The State-Dependent Degraded Broadcast Diamond Channel

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***Abstract*—A state-dependent degraded broadcast diamond channel is studied where the source-to-relays cut is modeled with two noiseless, finite-capacity digital links with a degraded broadcasting structure, while the relays-to-destination cut is a general multiple access channel controlled by a random state. It is assumed that the source has non-causal channel state information, the relays have no state information and the destination may or may not have state information. First, the capacity is found for the case where the destination has access to the state sequence. It is demonstrated that a joint message and state transmission scheme via binning is optimal. Next, for the case with state information at the source only, lower and upper bounds on the capacity are derived for the general discrete memoryless model. Achievable rates are then computed for the case in which the relays-to-destination cut is affected by an additive Gaussian state.**

I. Introduction

We consider a communication channel in which the source wishes to communicate to the destination via the help of two parallel relays and there is no direct link between the source and the destination, as shown in Fig. 1. The *first hop*, from the source to the relays, consists of two noiseless digital links of finite capacity: a common link of capacity C_1 (bits per channel use) from the source to both relays and a private link of capacity C_2 (bits per channel use) from the source to relay 2. The first hop has thus a degraded broadcast channel (BC) structure. The *second hop*, from the relays to the destination, is a general multiple access channel (MAC) controlled by a random state [1]. It is assumed that (*i*) the entire state sequence that affects the MAC is known to the source before transmission, (*ii*) the state is not available at the relays, and (*iii*) it may or may not be known at the destination. We term this channel model as the state-dependent degraded broadcast diamond channel (SD-DBDC) with non-causal channel state information (CSI) at the transmitter (i.e., CSIT) and with or without CSI at the receiver (CSIR).

The motivation to study this channel stems from the downlink of a distributed antenna system, in which a central unit controls two antennas, e.g., two pico-base stations, via backhaul links, for communication to an active user over a wireless channel, see for example [2]. The backhaul communication may be received by both antennas over a wireless broadcast channel modeled by C_1, or received by one of antennas via a dedicated optical fiber cable modeled by C_2. In such a

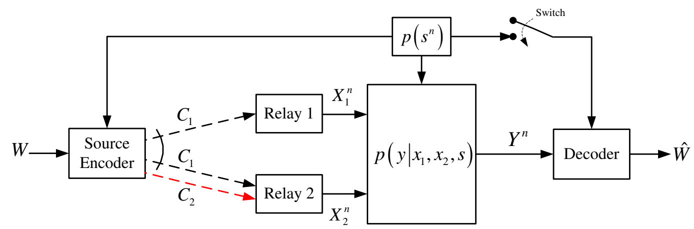

Fig. 1. A state-dependent degraded broadcast diamond channel (SD-DBDC) with non-causal channel state information (CSI) at the transmitter (CSIT) and with or without CSI at the receiver (CSIR, switch closed or open, respectively).

system, the state may model the fading coefficients for the MAC between the distributed antennas and the user, or an interference signal affecting this MAC. In the first case, the user can typically measure the fading channels of the MAC, thus obtaining CSIR, while the central unit may be informed about such fading channels, e.g., via dedicated feedback links, thus obtaining CSIT. The pico-base stations, serving as the relays, are not expected to decode the feedback signal from the user, due to a design choice or insufficient signal-to-noise ratio, and thus CSI is assumed to be unavailable at the relays. In the latter case of an interfering signal affecting the MAC, the interference signal may be communicated to the central unit via backhaul links from the interfering transmitters, e.g., another central unit, thus obtaining CSIT, while relays and the user are not informed, thus having no CSIR.

A. Related Work

The diamond channel, in which a source communicates to two relays via a general broadcast channel and the relays are connected to the destination via a state-independent MAC, was introduced by Schein and Gallager in [3] and has been widely studied ever since. A relevant special case of the diamond channel, obtained when the BC in the first hop is modeled as two orthogonal, noiseless digital links of finite capacity, is the orthogonal broadcast diamond channel (OBDC). The discrete memoryless (DM) OBDC was first studied by Traskov and Kramer in [4], where upper and lower bounds on the capacity were derived. Recently, Kang and Liu [5] proposed a single-letter upper bound for the OBDC with a Gaussian MAC and established the capacity for a subclass of Gaussian OBDCs. The SD-DBDC studied here is related to the

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OBDC, with the differences that the first hop is modeled as a *degraded* noiseless broadcast channel and that the MAC in the second hop is *state-dependent*. It is emphasized that, as discussed above, without the assumption of degradedness in the broadcast channel, the problem is currently open even with a state-independent MAC [4], [5].

