Age of Information Minimization in Wireless Powered Stochastic Energy Harvesting Networks

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***Abstract*—This paper studies age of information minimization for a large network, where transmitters harvest energy from power beacons and send time-sensitive updates to receivers. The distributions of power beacons and energy harvesting nodes are considered to be homogeneous Poisson point and binomial point processes, respectively. We focus on probabilistic update policies for energy harvesting nodes and derive a closed-form expression for the average age of information of a node in the network utilizing stochastic geometry. Specifically, the age of information minimization is formulated as a stochastic optimization problem, transformed to a convex program and solved by the projected gradient descent algorithm.**

I. Introduction

Timeliness of data delivery has become critical for various real-time applications, such as health care, infrastructure and environment monitoring, vehicular networks, etc. Requirements for such networks include, for example, periodically updating information among peer nodes on a short timescale in order to convey the states of the dynamic processes of the underlying system. As a novel metric to measure the freshness of information from a destination node's perspective, age of information (AoI) has been introduced recently [1].

The time elapsed since the generation of the latest successfully received information is considered as AoI measure in [1], and is characterized for single-source M/M/1, M/D/1, and D/M/1 queues with first-come-first-served (FCFS) service. The concept of peak AoI (PAoI) is introduced in [2], where a packet management policy of replacing an old packet with a new one is proposed to achieve a smaller average age in the FCFS M/M/1 queue. A more general form of AoI is proposed in [3], which measures the receiver's dissatisfaction of the information staleness. All of these works characterize AoI from queueing theoretic perspective and show that AoI minimization offers different insights from delay minimization.

Energy harvesting (EH) wireless networks [4] offer the possibility of energy sustainable perpetual operation which is imperative in many of the aforementioned monitoring applications. Given the intermittent and limited nature of harvested energy, these networks rely on carefully designed energy management strategies to deliver on this promise [5]. For energy harvesting systems, AoI minimization, taking into account the intermittency of energy harvesting, has been studied in several works recently [6]–[11]. Update policies for an EH node have been investigated in [6]. The results show that an optimal policy is lazy, leaving a certain idle period between updates instead of transmitting as frequently as possible. Reference [7] considers a single-hop model for offline and online schedules of updates. Reference [8] studies the optimal renewal policy for the case of finite battery, which is proved to hold an energy-dependent multi-threshold structure. An energy harvesting cognitive radio network (EH-CRN) is studied in [9], where the optimal sensing and update policy for the secondary user is solved by a partially observable Markov decision process formulation and proved to have a threshold structure. For wireless powered networks, where nodes harvest energy from dedicated wireless energy sources, AoI has been characterized and minimized in [10], [11]. In [10], energy and information exchange between an access point (AP) and an EH device are considered, where AoI and data rate are investigated. In [11], a single EH node is considered to harvest energy from a power beacon (PB). The expression of the average AoI in terms of the battery size is derived for the greedy policy, and the optimal battery size is solved for the minimum average AoI.

Stochastic-geometry has been utilized to analyze wireless energy harvesting cognitive radio networks in [12]–[14]. In these works, primary users (PUs) and secondary users (SUs) are distributed as independent point processes. SUs harvest energy from PUs and also experience interference from them. In [12] and [13], closed-form expressions for spatial throughput of SUs are derived using tools from stochastic geometry. In [14], AoI is considered as the performance metric for SUs which send updates to their destinations, and the average AoI is characterized under a greedy update policy.

Different from previous works on AoI in EH systems, in this work, we consider a network of EH nodes harvesting energy from PBs, where the PBs follow a homogeneous Poisson point process (HPPP) and the EH nodes follow a binomial point process (BPP). Our goal is to investigate the AoI for status updates of EH nodes when there are dedicated energy sources which offer energy beams to EH transmitters, albeit causing interference to their receivers. Considering random access update policies and small batteries for EH nodes, we derive a closed-form expression for the average charging time, which leads to the average AoI expression of a node at a randomly selected location in the network. The optimal policy is characterized by formulating the AoI minimization problem as a stochastic optimization one and transforming it to a convex program. Using the projected gradient descent

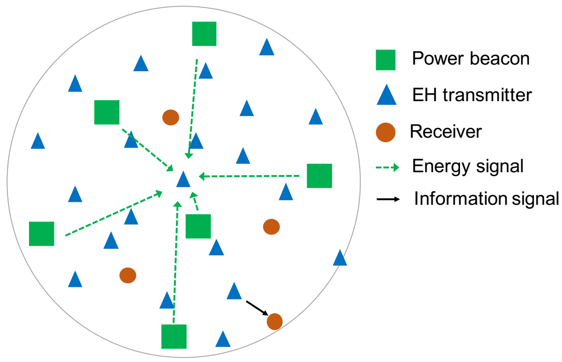

Fig. 1. System model.

(GD) algorithm, the optimal policy is found. Numerical results highlight the impact of PBs density, the number of EH nodes, and SINR threshold on the average AoI and demonstrate that the optimal policy achieves much lower average AoI compared to the always-update policy when the successful update probability is relatively small.

