E}_i + \sum_{i=m+1}^{m+k} \left(\hat{E}_i - \hat{X}_i^2\right) \ge 0\right\}\right\} \qquad (70)\end{aligned}$$

for each $m \in \mathbb{Z}_+$ where $\mathbb{P}\{\tau(m) = \infty\}$ denotes the escape probability.

In order to show (27), we first fix a $\gamma \in \mathbb{N}$ and let $\tau_1, \tau_2, \ldots, \tau_\gamma$ be γ independent copies of $\tau(1)$. Due to the construction of the blockwise save-and-transmit scheme with a saving phase of length mL, energy is saved but not consumed during the saving phase and each block of L consecutive mismatch events. Therefore, $\tau(m)$ serves as a lower bound on the number length-L blocks between the first length-L block in the transmission phase and the first block of mismatch events (excluding one block) and $\tau(1)$ serves as a lower bound on the number of blocks between two blocks of mismatch events (excluding one block). Fix any $w \in \mathcal{W}$ and consider

$$\begin{aligned}&\mathbb{P}\left\{|\mathcal{Q}^{(n)}(w)| \ge L\gamma + 1 \,\middle|\, W = w\right\}\\ &= \mathbb{P}\left\{\begin{matrix}\mathcal{Q}^{(n)}(w) \text{ contains at least } \gamma + 1 \text{ blocks of}\\ \text{mismatch events}\end{matrix} \,\middle|\, W = w\right\}\\ &\le \mathbb{P}\left\{\{\tau(m) < \infty\} \cap \bigcap_{k=1}^{\gamma}\{\tau_k < \infty\}\right\}\\ &= \mathbb{P}\{\tau(m) < \infty\}\left(\mathbb{P}\{\tau(1) < \infty\}\right)^\gamma. \qquad (71)\end{aligned}$$

In order to obtain an upper bound on $\mathbb{P}\{\tau(m) < \infty\}$, we first construct the following sequence denoted by $\{\hat{B}_k\}_{k=1}^\infty$. For each $k \in \mathbb{N}$, define $\hat{B}_k$ recursively as

$$\hat{B}_k \triangleq \begin{cases}\hat{E}_1 & \text{if } k = 1 \text{ and } m \ge 1,\\ \hat{E}_1 - \hat{X}_1^2 & \text{if } k = 1 \text{ and } m = 0,\\ \hat{B}_{k-1} + \hat{E}_k & \text{if } k \in \{2, 3, \ldots, m\},\\ \hat{B}_{k-1} + \hat{E}_k - \hat{X}_k^2 & \text{if } k \ge m+1 \text{ and } \hat{B}_{k-1} \ge 0,\\ \hat{B}_{k-1} & \text{if } k \ge m+1 \text{ and } \hat{B}_{k-1} < 0.\end{cases} \qquad (72)$$

By inspecting (72), we have

$$\{\hat{B}_\infty < 0\} = \bigcup_{k=1}^{\infty} \left\{ \sum_{i=1}^{m} \hat{E}_i + \sum_{i=m+1}^{m+k} (\hat{E}_i - \hat{X}_i^2) < 0 \right\} \quad (73)$$
$$= \{\tau(m) < \infty\}, \quad (74)$$

where each term in the union in (73) characterizes the event that the accumulated energy collected during the first $m + k$ energy blocks is insufficient to output the desired codeword symbols from block $m + 1$ to block $m + k$ during the transmission phase. It remains to obtain an upper bound on $\mathbb{P}\{\hat{B}_\infty < 0\}$. To this end, we first define for each $k \in \mathbb{N}$

$$\hat{U}_k \triangleq \begin{cases} \hat{B}_1 & \text{if } k = 1, \\ \hat{B}_k - \hat{B}_{k-1} & \text{otherwise} \end{cases}$$
$$= \begin{cases} \hat{E}_1 & \text{if } k = 1 \text{ and } m \geq 1, \\ \hat{E}_1 - \hat{X}_1^2 & \text{if } k = 1 \text{ and } m = 0, \\ \hat{E}_k & \text{if } k \in \{2, 3, \ldots, m\}, \\ \hat{E}_k - \hat{X}_k^2 & \text{if } k \geq m+1 \text{ and } \hat{B}_{k-1} \geq 0, \\ 0 & \text{if } k \geq m+1 \text{ and } \hat{B}_{k-1} < 0 \end{cases} \quad (75)$$

where (75) follows from (72). It then follows from (73) and (75) that

$$\mathbb{P}\{\hat{B}_\infty < 0\} = \mathbb{P}\left\{ \sum_{k=1}^{\infty} \hat{U}_k < 0 \right\}. \quad (76)$$