A related line of work is that on coded CSIT problems for point-to-point [6] or MAC [7] models, in which the encoders are informed about the state sequence via finite-capacity links by a third party, which is referred to as the genie [7]. Note that, unlike the problem we shall consider in this paper, in [7], the messages are already known at the encoders for the MAC. Therein, assuming full CSIR, inner and outer bounds on the capacity region were derived for the case where the genie-to-encoder channels consist of separate noiseless links. Instead, assuming that the genie-to-encoder channels are modeled as in Fig. 1 with degraded noiseless links, the capacity region was characterized. In [8], Permuter et al. derived the capacity region for a MAC where the encoders are connected by finite-capacity links to one another, and the MAC depends on two correlated state sequences, each known to only one encoder, and there is full CSIR.

With no CSIR, an upper bound on the capacity of the coded CSIT problem for a point-to-point channel was found in reference [9] and proved to be tight in some special cases. The state-dependent MACs with non-causal CSIT were studied, e.g., in [10], [11]. Relay channels with non-causal CSI at certain nodes have also been investigated, see for example, [12], [13].

B. Contributions

In this paper, we consider the SD-DBDC model illustrated in Fig. 1 with non-causal CSIT and with or without CSIR. Our contributions are summarized as follows:

• For the DM SD-DBDC with non-causal CSIT and CSIR, we find the capacity. The key ingredient of the achievability is a form of binning inspired by [8], whereby the source selects directly the codewords to be transmitted by the relays in such a way as to adapt them to the given realization of the state sequence. It is demonstrated, similar to [8], that such a joint message and state transmission scheme from the source to the relays is optimal and that it generally outperforms a simple scheme whereby the source sends separate message and state descriptions to the relays (Sec. III);

• For the DM SD-DBDC with non-causal CSIT and no CSIR, we first derive an upper bound on the capacity and then propose two achievable schemes. The corresponding lower bounds are derived in Sec. IV-B and IV-C, respectively. We also extend the results to the Gaussian SD-DBDC with an additive state, along with brief discussions on the relative performance between the proposed schemes in Sec. IV-D.

Notations: The probability distribution of a random variable X is denoted as $p_X(x) = \Pr[X = x]$, or as $p(x)$ when there is no ambiguity. Notation x^i denotes vector $[x_1, ..., x_i]$. For a positive real number l, the notation $[1 : 2^l]$ denotes the set of integers $\{1, ..., 2^{\lceil l \rceil}\}$, with $\lceil . \rceil$ be the ceiling function. $\mathcal{N}(0, \sigma^2)$ denotes a zero-mean Gaussian distribution with variance σ^2.

II. System Model and Main Definitions

In this section, we introduce the model studied in this work. Specifically, the SD-DBDC model (see Fig. 1) is denoted by the tuple $(C_1, C_2, \mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{S}, p(y\,|x_1, x_2, s\,), \mathcal{Y})$, where C_1 and C_2 are the capacities in bits per channel use of the common link from the source to both the relays, and the private link from the source to relay 2, respectively, $\mathcal{X}_1$ and $\mathcal{X}_2$ are the two input alphabets, $\mathcal{S}$ is the state alphabet, $\mathcal{Y}$ is the output alphabet and $p(y\,|x_1, x_2, s\,)$ represents the channel probability mass functions (PMFs) describing the MAC between the relays and the destination. The state sequence s^n is generated in an independently and identically distributed (i.i.d.) fashion according to a fixed PMF $p(s)$, i.e., $p(s^n) = \prod_{i=1}^{n} p(s_i)$. The channel is memoryless in the usual sense and the entire state sequence s^n is assumed to be non-causally known to the source node, i.e., we assume non-causal CSIT. However, sequence s^n may or may not be available at the decoder, i.e., we may or may not have CSIR.

Let W be the message that the source wishes to send to the destination, which is uniformly distributed over the set $\mathcal{W} = [1 : 2^{nR}]$. We define the code as follows.