II. System Model

A. Network Model

We consider a large-scale network consisting of N EH transmitters that are placed independent of each other, their receivers, and multiple PBs in a disk of radius R, as shown in Fig. 1. The EH nodes first harvest energy from the signals radiated by the PBs, and then send status updates to the associated receivers by consuming their harvested energy. The PBs are distributed as a HPPP with intensity λ_{b}, and the EH nodes are placed independently and uniformly in the area following a BPP with a fixed number N. The point processes for PBs and EH nodes are denoted by $\Phi_X = \{x\}$ and $\Phi_Y = \{y\}$, respectively, with $x, y \in \mathbb{R}^2$ representing the Cartesian coordinates of PBs and EH nodes. Each EH node has a dedicated receiver separated by a fixed distance d_{s} in a random direction. Time is slotted with the slot duration normalized to 1 second. In each time slot, the EH node either harvests energy or transmits an update with power P_{s}, while PBs transmit with continuously power P_{b}. We assume $P_{\mathrm{s}} \ll P_{\mathrm{b}}$ for practical applications with low-power EH sensors. Considering a flat fading channel with Rayleigh fading, the channel gain is an exponential random variable with unit mean, and independent and identically distributed (i.i.d.) over slots. Denote the channel gain associated with a PB at x by g_x for $x \in \Phi_X$, and denote the channel gain associated with an EH node at y by g_y for $y \in \Phi_Y$. The path loss are denoted by $|x|^{-\alpha}$ and $|y|^{-\alpha}$ for the PBs and the EH nodes, respectively, where $|\cdot|$ represents the euclidean distance to the origin and $\alpha \geq 2$ is the path loss exponent.

B. Energy Harvesting Model

We consider a finite battery for each EH node with the capacity equal to the amount of energy required for one transmission, which is denoted by E. This assumption is justified for networks of small sensors, for example in infrastructure, health, or environmental monitoring applications. The EH node receives energy beams from the PBs and converts them to energy to be stored in its battery. Let $P_{\mathrm{r}}(t)$ be the received power by an EH node located at the origin at slot t, that is,

$$P_{\mathrm{r}}(t) = \sum_{x \in \Phi_X} P_{\mathrm{b}} g_x |x|^{-\alpha}. \tag{1}$$

We assume that the energy conversion circuit can harvest a fraction of the received power. The harvested power at slot t is thus given by

$$P_{\mathrm{h}}(t) = \eta P_{\mathrm{r}}(t) = \sum_{x \in \Phi_X} \eta P_{\mathrm{b}} g_x |x|^{-\alpha}, \tag{2}$$

where $0 < \eta < 1$ is the energy conversion efficiency. The battery state at the beginning of slot t is

$$b(t) = \min\left\{b(t-1) - u(t)E + (1-u(t))P_{\mathrm{h}}(t), E\right\}, \tag{3}$$

where $u(t)$ is a binary variable indicating whether the EH node transmits in slot t or not, i.e., $u(t) = 1$ if the EH node transmits and $u(t) = 0$ otherwise.

C. Transmission Model

Each EH node has two modes of operation: it can harvest energy or it can transmit an update packet. We consider a probabilistic (random access) update policy, where we define the conditional update probability given a fully charged battery as ω, i.e., $\omega \triangleq \mathbb{P}[u=1|b=E]$. That is, once an EH node fully charges its battery, it updates with probability ω. Let $\rho_{\mathrm{t}} \triangleq \mathbb{P}[u=1, b=E]$ and $\rho_{\mathrm{f}} \triangleq \mathbb{P}[b=E]$, namely, the probability of update and the probability of full charge, respectively. Thus, we have $\rho_{\mathrm{t}} = \omega\rho_{\mathrm{f}}$. For the EH node at y_{s}, its intended receiver at y_o experiences inference from PBs and other transmitting EH nodes, which are denoted by I_1 and I_2, respectively.

$$I_1 = \sum_{x \in \Phi_X} P_{\mathrm{b}} g_x |x - y_o|^{-\alpha}, \tag{4}$$

$$I_2 = \sum_{y \in \Phi_Y \setminus y_{\mathrm{s}}} \mathbf{1}_y P_{\mathrm{s}} g_y |y - y_o|^{-\alpha}, \tag{5}$$

where $\mathbf{1}_y$ denotes the event that the EH node at y transmits and $\rho_{\mathrm{t}} = \mathbb{P}[\mathbf{1}_y = 1]$. The received signal to interference plus noise ratio (SINR) is given by

$$\gamma_{y_o} = \frac{P_{\mathrm{s}} g_{y_{\mathrm{s}}} d_{\mathrm{s}}^{-\alpha}}{\sigma^2 + I_1 + I_2}, \tag{6}$$

where σ^2 is the noise variance.