Following (76), we consider the chain of inequalities below for any $t > 0$:

$$\mathbb{P}\left\{ \sum_{k=1}^{\infty} \hat{U}_k < 0 \right\} = \mathbb{P}\left\{ \mathrm{e}^{-t\sum_{k=1}^{\infty} \hat{U}_k} > 1 \right\}$$
$$\leq \mathbb{E}\left[\mathrm{e}^{-t\sum_{k=1}^{\infty} \hat{U}_k} \right] \quad (77)$$

where the inequality follows from Markov's inequality. In order to simplify the RHS of (77), we use the convention $\hat{E}^0 = \hat{X}^0 = \hat{U}^0 = 0$ (useful only when $m = 0$) and consider the following chain of inequalities for each $i \in \{m + 1, m + 2, \ldots\}$:

$$\mathbb{E}\left[\mathrm{e}^{-t\sum_{k=1}^{i} \hat{U}_k}\right]$$
$$= \mathbb{E}\left[\mathbb{E}\left[\mathrm{e}^{-t\sum_{k=1}^{i} \hat{U}_k} \middle| \hat{U}^{i-1}\right]\right]$$
$$= \mathbb{E}\left[\mathbb{E}\left[\mathbb{E}\left[\mathrm{e}^{-t\hat{U}_i} \middle| \hat{U}^{i-1}\right] \mathrm{e}^{-t\sum_{k=1}^{i-1} \hat{U}_k} \middle| \hat{U}^{i-1}\right]\right]$$
$$\leq \mathbb{E}\left[\mathbb{E}\left[\max\left\{\mathbb{E}\left[\mathrm{e}^{-t(\hat{E}_i - \hat{X}_i^2)} \middle| \hat{U}^{i-1}\right], 1\right\} \cdot \mathrm{e}^{-t\sum_{k=1}^{i-1} \hat{U}_k} \middle| \hat{U}^{i-1}\right]\right] \quad (78)$$
$$= \max\left\{\mathbb{E}\left[\mathrm{e}^{-t(\hat{E}_i - \hat{X}_i^2)}\right], 1\right\} \mathbb{E}\left[\mathrm{e}^{-t\sum_{k=1}^{i-1} \hat{U}_k}\right] \quad (79)$$

where (78) is due to (75); (79) follows from the independence between $(\hat{E}_i, \hat{X}_i)$ and $\hat{U}^{i-1}$ due to the independence between $(\hat{E}_i, \hat{X}_i)$ and $(\hat{E}^{i-1}, \hat{X}^{i-1})$.

Combining (76), (77) and (79), we have

$$\mathbb{P}\left\{\hat{B}_\infty < 0\right\}$$
$$\leq \mathbb{E}\left[\mathrm{e}^{-t\sum_{k=1}^{m} \hat{U}_k}\right] \max\left\{\left(\mathbb{E}\left[\mathrm{e}^{-t(\hat{E} - \hat{X}^2)}\right]\right)^\infty, 1\right\}$$
$$= \left(\mathbb{E}\left[\mathrm{e}^{-t\hat{E}}\right]\right)^m \max\left\{\left(\mathbb{E}\left[\mathrm{e}^{-t(\hat{E} - \hat{X}^2)}\right]\right)^\infty, 1\right\},$$

which together with the definitions of $\hat{E}$ and $\hat{X}^2$ implies that

$$\mathbb{P}\left\{\hat{B}_\infty < 0\right\}$$
$$\leq \left(\mathbb{E}\left[\mathrm{e}^{-tLE}\right]\right)^m \max\left\{\left(\mathbb{E}\left[\mathrm{e}^{-t(LE - \sum_{j=1}^{L} X_j^2)}\right]\right)^\infty, 1\right\}. \quad (80)$$

In order to simplify the RHS of (80), we use the following two facts, whose proofs can be found in [2, Appendix]: For any $y \geq 0$,

$$1 + y \leq \mathrm{e}^y \leq 1 + y + \frac{y^2 \mathrm{e}^y}{2} \quad (81)$$

and

$$1 - y \leq \mathrm{e}^{-y} \leq 1 - y + \frac{y^2}{2}. \quad (82)$$

Let $t > 0$ be the positive solution of the quadratic equation

$$t = \frac{2(P - S)}{L\mathbb{E}[E^2] + 3S^2(1 + 63St)}. \quad (83)$$

Straightforward calculations reveal that

$$t = \frac{-(L\mathbb{E}[E^2]+3S^2)+\sqrt{(L\mathbb{E}[E^2]+3S^2)^2+1512S^3(P-S)}}{378S^3}$$
$$\leq \frac{\sqrt{42(P - S)}}{63\sqrt{S^3}} \quad (84)$$
$$= \frac{\sqrt{42\rho P}}{63\sqrt{S^3}}$$
$$< \frac{1}{6S} \quad (85)$$

where (84) is due to the fact that $\sqrt{a + b} \leq \sqrt{a} + \sqrt{b}$ for all $(a, b) \in \mathbb{R}_+^2$; (85) is due to (11) and (12). Using the definition of p_X in (7) and (85), we have

$$\mathbb{E}\left[X^4 \mathrm{e}^{tX^2}\right] = \frac{3S^2}{(1 - 2St)^{5/2}} < \infty. \quad (86)$$

In addition, using (85) and straightforward algebra, we obtain

$$\frac{1}{(1 - 2St)} \leq 1 + 3St \quad (87)$$

and

$$(1 + 3St)^{5/2} \leq (1 + 3St)^3$$
$$\leq 1 + 9St + 27S^2t^2 + 27S^3t^3$$
$$\leq 1 + 63St. \quad (88)$$

