Downlink Throughput Maximization for Interference Limited Multiuser Systems: TDMA versus CDMA

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***Abstract*— We consider the downlink throughput maximization problem for interference limited multiuser systems. Our goal is to characterize the optimum base station transmission strategy, i.e., whether the base station transmits to one-user (TDMA) or multiple users (CDMA). Specifically, we aim at determining the optimum number of users to be scheduled and finding the corresponding power allocation. We model the interference by the aid of the orthogonality factor, and determine the throughput maximizing transmission strategy for a range of the values of the orthogonality factor, and the channel gains, subject to a total power constraint. Although the resulting optimization problem may turn out to be non-convex, we show that valuable observations regarding the structure of the optimum solution can be obtained by examining the performance metric from an individual user's point of view. We propose an exact and a near-exact algorithm to determine whether one-user-transmission is the optimum strategy, or more than one user should be transmitted to. Numerical results to support our analysis, as well as the modifications to the proposed algorithms in the presence of individual power constraints are presented.**

***Index Terms*— Throughput maximization, scheduling, TDMA, CDMA.**

I. Introduction

Due to the scarcity of wireless resources, efficient resource management is crucial for high speed wireless communications. Power control is one important resource management technique [2]–[4]. For voice CDMA services, the main purpose of power control is to maintain the signal to interference ratio (SIR) to satisfy the minimum quality of service for all voice users constantly. Current and future wireless services are becoming more data centric, and often require higher data rate for downlink communications [5]–[7]. Data services typically require higher reliability but are delay tolerant. Conventional power control as described above is not efficient for data services in that, the transmit power would be wasted to compensate for constant interference. Instead, delay tolerance can be exploited by efficiently scheduling users in favorable channel conditions to increase overall system throughput [8].

Manuscript received July 27, 2005; revised December 26, 2006; accepted March 18, 2007. The editor coordinating the review of this paper and approving it for publication was A. Scaglione. This work was supported in part by NSF grant CCF (CAREER) 02-37727, and was presented in part at the Conference on Information Sciences and Systems, CISS'05, Baltimore, MD, March 2005 [1].

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Digital Object Identifier 10.1109/TWC.2007.05579.

In this paper, we consider the scheduling and power allocation problem for the downlink of delay tolerant CDMA systems, by taking the channels of the users into account. For the CDMA downlink, typically, orthogonal codes are used to communicate to each user [5]. However, due to multipath fading, orthogonality between codes is not preserved at the mobile side. In such cases, orthogonality factor is often used to represent the level of interference from other users [9]. This is precisely the scenario we focus on with the aim to find the optimum transmission policy, i.e., the user(s) the base station will transmit to and the corresponding transmit power levels so as to maximize the total system utility. The total system utility is defined simply as the sum of the individual utilities. The individual utility we consider is the transmission rate to a user, which is an increasing function of the user's transmit power.

Utility based power control for data services has previously been considered in [10]–[12], for the uplink, where the quality of service (QoS) of each user, e.g. signal to interference ratio (SIR), is defined as the utility. Power allocation problem for CDMA data services is considered in the uplink in [13], [14] and the downlink in [15]–[18]. References [13], [14], consider the jointly uplink power control and spreading gain allocation problem for the non-real time users. The jointly power and spreading gain control results in the optimum spreading gain being inversely proportional to the user's SIR, i.e., the optimum rate is proportional to the user's SIR. When no minimum spreading gain constraint exists, users are served in the order of decreasing channel gain [13].

Power allocation problem in the downlink differs from that of the uplink in that a total power constraint is required in downlink, while typically an individual power constraint is imposed in the uplink. In references [15], [16], all available power from the base station is transmitted to one user for the sake of increased throughput, and no compensation for the intracell interference is needed. To overcome the possible unfairness that may results from the one-user-transmission strategy, reference [17] imposes a minimum service rate requirement for each user, and considers the throughput maximization problem for downlink multirate CDMA system where the multirate is achieved by either multicode or orthogonal variable spreading factor (OVSF). The optimality of greedy power allocation is shown in this scenario [17]. Also provided in

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[17] are numerical results which demonstrate that the value of the orthogonality factor significantly affects the system performance.

References [15]–[17] promote a formulation where the transmission rate and the power have a *linear* relation. In references [18], [19], on the other hand, rate and power have a *logarithmic* relation. Reference [18] considers the optimization of the sum of the individual weighted throughput values in the downlink of a multirate CDMA system with orthogonal codes. It is assumed that the orthogonality is preserved at the receiver side. The resulting problem then becomes a convex program, which for the special case of the equal weight scenario, i.e., throughput maximization, produces one-user-transmission as the optimum policy [18]. Reference [19] investigates the multiuser scheduling gain, i.e., gain by transmissions of a fraction of users over transmissions of all users for a given orthogonality factor, with the assumption of equal power transmission for the scheduled users without the consideration of optimum scheduling and power allocation.

In this paper, we will adopt a model similar to [18], [19], but consider the downlink of an *interference limited* CDMA system. We determine the optimum number of users to be scheduled and find the corresponding power allocation. The key observation is that the optimum policy is a function of the orthogonality factor. Specifically, given the channel realization of the users and the orthogonality factor, the TDMA-mode, i.e., the base station transmits to one-user, or the CDMA-mode, i.e., the base station transmits to multiple users, can be the optimum strategy. If the CDMA-mode is optimum, that is, if more than one-user should be served by the base station, then the corresponding power allocation is also found. A similar approach for the effect of orthogonality factor is considered in uplink in [20] without the consideration of the optimum power allocation.

The paper is organized as follows. Section II describes the system model as well as the problem formulation and the performance metric. In Section III, we obtain the algorithm to find the optimum scheduling policy and the corresponding power allocation. This is done via examining the utility maximization problem from an individual user's perspective and reflecting our findings to the system-wide optimization problem. In Section IV, we extend our approach to the scenario where individual power constraints are in place. Section V provides numerical examples that support our analysis. Section VI summarizes the results and concludes the paper.

II. System Model and Problem Formulation

We consider a single cell CDMA system with K users. The base station has a maximum power budget, P_T. This power budget is assigned to users such that $\sum_{i=1}^{K} p_i \leq P_T$, where $p_i \geq 0$ is the allocated power for user i. Figure 1 depicts the model of the CDMA downlink. An orthogonal code is assigned to each user and is used to modulate the signal transmitted to that user. However, due to multipath fading, orthogonality between the codes (users) is lost at the receiver side. The degree of nonorthogonality is described by the *orthogonality factor*, α $(0 \leq \alpha \leq 1)$ [9]. Under the assumption of independent identical channels for all users, it is reasonable to work with the same average orthogonality factor for all users. The signal to interference ratio at the receiver of user i is expressed as

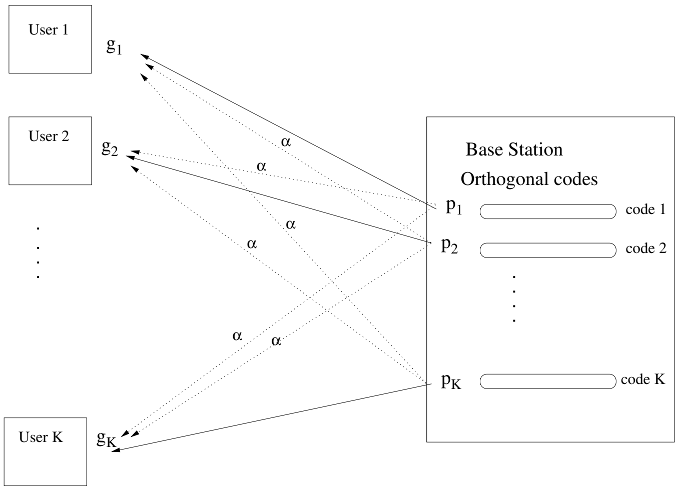

Fig. 1. Downlink system model.

