Non-Asymptotic Achievable Rates for Gaussian Energy-Harvesting Channels: Best-Effort and Save-and-Transmit

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***Abstract*—An additive white Gaussian noise (AWGN) energy-harvesting (EH) channel is considered where the transmitter is equipped with an infinite-sized battery which stores energy harvested from the environment. The energy arrival process is modeled as a sequence of independent and identically distributed (i.i.d.) random variables. The capacity of this channel is known and is achievable by the so-called best-effort and save-and-transmit schemes. This paper investigates the best-effort scheme in the finite blocklength regime and establishes the first non-asymptotic achievable rate for it. The first-order term of the non-asymptotic achievable rate equals the capacity, and the second-order term is proportional to $-\sqrt{\log n/n}$ where n denotes the blocklength. The proof technique involves analyzing the escape probability of a Markov process. In addition, we use this new proof technique to analyze the save-and-transmit and obtain a new non-asymptotic achievable rate for it, whose first-order and second-order terms achieve the capacity and the scaling $-1/\sqrt{n}$ respectively. For all sufficiently large signal-to-noise ratios (SNRs), our new achievable rate outperforms the existing ones.**

I. INTRODUCTION

In this paper, we consider communication over an energy-harvesting (EH) channel between a transmitted equipped with an infinite-capacity battery and a receiver as illustrated in Figure 1. The transmitter wants to transmit a message to the receiver through the EH channel, and the channel noise is modeled as an additive white Gaussian noise (AWGN). At each discrete time $k \in \{1, 2, \ldots\}$, a random amount of energy E_k arrives at the battery and the transmitter sends a symbol $X_k \in \mathcal{X}$ such that

$$\sum_{\ell=1}^{k} X_\ell^2 \leq \sum_{\ell=1}^{k} E_\ell \qquad \text{almost surely}$$

where X_ℓ^2 denote the energy consumed by transmitting X_ℓ. This implies that the total harvested energy $\sum_{\ell=1}^{k} E_\ell$ must be no smaller than the "energy" of the codeword $\sum_{\ell=1}^{k} X_\ell^2$ at every discrete time k for transmission to take place successfully. The receiver observes $Y_k = X_k + Z_k$ at each time k where Z_k is a standard normal random variable which is independent of X_k and $\{Z_k\}_{k=1}^{\infty}$ are independent. We assume that $\{E_\ell\}_{\ell=1}^{\infty}$ are independent and identically distributed (i.i.d.), where E_1 is a non-negative random variable. To simplify notation, we write $E \triangleq E_1$ if there is no ambiguity. Throughout the paper, we let $P \triangleq \mathbb{E}[E]$, the expected value of E, be the signal-to-noise ratio (SNR) of the channel.

For the AWGN EH channel described above, reference [1] showed that the capacity equals $\frac{1}{2}\log(1+P)$ and proposed two capacity-achieving schemes, namely *save-and-transmit* and *best-effort*.

The save-and-transmit scheme consists of an initial saving phase and a subsequent transmission phase. The transmitter remains silent in the saving phase so that energy will be accumulated within the battery. In the transmission phase the transmitter sends the symbols of a random Gaussian codeword with variance $P-\Delta$ as long as the battery has sufficient energy where $0 \leq \Delta < P$ denotes some small offset from P.

The best-effort scheme can be viewed as a save-and-transmit scheme without an initial saving phase. Therefore, information is sent right away and the transmitter uses every opportunity (as long as it has sufficient energy) to output the symbols of a random Gaussian codeword with variance $P - \Delta$ for some $0 \leq \Delta < P$.

Following reference [1], a number of non-asymptotic achievable rates for save-and-transmit schemes have been presented [2]–[4]. By contrast, no non-asymptotic achievable rate exists for the best-effort scheme except for a special discrete memoryless EH channel with finite battery studied in [5] and a special discrete memoryless EH channel with no battery studied in [6]. Therefore, we are motivated to prove the first non-asymptotic achievable rates for best-effort schemes over the AWGN EH channel.