D. Average AoI

The average AoI is adopted as the performance metric for the EH nodes. The AoI measures the time that elapsed since the latest received update was generated. More specifically, the AoI of a receiver at y_o associated with an EH node at y_{s} at current time t is defined as

$$a_{y_o}(t) = t - \theta(t), \tag{7}$$

where $\theta(t)$ is the generation time of the most recent successfully received update. We consider a target SINR γ^*, beyond

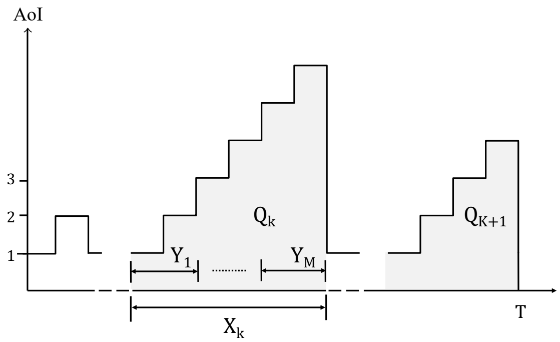

Fig. 2. AoI evolution.

which the update packet can be decoded successfully at the receiver, that is, the update is successful. Hence, the age keeps increasing when the EH node is harvesting energy or fails to update. The generation and transmission for each update takes one slot, assuming small packets. A successful update is achieved as long as $\gamma_{y_o} > \gamma^*$, thus, $a_{y_o}(t)$ drops to 1. The long-term average AoI of the EH node's receiver at y_o is

$$\Delta_{y_o} = \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} a_{y_o}(t). \tag{8}$$

III. Problem Formulation

A. AoI Analysis

Let X_k be the random variable denoting the time interval between the $(k-1)$-th and k-th successful updates. Also, we define Y to be the interval between two successive transmissions. Hence, $X_k = \sum_{i=1}^{M} Y_i$, where M denotes the number of transmissions till a successful update. Note that M follows a geometric distribution with parameter $\rho_{\mathrm{u}}(y_o) \triangleq \mathbb{P}[\gamma_{y_o} \geq \gamma^*]$, which is the successful update probability. Thus,

$$\mathbb{E}[X] = \mathbb{E}[M]\mathbb{E}[Y] = \frac{\mathbb{E}[Y]}{\rho_{\mathrm{u}}(y_o)}, \tag{9}$$

$$\begin{aligned}\mathbb{E}[X^2] &= \mathbb{E}[M]\mathbb{E}[Y^2] + (\mathbb{E}[M^2]-\mathbb{E}[M])\mathbb{E}[Y]^2\\ &= \frac{\mathbb{E}[Y^2]}{\rho_{\mathrm{u}}(y_o)} + \frac{2(1-\rho_{\mathrm{u}}(y_o))}{\rho_{\mathrm{u}}(y_o)^2}\mathbb{E}[Y]^2. \end{aligned}\tag{10}$$

Let K be the number of received updates up to time T. The average AoI, Δ_{T,y_o}, can be calculated by averaging the area of Fig. 2,

$$\Delta_{T,y_o} = \frac{K}{T}\frac{1}{K}\sum_{i=1}^{K}\frac{1}{2}X_i(X_i+1) + \frac{1}{T}Q_{K+1}. \tag{11}$$

As $T\to\infty$, $\frac{K}{T}\to\frac{1}{\mathbb{E}[X]}$ and $\frac{1}{T}Q_{K+1}\to 0$. Hence,

$$\begin{aligned}\Delta_{y_o} &= \lim_{T\to\infty}\Delta_{T,y_o} = \frac{1}{2}\frac{1}{\mathbb{E}[X]}(\mathbb{E}[X^2]+\mathbb{E}[X])\\ &= \frac{\mathbb{E}[Y^2]}{2\mathbb{E}[Y]} + \frac{1-\rho_{\mathrm{u}}(y_o)}{\rho_{\mathrm{u}}(y_o)}\mathbb{E}[Y] + \frac{1}{2}. \end{aligned}\tag{12}$$

Next, we derive expressions for $\mathbb{E}[Y]$ and $\mathbb{E}[Y^2]$. Between two successive transmissions, the EH node first fully recharge its battery, then transmits with probability ω. Let e_l be the accumulated energy over l slots after one transmission, and L be the random variable representing the number of slots for completely recharging the battery. Thus,

$$\begin{aligned}\mathbb{E}[Y] &= \sum_{k=2}^{\infty} k\sum_{l=1}^{k-1}\mathbb{P}[e_{l-1}<E, e_l \geq E](1-\omega)^{k-(l+1)}\omega\\ &= \sum_{l=1}^{\infty}\sum_{k\geq l+1}^{\infty} k\mathbb{P}[e_{l-1}<E, e_l \geq E](1-\omega)^{k-(l+1)}\omega\\ &= \sum_{l=1}^{\infty}\sum_{i=0}^{\infty}(l+i+1)\mathbb{P}[e_{l-1}<E, e_l \geq E](1-\omega)^{i}\omega\\ &= \mathbb{E}[L] + \frac{1}{\omega}. \end{aligned}\tag{13}$$

Similarly, the second moment of Y is given by

$$\begin{aligned}\mathbb{E}[Y^2] &= \sum_{k=2}^{\infty} k^2\sum_{l=1}^{k-1}\mathbb{P}[e_{l-1}<E, e_l \geq E](1-\omega)^{k-(l+1)}\omega\\ &= \mathbb{E}[L^2] + \frac{2\mathbb{E}[L]-1}{\omega} + \frac{2}{\omega^2} \end{aligned}\tag{14}$$