Definition 1: A $(2^{nR}, n)$ code for the SD-DBDC includes:

1) An encoding function at the source node
$$f : \mathcal{W} \times \mathcal{S}^n \to \left\{1, ..., 2^{nC_1}\right\} \times \left\{1, ..., 2^{nC_2}\right\}, \quad (1)$$
which maps the message and the state sequence into two indices M_1 and M_2 sent over the source-to-relays links;
2) Two encoding functions at the relays
$$h_1 : \left\{1, ..., 2^{nC_1}\right\} \to \mathcal{X}_1^n, \quad (2)$$
$$\text{and } h_2 : \left\{1, ..., 2^{nC_1}\right\} \times \left\{1, ..., 2^{nC_2}\right\} \to \mathcal{X}_2^n, \quad (3)$$
that map the information received by each relay, namely M_1 by relay 1 and (M_1, M_2) by relay 2, into the corresponding sequences transmitted by the two relays;
3) A decoding function at the destination. For the case of no CSIR, we have $g : \mathcal{Y}^n \to \mathcal{W}$, which maps the received sequence into a message estimate, while with CSIR, we have $g : \mathcal{Y}^n \times \mathcal{S}^n \to \mathcal{W}$, which maps the received sequence and the state sequence into a message estimate.

The average probability of error is defined in the usual sense and so is the concept of achievable rate and capacity C.

III. Non-causal CSIT and CSIR

In this section, the capacity is established for the DM SD-DBDC with non-causal CSIT and CSIR.

A. Capacity Result

Theorem 1: For the DM SD-DBDC model with non-causal CSIT and CSIR, the capacity is given by
$$C = \max_{\mathcal{P}} \min \begin{pmatrix} C_1 + C_2 - I(X_1, X_2; S), \\ C_1 - I(X_1; S) + I(X_2; Y\,|X_1, S\,), \\ I(X_1, X_2; Y\,|S\,) \end{pmatrix} \quad (4)$$

with the maximum taken over the distributions in the set

$$\mathcal{P} = \{p(s, x_1, x_2, y) : p(s)p(x_1, x_2 | s)p(y | x_1, x_2, s)\} \quad (5)$$

subject to $C_1 \geq I(X_1; S)$, and $C_1 + C_2 \geq I(X_1, X_2; S)$.

Proof: The achievability is based on a scheme in which the source encoder selects the codewords to be transmitted by the relays so as to adapt them to the given realization of the state sequence. This is accomplished via a binning strategy, inspired by [8], in which the codebooks for the transmitted signals X_1^n and X_2^n, are binned so that the bin index is identified by the message $w = (w_1, w_2)$ (where w_1, w_2 are independent and obtained from w at the source) to be delivered to the destination, and the codewords within the bin are chosen to "match" the state sequence. Moreover, given the degraded BC between source and relays, the codebooks for X_1^n and X_2^n are superimposed, so that the codeword for X_1^n is known at both relays, while the codeword for X_2^n is only transmitted, superimposed on X_1^n, by relay 2.

For the converse, the constraint on C_1 can be obtained starting from inequality $nC_1 \geq I(M_1; S^n)$, followed by standard arguments using the facts that the symbols S_i with $i \in [1 : n]$ are i.i.d. and $X_{1,i}$ is a deterministic function of M_1. Similarly, one can prove the constraint on $C_1 + C_2$. Considering now the first bound in (4), we get

$$\begin{aligned}
nR &= H(W | S^n) && (6)\\
&= H(W, M_1, M_2 | S^n) && (7)\\
&= H(M_1, M_2) - I(M_1, M_2; S^n) + H(W | M_1, M_2, S^n) &&\\
&= H(M_1, M_2) - \sum_{i=1}^n I(M_1, M_2, X_{1,i}, X_{2,i}, S^{i-1}; S_i) &&\\
&\qquad + H(W | M_1, M_2, S^n) && (8)\\
&\leq n(C_1 + C_2) - \sum_{i=1}^n I(X_{1,i}, X_{2,i}; S_i) &&\\
&\qquad + H(W | M_1, M_2, S^n) && (9)\\
&= n(C_1 + C_2) - \sum_{i=1}^n I(X_{1,i}, X_{2,i}; S_i) &&\\
&\qquad + H(W | M_1, M_2, S^n, Y^n) && (10)\\
&\leq n(C_1 + C_2) - \sum_{i=1}^n I(X_{1,i}, X_{2,i}; S_i) + n\epsilon_n && (11)
\end{aligned}$$

with $\epsilon_n \to 0$ as $n \to \infty$, where (6) is due to the independence between W and S^n; (7) holds because (M_1, M_2) is a deterministic function of (W, S^n); (8) follows from the facts that S_i is independent of S^{i-1} and $(X_{1,i}, X_{2,i})$ is fully determined given (M_1, M_2); (10) holds due to the Markov chain $W - (M_1, M_2, S^n) - Y^n$; and (11) follows from Fano's inequality. With similar arguments as above, one can prove the second rate bound in (4). The third bound in (4) is essentially the cut-set bound applied to the relays-to-destination cut. From (11) and the other bounds obtained, the converse is concluded by introducing an auxiliary variable Q uniformly distributed in the set $[1 : n]$ and then arguing that one can eliminate variable Q. Details of the proof can be found in [14]. ■