This paper contains two main results. First, we derive the first non-asymptotic achievable rate for the best-effort scheme. The derivation involves carefully designing the transmitted power to be $P - O\big(\sqrt{\log n/n}\big)$ so that we can effectively bound the number of mismatched positions between the desired transmitted codeword and the actual transmitted codeword for a fixed blocklength. Second, we propose a save-and-transmit scheme with a similar transmitted power $P -$

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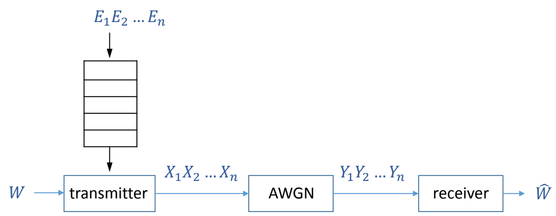

Fig. 1. The AWGN EH channel

$O\big(\sqrt{\log n/n}\big)$ and obtain a new non-asymptotic achievable rate which outperforms the best existing one for all sufficiently large SNRs $P = \mathbb{E}[X] > 0$.

Note that the results in this paper cease to hold if the size of the battery is finite. The channel capacity for the finite battery case is the subject of recent interest, see [7]–[9].

The rest of the paper is organized as follows. Section II describes the notation used in this paper. Section III presents the formulation of the AWGN EH channel. Section IV describes the best-effort scheme and presents the first main result. Section V discusses the save-and-transmit scheme and presents the second main result. Section VI concludes this paper and discusses the extension of the results to the block energy arrival model [4], [10].

II. Notation

We use $O(\cdot)$, $\Theta(\cdot)$, $\omega(\cdot)$ and $o(\cdot)$ to denote standard asymptotic Bachmann-Landau notations except our convention that they must be non-negative. The sets of natural numbers, real numbers and non-negative real numbers are denoted by $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{R}_+$ respectively. All logarithms are taken to base e throughout the paper.

Random variables are denoted by capital letters (e.g., X), and the realization and the alphabet of a random variable are denoted by the corresponding small letter (e.g., x) and calligraphic font (e.g., $\mathcal{X}$) respectively. We use X^n to denote a random tuple $(X_1, X_2, \dots, X_n)$, where all the elements X_k have the same alphabet $\mathcal{X}$. We let p_X and $p_{Y|X}$ denote the probability distribution of X and the conditional probability distribution of Y given X respectively for random variables X and Y. We let $p_X p_{Y|X}$ denote the joint distribution of (X, Y). For any function f whose domain contains $\mathcal{X}$, we use $\mathbb{E}_{p_X}[f(X)]$ to denote the expectation of $f(X)$ where X is distributed according to p_X. For simplicity, we omit the subscript of a notation when there is no ambiguity. The distribution of a Gaussian random variable Z whose mean and variance are μ and σ^2 respectively is denoted by $\mathcal{N}(z; \mu, \sigma^2) \triangleq \frac{1}{\sqrt{2\pi\sigma^2}} \mathrm{e}^{-\frac{(z-\mu)^2}{2\sigma^2}}$.

III. The AWGN EH Channel

A. Problem Formulation

The AWGN EH channel, as illustrated in Figure 1, consists of one transmitter and one receiver. Energy harvesting and communication occur in n time slots, i.e., channel uses. In each time slot, a random amount of energy E, $\mathcal{E} = \mathbb{R}_+$, is harvested where $\mathbb{E}[E] > 0$ and $\mathbb{E}[E^2] < \infty$. The energy-harvesting process is characterized by n independent copies of E denoted by $E_1, E_2, \dots, E_n$. Before the n time slots, the transmitter chooses a message W. For each $k \in \{1, 2, \dots, n\}$, the transmitter consumes X_k^2 units of energy to transmit $X_k \in \mathbb{R}$ based on (W, E^k) and the receiver observes $Y_k \in \mathbb{R}$ in time slot k. The energy state information E_k is known by the transmitter at time k before encoding X_k, but the receiver has no access to E_k. For each $k \in \{1, 2, \dots, n\}$, we have:

(i) E_k and $(W, E^{k-1}, X^{k-1}, Y^{k-1})$ are independent, i.e.,

$$p_{W,E^k,X^{k-1},Y^{k-1}} = p_{E_k} p_{W,E^{k-1},X^{k-1},Y^{k-1}}. \tag{1}$$

(ii) For $w \in \mathcal{W}$ and every $e^n \in \mathbb{R}_+^n$, a transmitted codeword X^n should satisfy

$$\mathbb{P}\left\{ \sum_{\ell=1}^{k} X_\ell^2 \le \sum_{\ell=1}^{k} e_\ell \,\middle|\, W = w, E^n = e^n \right\} = 1 \tag{2}$$

for each $k \in \{1, 2, \dots, n\}$.