$$SIR_i = \frac{p_i g_i}{\alpha \sum_{j \neq i} p_j g_i + I} \tag{1}$$

where g_i denotes channel gain of user i. I denotes the sum of the thermal noise and the intercell interference. We define the utility of user i as follows:

$$U_i(\mathbf{p}) = \log\left(1 + k\frac{p_i g_i}{\alpha \sum_{j \neq i} p_j g_i + I}\right) \tag{2}$$

where $\log$ denotes the natural logarithm. We note that the individual utility is a function of the power vector $\mathbf{p} = [p_1, p_2, ..., p_K]$. The utility is an achievable rate for user i, specifically by treating the interference as noise [15]. It is assumed that adaptive modulation is employed to enable the rate determined for each user [21]. Following reference [22], we denote the factor, that captures the product of the SNR gap Γ and the processing gain N, $k = \Gamma \times N$, where Γ is derived from the target bit error rate (BER), $\Gamma = \frac{-\ln(5BER)}{1.5}$. We should note that, in a practical system employing M-ary modulation, the transmission rate (utility) is a discrete quantity. For simplicity of analysis, and similar to previous work [13], [14], we will assume continuous rate values. We will examine the effect of this assumption in the numerical results.

The problem we consider is to determine the optimum allocation of the total transmit power from the base station to the users so as to maximize the overall system utility, i.e., sum of individual utilities. Formally, the optimization problem is formulated as:

$$\begin{aligned} \max_{\mathbf{p}} \quad & \sum_{i=1}^{K} \log\left(1 + k\frac{p_i g_i}{\alpha \sum_{j \neq i} p_j g_i + I}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{K} p_i \leq P_T, \quad p_i \geq 0,\ i = 1, \cdots, K \end{aligned} \tag{3}$$

where we assume that $g_1 > g_2 > .... > g_K$, such that user with the lower index has a higher channel gain. We note

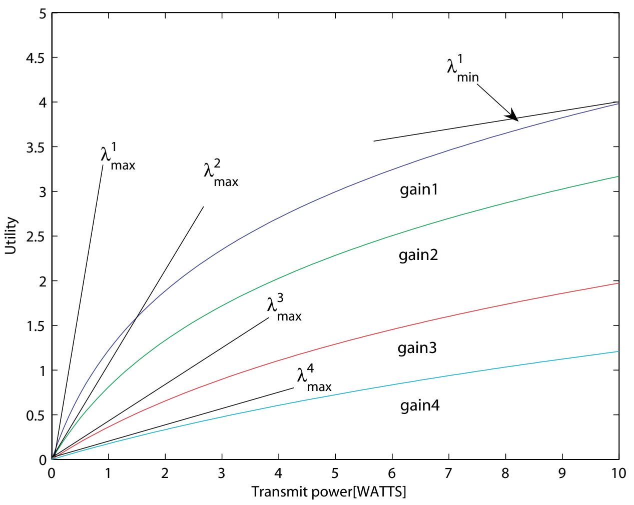

Fig. 2. Utility versus transmit power. $gain_1 > gain_2 > gain_3 > gain_4$. An example where individual utility functions for all users are concave.

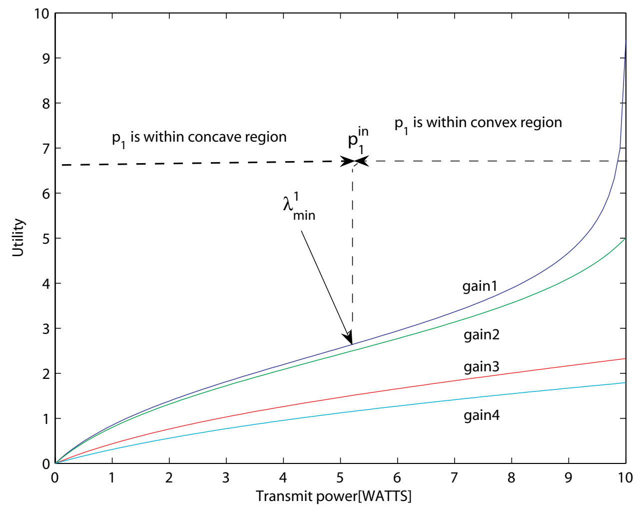

Fig. 3. Utility versus transmit power. $gain_1 > gain_2 > gain_3 > gain_4$. An example where individual utility functions for some users are not concave.

that (3) considers only the total power constraint. This is the problem we will consider in Section III. In Section IV, we will consider additional individual power constraints.

The actual outcome of optimization problem in (3) is the power vector. The optimum transmission strategy is implicitly included in the optimum power vector in that the users that belong to the subset of users that the base station transmits to, end up with non-zero power, and the rest with zero power. If the TDMA-mode turns out to be optimum, the power allocation is such that transmission power to only one user is positive. If the CDMA-mode turns out to be optimum, the power allocation is such that transmitted power values to multiple users are positive.

We first note that any power vector $\overline{\mathbf{p}}$ with $\sum_{i=1}^{K} \overline{p}_i < P_T$ can not be the optimum power vector. Let us define a power vector $\mathbf{p}$ with $p_i = \beta \overline{p}_i \quad (\beta > 1), i = 1, \cdots, K$ such that $\sum_{i=1}^{K} \beta p_i = P_T$. It is easy to see that $\mathbf{p}$ increases all individual utilities, and hence the system utility, as compared to $\overline{\mathbf{p}}$. Thus, (3) can be rewritten as:

$$\max_{\mathbf{p}} \quad \sum_{i=1}^{K} \log\left(1 + k\frac{p_i}{\alpha(P_T - p_i) + \frac{I}{g_i}}\right) \tag{4}$$
$$\text{s.t.} \quad \sum_{i=1}^{K} p_i = P_T, \quad p_i \geq 0, \; i = 1, \cdots, K.$$

Observe that the utility of user i is a function of the power for user i only, i.e., $U_i(\mathbf{p}) = U_i(p_i)$. Also note that, although the utility is concave in SIR, it need not be concave in power. Therefore, the optimization problem is in general not a convex program. Figure 3 emphasizes this point by plotting the individual utilities for users with different channel gains.

III. Optimum Transmit Strategy and Power Allocation

At the outset, (4) does not appear to be easy to solve. In an effort to gain understanding towards its optimum solution, we first consider the behavior of the individual utility by observing its derivative. Our observations motivate us to consider the system-wide approach, in which we investigate the system utility in terms of the best user's power. The investigation provided in Section III-A and III-B leads to the algorithms in Section III-C.