B. The Suboptimality of Separate Message-State Transmission

For comparison with the joint message-state transmission strategy discussed above, we consider a scheme in which the source encoder sends message and state information to the relays separately. The suboptimality of such an approach for a related model was discussed in [8]. We emphasize, however, that, while related, the model considered here is not subsumed by, nor does it subsume, the model in [8]. To elaborate, assume that the source splits the message as $w = (w_1, w_2)$, and describes the state sequence using a successive refinement code (S_1, S_2) [15]. Then, message w_1 and the base state description S_1 are sent to both relays, while message w_2 and the refined state description S_2 are sent only to relay 2. The input codewords sent by each relay can now be generated conditioned on the quantized state(s) available, similar to the coding scheme of Theorem 1 in [7]. The corresponding achievable rate R_{separate} is equal to

$$\max_{\mathcal{P}'} \min \begin{pmatrix} C_1 + C_2 - I(S_1, S_2; S), \\ C_1 - I(S_1; S) + I(X_2; Y | X_1, S, S_1, S_2), \\ I(X_1, X_2; Y | S, S_1, S_2) \end{pmatrix} \quad (12)$$

with the maximum taken over the distributions in the set

$$\mathcal{P}' = \left\{ \begin{array}{l} p(s, s_1, s_2, x_1, x_2, y) : p(s)p(s_1, s_2 | s) \\ \qquad p(x_1 | s_1)p(x_2 | x_1, s_1, s_2)p(y | x_1, x_2, s) \end{array} \right\} \quad (13)$$

subject to $C_1 \geq I(S_1; S)$, and $C_1 + C_2 \geq I(S_1, S_2; S)$, where the auxiliary alphabets $\mathcal{S}_1$ and $\mathcal{S}_2$ satisfy $|\mathcal{S}_1| \leq |\mathcal{S}| + 3$ and $|\mathcal{S}_2| \leq |\mathcal{S}| (|\mathcal{S}| + 3) + 2$.

We now show that we have in general $R_{\text{separate}} \leq C$ and that this inequality can be strict. In particular, for a fixed $p(s)$ and channel PMF $p(y | x_1, x_2, s)$, considering any PMF in the set $\mathcal{P}'$ of (13), we have the following Markov chains: $S - S_1 - X_1$, $S - (S_1, S_2) - (X_1, X_2)$ and $(S_1, S_2) - (S, X_1, X_2) - Y$. Based on these chains, we can prove the following inequalities

$$\begin{aligned}
C_1 &\geq I(S_1; S) \geq I(X_1; S), && (14)\\
C_1 + C_2 &\geq I(S_1, S_2; S) \geq I(X_1, X_2; S), && (15)\\
I(X_2; Y | X_1, S, S_1, S_2) &\leq I(X_2; Y | X_1, S), && (16)\\
\text{and } I(X_1, X_2; Y | S, S_1, S_2) &\leq I(X_1, X_2; Y | S), && (17)
\end{aligned}$$

which imply that $R_{\text{separate}} \leq C$. We now show with an example that this inequality can be strict.

For the example, we consider the special case of our model obtained with $C_1 = 0$ and X_1 taken as a constant, so that the model reduces to the two-hop line network, consisting of the source, relay 2 and the destination (studied also in [8], see Fig. 2 of [8] if $R_2 = 0$ and $p(y | x_1, x_2, s) = p(y | x_2, s)$). Inspired by the example considered in [8] in a slightly different context, we then concentrate on the binary model described by

$$Y = SX_2 \oplus Z, \quad (18)$$

where the state $S \sim \text{Bern}(\frac{1}{2})$, the noise $Z \sim \text{Bern}(p_z)$ with $p_z \triangleq \Pr[Z = 1] \in [0, \frac{1}{2}]$, independent of S, and $\oplus$ denotes the modulo-sum operation. We further impose a cost constraint on the binary input X_2 at relay 2 as $\frac{1}{n} \sum_{i=1}^n \mathbb{E}[X_{2,i}] \leq p_{x_2}$ with $p_{x_2} \in [0, \frac{1}{2}]$, where $\mathbb{E}[.]$ denotes the expectation operation. The