After n time slots, the receiver declares $\hat{W}$ based on Y^n to be the transmitted W.

B. Standard Definitions

Formally, we define a code as follows:

Definition 1: An (n, M)-*code* consists of the following:

1) A message set $\mathcal{W} \triangleq \{1, 2, \dots, M\}$, where W is uniform on $\mathcal{W}$.
2) A sequence of encoding functions $f_k : \mathcal{W} \times \mathbb{R}_+^k \to \mathbb{R}$ for each $k \in \{1, 2, \dots, n\}$, where f_k is used by the transmitter at time slot k for encoding X_k according to $X_k = f_k(W, E^k)$.
3) A decoding function $\varphi : \mathbb{R}^n \to \mathcal{W}$, for decoding W at the receiver, i.e., $\hat{W} = \varphi(Y^n)$.

If the sequence of encoding functions f_i satisfies (2), the code is also called an (n, M)-*EH code*.

Definition 2: The *AWGN EH channel* is characterized by a conditional probability distribution $q_{Y|X}(y|x) \triangleq \mathcal{N}(y; x, 1)$ such that the following holds for any (n, M)-code: For each $k \in \{1, 2, \dots, n\}$,

$$p_{W,E^k,X^k,Y^k} = p_{W,E^k,X^k,Y^{k-1}} p_{Y_k|X_k}$$

where

$$p_{Y_k|X_k}(y_k|x_k) = q_{Y|X}(y_k|x_k) = \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{(y_k - x_k)^2}{2}} \tag{3}$$

for all $x_k \in \mathcal{X}$ and $y_k \in \mathcal{Y}$.

For any (n, M)-code defined on the AWGN EH channel, let $p_{W,E^n,X^n,Y^n,\hat{W}}$ be the joint distribution induced by the code. We can factorize $p_{W,E^n,X^n,Y^n,\hat{W}}$ as

$$p_{W,E^n,X^n,Y^n,\hat{W}} = p_W \left(\prod_{k=1}^{n} p_{E_k} p_{X_k|W,E^k} p_{Y_k|X_k} \right) p_{\hat{W}|Y^n}, \tag{4}$$

which follows from the i.i.d. assumption of the EH process E^n in (1), the fact by Definition 1 that X_i is a function of (W, E^i) and the memoryless property of the channel $q_{Y|X}$ described in Definition 2.

Definition 3: For an (n, M)-code defined on the AWGN EH channel, we can calculate according to (4) the *average probability of decoding error* defined as $\mathbb{P}\{\hat{W} \neq W\}$. We call an (n, M)-EH code with average probability of decoding error no larger than ε an *(n, M, ε)-EH code.*

Definition 4: Let $\varepsilon \in (0, 1)$ be a real number. A rate R is said to be *ε-achievable* for the EH channel if there exists a sequence of (n, M_n, ε)-EH codes such that $\liminf_{n\to\infty} \frac{1}{n} \log M_n \geq R$.

Definition 5: The *ε-capacity* of the AWGN EH channel, denoted by C_ε, is defined to be $C_\varepsilon \triangleq \sup\{R : R \text{ is } \varepsilon\text{-achievable for the EH channel}\}$. The *capacity* of the AWGN EH channel is $C \triangleq \inf_{\varepsilon>0} C_\varepsilon$.

Define the capacity function $\mathrm{C}(x) \triangleq \frac{1}{2}\log(1 + x)$ for all $x \geq 0$ and define $P \triangleq \mathbb{E}[E]$. It was shown in [2] that $C_\varepsilon = C = \mathrm{C}(P)$ for all $\varepsilon \in (0, 1)$ where $P = \mathbb{E}[X]$ can be viewed as the signal-to-noise ratio (SNR) of the AWGN EH channel.