Following (80), we use the two facts (81) and (82) to obtain

$$\mathbb{E}\left[\mathrm{e}^{-tLE}\right] \leq 1 - tLP + \frac{t^2L^2\mathbb{E}[E^2]}{2} \leq \mathrm{e}^{-tLP + \frac{t^2L^2\mathbb{E}[E^2]}{2}} \quad (89)$$

and

$$\mathbb{E}\left[\mathrm{e}^{tX^2}\right] \leq 1 + tS + \frac{t^2\mathbb{E}[X^4\mathrm{e}^{tX^2}]}{2} \leq \mathrm{e}^{tS + \frac{t^2\mathbb{E}[X^4\mathrm{e}^{tX^2}]}{2}},$$

which implies that

$$\mathbb{E}\left[\mathrm{e}^{-tLE}\right]\mathbb{E}\left[\mathrm{e}^{t\sum_{j=1}^{L} X_j^2}\right] \leq \mathrm{e}^{-tL(P-S) + \frac{t^2L}{2}\left(L\mathbb{E}[E^2] + \mathbb{E}[X^4\mathrm{e}^{tX^2}]\right)}$$
$$\leq 1 \quad (90)$$

where (90) follows from the fact due to (83), (86), (87) and (88) that

$$t \le \frac{2(P-S)}{L\mathbb{E}[E^2] + \mathbb{E}\left[X^4 \mathrm{e}^{tX^2}\right]}.$$

Using (80), (89) and (90), we obtain

$$\mathbb{P}\left\{\hat{B}_\infty < 0\right\} \le \mathrm{e}^{-tLP + \frac{t^2L^2\mathbb{E}[E^2]}{2}}. \tag{91}$$

Using (74) and (91), we obtain

$$\mathbb{P}\{\tau(m) < \infty\} = \mathbb{P}\left\{\hat{B}_\infty < 0\right\} \le \mathrm{e}^{-m\left(tLP - \frac{t^2L^2\mathbb{E}[E^2]}{2}\right)}. \tag{92}$$

Combining (71) and (92), we have

$$\mathbb{P}\left\{|\mathcal{Q}^{(n)}(w)| \ge L\gamma + 1 \middle| W = w\right\} \le \mathrm{e}^{-(m+\gamma)\left(tLP - \frac{t^2L^2\mathbb{E}[E^2]}{2}\right)}. \tag{93}$$

In order to obtain an upper bound on the RHS of (93), we define α and β as in (25) and (26) respectively and use the following two facts due to (83) and (12):

$$t \le \frac{2\rho P}{L\mathbb{E}[E^2] + 3S^2} = \alpha \tag{94}$$

and hence

$$t \ge \frac{2\rho P}{L\mathbb{E}[E^2] + 3S^2(1 + 63\alpha S)} \ge \beta. \tag{95}$$

Combining (93), (94) and (95), we conclude that (27) holds for any natural number γ. It remains to show that (27) also holds if γ is an arbitrary positive real number, which holds true due to the simple fact that

$$\begin{aligned}&\mathbb{P}\left\{|\mathcal{Q}^{(n)}(w)| \ge L\gamma + 1 \middle| W = w\right\}\\ &= \mathbb{P}\left\{|\mathcal{Q}^{(n)}(w)| \ge \lceil L\gamma + 1\rceil \middle| W = w\right\}\end{aligned}$$

for any $\gamma \in \mathbb{R}_+$.

Appendix B
Proofs of Lemma 3 and Lemma 7

Since save-and-transmit defined in Section III-A is a special case of blockwise save-and-transmit defined in Section IV-B with $L = 1$ and Lemma 3 is a special case of Lemma 7 with $L = 1$, it suffices to prove Lemma 7.

Suppose we are given a blockwise save-and-transmit (n, M)-EH code. Fix an $L < n/m$, a $\gamma \ge 0$, a $\delta > 0$ and an $M \in \mathbb{N}$. We would like to obtain an upper bound on

$$\mathbb{P}\left\{\begin{array}{l}\left\{\log \frac{p_{Y^n|X^n}(\tilde{Y}^n(1)|X^n(2))}{p_{Y^n}(\tilde{Y}^n(1))} > \log M + \delta\right\}\\ \cap \left\{|\mathcal{Q}^{(n)}(1)| < L\gamma + 1\right\}\end{array} \middle| W = 1\right\}$$

by a change-of-measure argument. To this end, we let $X^n = X^n(1)$, $\tilde{X}^n = \tilde{X}^n(1)$, $Y^n = Y^n(1)$ and $\tilde{Y}^n = \tilde{Y}^n(1)$ and use the definition of blockwise save-and-transmit in Section IV-B and the definition of $\mathcal{Q}^{(n)}(w)$ in (10) to obtain

$$\begin{aligned}&p_{E^n,X^n,Y^n,\tilde{X}^n,\tilde{Y}^n,\mathcal{Q}^{(n)}(1)|W=1}\\ &= \left(\prod_{k=1}^n p_{E_k,X_k,Y_k|W=1} \times p_{\tilde{X}_k|X_k,E^k,\tilde{X}^{k-1},W=1} \times p_{\tilde{Y}_k|\tilde{X}_k}\right)\\ &\quad \times p_{\mathcal{Q}^{(n)}(1)|X^n,\tilde{X}^n,W=1}\end{aligned} \tag{96}$$

where

$$p_{\tilde{Y}_k|\tilde{X}_k}(\tilde{y}_k|\tilde{x}_k) \equiv q_{Y|X}(\tilde{y}_k|\tilde{x}_k), \tag{97}$$

$p_{\tilde{X}_k|X_k,E^k,\tilde{X}^{k-1},W=1}$ is some distribution readily determined by the encoding function (22), and

$$\begin{aligned}&p_{\mathcal{Q}^{(n)}(1)|X^n,\tilde{X}^n,W=1}(\mathcal{A}|x^n,\tilde{x}^n)\\ &\equiv \begin{cases}1 & \text{if } \mathcal{A} = \{i \in \{m+1, m+2, \ldots, n\} |\, \tilde{x}_i \ne x_i\},\\ 0 & \text{otherwise.}\end{cases}\end{aligned}$$