To obtain the first and second moments of L in (13) and (14), we characterize the probability mass function (PMF) of L for $\alpha = 4$, which can be expressed as

$$\mathbb{P}[L=k] = \mathrm{erf}\left(\frac{\pi^2\lambda_{\mathrm{b}}}{4\sqrt{z_k E}}\right) - \mathrm{erf}\left(\frac{\pi^2\lambda_{\mathrm{b}}}{4\sqrt{z_{k-1} E}}\right), \tag{15}$$

for $k = 1, 2, \ldots$, where $z_k \triangleq \frac{1}{k^{\frac{\alpha}{2}}\eta P_{\mathrm{b}}}$ and $\mathrm{erf}(x) = \frac{1}{\sqrt{\pi}}\int_{-x}^{x}\exp(-t^2)dt$. The derivation is given in Appendix A. Thus, $\mathbb{E}[L]$ and $\mathbb{E}[L^2]$ can be calculated and we denote them by μ_{l} and μ_{l^2}, respectively.

B. Probability of a Successful Update

As discussed, an update for a receiver at y_o connected to an EH node located at y_{s} is received reliably when $\gamma_{y_o} \geq \gamma^*$, and $\rho_{\mathrm{u}}(y_o) = \mathbb{P}[\gamma_{y_o} \geq \gamma^*]$ is the probability of a successful update. In this section, following [14], we derive $\rho_{\mathrm{u}}(y_o)$.

$$\begin{aligned}\rho_{\mathrm{u}}(y_o) &= \mathbb{P}\Big[\frac{P_{\mathrm{s}} g_{y_{\mathrm{s}}} d_{\mathrm{s}}^{-\alpha}}{\sigma^2+I_1+I_2} \geq \gamma^*\Big] = \mathbb{P}\Big[g_{y_{\mathrm{s}}} \geq \frac{\gamma^* d_{\mathrm{s}}^{\alpha}}{P_{\mathrm{s}}}(\sigma^2+I_1+I_2)\Big]\\ &\overset{(a)}{=} \mathbb{E}\Big[\exp\Big(-\frac{\gamma^* d_{\mathrm{s}}^{\alpha}(\sigma^2+I_1+I_2)}{P_{\mathrm{s}}}\Big)\Big]\\ &\overset{(b)}{=} \exp\left(-s\sigma^2\right)\mathcal{L}_{I_1}(s)\mathcal{L}_{I_2}(s), \end{aligned}\tag{16}$$

where (a) is from the CDF of $g_{y_{\mathrm{s}}}$ and (b) is from the definition of Laplace transform for $s \triangleq \frac{\gamma^* d_{\mathrm{s}}^{\alpha}}{P_{\mathrm{s}}}$. By the homogeneity of the HPPP distribution of PBs and the displacement theorem [15], $\mathcal{L}_{I_1}(s)$ can be obtained by considering a receiver at the origin, i.e., letting $y_o = (0,0)$ in (4). Thus, by Eq. 3.21 in [15],

$$\mathcal{L}_{I_1}(s) = \exp\left(\frac{-\pi\lambda_{\mathrm{b}}}{\mathrm{sinc}(\frac{2}{\alpha})}(P_{\mathrm{b}} s)^{\frac{2}{\alpha}}\right) \tag{17}$$

For the aggregate interference I_2 in (5),

$$\begin{aligned}\mathcal{L}_{I_2}(s) &= \mathbb{E}\left[\prod_{y\in\Phi_Y\setminus y_s}\exp\left(-s\mathbf{1}_y P_s g_y|y-y_o|^{-\alpha}\right)\right]\\ &\overset{(c)}{=}\mathbb{E}\left[\prod_{y\in\Phi_Y\setminus y_s}\frac{1}{1+s\mathbf{1}_y P_s|y-y_o|^{-\alpha}}\right]\\ &\overset{(d)}{=}\mathbb{E}\left[\frac{1}{1+s\mathbf{1}_y P_s|y-y_o|^{-\alpha}}\right]^{N-1}\\ &\overset{(e)}{=}\left(1-\rho_t+\rho_t\mathbb{E}\left[(1+sP_s|y-y_o|^{-\alpha})^{-1}\right]\right)^{N-1}, \quad (18)\end{aligned}$$

where (c) is from the Laplace transform of g_y, (d) is by the independency of the EH nodes placement, (e) is by the expectation of $\mathbf{1}_y$. Due to the rotation invariance property of uniform BPP, we only need to consider the distance between the receiver and the origin, which is denoted by the random variable v. Let v_o be a realization of v. From [16], the conditional probability density function (PDF) of the distance between a random point to a reference point located at a distance v from the origin, given $v = v_o$, is given by

$$f(q|v=v_o)=\begin{cases}\frac{2q}{R^2}, & 0\le q\le R-v_o,\\ \frac{2q}{\pi R^2}\,cos^{-1}\!\left(\frac{q^2+v_o^2-R^2}{2qv_o}\right), & R-v_o\le q\le R+v_o.\end{cases} \quad (19)$$

Hence, for a receiver located at a distance v_o from the origin,

$$\begin{aligned}&\mathbb{E}\left[(1+sP_s|y-y_o|^{-\alpha})^{-1}\right]\\ &=\int_0^{R+v_o}\frac{1}{1+sP_s q^{-\alpha}}f(q|v=v_o)dq=\phi(v_o). \quad (20)\end{aligned}$$

As $0 < \frac{1}{1+sP_s q^{-\alpha}} < 1$, and by the monotonicity property of the expectation, we can conclude that $0<\phi(v_o)<1$ for every $v_o \in [0, R]$. By substituting (18) in (16), we have

$$\rho_u(v_o) = A(1-\omega\rho_f(1-\phi(v_o)))^{N-1}, \quad (21)$$

where $A \triangleq \exp(-s\sigma^2)\mathcal{L}_{I_1}(s)$, $\rho_f = \frac{1}{\mu_l} < 1$.