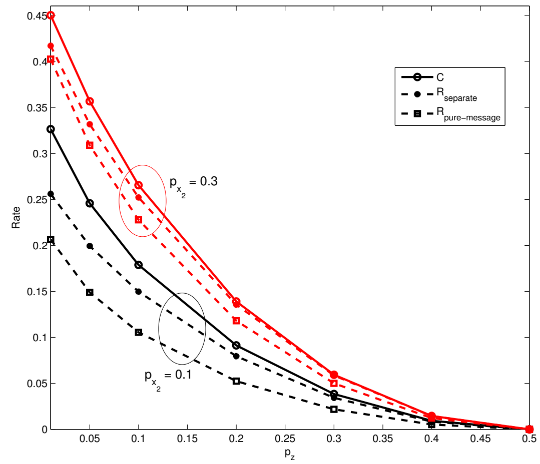

Fig. 2. Performance comparison between C, R_{separate}, and $R_{\text{pure-message}}$ for $C_2 = 0.5$, and $p_{x_2} = 0.1$ or 0.3 in the binary example of Sec. III-B.

capacity C of this example can be derived from Theorem 1 along with the additional input constraint and is given by

$$\max\min\left(\begin{array}{l} C_2 - H_b(\frac{1}{2}(p_0+p_1)) + \frac{1}{2}H_b(p_0) + \frac{1}{2}H_b(p_1), \\ \frac{1}{2}H_b(p_1 * p_z) - \frac{1}{2}H_b(p_z) \end{array}\right)$$

subject to $H_b(\frac{1}{2}(p_0+p_1)) - \frac{1}{2}H_b(p_0) - \frac{1}{2}H_b(p_1) \leq C_2$ and $\frac{1}{2}(p_0+p_1) \leq p_{x_2}$, where $p_0 \triangleq \Pr[X_2 = 1 | S = 0] \in [0,1]$, $p_1 \triangleq \Pr[X_2 = 1 | S = 1] \in [0,1]$, $H_b(p) \triangleq -p\log_2(p) - (1-p)\log_2(1-p)$, and $p_1 * p_z \triangleq p_1(1-p_z) + (1-p_1)p_z$. Similarly, rate R_{separate} can be obtained from (12). We also consider a special case of the "separate" scheme, in which only message information is sent to the relays, so that we set S_1, S_2 to a constant in (12) (rate $R_{\text{pure-message}}$ in the figure).

Numerical results are provided in Fig. 2, where C, R_{separate} and $R_{\text{pure-message}}$ are plotted versus p_z for $C_2 = 0.5$, $p_{x_2} = 0.1$ or 0.3, and the cardinality of $\mathcal{S}_2$ is assumed to be $m = 2$ in R_{separate} (increasing m to 3, 4 or 5 did not boost the numerical rates of R_{separate}). It is clearly seen that C strictly improves upon R_{separate} and the latter strictly outperforms $R_{\text{pure-message}}$ for a wide range of p_z.

IV. Non-causal CSIT and No CSIR

In this section, we turn to the SD-DBDC with non-causal CSIT and without CSIR.

A. An Upper Bound

Proposition 1: For the DM SD-DBDC model with non-causal CSIT and no CSIR, the capacity is upper bounded by

$$R_{\text{upp}} = \max_{\mathcal{P}_{\text{upp}}}\min\left(\begin{array}{l} C_1 + C_2 - I(X_1, X_2; S), \\ C_1 - I(X_1; S) + I(X_2; Y | X_1, S), \\ I(U; Y) - I(U; S) \end{array}\right) \tag{19}$$

with the maximization taken over the distributions in the set

$$\mathcal{P}_{\text{upp}} = \left\{\begin{array}{l} p(s, u, x_1, x_2, y): \\ p(s)p(u|s)p(x_1, x_2|u, s)p(y|x_1, x_2, s) \end{array}\right\}.$$

Proof: Since the capacity with CSIR cannot be smaller than without CSIR, the first two bounds follows from the converse proof of Proposition 1. The third bound in (19) is instead obtained by providing message and state information to the relays and thus the proof can be derived as in [1]. ■

B. Achievable Scheme 1: Gel'fand-Pinsker (GP) Coding With Quantized States At The Relays

In the absence of CSIR, the source can provide information about the state to the relays so as to allow the latter to perform GP coding [1]. Following this idea and an appropriate combination of message splitting, superposition coding and successive refinement [15], we can devise a coding scheme sketched below, which is referred to as GP coding with quantized states at the relays (GP-QS).