A. User Centric Approach

Consider the utility function of user i and its first and second derivative in terms of p_i, the power of user i:

$$U_i(p_i) = \log\left(1 + k\frac{p_i}{\alpha(P_T - p_i) + \frac{I}{g_i}}\right) \tag{5}$$

$$\frac{\partial U_i(p_i)}{\partial p_i} = k\frac{\alpha p_T + \frac{I}{g_i}}{(\alpha(p_T - p_i) + \frac{I}{g_i} + kp_i)(\alpha(p_T - p_i) + \frac{I}{g_i})} > 0 \tag{6}$$

$$\frac{\partial^2 U_i(p_i)}{\partial p_i^2} = k(\alpha p_T + \frac{I}{g_i})\frac{A_i}{B_i} \tag{7}$$

where A_i and B_i are defined as

$$A_i = (\alpha - k)(\alpha(p_T - p_i) + \frac{I}{g_i}) + \alpha(\alpha(p_T - p_i) + \frac{I}{g_i} + kp_i) \tag{8}$$

$$B_i = (\alpha(p_T - p_i) + \frac{I}{g_i} + kp_i)^2(\alpha(p_T - p_i) + \frac{I}{g_i})^2. \tag{9}$$

Clearly, $U_i(p_i)$ is an increasing function of p_i. We are interested in whether the utility $U_i(p_i)$ is an increasing concave or convex in $p_i \in [0, P_T]$. Whether $U_i(p_i)$ is concave ($\frac{\partial^2 U_i(p_i)}{\partial p_i^2} < 0$) or convex ($\frac{\partial^2 U_i(p_i)}{\partial p_i^2} > 0$) depends on the $sign(A_i)$, the numerator of $\frac{\partial^2 U_i(p_i)}{\partial p_i^2}$, defined in (8). By letting $I_{ti} = \alpha(p_T - p_i) + \frac{I}{g_i}$, we have $A_i = (\alpha - k)I_{ti} + \alpha(I_{ti} + kp_i)$. Thus, $U_i(p_i)$ is an increasing convex function of p_i if

$$\frac{p_i}{I_{ti}} > \frac{1}{\alpha} - \frac{2}{k} \tag{10}$$

and an increasing concave function of p_i otherwise. Clearly, by examining $\frac{\partial^2 U_i(p_i)}{\partial p_i^2}|_{p_i = P_T}$, we can identify the behavior of function $U_i(p_i)$. When $\frac{\partial^2 U_i(p_i)}{\partial p_i^2}|_{p_i = P_T} < 0$, $U_i(p_i)$ is an increasing concave function in $0 \leq p_i \leq P_T$ as depicted in Figure 2. When $\frac{\partial^2 U_i(p_i)}{\partial p_i^2}|_{p_i = P_T} > 0$, $U_i(p_i)$ is an increasing concave function in $0 \leq p_i \leq p_i^{in}$, while it is an increasing convex function in $p_i^{in} \leq p_i \leq P_T$, with p_i^{in} denoting the inflection point in $U_i(p_i)$. Figure 2 shows the individual utilities corresponding to users with different channel gains. In this example, the individual utilities of all users are concave in power.

Observe that when each individual utility is concave in power, i.e., the case shown in Figure 2, rather than allocating the entire power P_T to one user, sharing the power P_T among multiple users will yield a higher total utility. In this case, the optimization problem in (4) is a convex program. However, this is no longer the case when the individual utilities of some users are not concave, as depicted in Figure 3. In this case, we need to have a closer look at the components that contribute to the system utility. The following terminology is used extensively in the sequel.

- $U_i(p_i)$ is convex at p_i if $\frac{\partial^2 U_i(p_i)}{\partial p_i^2} > 0$ for $0 \leq p_i \leq P_T$
- $U_i(p_i)$ is concave/convex if $\frac{\partial^2 U_i(p_i)}{\partial p_i^2}|_{p_i=0} < 0$ and $\frac{\partial^2 U_i(p_i)}{\partial p_i^2}|_{p_i=P_T} > 0$
- $U_i(p_i)$ is concave if $\frac{\partial^2 U_i(p_i)}{\partial p_i^2}|_{p_i=P_T} < 0$
- p_i^* is said to be within the concave region of $U_i(p_i)$, if $\frac{\partial^2 U_i(p_i)}{\partial p_i^2}|_{p_i=p_i^*} < 0$
- p_i^* is said to be within the convex region of $U_i(p_i)$, if $\frac{\partial^2 U_i(p_i)}{\partial p_i^2}|_{p_i=p_i^*} > 0$

Figure 3 shows the two possible power regions for user 1 (a concave/convex user).

Following from above, we define

$$\lambda_{min}^1 = \begin{cases} \frac{\partial U_1(p_1)}{\partial p_1}|_{p_1=p^*} & \text{if } U_1(p_1) \text{ is concave/convex,} \\ & \text{and } \frac{\partial^2 U_1(p_1)}{\partial p_1^2}|_{p_1=p^*} = 0 \\ \frac{\partial U_1(p_1)}{\partial p_1}|_{p_1=p_T} & \text{if } U_1(p_1) \text{ is concave} \end{cases} \tag{11}$$

$$\lambda_{max}^i = \frac{\partial U_i(p_i)}{\partial p_i}|_{p_i=0} \tag{12}$$

and,

$$p_i(\lambda) = \begin{cases} \arg_{p_i}(\frac{\partial U_i(p_i)}{\partial p_i}|_{p_i=p_i(\lambda)} = \lambda) & \text{if } \lambda < \lambda_{max}^i \\ 0 & \text{if } \lambda > \lambda_{max}^i \end{cases} \tag{13}$$

where λ_{min}^1 is the minimum derivative value of utility of user 1, while λ_{max}^i denotes the maximum derivative value of utility of user i in the concave region of $U_i(.)$. Figures (2) and (3) depict λ_{min}^1 and λ_{max}^i.

Having set the stage with these definitions, we now move to presenting our findings. The optimum scheduling policy is a function of the orthogonality factor as well as users' channel gains. Below we state the propositions that lead to characterizing the optimum policy.

Our first observation states that when the orthogonality factor is larger than a certain threshold, TDMA-mode is always optimum, regardless of users' channel gains:

Proposition 3.1: If $\alpha \geq \frac{k}{2}$, TDMA-mode with $p_1 = P_T$ yields the maximum system utility.

Proof: $\alpha \geq \frac{k}{2}$ implies $U_i(p_i)$ is a convex function for $\forall i$, because $\frac{\partial^2 U_i}{\partial p_i^2} > 0$ for all $0 \leq p_i \leq P_T$. When $U_i(p_i)$ is convex for $\forall i$, the overall utility is convex. In this case, the maximizer of the overall utility function is at $p_1 = P_T$, and $p_i = 0, i > 1$. ■

While, it is clear that the TDMA-mode is optimum when $\alpha \geq \frac{k}{2}$, and no further power allocation is necessary, when $\alpha \leq \frac{k}{2}$, we need to characterize the optimum policy which consists of the transmission strategy, i.e., TDMA-mode or CDMA-mode, and the corresponding power allocation. The following propositions help in that regard.

Proposition 3.2: Suppose $\mathbf{p}^* = (p_1^*, p_2^*, ..., p_K^*)$ is the optimum power allocation where the n users are scheduled, i.e. $p_i^* > 0$ for $i = 1, \cdots, n$ and $p_i = 0$ for $i = n+1, \cdots, K$. Then, $p_1^* > p_2^* > ... > p_n^*$.

Proof: Suppose the optimum power allocation is such that $p_i^* < p_j^*$ $(g_i > g_j)$, for $i, j \in \{1, \cdots, n\}$. First, we note that $\frac{\partial U_i(p_i)}{\partial p_i}|_{p_i=p} > \frac{\partial U_j(p_j)}{\partial p_j}|_{p_j=p}$, i.e., at a given power level, the rate of change in the utility of the user with the better channel is larger. This can be shown simply by evaluating (6) for user i and user j at $p_i = p_j = p > 0$, and observing that the difference of the two terms is strictly positive. This in turn means that by exchanging the power between user i and j, we get $U_i(p_j^*) + U_j(p_i^*) > U_i(p_i^*) + U_j(p_j^*)$, i.e., we can always increase the system utility. Therefore, the optimum power allocation $\mathbf{p}^*$ must be such that the better channel user gets assigned a higher power value among the users with non-zero power. ■

When the individual utilities of multiple users are concave/convex as in Figure 3, the following holds.

Proposition 3.3: At $\mathbf{p}^*$, the optimum power allocation, at most one user's power is within the convex region of its utility function. This is the user with the best channel gain.