C. Average AoI Minimization

The average AoI at a receiver node located at a distance v_o from the origin can be written as a function of ω and v_o. That is $\Delta(\omega, v_o) = \Delta_1(\omega) + \Delta_2(\omega, v_o) + \frac{1}{2}$, where

$$\Delta_1(\omega) = \frac{\frac{2}{\omega^2}+\frac{2\mu_l-1}{\omega}+\mu_{l^2}}{2(\mu_l+\frac{1}{\omega})}, \quad (22)$$

$$\Delta_2(\omega, v_o) = \left(\frac{(1-\omega\rho_f(1-\phi(v_o)))^{1-N}}{A}-1\right)\left(\mu_l+\frac{1}{\omega}\right). \quad (23)$$

We aim to find the optimal random access update probability ω such that the long-term average AoI for a random located transmitter-receiver pair is minimized. By taking the expectation of the receiver's location, the optimization problem is described as

$$\min_{0<\omega\le 1} \mathbb{E}_v[\Delta(\omega, v)]. \quad (24)$$

Since the EH nodes are uniformly distributed, so are the associated receivers. Hence, the PDF of v is $f_v(v_o) = \frac{2v_o}{R^2}, 0 \le v_o \le R$.

IV. Optimal Transmission Policy

In this section, we transform $\Delta(\omega, v)$ into an equivalent convex function and solve (24). Define $\tilde{\omega} \triangleq \frac{1}{\omega}$, then the expressions in (22) and (23) can be written as

$$\bar{\Delta}_1(\tilde{\omega}) = \frac{2\tilde{\omega}^2+(2\mu_l-1)\tilde{\omega}+\mu_{l^2}}{2(\mu_l+\tilde{\omega})}, \quad (25)$$

$$\bar{\Delta}_2(\tilde{\omega}, v) = \left(\frac{1}{A}\left(1-\frac{B(v)}{\tilde{\omega}}\right)^{1-N}-1\right)(\tilde{\omega}+\mu_l), \quad (26)$$

where $B(v)\triangleq\rho_f(1-\phi(v))<\rho_f$. We now consider the problem

$$\min_{1\le\tilde{\omega}<\infty} \mathbb{E}_v[\bar{\Delta}(\tilde{\omega}, v)]=\mathbb{E}_v[\bar{\Delta}_1(\tilde{\omega})+\bar{\Delta}_2(\tilde{\omega}, v)]+\frac{1}{2}. \quad (27)$$

By using the facts that $0<B(v)<\rho_f$ and $\mu_l>1$, the second derivatives of $\bar{\Delta}_1(\tilde{\omega})$ and $\bar{\Delta}_2(\tilde{\omega}, v)$ can be easily shown to be non-negative. Hence, $\bar{\Delta}(\tilde{\omega}, v)$ is convex with respect to $\tilde{\omega}$, and $\mathbb{E}_v[\bar{\Delta}(\tilde{\omega}, v)]$ is convex as a nonnegative weighted sum of $\bar{\Delta}(\tilde{\omega}, v)$ [17]. We conclude that (27) is a convex program. From (25) and (26) we have,

$$\begin{aligned}|g(\tilde{\omega}, v)| &\triangleq |\frac{\partial\bar{\Delta}(\tilde{\omega}, v)}{\partial\tilde{\omega}}| = |\frac{2\tilde{\omega}^2+\mu_l(-1+4\tilde{\omega}+2\mu_l)-\mu_{l^2}}{2(\mu_l+\tilde{\omega})^2}\\ &+\frac{\tilde{\omega}(\tilde{\omega}-B(v)N)+\frac{B(v)(1-N)}{\mu_l^{-1}}}{\left(1-\frac{B(v)}{\tilde{\omega}}\right)^N A\tilde{\omega}^2}| \le \frac{2\tilde{\omega}^2+\mu_l(4\tilde{\omega}+2\mu_l)+\mu_{l^2}}{2\tilde{\omega}^2}\\ &+\frac{\tilde{\omega}(\tilde{\omega}+B(v)N)+\frac{B(v)(1+N)}{\mu_l^{-1}}}{\left(1-\frac{B(v)}{\tilde{\omega}}\right)^N A\tilde{\omega}^2} \triangleq h(\tilde{\omega}, v). \quad (28)\end{aligned}$$