IV. An Achievable Rate for Best-Effort

A. Best-Effort Scheme

Fix a blocklength n. Choose a positive real number $S_n < P = \mathbb{E}[E]$ and let

$$p_X(x) \equiv \mathcal{N}(x; 0, S_n) \tag{5}$$

such that

$$S_n = \mathbb{E}_{p_X}[X^2]. \tag{6}$$

The codebook consists of M mutually independent random codewords, which are constructed as follows. For each message $w \in \mathcal{W}$, a length-n codeword $X^n(w) \triangleq (X_1(w), X_2(w), \ldots X_n(w))$ consisting of n i.i.d. symbols is constructed where $X_1(w) \sim p_X$. Suppose $W = w$ and $E^n = e^n$, i.e., the transmitter chooses message $w \in \mathcal{W}$ and the realization of E^n is $e^n \in \mathbb{R}_+^n$. Then, the transmitter uses the following *best-effort (n, M)-EH code* with encoding functions $\{f_k^{\text{best}}\}_{k=1}^n$ and decoding function φ^{best}. Define $f_1^{\text{best}}, f_2^{\text{best}}, \ldots, f_n^{\text{best}}$ in a recursive manner where

$$f_k^{\text{best}}(w, e^k) \triangleq \begin{cases} X_k(w) & \text{if } \big(X_k(w)\big)^2 \leq e_k + \sum\limits_{\ell=1}^{k-1}\left(e_\ell - \left(f_\ell^{\text{best}}(w, e^\ell)\right)^2\right), \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

For each $k \in \{1, 2, \ldots, n\}$, the transmitter sends

$$\tilde{X}_k(W) \triangleq f_k^{\text{best}}(W, E^k). \tag{8}$$

By construction,

$$\mathbb{P}\left\{\sum_{\ell=1}^k \big(\tilde{X}_\ell(W)\big)^2 \leq \sum_{\ell=1}^k E_\ell\right\} = 1. \tag{9}$$

Upon receiving $\tilde{Y}^n(W) \triangleq (\tilde{Y}_1(W), \tilde{Y}_2(W), \ldots, \tilde{Y}_n(W))$ where $\tilde{Y}_k(W)$ is generated according to

$$\mathbb{P}\{\tilde{Y}_k(W) = b \,|\, \tilde{X}_k(W) = a\} \equiv q_{Y|X}(b|a), \tag{10}$$

the receiver declares that $\varphi^{\text{best}}(\tilde{Y}^n(W)) = j$ if j is the unique integer in $\mathcal{W}$ that satisfies

$$\sum_{k=1}^n \log \frac{q_{Y|X}(\tilde{Y}_k(W)|X_k(j))}{p_Y(\tilde{Y}_k(W))} \geq \log \xi, \tag{11}$$

where p_Y is the marginal distribution of $p_X q_{Y|X}$ and $\log \xi$ is an arbitrary threshold. Otherwise, the receiver chooses $\varphi^{\text{best}}(\tilde{Y}^n(W)) \in \mathcal{W}$ according to the uniform distribution.

The code described above is said to be best-effort because according to (7) and (18), the transmitter tries its best to output the desired symbol $X_k(W)$ whenever the battery contains enough energy for transmitting $X_k(W)$.

B. Preliminaries

An important quantity that determines the performance of the best-effort (n, M)-EH code is

$$\mathcal{Q}^{(n)} \triangleq \left\{k \in \{1, 2, \ldots, n\} \,\middle|\, \tilde{X}_k(W) \neq X_k(W)\right\}, \tag{12}$$

which is a random set that specifies the mismatched positions between $\tilde{X}^n(W)$ and $X^n(W)$. The following lemma concerns the number of mismatched positions between $\tilde{X}^n(W)$ and $X^n(W)$.

Lemma 1: Fix any n and any $\rho_n \in (0, 1)$ such that

$$\frac{\sqrt{42\rho_n}}{21} < \frac{\sqrt{1-\rho_n}}{2}, \tag{13}$$

and fix a best-effort (n, M)-EH code with

$$S_n \triangleq P(1 - \rho_n). \tag{14}$$

Define

$$\alpha_n \triangleq \frac{2\rho_n P}{\mathbb{E}[E^2] + 3S_n^2} \tag{15}$$

and

$$\beta_n \triangleq \frac{\alpha_n}{1 + 63\alpha_n S_n}. \tag{16}$$

For any $\gamma \in \mathbb{R}_+$, we have

$$\mathbb{P}\left\{|\mathcal{Q}^{(n)}| \geq \gamma + 1\right\} \leq \mathrm{e}^{-\gamma\left(P\beta_n + \frac{\alpha_n^2 \mathbb{E}[E^2]}{2}\right)}. \tag{17}$$

Remark 1: The proof of Lemma 1 can be found in [11, Appendix A]. An important step in the proof is analyzing the escape probability $\mathbb{P}\{\tau = \infty\}$ of the Markov process $\left\{E_1 + \sum_{\ell=2}^k \left(E_\ell - X_\ell^2\right)\right\}_{k=2}^\tau$ where τ is the stopping time when the value of the Markov process hits any negative number $a < 0$. In particular, $\mathbb{P}\{\tau = \infty\} \geq 1 - \mathrm{e}^{-\left(P\beta_n + \frac{\alpha_n^2 \mathbb{E}[E^2]}{2}\right)}$.