Proof: Suppose the optimum power allocation is such that two users' power values are within the convex region of their utility functions, i.e., $\mathbf{p}^* = [p_1^*, p_2^*, p_3^*, ..., p_K^*]$ where p_1^* and p_2^* are within the convex region of $U_1(p_1)$ and $U_2(p_2)$. Consider an alternative power allocation $\mathbf{p}^{**} = [p_1^* + \triangle p, p_2^* - \triangle p, p_3^*, ..., p_K^*]$, i.e., the power of user 1 and 2 are increased by $\triangle p$ and $-\triangle p$, respectively, while the rest of users' power values remain the same. Since, by assumption, $U_1(p_1)$ and $U_2(p_2)$ are convex at p_1^* and p_2^*, and as explained in the proof of Proposition 3.2, $\frac{\partial U_1(p_1)}{\partial p_1}|_{p_1=p} > \frac{\partial U_2(p_2)}{\partial p_2}|_{p_2=p}$, it follows that $U_1(p_1^* + \triangle p) + U_1(p_2^* - \triangle p) > U_1(p_1^*) + U_1(p_2^*)$. Thus, when the power p_2^* is within the convex region of $U_2(p_2)$, we can always increase the total utility. Therefore, this cannot be the optimum policy, and indeed at the optimum point, only the best user's power p_1 can be within the convex region of $U_1(p_1)$. ■

From the preceding two propositions, we conclude that the optimum power allocation $\mathbf{p}^* = [p_1^*, p_2^*, ..., p_K^*]$ has to be one of two cases in terms of the best user, i.e., p_1^* can be either within the concave region or within the convex region of $U_1(p_1)$[1], while the rest of the power values cannot be in the convex region of their respective utility. This fact motivates our approach to focus on the the system utility in terms of the best user.

B. System-Wide Approach

Let us rewrite the system utility in terms of the power allocated to the best user, i.e., p_1:

[1] This possibility includes $p_1^* = P_T$

$$J(p_1) = U_1(p_1) + Z(p_1) \quad (14)$$

$$Z(p_1) = \max[\sum_{j=2}^{K} U_j(p_j)]_{\sum_{j=2}^{K} p_j = P_T - p_1}. \quad (15)$$

We note that $J(p_1)$ expends all available power, P_T, p_1 for user 1 and $P_T - p_1$ for the rest of the users. Our aim is to find the maximizer of $J(p_1)$ over $0 \leq p_1 \leq P_T$. Proposition 3.3 guarantees that $\{U_j(p_j)\ (j \geq 2)\}$ are all in their concave region at the optimum power allocation. Hence, $Z(p_1)$ can be maximized with the power constraint $\sum_{j=2}^{K} p_j(\lambda) = P_T - p_1$ where λ is the optimum Lagrange multiplier. We note that no power should be assigned to the user i when $\lambda_{max}^i < \lambda$. Depending on user 1's channel condition, $U_1(p_1)$ can be either concave or concave/convex. When $U_1(p_1)$ is concave, the resulting optimization problem is convex and the optimum power allocation is such that $\sum_{j=1}^{K} p_j(\lambda) = P_T$. If $U_1(p_1)$ is concave/convex as in Figure 3, we have the following observation.

***Observation** 3.4:* If $U_1(p_1)$ is concave/convex, the optimum power allocation is one of three local optimum solutions: (i) p_1 is within the concave region with $0 \leq p_1 < p_1(\lambda_{min}^1)$, (ii) p_1 is within the convex region with $p_1(\lambda_{min}^1) < p_1 < P_T$, or (iii) $p_1 = P_T$.

The transmission strategy for the first two cases is CDMA, while the third case ($p_1 = P_T$) is TDMA. Given the fact that when $U_1(p_1)$ is concave/convex, the resulting optimization problem is non-convex, the three possible cases described above need to be considered in detail to maximize $J(p_1)$.

Consider $\sum_{i=1}^{K} p_i(\lambda)$, i.e., the sum of the powers transmitted to all users, given λ. Observing whether $\sum_{i=1}^{K} p_i(\lambda) < P_T$, or $\sum_{i=1}^{K} p_i(\lambda) > P_T$ provides us with the information whether $J(p_1)$ is increasing or decreasing at $p_1 = p_1(\lambda)$:

***Proposition** 3.5:* If $\sum_{i=1}^{K} p_i(\lambda) < P_T$ for a given λ, $J(p_1)$ is an increasing function at $p_1 = p_1(\lambda)$.

Proof: We can find λ^* such that $p_1(\lambda) + \sum_{j=2}^{K} p_j(\lambda^*) = P_T$ where $\lambda^* < \lambda$, because $U_j(p_j)\ (j \geq 2)$ is increasing concave at the optimum power values as depicted by proposition 3.3. This implies we can increase the system utility by increasing p_1 and decreasing $p_j\ (j > 2)$, while maintaining $\sum_{j=1}^{K} p_j(\lambda) = P_T$. ■

***Proposition** 3.6:* If $\sum_{i=1}^{K} p_i(\lambda) > P_T$ for a given λ, $J(p_1)$ is a decreasing function at $p_1 = p_1(\lambda)$.

Proof: This is the opposite situation to the previous case. We can find λ^* such that $p_1(\lambda) + \sum_{j=2}^{K} p_j(\lambda^*) = P_T$ where $\lambda^* > \lambda$. This implies we can increase the system utility by decreasing p_1 and increasing the $p_j (j > 2)$, while maintaining $\sum_{j=1}^{K} p_j(\lambda) = P_T$. ■

It follows from the previous two propositions, by comparing $\sum_{i=1}^{K} p_i(\lambda)$ with P_T, we can always determine whether $J(p_1)$ is increasing or decreasing at $p_1 = p_1(\lambda)$. In particular, if $\sum_{i=1}^{K} p_i(\lambda_{min}^1) > P_T$, $J(p_1)$ is a decreasing function at $p_1 = p_1(\lambda_{min}^1)$ and the optimum power allocation is one of three possibilities as described by Observation 3.4. If, on the other hand, $\sum_{i=1}^{K} p_i(\lambda_{min}^1) < P_T$, this implies that $\sum_{i=1}^{K} p_i(\lambda) < P_T$ for all $\lambda > \lambda_{min}^1$ where $p_1(\lambda)$ is within the concave region[2]. In this case, $J(p_1)$ is an increasing function when p_1 is within the concave region, i.e., $0 \leq p_1 \leq p_1(\lambda_{min}^1)$. This means that, no p_1 value within the concave region can be optimum.

We know that the optimum power allocation dictates that p_1 be either $p_1(\lambda_{min}^1) < p_1 < P_T$, or $p_1 = P_T$, and that we would have to find the local optimum solution of the sum utility function, $\sum_{i=1}^{K} U_i(p_i)$, within the convex region of $U_1(p_1)$. Given that the overall utility function is not necessarily unimodal, this task in general requires exhaustive search and the computational complexity associated with this task may be prohibitive since locating the optimum λ would require a search with fine resolution. Thus, if we find a computationally inexpensive way to identify whether $p_1^* = P_T$ is the optimum strategy, i.e., the optimality of TDMA, we can reduce the complexity of identifying the optimum policy. Next, we set out to accomplish this task, and make the following observation.

***Observation** 3.7:* If $\sum_{i=1}^{K} p_i(\lambda) < P_T$ for $\forall \lambda$ where $p_1(\lambda)$ is within the convex region of $U_1(p_1)$, then $p_1 = P_T$ is optimum power allocation.