One can easily find that

$$\mathbb{E}_v[h(\tilde{\omega}, v)] \overset{(f)}{<} \frac{2\tilde{\omega}^2+\frac{4\tilde{\omega}+2\mu_l}{\mu_l^{-1}}+\mu_{l^2}}{2\tilde{\omega}^2}+\frac{\frac{\tilde{\omega}+\rho_f N}{\tilde{\omega}^{-1}}+\frac{\rho_f(1+N)}{\mu_l^{-1}}}{\left(1-\frac{\rho_f}{\tilde{\omega}}\right)^N A\tilde{\omega}^2}<\infty, \quad (29)$$

where (f) is by replacing $B(v)$ with its upper bound ρ_f and evaluating the integral, while the last inequality is by the facts that the expression is bounded for $\tilde{\omega} \in [1, \infty)$ and $\rho_f < 1$. From (28) and (29), it can be found that the conditions of the dominated convergence theorem are satisfied and hence,

$$\frac{\partial\mathbb{E}_v[\bar{\Delta}(\tilde{\omega}, v)]}{\partial\tilde{\omega}} = \mathbb{E}_v\left[\frac{\partial\bar{\Delta}(\tilde{\omega}, v)}{\partial\tilde{\omega}}\right] = \mathbb{E}_v\Big[g(\tilde{\omega}, v)\Big]. \quad (30)$$

From (30) and due to the convexity of (27), one can apply the projected GD algorithm to find the optimal policy ω^* as in Algorithm 1, where a_n is the step size.

V. Numerical Results

In this section, we present the numerical results for $d_s = 2$ m, σ^2= -115 dBm, P_b = 1 W, P_s = 0.2 W, α=4, η=0.1, R=50 m and $a_n = \frac{0.5}{n+100}$. The impact of the number of EH nodes N, the PBs density λ_b, and the SINR threshold γ^* is demonstrated. We compare the optimal update policy with the always-update policy, i.e., $\omega = 1$.

Algorithm 1 Projected GD algorithm

Initialize: $\tilde{\omega}_1 = 1$, $n = 1$.
repeat
  $\tilde{\omega}_{n+1} = \max\{1, \tilde{\omega}_n - a_n \mathbb{E}_v[g(\tilde{\omega}_n, v)]\}$
  $n = n + 1$
until convergence
$\omega^* = \frac{1}{\tilde{\omega}_n}$

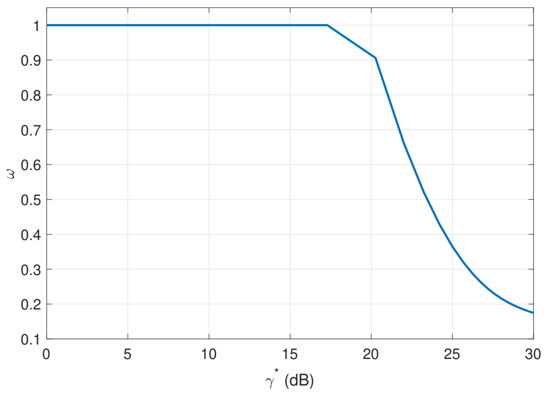

Fig. 3. Optimal policy vs. SINR threshold, N=500, $\lambda_{\mathrm{b}} = 0.005$.

A. The SINR Threshold

Fig. 3 and Fig. 4 demonstrate that the optimal policy is always-update when γ^* is small. For large γ^*, the optimal policy tends to be a probabilistic one and achieves a performance gain compared to the always-update one. This is because when γ^* is high and thus ρ_{u} is low, EH nodes prefer to take probabilistic update actions in order to decrease the interference to peer nodes and not waste the harvested energy.

B. The Number of EH Nodes

In Fig. 5, we observe that for small γ^*, the optimal policy is to always update for small number of EH nodes, and a probabilistic one as N increases. Since the interference from peer EH nodes for small N is small, the successful update probability ρ_{u} is high. Thus, the nodes prefer to update frequently to decrease the age. However, when N increases, probabilistic updates take place to avoid high interference and conserve energy. Furthermore, as γ^* increases, we see that the optimal policy is more likely to be a probabilistic one since ρ_{u} decreases on higher γ^*. Fig. 6 shows the average AoI resulting from the optimal and always-update policies. The optimal policy outperforms the always-update one significantly for large N and γ^*, as in this regime always-updating policy results in excessive unsuccessful updates, leading to waste of energy and aging of information.

C. The Density of Power Beacons

The impact of the PB density on the optimal policy and the average AoI are shown in Fig. 7 and Fig. 8, respectively. We see that in Fig. 7 the always-update policy is optimal for small γ^*, similar to Fig. 5. For large γ^*, the optimal update probability decays upon increasing λ_{b}. This is because

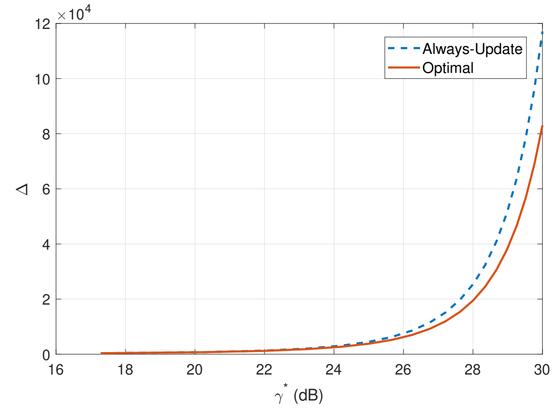

Fig. 4. Average AoI vs. SINR threshold, N=500, $\lambda_{\mathrm{b}} = 0.005$.