Proposition 2: For the DM SD-DBDC model with non-causal CSIT and no CSIR, a lower bound on the capacity, $R_{\text{GP-QS}}$, attained via the GP-QS scheme, is equal to

$$\max_{\mathcal{P}_1}\min\left(\begin{array}{l} C_1 + C_2 - I(S_1, S_2; S), \\ C_1 - I(S_1; S) \\ \quad + I(U_2; Y | U_1) - I(U_2; S_1, S_2 | U_1), \\ I(U_1, U_2; Y) - I(U_1; S_1) - I(U_2; S_1, S_2 | U_1) \end{array}\right),$$

with the maximum taken over the distributions in the set

$$\mathcal{P}_1 = \left\{\begin{array}{l} p(s, s_1, s_2, u_1, u_2, x_1, x_2, y): \\ p(s)p(s_1, s_2|s)p(u_1|s_1)p(u_2|u_1, s_1, s_2) \\ p(x_1|u_1, s_1)p(x_2|x_1, u_1, u_2, s_1, s_2)p(y|x_1, x_2, s) \end{array}\right\}$$

subject to $I(S_1; S) \leq C_1$, and $I(S_1, S_2; S) \leq C_1 + C_2$.

Proof: As done in the "separate" strategy discussed in Sec. III-B, the source encoder splits the message as $w = (w_1, w_2)$ and describes the state sequence via a successive refinement code (S_1, S_2), with (w_1, S_1) to be delivered to both relays and (w_2, S_2) to be delivered to relay 2 only. Given the messages and quantized states, GP coding is performed by the relays. Specifically, relay 1 and relay 2 first encode w_1 via GP codeword U_1^n, binned against the common quantized state. Next, relay 2 encodes message w_2 via GP codeword U_2^n, superimposed on codeword U_1^n and binned against both the common and private quantized states. Appropriate channel inputs are then formed by each relay, based on the binning codeword(s) selected and the available quantized state(s). At the destination, the decoding is done by looking for a unique pair of codewords that are jointly typical with the channel output, and the message estimates $(\hat{w}_1, \hat{w}_2)$ are assigned as the indices of the bins to which such codewords belong. ■

C. Achievable Scheme 2: Quantized GP Coding

In the GP-QS scheme above, a separate description of state and message is conveyed to the relays. Based on the results with CSIR in Section III, one might envision that a scheme in which selection of the relays' codewords is done directly at the source based on both message and state information could be instead advantageous. One such scheme is described here. As further discussed below, however, without CSIR, this scheme is generally not optimal and might even be outperformed by the "separate" GP-QS strategy.

In the second scheme, inspired by [13], [16], GP coding is done by the source encoder, as if the source encoder had direct access to the relays. Given the finite-capacity links between source and relays, the source encoder then quantizes the resulting GP sequence using a successive refinement code, and conveys a common description to both relays and a private description to relay 2. Upon receiving the descriptions and hence having the reconstructed sequences, the relays simply forward them to the destination. Observing the channel output, the decoder looks for a GP codeword that is jointly typical with the received sequence, and obtains the message estimate as the index of the bin to which such codeword belongs. This scheme is referred to as the quantized GP coding (QGP).

Proposition 3: For the DM SD-DBDC model with non-causal CSIT and no CSIR, a lower bound on the capacity, attained via the QGP scheme, is given by

$$R_{\mathrm{QGP}} = \max_{\mathcal{P}_2}(I(U;Y) - I(U;S)) \tag{20}$$

with the maximum taken over the distributions in the set

$$\mathcal{P}_2 = \left\{ \begin{array}{l} p(s,u,v,x_1,x_2,y): \\ p(s)p(u\,|s)p(v\,|u,s)p(x_1,x_2\,|v)p(y\,|x_1,x_2,s) \end{array} \right\}$$

subject to $I(X_1;V) \leq C_1$, and $I(X_1,X_2;V) \leq C_1 + C_2$.

Remark 1: While a general performance comparison between the GP-QS and QGP schemes does not seem to be easy to establish, it can be seen that when the link capacities are arbitrarily large, either the state sequence or the GP sequence can be perfectly conveyed to the relays, and thus both the GP-QS and QGP achieve the upper bound (19), and specifically the third bound in (19), thus establishing the capacity. □

D. Gaussian SD-DBDC

We now study a Gaussian SD-DBDC. In particular, we assume that the destination output Y_i at time instant i is related to the channel inputs $X_{1,i}, X_{2,i}$ at the relays and the channel state S_i as $Y_i = X_{1,i} + X_{2,i} + S_i + Z_i$, where $S_i \sim \mathcal{N}(0, P_S)$ and $Z_i \sim \mathcal{N}(0, N_0)$, are i.i.d., mutually independent sequences. The channel inputs at the relays satisfy the average power constraints $\frac{1}{n}\sum_{i=1}^{n}\mathbb{E}[X_{k,i}^2] \leq P_k, k = 1,2$. The code definition follows Definition 1 except that the input codewords are required to guarantee the power constraints.