Observation 3.7 follows from the fact that, in this case, $J(p_1)$ is an increasing function for $0 \leq p_1 \leq P_T$. Although the condition in observation 3.7 guarantees the optimality of TDMA, it still requires considerable computational complexity to verify. Notice that, as described above, when $\sum_{i=1}^{K} p_i(\lambda_{min}^1) < P_T$, the optimum power allocation does not fall within the concave region of $U_1(p_1)$. Thus, by adopting TDMA as the optimum policy whenever $\sum_{i=1}^{K} p_i(\lambda_{min}^1) < P_T$, we can significantly reduce computational complexity in finding the local optimum power allocation in the convex region of $U_1(p_1)$. In the next section, we will term this shortcut, the *near-exact algorithm*. The numerical results in Section V validate the accuracy of near-exact algorithm.

C. Algorithms for Transmit Strategy and Power Allocation

Following the preceding discussion in Section III, we conclude that the optimality of TDMA is determined by checking $\sum_{i=1}^{K} p_i(\lambda) < P_T$ for all λ where $p_1(\lambda)$ is within the convex region of $U_1(p_1)$. Instead of this potentially computationally intensive search, we propose to simply check $\sum_{i=1}^{K} p_i(\lambda_{min}^1) < P_T$, and declare that $p_1^* = P_T$ and $p_i = 0, i \geq 2$ whenever this condition is satisfied.

This action, i.e., deciding that $p_1 = P_T$ is optimum whenever $\sum_{i=1}^{K} p_i(\lambda_{min}^1) < P_T$, regardless of $\max_{p_1} J(p_1)$ for $p(\lambda_{min}^1) < p_1 \leq P_T$ results in sub-optimum performance whenever the optimum power allocation is such that $p_1(\lambda_{min}^1) < p_1 < P_T$. In our numerical results, we have observed that cases where the optimum power allocation is such that $p_1(\lambda_{min}^1) < p_1 < P_T$, are rare and that in most cases, the optimum power allocation is either $0 < p_1 < p_1(\lambda_{min}^1)$ or $p_1 = P_T$.

We should note that there may be the cases when even though $\sum_{j=1}^{K} p_j(\lambda_{min}^1) > P_T$, TDMA can be optimum. However, as the orthogonality factor increases, the condition $\sum_{i=1}^{K} p_i(\lambda_{min}^1) < P_T$ accounts for almost all of the cases where TDMA is optimum, as demonstrated by numerical results in Section V. The following describes the steps of the proposed algorithm to maximize the system utility, which we term *the system utility maximizer*, A-SUM.

[2]Recall that $p_i(\lambda_{min}^1) > p_i(\lambda)$ when $U_i(p_i)$ is increasing concave.

A-SUM:

STEP 1. If $\alpha \geq \frac{k}{2}$, then TDMA mode is optimum, $p_1 = P_T$, **STOP**.

STEP 2. Find λ_{max}^i for $i = 1, \cdots, K$, and λ_{min}^1.

STEP 3. (convex region) If $\sum_{j=1}^{K} p_j(\lambda_{min}^1) < P_T$, then, declare that TDMA mode is optimum, **STOP**.

STEP 4. (concave region) Find the power allocation $J^*(p_1(\lambda)) = \max(J(p(\lambda)))$, $0 \leq p_1(\lambda) \leq p_1(\lambda_{min}^1)$, using **A-PACR**.

STEP 5. Choose $\max(J^*(p_1(\lambda)), J(P_T))$.

We re-emphasize that the reduction in the computational complexity of the *near*-exactness of the algorithm arises from the STEP 3, instead of solving for $\max_{p_1} J(p_1)$, $p_1(\lambda_{min}^1) < p_1 < P_T$.

The optimum power allocation (STEP 4) entails selecting the users to which the base station transmits with positive power. If zero power is assigned to one user, that user is not *scheduled* for transmission. The algorithm to determine the power allocation in the concave region (A-PACR) is given below. It is assumed in A-PACR that $\sum_{j=1}^{K} p_j(\lambda_{min}^1) > P_T$. If this is not the case, there is no solution in the concave region, and A-PACR simply determines $p_1 = P_T$.

A-PACR:

STEP 1. Find $\hat{k} = \arg_k \max \sum_{j=1}^{k} p_j(\lambda_{max}^k)$, s.t. $\sum_{j=1}^{k} p_j(\lambda_{max}^k) \leq P_T$, $k = 1, \cdots, K$.

STEP 2. Find $p_j(\lambda)$ $(1 \leq j \leq \hat{k})$ s. t. $\sum_{j=1}^{\hat{k}} p_j(\lambda) = P_T$ and $\lambda_{min}^1 < \lambda < \lambda_{max}^{\hat{k}}$.

The optimum number of users to which the base station transmits is selected in STEP 1. Then, optimum power values are allocated in STEP 2. Note that a numerical method, e.g., bisection [23] should be used to determine the value of λ.

IV. Downlink Transmission Policies with Individual Power Constraints

In this section, we consider the effect of having minimum and maximum limits on the powers that the base station transmits to individual users. Specifically, we extend our approach and present modifications to the algorithms presented in Section III. We assume that each *scheduled* user, i.e., each user that the base station is designated to transmit with non-zero power, has a maximum and/or a minimum transmit power constraint. We emphasize this to convey the fact that we still may end up with non-scheduled users. This is in contrast to having a minimum power constraint for each user, which automatically sets the transmission policy to CDMA with all users simultaneously transmitted to by at least the minimum power. As a result, the optimization problem (4) now has additional constraints,

$$p_i^{min} \leq p_i \leq p_i^{max} \qquad \forall i \quad \exists\, p_i > 0 \tag{16}$$

where p_i^{min} ensures all users selected for transmission to achieve minimum required rate U^{min}, while p_i^{max} signifies a practical limit, i.e., an arbitrary high data rate cannot be achieved, even if a user has a very high channel gain. The limits would be obtained from the minimum and maximum rate, U^{min} and U^{max}, via

$$U^{min} \leq U_i(\mathbf{p}) \leq U^{max} \tag{17}$$

where

$$U^{min} = \log\left(1 + k\frac{p_i^{min}}{\alpha(P_T - p_i^{min}) + \frac{I}{g_i}}\right), \tag{18}$$

and

$$U^{max} = \log\left(1 + k\frac{p_i^{max}}{\alpha(P_T - p_i^{max}) + \frac{I}{g_i}}\right). \tag{19}$$

Observe the above assumes that full power is expended at the base station. We will elaborate on this issue in the next section.

A. Observations on the Constrained Problem

First, we consider the case when only maximum power constraints exist.

Observation 4.1: When only maximum power constraints exist and the base station transmits to more than one user, full power, P_T should be expended for the optimum power allocation.

Proof: Suppose the optimum power allocation expends $\hat{P}_T$ $(\hat{P}_T = \sum_{i=1}^{K} p_i^* < P_T)$, while one user, say user 1, is at its maximum allowable utility value, i.e. $U_1 = U^{max}$. Observe that, we can concentrate on this case, without loss of generality, because if $\sum_{i=1}^{K} p_i^* < P^T$ and $U_1 < U^{max}$, by proportionally increasing transmit power of all scheduled users such that U_1 reaches U^{max}, the utility of each scheduled user is further increased. Thus, $\sum_{i=1}^{K} p_i^* < P^T$ and $U_1 < U^{max}$ cannot be the condition for optimum power allocation.

Note that, from (19), it follows that by expending P_T, $U_1 = U^{max}$ would be achieved with $p^{max} < p_1^*$. Specifically,

$$\begin{aligned} U^{max} &= \log\left(1 + k\frac{p_1^{max}}{\alpha(P_T - p_1^{max}) + \frac{I}{g_1}}\right) \\ &= \log\left(1 + k\frac{p_1^*}{\alpha(\hat{P}_T - p_1^*) + \frac{I}{g_1}}\right). \end{aligned} \tag{20}$$

Since, $\hat{P}_T < P_T$, (20) implies that $p_1^{max} < p_1^*$. This means that with leftover power, $(P_T - \hat{P}_T) + (p_1^* - p_1^{max})$, by proportionally increasing the transmit power of all other scheduled users such that $\sum_{j=2}^{K} \beta p_j^* = P_T - p_1^{max}$, the utilities of these scheduled users are increased, while U_1 is kept at U^{max}. Thus, if $\hat{P}_T = \sum_{i=1}^{K} p_i^* < P_T$ and $U_1 = U^{max}$, the system utility can be further increased by expending full power P_T. Therefore, full power P_T should be expended for the optimum power allocation. ■

Observation 4.2: When both the maximum and the minimum power constraints exist, the optimum policy can be such that the base station transmits less than full power.