Using (96) and (97), we obtain

$$\begin{aligned}&p_{E^n,X^n,Y^n,\tilde{X}^n,\tilde{Y}^n,\mathcal{Q}^{(n)}(1)|W=1}(e^n,x^n,y^n,\tilde{x}^n,\tilde{y}^n,\mathcal{A})\\ &\le \Bigg(\prod_{k=1}^n p_{E_k,X_k,Y_k|W=1}(e_k,x_k,y_k)\\ &\qquad \times p_{\tilde{X}_k|X_k,E^k,\tilde{X}^{k-1},W=1}(\tilde{x}_k|x_k,e^k,\tilde{x}^{k-1})\Bigg)\\ &\quad \times \left(\prod_{k\in\mathcal{A}} q_{Y|X}(\tilde{y}_k|0)\right)\left(\prod_{k\notin\mathcal{A}} q_{Y|X}(\tilde{y}_k|x_k)\right),\end{aligned}$$

for each $(e^n, x^n, y^n, \tilde{x}^n, \tilde{y}^n)$ and each $\mathcal{A} \subseteq \{1, 2, \ldots, n\}$, which implies by summing over $(e^n, x^n, y^n, \tilde{x}^n)$ that

$$\begin{aligned}&p_{\tilde{Y}^n,\mathcal{Q}^{(n)}(1)|W=1}(\tilde{y}^n,\mathcal{A})\\ &\le \left(\prod_{k\in\mathcal{A}} \mathcal{N}(\tilde{y}_k; 0, 1)\right)\left(\prod_{k\notin\mathcal{A}} \mathcal{N}(\tilde{y}_k; 0, S+1)\right)\end{aligned} \tag{98}$$

for all $(\tilde{y}^n, \mathcal{A})$. Consider the chain of inequalities leading to (101) as shown at the top of next page for each $\mathcal{A} \subseteq \{1, 2, \ldots, n\}$, where

- (99), as shown at the top of the next page, is due to (98) and the fact that
$$\frac{\mathcal{N}(y; 0, 1)}{\mathcal{N}(y; 0, S+1)} \le \sqrt{S+1}$$
for all $y \in \mathbb{R}$;
- (100), as shown at the top of the next page, follows from Markov's inequality where the expectation is evaluated with respect to the distribution
$$p_{X^n(2)} \times p_{\tilde{Y}^n(1)|\mathcal{Q}^{(n)}(1)=\mathcal{A},W=1};$$
- (101) is due to simplifying the expectation term by first principles.

Consequently,

$$\begin{aligned}&\mathbb{P}\left\{\begin{array}{l}\left\{\log \frac{p_{Y^n|X^n}(\tilde{Y}^n(1)|X^n(2))}{p_{Y^n}(\tilde{Y}^n(1))} > \log M + \delta\right\}\\ \cap \left\{|\mathcal{Q}^{(n)}(1)| < L\gamma + 1\right\}\end{array} \middle| W = 1\right\}\\ &\le \sum_{\substack{\mathcal{A}\subseteq\{mL+1,\ldots,n\}:\\ |\mathcal{A}|\le L\gamma+1}} \mathbb{P}\left\{\begin{array}{l}\log \frac{p_{Y^n|X^n}(\tilde{Y}^n(1)|X^n(2))}{p_{Y^n}(\tilde{Y}^n(1))}\\ \quad > \log M + \delta\end{array} \middle| \begin{array}{l}W = 1,\\ \mathcal{Q}^{(n)}(1) = \mathcal{A}\end{array}\right\}\\ &\quad \times \mathbb{P}\{\mathcal{Q}^{(n)}(1) = \mathcal{A}|W = 1\}\\ &\le \frac{\mathrm{e}^{-\delta}}{M} \times (S+1)^{\frac{L\gamma+1}{2}}\\ &\quad \times \left|\mathcal{A} \subseteq \{mL+1, mL+2, \ldots, n\} : |\mathcal{A}| \le L\gamma + 1\right|\end{aligned} \tag{102}$$