The following lemma [12] is standard for proving achievability results in the finite blocklength regime and its proof can be found in [13, Th. 3.8.1].

Lemma 2 (Implied by Shannon's bound [12]): Let p_{X^n,Y^n} be the probability distribution of a pair of random variables (X^n, Y^n). Suppose $(X^n(1), Y^n(1)) \sim p_{X^n,Y^n}$ and

$(X^n(2), Y^n(2)) \sim p_{X^n,Y^n}$ are independent. Then for each $\delta > 0$ and each $M \in \mathbb{N}$, we have

$$\mathbb{P}\left\{\log \frac{p_{Y^n|X^n}(Y^n(1)|X^n(2))}{p_{Y^n}(Y^n(1))} > \log M + n\delta\right\} \leq \frac{1}{M}\,\mathrm{e}^{-n\delta}.$$

The following lemma is a modification of the Shannon's bound stated in the previous lemma, and its proof can be found in [11, Appendix B].

Lemma 3: Suppose we are given a best-effort (n, M)-EH code as described in Section IV-A. Then for each $\gamma \in \mathbb{R}_+$, each $\delta > 0$ and each $M \in \mathbb{N}$, we have

$$\mathbb{P}\left\{\begin{array}{l}\left\{\log \frac{p_{Y^n|X^n}(\tilde{Y}^n(1)|X^n(2))}{p_{Y^n}(\tilde{Y}^n(1))} > \log(M\mathrm{e}^{n\delta})\right\} \\ \cap \left\{|\mathcal{Q}^{(n)}| < \gamma + 1\right\}\end{array}\right\} \leq \frac{(S_n+1)^{\frac{\lfloor \gamma+1 \rfloor}{2}}}{M\mathrm{e}^{n\delta}}.$$

C. An Achievable Rate for the Best-Effort Scheme

The following theorem is the first main result of this paper. The proof relies on Lemma 1 and Lemma 3, and it is contained in [11, Sec. IV and Appendix B].

Theorem 1: Fix an $\varepsilon \in (0, 1)$, and fix any $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that $\varepsilon_1 + \varepsilon_2 \leq \varepsilon$. There exists a constant $\kappa > 0$ which does not depend on n such that for all sufficiently large n, there exists a best-effort (n, M, ε)-EH code with

$$\rho_n \triangleq \frac{\sqrt{11(P+1)(\mathbb{E}[E^2] + 3P^2)\log\frac{1}{\varepsilon_2}}}{P} \times \sqrt{\frac{\log n}{n}},$$

$$S_n = P(1-\rho_n) = P - \Theta\left(\sqrt{\frac{\log n}{n}}\right)$$

and some choice of

$$\log \xi = \log M + O(\sqrt{n \log n})$$

where

$$\begin{aligned}\frac{1}{n}\log M \geq{}& \frac{1}{2}\log(1+P) \\ &- \sqrt{\frac{11(\mathbb{E}[E^2] + 3P^2)\log\frac{1}{\varepsilon_2}}{P+1}} \times \sqrt{\frac{\log n}{n}} \\ &- \sqrt{\frac{P}{(P+1)n}}\,\Phi^{-1}(\varepsilon_1) - \frac{\kappa \log n}{n}.\end{aligned}$$

In particular, the probability of seeing more than $\Theta(\sqrt{n/\log n})$ mismatch events can be bounded as

$$\mathbb{P}\{|\mathcal{Q}^{(n)}| \geq \gamma_n + 1\} \leq \varepsilon_2$$

where

$$\gamma_n \triangleq \frac{\log\frac{1}{\varepsilon_2}}{P\beta_n + \frac{\alpha_n^2 \mathbb{E}[E^2]}{2}} = \Theta\left(\sqrt{\frac{n}{\log n}}\right).$$

V. An Achievable Rate for Save-and-Transmit

In Section IV-A, we have described the construction of the codebook of a best-effort scheme. In this section, we would like to investigate a save-and-transmit strategy that uses a similar codebook.