To see why this observation is valid, consider the following argument. For simplicity of analysis, assume each scheduled user experiences interference as if full power P_T is transmitted from the base station. Then, the utility of each scheduled user is a function of its power only and the minimum and maximum power of user i, p_i^{min} and p_i^{max} are obtained for each scheduled user i from (19).

Consider the case that the optimum policy schedules the first n users to be transmitted with their maximum power corresponding to the maximum utility value U^{max}, while satisfying $\sum_{i=1}^{n} p_i^{max} < P_T$. In this case, the optimum policy may try to schedule the next user with leftover power $P_T - \sum_{i=1}^{n} p_i^{max}$. If this action violates the total power constraint, $\sum_{i=1}^{n} p_i^{max} + p_{n+1}^{min} > P_T$, the transmission policy ends up with expending less than full power. However, we note that p_i^{max} $(i = 1, \cdots, n)$ values were obtained with the the assumption that the base station transmits full power, P_T. Noting the fact that less than full power transmission is optimum, we can see that the actual interference scheduled user i experiences is in fact less than $\alpha(P_T - p_i^{max}) + \frac{I}{g_i}$. In turn, the maximum utility U^{max} can be achieved by user i with power value $p_i^{opt} < p_i^{max}$. Fortunately, the power vector $\mathbf{p}^{opt} = [p_1^{opt}, \cdots, p_n^{opt}]$ simply is the downlink power vector that satisfies a given SIR value, i.e. $\gamma_i = \frac{e^{U^{max}}-1}{k}$ $(i = 1, \cdots, n)$, and can be easily determined [4]. For convenience, we term this Power Control for Fixed Rate (PCFR).

B. Utility Maximization with Individual Constraints

Proposition 3.3 where no power constraint is assumed states that at most one user can be within the convex region of the utility function. By imposing a maximum power constraint, we may have more than one user within the convex region. In this case, the utility with higher channel gain can be increased up to the maximum utility value by increasing the power of the user with higher channel gain, and decreasing the power of the user with lower channel gain, while the sum of the power of these two users remains the same. After the utility with higher channel gain reaches the maximum utility value, the power of the user with lower channel gain can be either within the concave or the convex region. This implies that, under the maximum power constraint, there can be more than one user within the convex region. However, *at most one user* expends less than maximum power within the convex region at the optimum power allocation. Otherwise, the system utility can be increased as described by Proposition 3.3.

The observation below follows from the fact that by $\frac{\partial U_i(p_i)}{\partial p_i}|_{p_i=p_i^{min}} > \frac{\partial U_j(p_j)}{\partial p_j}|_{p_j=p_j^{min}}$ $(g_i > g_j)$, and the fact that at most one user has less than maximum power within the convex region.

***Observation** 4.3:* If p_i^{min} for some users are within the convex region, the optimum policy employs greedy packing of these users in the order of decreasing channel gains, i.e., allocation of maximum allowable power in the order of decreasing channel gains.

We term such users *greedy users* and the resulting process *convex greedy packing*. If all available power is expended in this process, then the resulting power allocation *is* the optimum power allocation. If there is a leftover power, but is not enough to support the next user with the minimum required power, the maximum system utility is achieved with less than full power and with convex greedy packing. After greedy packing, if the leftover power is enough to support the next user with minimum required power, we need to consider the optimization problem in terms of each concave/convex user by allowing each concave/convex user to have the maximum power in the order of decreasing channel gains. To that end, we modify (14) and (15) such that

$$J(p_j)_M = \sum_{i<j} U_i(p_i^{max}) + U_j(p_j) + Z(p_j) \quad (21)$$

$$Z(p_j) = \max[\sum_{l>j}^{K} U_l(p_l)]_{\sum p_l = P_T - \sum_i p_i^{max} - p_j, p_l \geq p_l^{min}} \quad (22)$$

where the notation $J(p_j)_M$ denotes the fact that we have M users with minimum power requirement. The first term in (21) includes users which expend their maximum power p_i^{max} within the convex region. Similar to (15), $J(p_j)_M$ is optimized over p_j with remaining power $P_T - \sum_i p_i^{max} - \sum_{l>i}^{M} p_l^{min}$. The resulting solution $\max J(p_j)_M$ only considers the case when $p_i = p_i^{max}$ $(i < j)$. Recall that that at most one user expends less than maximum power within the convex region at the optimum power allocation. Hence, by solving $\max J(p_j)_M$ for all concave/convex users, we can find the optimum solution. This means that we need to solve

$$\max_j \left(\max_{p_j} J(p_j)_M \right) \qquad j = k^* + 1, \cdots, \min(k^{**}, M) \quad (23)$$

where k^*, k^{**} denotes the number of greedy users and concave/convex users, respectively. If all individual utilities are concave, the resulting optimization problem is convex, and the solution of (23) is given in Section III. Note that, due to the minimum power constraint, the following modifications in our previous definition (Section III) are needed:

$$\lambda_{max}^i = \frac{\partial U_i(p_i)}{\partial p_i}|_{p_i=p_i^{min}} \quad (24)$$

$$\lambda_{min}^i = \frac{\partial U_i(p_i)}{\partial p_i}|_{p_i=p^*}, \text{when} \frac{\partial^2 U_i(p_i)}{\partial p_i^2}|_{p_i=p^*} = 0 \text{ and } U_i(p_i) \text{ is concave/convex.} \quad (25)$$

$$p_i(\lambda) = \begin{cases} \arg_{p_i}(\frac{\partial U_i(p_i)}{\partial p_i}|_{p_i=p_i(\lambda)} = \lambda) & \text{if } \lambda < \lambda_{max}^i \\ p_i^{min} & \text{if } \lambda > \lambda_{max}^i \end{cases} \quad (26)$$

C. Algorithms with Individual Power Constraints

Following the observations in Section IV-A, we propose modified versions of A-SUM and A-PACR in the presence of power constraints, A-SUMPC and A-PACRPC, respectively. Let k^* and k^{**} $(k^{**} \geq k^*)$ denote the number of greedy users and the number of concave/convex users respectively.

A-SUMPC:

STEP 1. Find λ_{max}^i $i = 1, \cdots, K$ and λ_{min}^i; find $T_1 = U_1(p_1 = min(p_1^{max}, P_T)$, $p_2 = 0, \cdots, p_K = 0)$, set $M = k^* + 1$.

STEP 2. If $k^* > 0$, Apply **Convex Greedy Packing**.

STEP 3. (M-user system) If $\sum_{i=1}^{k^*} p_i^{max} + \sum_{j=k^*+1}^{M} p_j^{min} > P_T$, $M = M - 1$, **GO TO STEP 5**. $T_M = \max_j \left(\max_{p_j} J(p_j)_M\right)$, $j = k^* + 1, \cdots, \min(M, k^{**})$, via **A-PACRPC**.

STEP 4. If $M < K$, $M = M + 1$ and **GO TO STEP 3**.

STEP 5. $max_{1 \leq i \leq M} T_i$.