$$
\begin{aligned}
&\mathbb{P}\left\{\log\frac{p_{Y^n|X^n}(\tilde{Y}^n(1)|X^n(2))}{p_{Y^n}(\tilde{Y}^n(1))} > \log M + \delta \,\middle|\, \mathcal{Q}^{(n)}(1)=\mathcal{A}, W=1\right\} \\
&= \int_{\mathbb{R}^n}\int_{\mathbb{R}^n} p_{X^n(2)}(x^n)\, p_{\tilde{Y}^n(1)|\mathcal{Q}^{(n)}(1),W=1}(\tilde{y}^n|\mathcal{A}) \\
&\quad \times \mathbf{1}\left\{\frac{p_{\tilde{Y}^n(1),\mathcal{Q}^{(n)}(1)|W=1}(\tilde{y}^n,\mathcal{A})}{p_{Y^n}(\tilde{y}^n)} \times \frac{p_{Y^m}(\tilde{y}^m)\prod_{k=m+1}^{n} p_{Y_k|X_k}(\tilde{y}_k|x_k)}{p_{\tilde{Y}^n(1),\mathcal{Q}^{(n)}(1)|W=1}(\tilde{y}^n,\mathcal{A})} > M\mathrm{e}^{\delta}\right\} \mathrm{d}\tilde{y}^n \mathrm{d}x^n \\
&\le \int_{\mathbb{R}^n}\int_{\mathbb{R}^n} p_{X^n(2)}(x^n)\, p_{\tilde{Y}^n(1)|\mathcal{Q}^{(n)}(1),W=1}(\tilde{y}^n|\mathcal{A}) \times \mathbf{1}\left\{(S+1)^{\frac{|\mathcal{A}|}{2}} \times \frac{p_{Y^m}(\tilde{y}^m)\prod_{k=m+1}^{n} p_{Y_k|X_k}(\tilde{y}_k|x_k)}{p_{\tilde{Y}^n(1),\mathcal{Q}^{(n)}(1)|W=1}(\tilde{y}^n,\mathcal{A})} > M\mathrm{e}^{\delta}\right\} \mathrm{d}\tilde{y}^n \mathrm{d}x^n \qquad (99) \\
&\le \frac{\mathrm{e}^{-\delta}}{M p_{\mathcal{Q}^{(n)}(1)|W=1}(\mathcal{A})} \times (S+1)^{\frac{|\mathcal{A}|}{2}} \times \mathbb{E}\left[\frac{p_{Y^m}(\tilde{Y}^m(1))\prod_{k=m+1}^{n} p_{Y_k|X_k}(\tilde{Y}_k(1)|X_k(2))}{p_{\tilde{Y}^n(1)|\mathcal{Q}^{(n)}(1)=\mathcal{A},W=1}(\tilde{Y}^n(1))}\right] \qquad (100) \\
&= \frac{\mathrm{e}^{-\delta}}{M p_{\mathcal{Q}^{(n)}(1)|W=1}(\mathcal{A})} \times (S+1)^{\frac{|\mathcal{A}|}{2}} \qquad (101)
\end{aligned}
$$

---

where the last inequality is due to (101). Since the mismatched positions occur in blocks of L symbols except for the last block whose length is no larger than L, we have

$$
\begin{aligned}
&\big|\mathcal{A}\subseteq\{mL+1, mL+2,\ldots,n\} : |\mathcal{A}| \le L\gamma+1\big| \\
&\le \sum_{i=0}^{\lceil \gamma+1/L\rceil}\binom{\lceil (n-mL)/L\rceil}{i} \\
&\le \sum_{i=0}^{\lceil \gamma+1/L\rceil}\left(\left\lceil\frac{n-mL}{L}\right\rceil\right)^i \\
&\le \sum_{i=0}^{\lceil \gamma+1/L\rceil}(n-mL)^i \\
&\le \frac{(n-mL)^{\gamma+2}}{n-mL-1} \\
&\le 2(n-mL)^{\gamma+1}. \qquad (103)
\end{aligned}
$$

Combining (102) and (103), we obtain (28).

Appendix C
Proofs of Corollary 4 and Corollary 8

Since save-and-transmit defined in Section III-A is a special case of blockwise save-and-transmit defined in Section IV-B with $L=1$ and Corollary 4 is a special case of Corollary 8 with $L=1$, it suffices to prove Corollary 8.

Fix an $\varepsilon\in(0,1/2)$, and fix any $\varepsilon_1>0$ and $\varepsilon_2>0$ such that $\varepsilon_1+\varepsilon_2=\varepsilon$. Define ρ, S, α, β and m as in (35), (12), (25), (26) and (36) respectively, and define $\gamma(\varepsilon_2)=0$ as in (32). To simplify notation, we do not explicitly specify the dependence on n for ρ, S, α, β and m. Let $p_X=\mathcal{N}(x;0,S)$ and let $p_Y=\mathcal{N}(y;0,S+1)$ be the marginal distribution of $p_X q_{Y|X}$, and let σ^2 and T denote the variance and the third absolute moment of $\log\frac{q_{Y|X}(Y|X)}{p_Y(Y)}$ respectively. Fix any sufficiently large n and any $L<n$ such that $\rho\in(0,1)$, (29) and (11) simultaneously hold. Then, Theorem 2 implies that there exists a blockwise save-and-transmit (n,M,ε)-EH code which satisfies (30) and (33). We would like to show that (34) holds for the blockwise save-and-transmit code by obtaining a lower bound on the RHS of (30). To this end, we fix a sufficiently large n such that (30) holds for the blockwise save-and-transmit code. By construction, we have $S=P(1-\rho)$, $\rho=\Theta_n(\sqrt{L/n})$ and $m=\Theta_n(\sqrt{n/L})$, and we use Taylor's theorem to conclude that there exist some $\kappa_1>0$ and $\kappa_2>0$ which do not depend on n such that

$$\frac{1}{2}\log(1+S)\ge\frac{1}{2}\left(\log(1+P)-\frac{\rho P}{1+P}-\frac{\kappa_1 L}{n}\right) \qquad (104)$$

and

$$\Phi^{-1}(\varepsilon_1)-\frac{\kappa_2}{\sqrt{n}}\le\Phi^{-1}\left(\varepsilon_1-\frac{T}{\sigma^3\sqrt{n_m}}-\frac{4}{\sqrt{n_m}}\right)<0. \qquad (105)$$