We apply the GP-QS and QGP schemes discussed above to the given Gaussian model and obtain the corresponding achievable rates. Details of the proof can be found in [14].

Proposition 4: Let $\mathcal{C}(x) = \frac{1}{2}\log_2(1+x)$. For the Gaussian SD-DBDC, the rate $R^{\mathrm{G}}_{\mathrm{GP-QS}}$ of scheme GP-QS is given by

$$\max_{\substack{0\leq\rho\leq 1,\\ (D_1,D_2)\in\mathcal{A}}} \min \left(\begin{array}{l} C_1 + C_2 - \frac{1}{2}\log_2(\frac{P_S}{D_2}), \\ C_1 - \frac{1}{2}\log_2(\frac{P_S}{D_1}) + \mathcal{C}\left(\frac{\bar{\rho}P_2}{D_2+N_0}\right), \\ \mathcal{C}\left(\frac{(\sqrt{P_1}+\sqrt{\rho P_2})^2}{\bar{\rho}P_2+D_1+N_0}\right) + \mathcal{C}\left(\frac{\bar{\rho}P_2}{D_2+N_0}\right) \end{array} \right),$$

where $\bar{\rho} = 1-\rho$ and the set of $\mathcal{A}$ is defined as

$$\mathcal{A} \triangleq \left\{ \begin{array}{l} (D_1,D_2): P_S \geq D_1 \geq D_2 \geq 0, \\ \quad D_1 \geq P_S 2^{-2C_1}, D_2 \geq P_S 2^{-2(C_1+C_2)} \end{array} \right\}; \tag{21}$$

while, the rate $R^{\mathrm{G}}_{\mathrm{QGP}}$ of scheme QGP is given by

$$\mathcal{C}\left(\frac{\left(\sqrt{P_1(1-2^{-2C_1})} + \sqrt{P_2\left(1-2^{-2(C_1+C_2)}\right)}\right)^2}{P_1 2^{-2C_1} + P_2 2^{-2(C_1+C_2)}\left(1+2\sqrt{\frac{P_1(1-2^{-2C_1})}{P_2\left(1-2^{-2(C_1+C_2)}\right)}}\right) + N_0} \right).$$

For reference, a natural lower bound on the capacity is obtained when the source transmits pure message to the relays, so that the model at hand is converted into a Gaussian MAC with degraded messages. The decoder simply treats the state as noise. This lower bound, denoted by $R^{\mathrm{G}}_{\text{no SI}}$, is equal to

$$\max_{0\leq\rho\leq 1} \min \left(\begin{array}{c} C_1 + C_2,\ \mathcal{C}\left(\frac{(1-\rho^2)P_2}{N_0+P_S}\right) + C_1, \\ \mathcal{C}\left(\frac{P_1+P_2+2\rho\sqrt{P_1P_2}}{N_0+P_S}\right), \end{array} \right). \tag{22}$$

A simple upper bound $R^{\mathrm{G}}_{\mathrm{upp}}$, instead obtained by providing the decoder with the interference sequence so that it can be cancelled, is given by (22) with N_0 in lieu of $N_0 + P_S$.

Remark 2: As the link capacity C_1 becomes arbitrarily large, it is seen that both the GP-QS and QGP schemes attain $R^{\mathrm{G}}_{\mathrm{upp}}$, leading to the capacity $C = \mathcal{C}\left(\frac{P_1+P_2+2\sqrt{P_1P_2}}{N_0}\right)$. Note that the capacity is the same as if the interference at the destination was not present and if full cooperation was possible at the relays. The benefit of utilizing the non-causal CSIT is hence evident. We also emphasize that letting capacity C_2 alone grow to infinity is not enough to achieve $R^{\mathrm{G}}_{\mathrm{upp}}$, as in this case only relay 2 can be fully informed by the source. □