A. Save-and-Transmit Scheme

Fix a blocklength n. Choose a positive real number $S_n < P = \mathbb{E}[E]$ and let p_X and S_n as defined in (5) and (6) respectively. The codebook consists of M mutually independent random codewords denoted by $\{X^n(w) \,|\, w \in \mathcal{W}\}$, which are constructed as described in Section IV-A. Suppose $W = w$ and $E^n = e^n$, i.e., the transmitter chooses message $w \in \mathcal{W}$ and the realization of E^n is $e^n \in \mathbb{R}_+^n$. Then, the transmitter uses the following *save-and-transmit* (n, M)*-EH code* with encoding functions $\{f_k^{\text{save}}\}_{k=1}^n$ and decoding function φ^{save}. The save-and-transmit code consists of an initial saving phase and a subsequent transmission phase. Define γ_n to be the number of time slots in the initial saving phase during which no energy is consumed and hence no information is conveyed. Define $f_1^{\text{save}}, f_2^{\text{save}}, \ldots, f_n^{\text{save}}$ in a recursive manner where

$$f_k^{\text{save}}(w, e^k) \triangleq \begin{cases} X_k(w) & \text{if } k > \gamma_n \text{ and} \\ & \big(X_k(w)\big)^2 \leq e_k + \sum\limits_{\ell=1}^{k-1}\Big(e_\ell - \big(f_\ell^{\text{save}}(w, e^\ell)\big)^2\Big), \\ 0 & \text{otherwise.}\end{cases}$$

For each $k \in \{1, 2, \ldots, n\}$, the transmitter sends

$$\tilde{X}_k(W) \triangleq f_k^{\text{save}}(W, E^k). \tag{18}$$

By construction,

$$\mathbb{P}\left\{\sum_{\ell=1}^{k}\big(\tilde{X}_\ell(W)\big)^2 \leq \sum_{\ell=1}^{k} E_\ell\right\} = 1. \tag{19}$$

Upon receiving $\tilde{Y}^n(W) \triangleq (\tilde{Y}_1(W), \tilde{Y}_2(W), \ldots, \tilde{Y}_n(W))$ where $\tilde{Y}_k(W)$ is generated according to (10), the receiver declares that $\varphi^{\text{save}}(\tilde{Y}^n(W)) = j$ if j is the unique integer in $\mathcal{W}$ that satisfies

$$\sum_{k=\gamma+1}^{n} \log \frac{q_{Y|X}(\tilde{Y}_k(W)|X_k(j))}{p_Y(\tilde{Y}_k(W))} \geq \log \xi, \tag{20}$$

where p_Y is the marginal distribution of $p_X q_{Y|X}$ and $\log \xi$ is an arbitrary threshold. Otherwise, the receiver chooses $\varphi^{\text{save}}(\tilde{Y}^n(W)) \in \mathcal{W}$ according to the uniform distribution.

The following lemma states an upper bound on the probability of a mismatch event occurring in the transmission phase given that the saving phase lasts for γ time slots. The proof of Lemma 4 is quite similar to the proof of Lemma 1 established for the best-effort scheme, and it can be found in [11, Appendix E].

Lemma 4: Fix any n and any $\rho_n \in (0, 1)$ such that (13) holds, and fix a save-and-transmit (n, M)-EH code with S_n, α_n and β_n being defined as in (14), (15) and (16) respectively. For any $\gamma \in \mathbb{N}$, we have

$$\mathbb{P}\left\{\bigcup_{k=\gamma+1}^{n}\left\{\sum_{i=1}^{k} E_i < \sum_{i=\gamma+1}^{k} X_i^2\right\}\right\} \leq \mathrm{e}^{-\gamma\left(P\beta_n + \frac{\alpha_n^2 \mathbb{E}[E^2]}{2}\right)}. \tag{21}$$

B. An Achievable Rate for the Save-and-Transmit Scheme

The following theorem is the second main result of this paper. The proof relies on Lemma 4 and Lemma 2, and it is contained in [11, Sec. VI and Appendix F].