Combining (30), (104) and (105) and using the facts that $\rho=\Theta_n\big(\sqrt{L/n}\big)$, $m=\Theta_n\big(\sqrt{n/L}\big)$ and

$$\sqrt{n\sigma^2}-\sqrt{mL\sigma^2}\le\sqrt{(n-mL)\sigma^2},$$

we obtain

$$
\begin{aligned}
&\frac{1}{n}\log M \\
&\ge \frac{n-mL}{2n}\log(1+S) \\
&\quad +\frac{\sqrt{(n-mL)\sigma^2}}{n}\Phi^{-1}\left(\varepsilon_1-\frac{T}{\sigma^3\sqrt{n_m}}-\frac{4}{\sqrt{n_m}}\right)-\frac{\log\sqrt{n}+1}{n} \\
&\quad -\frac{1}{n}\left(L\left(2S\log 2+\frac{\log(1+S)}{2}\right)+(8S+1)\log n_m\right) \\
&\ge \frac{1}{2}\log(1+P)-\frac{\rho P}{2(1+P)}-\frac{mL}{2n}\log(1+P)
\end{aligned}
$$

$$+\sqrt{\frac{\sigma^2}{n}}\,\Phi^{-1}(\varepsilon_1) - \kappa_3 \max\left\{\frac{L^{1/4}}{n^{3/4}}, \frac{L}{n}\right\} \tag{106}$$

for some $\kappa_3 > 0$ which does not depend on n. In order to bound the second term on the RHS of (106), we obtain from the definition of ρ in (35) that

$$\begin{aligned}\frac{\rho P}{2(1+P)} &= \frac{\sqrt{(L\mathbb{E}[E^2]+3P^2)\log(1+P)\log\frac{1}{\varepsilon_2}}}{2\sqrt{2nP(1+P)}}\\ &= \Theta_n\big(\sqrt{L/n}\big). \end{aligned}\tag{107}$$

In order to bound the third term on the RHS of (106), we first recall the definition of

$$\alpha = \Theta_n(\rho/L) = \Theta_n\big(1/\sqrt{Ln}\big) \tag{108}$$

in (25), the definition of β in (26) and the definition of $\gamma(\varepsilon_2)$ in (32). Consider the following three bounds where κ_4 and κ_5 are some positive constants that do not depend on n:

$$\alpha \geq \frac{2\rho P}{L\mathbb{E}[E^2]+3P^2} \tag{109}$$

where (109) follows from the definition of ρ in (35);

$$\begin{aligned}\beta &\geq \frac{\alpha}{1+63\alpha P}\\ &\geq \frac{2\rho P}{L\mathbb{E}[E^2]+3P^2} - \frac{\kappa_4}{Ln}\end{aligned}\tag{110}$$

where (110) is due to (109) and (108);

$$\begin{aligned}m &= \left\lceil \frac{\log\frac{1}{\varepsilon_2}}{LP\beta + \frac{L^2\alpha^2\mathbb{E}[E^2]}{2}}\right\rceil\\ &\leq \frac{\log\frac{1}{\varepsilon_2}}{LP\beta}+1\\ &= \frac{(L\mathbb{E}[E^2]+3P^2)\log\frac{1}{\varepsilon_2}}{2\rho LP^2}\times\frac{\frac{2\rho P}{L\mathbb{E}[E^2]+3P^2}}{\beta}+1\\ &\leq \frac{(L\mathbb{E}[E^2]+3P^2)\log\frac{1}{\varepsilon_2}}{2\rho LP^2}+\frac{\kappa_5}{L}\end{aligned}\tag{111}$$

where (111) is due to (110) and the definition of ρ in (35). Using (111) and the definition of ρ in (35), we have

$$m \leq \frac{1}{L}\sqrt{\frac{n(L\mathbb{E}[E^2]+3P^2)\log\frac{1}{\varepsilon_2}}{2P(P+1)\log(1+P)}}+\frac{\kappa_5}{L}. \tag{112}$$

Combining (106), (107) and (112), we conclude that (34) holds for any sufficiently large n where $\kappa > 0$ is some constant which does not depend on n.

In addition, (37) follows from the following inequality due to (33), the definition of $\mathcal{Q}^{(n)}(w)$ in (10) and our choice for $\gamma(\varepsilon_2)$ that $\gamma(\varepsilon_2)=0$:

$$\begin{aligned}&\mathbb{P}\left\{\bigcup_{k=mL+1}^{n}\left\{\sum_{i=1}^{k}E_i < \sum_{i=m+1}^{k}X_i^2\right\}\right\}\\ &\quad= \mathbb{P}\big\{|\mathcal{Q}^{(n)}(W)|\geq 1\big\}\\ &\quad= \mathbb{P}\big\{|\mathcal{Q}^{(n)}(W)|\geq L\gamma(\varepsilon_2)+1\big\}\\ &\quad\leq \varepsilon_2.\end{aligned}$$

Appendix D
Proofs of Corollary 5 and Corollary 9

Since best-effort defined in Section III-D is a special case of blockwise best-effort defined in Section IV-D with $L=1$ and Corollary 5 is a special case of Corollary 9 with $L=1$, it suffices to prove Corollary 9.