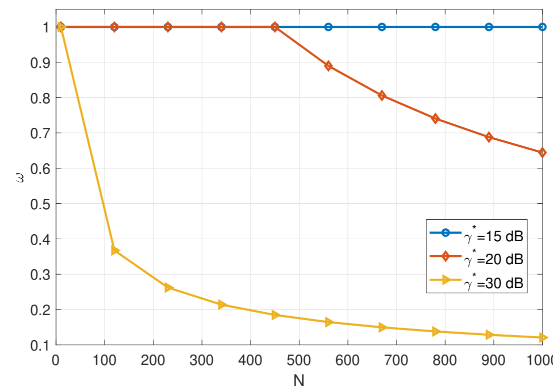

Fig. 5. Optimal policy vs. number of EH nodes, $\lambda_{\mathrm{b}} = 0.005$.

when more PBs exist, the EH nodes suffer from more interference which decreases the probability of successful decoding per node. Hence, the nodes prefer to have a probabilistic transmission in order not to waste the harvested energy and decrease the interference. In Fig. 8, we observe that the two policies have similar AoI trends versus λ_b. The optimal policy performs better when there are many PBs interfering with EH transmissions, in which case thus conservative updates results in better average AoI. In particular, a best operating point in PB density λ_b can be observed for moderate SINR target values, e.g., $\gamma^* = 15dB$, which balances the wireless power supply to EH nodes and the interference to the receivers. For very high target SINR values, e.g., $\gamma^* = 20dB$, we can see that densifying PBs does not improve the AoI performance.

VI. Conclusion

In this paper, we have studied age of information (AoI) optimization in a wireless powered stochastic network with power beacons. Considering small batteries and a random access transmission policy for the energy harvesting transmit nodes, we have derived the average AoI, utilizing stochastic geometry tools. The minimum average AoI policy is found by solving a stochastic optimization problem. The optimal policy is shown to be persistent updates in the regime where the update success probability is high. By contrast when high QoS requirements on reliable reception of updates and high levels of interference from power beacons and/or other nodes

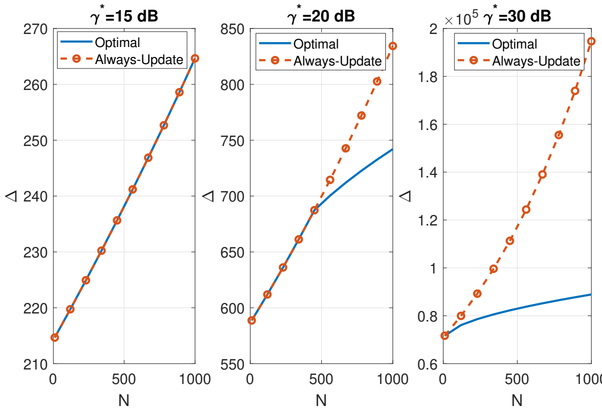

Fig. 6. Average AoI vs. number of EH nodes, $\lambda_{\rm b} = 0.005$.

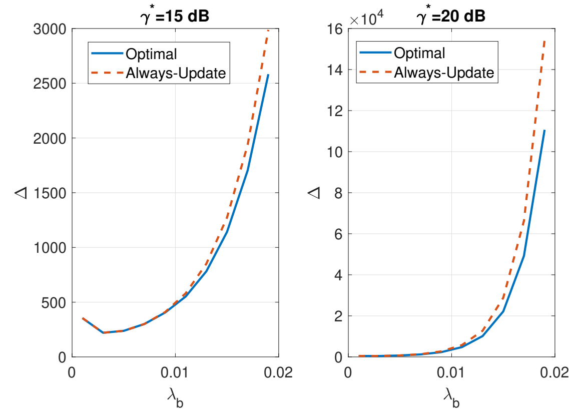

Fig. 8. Average AoI vs. PBs density, N=500.

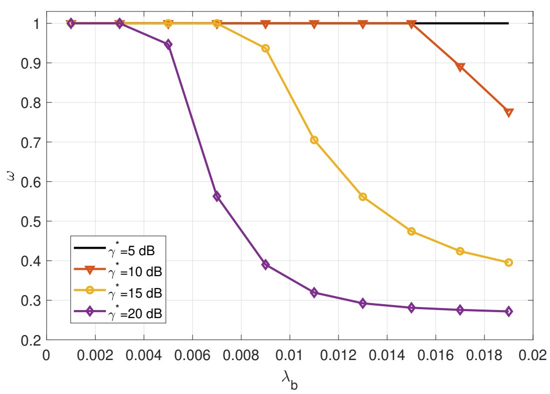

Fig. 7. Optimal transmission policy vs. PBs density, N=500.

result in small success probabilities for updates, choosing not to update even when the node has energy is beneficial.