Theorem 2: Fix an $\varepsilon \in (0,1)$, and fix any $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that $\varepsilon_1 + \varepsilon_2 \le \varepsilon$. There exists a constant $\kappa > 0$ which does not depend on n such that for all sufficiently large n, there exists a save-and-transmit (n, M, ε)-EH code with

$$\rho_n \triangleq \frac{\sqrt{(P+1)(\mathbb{E}[E^2]+3P^2)\log(1+P)\log\frac{1}{\varepsilon_2}}}{P\sqrt{2nP}},$$

$$S_n = P(1-\rho_n) = P - \Theta(1/\sqrt{n})$$

and some choice of

$$\log \xi = \log M + O(\log n)$$

where

$$\begin{aligned}\frac{1}{n}\log M \ge{}& \frac{1}{2}\log(1+P) \\ &- \sqrt{\frac{(\mathbb{E}[E^2]+3P^2)\log(1+P)\log\frac{1}{\varepsilon_2}}{2nP(P+1)}} \\ &+ \sqrt{\frac{P}{(P+1)n}}\,\Phi^{-1}(\varepsilon_1) - \frac{\kappa}{n^{3/4}}. \end{aligned} \tag{22}$$

In particular, the probability of seeing a mismatch event in the transmission phase can be bounded as

$$\mathbb{P}\left\{\bigcup_{k=\gamma_n+1}^{n}\left\{\sum_{i=1}^{k} E_i < \sum_{i=\gamma_n+1}^{k} X_i^2\right\}\right\} \le \varepsilon_2$$

where

$$\gamma_n \triangleq \left\lceil \frac{\log\frac{1}{\varepsilon_2}}{P\beta_n + \frac{\alpha_n^2\mathbb{E}[E^2]}{2}} \right\rceil = \Theta(\sqrt{n}).$$

The parameters ρ_n and γ_n in Theorem 2 have been optimized to achieve the second-order scaling $-O(1/\sqrt{n})$. Fix any $\varepsilon > 0$. The best existing lower bound on the second-order term of $\frac{1}{n}\log M$ was derived in [4, Th. 1], which states that there exists a save-and-transmit (n, M, ε)-EH code that satisfies

$$\begin{aligned}&\liminf_{n\to\infty}\frac{1}{\sqrt{n}}\left(\log M - \frac{n}{2}\log(1+P)\right) \\ &\ge -\frac{\log(1+P)}{2P}\sqrt{(\mathbb{E}[E^2]+P^2)\log\frac{1}{\varepsilon_2}} + \sqrt{\frac{P}{P+1}}\,\Phi^{-1}(\varepsilon_1)\end{aligned} \tag{23}$$

for any $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that $\varepsilon_1 + \varepsilon_2 \le \varepsilon$. Note that the second-order term of the best existing lower bound as stated on the RHS of (23) decays as $-\frac{1}{2}\log(1+P)\sqrt{(1+\frac{\mathbb{E}[E^2]}{P^2})\log\frac{1}{\varepsilon_2}} + \Phi^{-1}(\varepsilon_1)$ as P tends to ∞. On the other hand, it follows from (22) in Theorem 2 that the second-order term of our lower bound decays as $-\sqrt{\frac{1}{2}(3+\frac{\mathbb{E}[E^2]}{P^2})\log(1+P)\log\frac{1}{\varepsilon_2}} + \Phi^{-1}(\varepsilon_1)$ as P tends to ∞. Consequently, the second-order term achievable by the save-and-transmit scheme guaranteed by Theorem 2 is strictly larger (less negative) than the best existing bound for all sufficiently large $P > 0$.

VI. Concluding Remarks

In this work, we prove the first non-asymptotic achievable rate for the best-effort scheme over the AWGN EH channel. The second-order scaling of the non-asymptotic rate for best-effort is $-O(\sqrt{\log n/n})$. Then, we use a similar proof technique and obtain a new non-asymptotic achievable rate for save-and-transmit over the same channel. The second-order scaling of the non-asymptotic rate for save-and-transmit is $-O\big(1/\sqrt{n}\big)$. The achievable rates for best-effort and save-and-transmit have been extended to the block energy arrival model in the long version of this paper [11] where the energy arrives in a block i.i.d. fashion [4], [10]. If the length of each energy block L grows sublinearly in n, i.e., $L = o(n)$, we show in [11] that best-effort and save-and-transmit achieve the second-order scalings $-O(\sqrt{\max\{\log n, L\}/n})$ and $-O(\sqrt{L/n})$. A future direction may improve the second-order scaling $-O\big(\sqrt{\log n/n}\big)$ for $L = 1$ for best-effort schemes by possibly proving a sharper probability bound than (17) in Lemma 1. Another interesting direction is to explore the finite-battery case and complement existing results [8], [9] by deriving new non-asymptotic achievable rates for best-effort and save-and-transmit with finite battery.

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