Fix an $\varepsilon \in (0,1/2)$, and fix any $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that $\varepsilon_1+\varepsilon_2=\varepsilon$. Define ρ, S, α and β as in (40), (12), (25) and (26) respectively. In addition, let $m \triangleq 0$ and define $\gamma(\varepsilon_2)$ as in (42). To simplify notation, we do not explicitly specify the dependence on n for ρ, S, α, β and $\gamma(\varepsilon_2)$. Let $p_X = \mathcal{N}(x;0,S)$ and let $p_Y = \mathcal{N}(y;0,S+1)$ be the marginal distribution of $p_X q_{Y|X}$, and let σ^2 and T denote the variance and the third absolute moment of $\log\frac{q_{Y|X}(Y|X)}{p_Y(Y)}$ respectively. Fix any sufficiently large n such that $\rho\in(0,1)$, $\varepsilon_1 - \frac{T}{\sigma^3\sqrt{n}} - \frac{4}{\sqrt{n}} > 0$ and (11) simultaneously hold. Then, Theorem 2 implies that there exists a blockwise best-effort (n,M,ε)-EH code which satisfies

$$\begin{aligned}\log M &\geq \frac{n}{2}\log(1+S)+\sqrt{n\sigma^2}\,\Phi^{-1}\left(\varepsilon_1-\frac{T}{\sigma^3\sqrt{n}}-\frac{4}{\sqrt{n}}\right)\\ &\quad-(\lambda_1 L+\lambda_2\log n)\,(\gamma(\varepsilon_2)+1)-\log\sqrt{n}-1\end{aligned}\tag{113}$$

and (33) where λ_1 and λ_2 are as defined in (23) and (24) respectively. In the rest of the proof, we will derive (41) from (113). By construction, we have $S=P(1-\rho)$ and $\rho=\Theta_n\big(\sqrt{\max\{\log n, L\}/n}\big)$, and we use Taylor's theorem to conclude that there exist some $\kappa_1>0$ and $\kappa_2>0$ which do not depend on n such that

$$\frac{1}{2}\log(1+S)\geq\frac{1}{2}\left(\log(1+P)-\frac{\rho P}{1+P}-\frac{\kappa_1\log n}{n}\right) \tag{114}$$

and

$$\Phi^{-1}\left(\varepsilon_1-\frac{T}{\sigma^3\sqrt{n}}-\frac{4}{\sqrt{n}}\right)\geq\Phi^{-1}(\varepsilon_1)-\frac{\kappa_2}{\sqrt{n}}. \tag{115}$$

Combining (113), (114) and (115), we obtain

$$\begin{aligned}&\frac{1}{n}\log M\\ &\geq\frac{1}{2}\log(1+P)-\frac{\rho P}{2(1+P)}-\frac{(\lambda_1 L+\lambda_2\log n)(\gamma(\varepsilon_2)+1)}{n}\\ &\quad-\sqrt{\frac{\sigma^2}{n}}\,\Phi^{-1}(\varepsilon_1)-\frac{\kappa_2\sqrt{\sigma^2}}{n}-\frac{\log\sqrt{n}+1}{n}-\frac{\kappa_1\log n}{2n}\\ &\geq\frac{1}{2}\log(1+P)-\frac{\rho P}{2(1+P)}-\frac{(\lambda_1 L+\lambda_2\log n)\gamma(\varepsilon_2)}{n}\\ &\quad-\sqrt{\frac{\sigma^2}{n}}\,\Phi^{-1}(\varepsilon_1)-\frac{\kappa_3\max\{\log n, L\}}{n}\end{aligned}\tag{116}$$

for some $\kappa_3>0$ which does not depend on n. In order to bound the second term on the RHS of (116), we obtain from the definition of ρ in (40) that

$$\begin{aligned}\frac{\rho P}{2(1+P)}&=\frac{1}{2}\sqrt{\frac{(\lambda_1 L+\lambda_2\log n)(L\mathbb{E}[E^2]+3P^2)\log\frac{1}{\varepsilon_2}}{LP(P+1)n}}\\ &=\Theta_n\big(\sqrt{\max\{\log n, L\}/n}\big).\end{aligned}\tag{117}$$

In order to bound the third term on the RHS of (116), we first recall the definition of

$$\alpha = \Theta_n(\rho/L) = \Theta_n\left(\sqrt{\max\{\log n, L\}/(L^2 n)}\right) \tag{118}$$

in (25), the definition of β in (26) and the definition of $\gamma(\varepsilon_2)$ in (42). Consider the following three bounds where κ_4 and κ_5 are some positive constants that do not depend on n:

$$\alpha \geq \frac{2\rho P}{L\mathbb{E}[E^2] + 3P^2} \tag{119}$$

where (119) follows from the definition of ρ in (40);

$$\begin{aligned}\beta &\geq \frac{\alpha}{1 + 63\alpha P} \\ &\geq \frac{2\rho P}{L\mathbb{E}[E^2] + 3P^2} - \frac{\kappa_4 \max\{\log n, L\}}{L^2 n}\end{aligned} \tag{120}$$

where (120) is due to (119) and (118);

$$\begin{aligned}\gamma(\varepsilon_2) &= \frac{\log \frac{1}{\varepsilon_2}}{PL\beta + \frac{L^2\alpha^2\mathbb{E}[E^2]}{2}} \\ &\leq \frac{\log \frac{1}{\varepsilon_2}}{PL\beta} \\ &= \frac{(L\mathbb{E}[E^2] + 3P^2)\log \frac{1}{\varepsilon_2}}{2\rho L P^2} \times \frac{\frac{2\rho P}{L\mathbb{E}[E^2]+3P^2}}{\beta} \\ &\leq \frac{(L\mathbb{E}[E^2] + 3P^2)\log \frac{1}{\varepsilon_2}}{2\rho L P^2} + \frac{\kappa_5}{L}\end{aligned} \tag{121}$$

where (121) is due to (120) and the definition of ρ in (40). Using (121) and the definition of ρ in (40), we have

$$\gamma(\varepsilon_2) \leq \frac{1}{2}\sqrt{\frac{n(L\mathbb{E}[E^2] + 3P^2)\log \frac{1}{\varepsilon_2}}{LP(P+1)(\lambda_1 L + \lambda_2 \log n)}} + \frac{\kappa_5}{L}. \tag{122}$$

Combining (116), (117) and (122), we conclude that (41) holds for any sufficiently large n where $\kappa > 0$ is some constant which does not depend on n.