In the algorithm above, T_i denotes the maximum system utility of the i-user system. In STEP 2, convex greedy packing (given below) is applied to greedy users. Feasibility of the M-user system is checked in STEP 3, then optimum power allocation of M-users system is obtained via A-PACRPC. STEP 5 selects the largest among T_i $(i = 1, \cdots, M)$. When the optimum power allocation expends less than full power P_T, the actual optimum power vector is found via solving M linear equations, i.e. PCFR.

Convex Greedy Packing:

STEP 1. $P_{re} = P_T$. $i = 1$.
STEP 2. $p_i^* = \min(p_i^{max}, P_{re})$, $P_{re} = P_{re} - p_i^*$
If $p_i^* < p_i^{min}$, $p_i^* = 0$.
STEP 3. If $P_{re} = 0$ or $p_i^* < p_i^{min}$, **STOP**.
STEP 4. If $i < k^*$, then $i = i + 1$, **GO TO STEP 2**.

In STEP 2 of Convex Greedy Packing, the maximum allowable power is assigned to greedy users in the order of decreasing channel gains. If the algorithm stops in STEP 3, and $P_{re} = 0$, then the resulting solution is optimum power. If $P_{re} \neq 0$, the actual power vector is obtained via PCFR.

A-PACRPC:

STEP 1. $i = k^* + 1$.
STEP 2. (convex region)

$$p_i^* = \min(p_i^{max}, P_T - \sum_{j=1}^{i-1} p_j^{max} - \sum_{j=i+1}^{M} p_j^{min}),$$

Find $p_j(\lambda)$ $(i < j \leq M)$
s.t. $\sum_{j=1}^{i-1} p_j^{max} + p_i^* + \sum_{j=i+1}^{M} p_j(\lambda) = P_T$,
and the corresponding system utility T_{Mi}^{convex}.
If $\sum_{j=1}^{i-1} p_j^{max} + \sum_{j=i}^{M} p_j(\lambda_{min}^i) < P_T$,
$T_{Mi}^{concave} = 0$ and **GO TO STEP 4**.
STEP 3. (concave region)
Find $p_j(\lambda)$ $(i \leq j \leq M)$
s.t. $\sum_{j=1}^{i-1} p_j^{max} + \sum_{j=i}^{M} p_j(\lambda) = P_T$
and the corresponding system utility $T_{Mi}^{concave}$.
STEP 4. $T_{Mi} = \max(T_{Mi}^{convex}, T_{Mi}^{concave})$
If $i < \min(M, k^{**})$, $i = i + 1$, **GO TO STEP 2**.
STEP 5. $T_M = \max_{1 \leq j \leq \min(M,k^{**})} T_{Mj}$.

Note that the near exactness of the algorithm A-PACRPC arises from the fact that in STEP 2, we simply take $p_i^* = \min(p_i^{max}, P_T - \sum_{j=1}^{i-1} p_j^{max} - \sum_{j=i+1}^{M} p_j^{min})$, instead of solving $\max_{p_i} J(p_i)_M$ over all p_i within the convex region.

V. Numerical Results

We provide simulation results for a range of the orthogonality factor values. The total power at the base station is $P_T = 10$ watts. Total noise and intercell interference is $I = 10^{-11}$ watts. Factor k in the utility function is $k = 1.2$, corresponding to $\Gamma = 0.15$ and processing gain $N = 8$. Thus, when $\alpha \geq 0.6$, the individual utilities of all users are convex in power, and the TDMA-mode is optimum, regardless of the channel gains. Channel gain from the base station to mobile i is modeled as $g_i = \frac{r_i}{d_i^4}$ where d_i denotes the distance between mobile i and the base station, which is uniformly distributed between 100m and 1000m, and r_i is the realization of the lognormal fading coefficient with variance $8dB$.

We have first examined the accuracy of the near-exact algorithm determining TDMA optimality. Our experiments over 10,000 channel gain realizations have demonstrated that the near-exact algorithm correctly determines TDMA optimality 97.5%, 99.6%, 99.9% of the time for $\alpha = 0.1, 0.2, 0.3$, respectively. As α increases, as expected, the accuracy gets better since the likelihood of the case when the optimum power allocation is such that $p_1(\lambda_{min}^1) \leq p_1 < P_T$ gradually disappears. To give the idea about the savings in complexity of the near-exact algorithm versus the optimum solution, we were able to locate the optimum solution by evaluating 1000 λ values within the convex region of $U_1(p_1)$ in STEP 3 of A-SUM. On the other hand, the near-exact algorithm requires a single evaluation at λ_{min}.

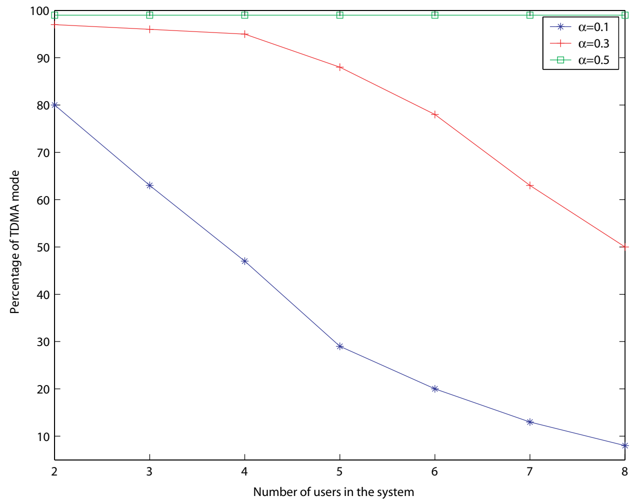

Fig. 4. Percentage of channel realizations in which the TDMA mode is selected.

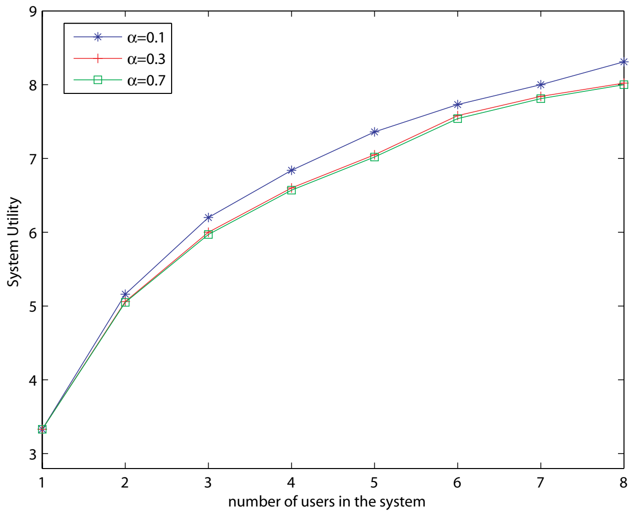

Fig. 5. System Utility versus number of users in the system.

Figure 4 shows the percentage of channel realizations in which the policy our proposed algorithm finds is TDMA. As the number of users in the system increases, the possibility that multiple users may have high channel gain values increases. Accordingly, the percentage decreases. As expected, the frequency of TDMA being selected increases as the orthogonality factor increases.

Figure 5 shows the system utility, given α and the number of users K in the system. The system utility is averaged over 10,000 channel gain realizations. The optimum policy selects the number of users and the power levels for the scheduled users. In general, not all users in the system are simultaneously

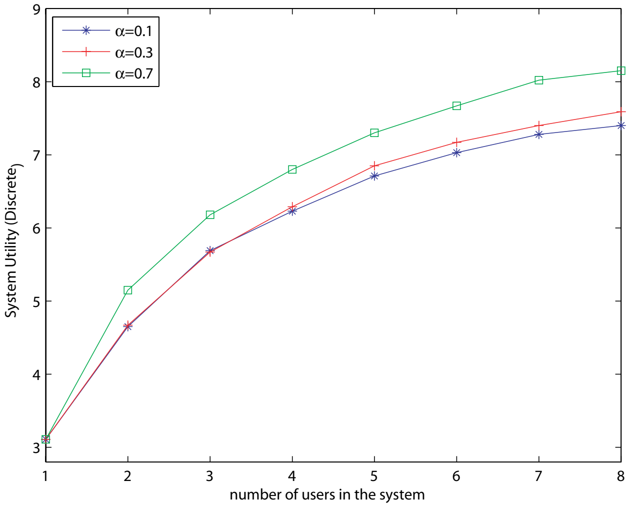

Fig. 6. System Utility versus number of users in the system.