Appendix A
Distribution of Charging Time L

Since e_k is the amount of energy harvested up to slot k, $e_k = \sum_{i=1}^{k} P_{\rm h}(i)$. Note that $P_{\rm h}(i)$, $i = 1, 2, \dots$, are i.i.d. over slots. Thus, for $k = 1, 2, \dots$,

$$\begin{aligned}\mathbb{P}[L = k] &= \mathbb{P}[e_k \geq E, e_{k-1} < E] \\ &= \mathbb{P}[e_k \geq E | e_{k-1} < E]\mathbb{P}[e_{k-1} < E] \\ &= \mathbb{P}[e_{k-1} < E] - \mathbb{P}[e_k < E], \end{aligned} \tag{31}$$

where $\mathbb{P}[e_0 < E] = 1$. Since the PBs are assumed to be HPPP distributed with intensity $\lambda_{\rm b}$, referring to [15], the Laplace transform of e_k is given by

$$\mathcal{L}_{e_k}(s) = (\mathcal{L}_{P_{\rm h}}(s))^k = \exp\left(\frac{-\pi\lambda_{\rm b}}{\text{sinc}(\frac{2}{\alpha})}(\frac{s}{z_k})^{\frac{2}{\alpha}}\right), \tag{32}$$

where $z_k = \frac{1}{k^{\frac{\alpha}{2}}\eta P_{\rm b}}$. The PDF of e_k can be obtained by the inverse Laplace transform. By Eq. 3.22 in [15],

$$f_{e_k}(x) = \frac{\lambda_{\rm b} z_k}{4}\left(\frac{\pi}{z_k x}\right)^{\frac{3}{2}} \exp\left(-\frac{\pi^4\lambda_{\rm b}^2}{16 z_k x}\right), \tag{33}$$

which gives the CDF of e_k,

$$F_{e_k}(x) = 1 - \text{erf}\left(\frac{\pi^2\lambda_{\rm b}^2}{4\sqrt{z_k x}}\right). \tag{34}$$

By substituting (34) in (31), the PMF of L is derived.

References

[1] S. Kaul, R. Yates, and M. Gruteser, "Real-time status: How often should one update?," in *Proc. IEEE INFOCOM*, pp. 2731–2735, 2012.

[2] M. Costa, M. Codreanu, and A. Ephremides, "On the age of information in status update systems with packet management," *IEEE Trans. Inf. Theory*, vol. 62, no. 4, pp. 1897–1910, 2016.

[3] Y. Sun, E. Uysal-Biyikoglu, R. D. Yates, C. E. Koksal, and N. B. Shroff, "Update or wait: How to keep your data fresh," *IEEE Trans. Inf. Theory*, vol. 63, no. 11, pp. 7492–7508, 2017.

[4] S. Ulukus, A. Yener, E. Erkip, O. Simeone, M. Zorzi, P. Grover, and K. Huang, "Energy harvesting wireless communications: A review of recent advances," *IEEE J. Select. Areas Commun.*, vol. 33, no. 3, pp. 360–381, 2015.

[5] K. Tutuncuoglu and A. Yener, "Optimum transmission policies for battery limited energy harvesting nodes," *IEEE Trans. Wireless Commun.*, vol. 11, no. 3, pp. 1180–1189, 2012.

[6] R. D. Yates, "Lazy is timely: Status updates by an energy harvesting source," in *Proc. IEEE Intern. Sympos. on Inf. Theory*, pp. 3008–3012, 2015.

[7] B. T. Bacinoglu, E. T. Ceran, and E. Uysal-Biyikoglu, "Age of information under energy replenishment constraints," in *Proc. IEEE Inf. Theory Workshop*, pp. 25–31, 2015.

[8] A. Arafa, J. Yang, S. Ulukus, and H. V. Poor, "Age-minimal transmission for energy harvesting sensors with finite batteries: Online policies," *IEEE Trans. on Information Theory*, vol. 66, no. 1, pp. 534–556, 2020.

[9] S. Leng and A. Yener, "Age of information minimization for an energy harvesting cognitive radio," *IEEE Trans. Cognitive Commun. and Networking*, vol. 5, no. 2, pp. 427–439, 2019.

[10] Y. Dong, Z. Chen, and P. Fan, "Uplink age of information of unilaterally powered two-way data exchanging systems," in *Proc. IEEE INFOCOM WKSHPS*, pp. 559–564, 2018.

[11] I. Krikidis, "Average age of information in wireless powered sensor networks," *IEEE Commun. Lett.*, vol. 8, no. 2, pp. 628–631, 2019.

[12] S. Lee, R. Zhang, and K. Huang, "Opportunistic wireless energy harvesting in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 12, no. 9, pp. 4788–4799, 2013.

[13] E. Demarchou, C. Psomas, and I. Krikidis, "Asynchronous ad hoc networks with wireless powered cognitive communications," *IEEE Trans. Cognitive Commun. and Networking*, vol. 5, no. 2, pp. 440–451, 2019.

[14] S. Leng, X. Ni, and A. Yener, "Age of information for wireless energy harvesting secondary users in cognitive radio networks," in *Proc. of the 16th IEEE Intern. Conf. on Mobile Ad-Hoc and Smart Systems (MASS)*, 2019.

[15] M. Haenggi, R. K. Ganti, *et al.*, "Interference in large wireless networks," *Foundations and Trends in Networking*, vol. 3, no. 2, pp. 127–248, 2009.

[16] M. Afshang and H. S. Dhillon, "Fundamentals of modeling finite wireless networks using binomial point process," *IEEE Trans. Wireless Commun.*, vol. 16, no. 5, pp. 3355–3370, 2017.

[17] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.