In addition, (43) follows from (33) and our choice for m that $m = 0$.

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Silas L. Fong (M'15) received the B.Eng., M.Phil. and Ph.D. degrees in Information Engineering from The Chinese University of Hong Kong (CUHK), Hong Kong, in 2005, 2007 and 2011 respectively. He is currently a Postdoctoral Fellow with the Department of Electrical and Computer Engineering at University of Toronto, Toronto, ON, Canada.

From 2011 to 2013, Dr. Fong was a Postdoctoral Fellow with the Department of Electronic Engineering at City University of Hong Kong, Hong Kong. From 2013 to 2014, he was a Postdoctoral Associate with the Department of Electrical and Computer Engineering at Cornell University, Ithaca, NY. From 2014 to 2017, he was a Research Fellow with the Department of Electrical and Computer Engineering at National University of Singapore (NUS), Singapore. His research interests include information theory and its applications to communication systems such as relay networks, wireless networks, and energy-harvesting channels.

Jing Yang (S'08–M'10) received the B.S. degree in Electrical Engineering from the University of Science and Technology of China and the M.S. and Ph.D. degrees in Electrical Engineering from the University of Maryland at College Park. She is an Assistant Professor of Electrical Engineering with the Pennsylvania State University. She was a Post-Doctoral Fellow with the University of Wisconsin–Madison, and an Assistant Professor with the Department of Electrical Engineering, University of Arkansas. Her research interests are in wireless communications and networking, statistical learning and signal processing, and information theory. She was a recipient of the NSF CAREER Award in 2015. She served as the Co-Chair for the MAC and Cross-Layer Design Track of the IEEE International Symposium on Personal Indoor, and Mobile Radio Communications in 2014, and a Session Co-Organizer for IEEE Communication Theory Workshop in 2015. She is currently serving as an Editor for the IEEE TRANSACTIONS ON GREEN COMMUNICATIONS AND NETWORKING.

Aylin Yener (S'91–M'01–SM'14–F'15) received the B.Sc. degree in Electrical and Electronics Engineering and the B.Sc. degree in Physics from Bogazici University, Istanbul, Turkey, and the M.S. and Ph.D. degrees in Electrical and Computer Engineering from the Wireless Information Network Laboratory (WINLAB), Rutgers University, New Brunswick, NJ, USA. She is a Distinguished Professor of Electrical Engineering at The Pennsylvania State University, University Park, PA, USA, where she joined the faculty as an assistant professor in 2002. Since 2017, she is also a Dean's Fellow in the College of Engineering at The Pennsylvania State University. She was a visiting professor of Electrical Engineering at Stanford University in 2016–2018 and a visiting associate professor in the same department in 2008–2009. Her current research interests are in information security, green communications, caching systems, and more generally in the fields of information theory, communication theory and networked systems. She received the NSF CAREER Award in 2003, the Best Paper Award in Communication Theory from the IEEE International Conference on Communications in 2010, the Penn State Engineering Alumni Society (PSEAS) Outstanding Research Award in 2010, the IEEE Marconi Prize Paper Award in 2014, the PSEAS Premier Research Award in 2014, the Leonard A. Doggett Award for Outstanding Writing in Electrical Engineering at Penn State in 2014, the IEEE Women in Communications Engineering Outstanding Achievement Award in 2018, and the IEEE Communications Society Best Tutorial Paper Award in 2019. She is a distinguished lecturer for the IEEE Information Theory Society (2019–2020), the IEEE Communications Society (2018–2020) and the IEEE Vehicular Technology Society (2017–2019).

Dr. Yener is serving as the vice president of the IEEE Information Theory Society in 2019. Previously she was the second vice president (2018), member of the Board of Governors (2015–2018) and the treasurer (2012–2014) of the IEEE Information Theory Society. She served as the Student Committee Chair for the IEEE Information Theory Society (2007–2011), and was the co-Founder of the Annual School of Information Theory in North America in 2008. She was a Technical (Co)-Chair for various symposia/tracks at the IEEE ICC, PIMRC, VTC, WCNC, and Asilomar in 2005, 2008–2014 and 2018. Previously, she served as an Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS (2009-2012), an Editor for the IEEE TRANSACTIONS ON MOBILE COMPUTING (2017–2018), and an Editor and an Editorial Advisory Board Member for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS (2001–2012). She also served a Guest Editor for the IEEE TRANSACTIONS ON INFORMATION FORENSICS AND SECURITY in 2011, and the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS in 2015. Currently, she serves as a Senior Editor for the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS and is on the inaugural Senior Editorial Board of the IEEE JOURNAL ON SELECTED AREAS IN INFORMATION THEORY.