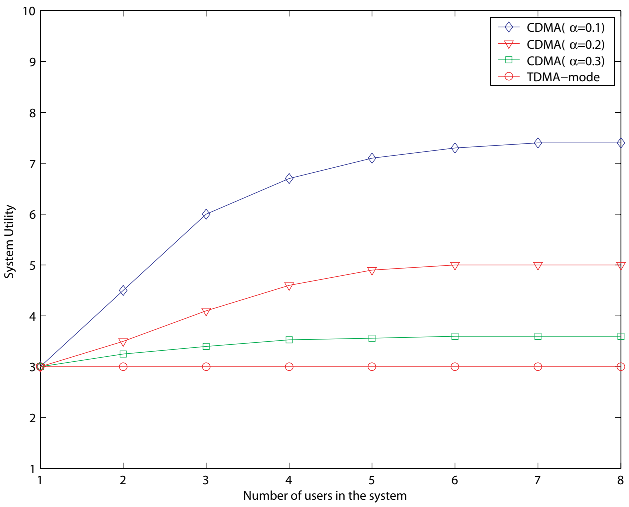

Fig. 7. CDMA-mode gain over TDMA-mode for different α values.

scheduled. When $\alpha = 0.1$, i.e., a low orthogonality factor, simultaneous transmissions do not result in much interference, and the optimum policy chooses the CDMA-mode. However, if the best user has a much higher channel gain as compared to the rest, the optimum policy may still end up to allocating all power to that best user. As α increases, more than one user transmissions result in higher interference, and TDMA-mode becomes the preferred mode so as to avoid the interference. For example, for $\alpha > 0.5$, TDMA-mode is optimum in most cases. For any value of $\alpha > 0.6$, for example, when $\alpha = 0.7$, TDMA-mode is always optimum: Figure 5 shows that when $\alpha = 0.7$ the maximum system utility is lower than the maximum system utility values achieved for smaller α values that facilitate transmissions to multiple users. The gap between the system utility for α ($\alpha < 0.6$) and the system utility for $\alpha = 0.7$ can be interpreted as the gain of optimum policy that results in hybrid CDMA/TDMA over the one that results in TDMA, as a result of the difference in the orthogonality factor values. As the number of users in the system increases, the chance of selecting users with higher channel gains, multiuser diversity, increases. Thus, the system utility increases.

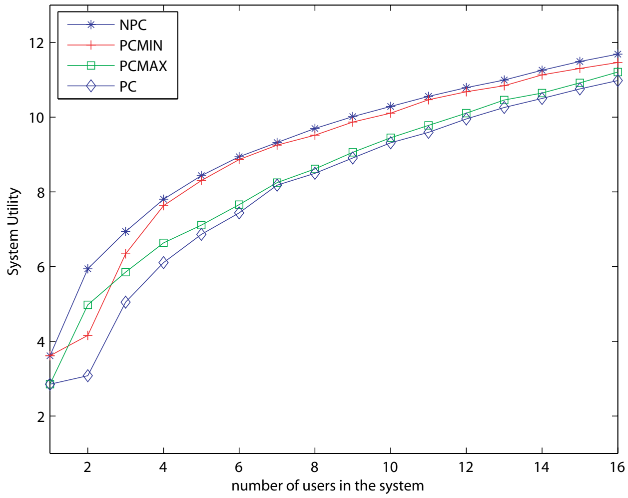

Fig. 8. System utility with no power constraint (NPC), minimum power constraint (PCMIN), maximum power constraint (PCMAX) and maximum and minimum power constraints (PC). $k = 2.4$ ($\Gamma = 0.15, N = 16$), $\alpha = 0.1$.

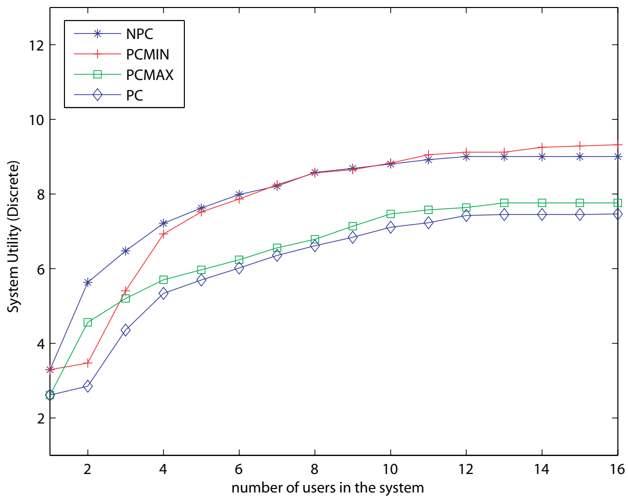

Fig. 9. Discrete system utilities with no power constraint (NPC), minimum power constraint (PCMIN), maximum power constraint (PCMAX) and maximum and minimum power constraints (PC). $k = 2.4$ ($\Gamma = 0.15, N = 16$), $\alpha = 0.1$.

Figure 6 shows the system utility where the resulting individual utility is discretized to the integer value. With this, we attempt to investigate the performance of the proposed algorithm in a practical setting where only discrete rates are available. We observe that discretization does not lead to a significant loss in system utility especially for large α values. This is because for large α, TDMA results most often, and the quantization loss in utility is due to one user only.

Figure 7 shows the system utility gain of CDMA-mode over TDMA-mode. When the orthogonality factor is low, multiple-user transmissions has a gain over the one user transmission. The gain increases as the number of users in the system increases. As the orthogonality factor increases, however, we observe that the gain disappears, as eventually TDMA-mode becomes optimum.

Figures 8 and 9 show the system utilities and the discretized system utilities with no power constraint (NPC), minimum power constraint only (PCMIN), maximum power constraint only (PCMAX) and minimum and maximum power constraint (PC) for $\alpha = 0.1$ and $k = 2.4$ ($\Gamma = 0.15$ and $N = 16$). Note the loss in total system utility we have due to the presence of the maximum power constraints, because these constraints limit the throughput values of the users with high channel gains.

VI. Conclusion

In this paper, we have considered an interference limited downlink and investigated the total utility (throughput) maximizing policy which consists of the user(s) the base station selects to transmit to, and the corresponding power allocation. Depending on the channel conditions, i.e., the orthogonality factor, and the users' channel gains, the optimum policy chooses either TDMA-mode or CDMA-mode and the corresponding power allocation. The higher the orthogonality factor, the larger the interference caused by multiple-user transmissions, rendering TDMA-mode being optimal. In contrast, multiple-user-transmissions yield a higher overall utility when the orthogonality factor is low. We observed that depending on the channel conditions, the overall system utility may or may not be a concave function and care must be given to characterizing the solution when it is not. In particular, we took the approach of examining the system utility in terms of the best user, and identified properties that helped us design an algorithmic method of finding the optimal policy. Since identifying the optimum policy may be prohibitively complex, we also identified a near-exact algorithm with reduced complexity, which is observed to find the optimum policy in an overwhelming majority of numerical examples.We also considered the effect of imposing minimum and maximum individual power constraints and observed that CDMA-mode becomes the choice more often when maximum power constraints are imposed.

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