Remark 3: The rate $R^{\mathrm{G}}_{\mathrm{GP-QS}}$ is generally dependent on the interference power P_S, while the rate $R^{\mathrm{G}}_{\mathrm{QGP}}$ is not. This is because in the GP-QS scheme, the state sequence needs to be described to the relays on the finite-capacity links, and thus the stronger is the power P_S of the state, the larger are the feasible distortions (D_1, D_2) in (21) for reproducing the state sequence at the relays. As a result, in the extreme case in which P_S becomes arbitrarily large, the rate $R^{\mathrm{G}}_{\mathrm{GP-QS}}$ reduces to rate $R^{G}_{\text{no SI}}$ of (22) obtained by treating the state as noise. On the other hand, in the QGP scheme, the source compresses directly the appropriate GP sequence, whose power does not depend on P_S. Given the fact that the performance of QGP is not dependent on P_S, it is expected that scheme QGP outperforms scheme GP-QS in case P_S is sufficiently large. But in general, scheme QGP is not optimal and might even be outperformed by scheme GP-QS, e.g., when P_S is relatively small, as seen from the numerical results provided in [14]. □

V. Concluding Remarks

In this work, we have studied a state-dependent diamond channel, in which the BC between source and relays is defined by a noiseless degraded BC, and the MAC between relays and destination is state-dependent. For the case with non-causal CSIT and CSIR, we have established the capacity and shown that a joint message and state transmission scheme via binning is optimal and superior to the scheme that performs separate message and state description transmission. For the case without CSIR, we have proposed an upper bound and two transmission schemes, and applied the results to a Gaussian model with an additive Gaussian state.

REFERENCES

[1] S. I. Gel'fand and M. S. Pinsker, "Coding for channel with random parameters," *Problems of Control and Information Theory*, vol. 9, no. 1, pp. 19–31, 1980.

[2] D. Gesbert, S. Hanly, H. Huang, S. Shamai, O. Simeone, and W. Yu, "Multi-cell MIMO cooperative networks: A new look at interference," *IEEE Journal on Selected Areas in Communications*, vol. 28, no. 9, pp. 1380–1408, 2010.

[3] B. E. Schein, "Distributed coordination in network information theory," Ph.D. dissertation, Massachusetts Institute of Technology, 2001.

[4] D. Traskov and G. Kramer, "Reliable communication in networks with multi-access interference," in *Proceedings of IEEE Information Theory Workshop*, Lake Tahoe, California, USA, September 2007, pp. 343–348.

[5] W. Kang and N. Liu, "The Gaussian multiple access diamond channel," in *Proceedings of IEEE International Symposium on Information Theory*, Saint Petersburg, Russia, August 2011, pp. 1499–1503.

[6] C. Heegard and A. El Gamal, "On the capacity of computer memory with defects," *IEEE Transactions on Information Theory*, vol. 29, no. 5, pp. 731–739, September 1983.

[7] Y. Cemal and Y. Steinberg, "The multiple-access channel with partial state information at the encoders," *IEEE Transactions on Information Theory*, vol. 51, no. 11, pp. 3992–4003, November 2005.

[8] H. Permuter, S. Shamai, and A. Somekh-Baruch, "Message and state cooperation in multiple access channels," *IEEE Transactions on Information Theory*, vol. 57, no. 10, pp. 6379–6396, October 2011.

[9] R. Tandon and S. Ulukus, "On the rate-limited Gel'fand-Pinsker problem," in *Proceedings of IEEE International Symposium on Information Theory*, Seoul, Korea, June 2009, pp. 1963–1967.

[10] A. Somekh-Baruch, S. Shamai, and S. Verdu, "Cooperative multiple-access encoding with states available at one transmitter," *IEEE Transactions on Information Theory*, vol. 54, no. 10, pp. 4448–4469, October 2008.

[11] S. P. Kotagiri and J. N. Laneman, "Multiaccess channels with state known to some encoders and independent messages," *EURASIP Journal on Wireless Communications and Networking*, vol. 2008, pp. 1–14, January 2008.

[12] A. Zaidi, S. P. Kotagiri, J. N. Laneman, and L. Vandendorpe, "Cooperative relaying with state available noncausally at the relay," *IEEE Transactions on Information Theory*, vol. 56, no. 5, pp. 2272–2298, May 2010.

[13] A. Zaidi, S. Shamai, P. Piantanida, and L. Vandendorpe, "Bounds on the capacity of the relay channel with noncausal state at source," April 2011, available at http://arxiv.org/abs/1104.1057.

[14] M. Li, O. Simeone, and A. Yener, "Degraded broadcast diamond channels with non-causal state information at the source," February 2012, submitted to *IEEE Transactions on Information Theory*, available at http://arxiv.org/abs/1203.1869.

[15] B. Rimoldi, "Successive refinement of information: Characterization of the achievable rates," *IEEE Transactions on Information Theory*, vol. 40, no. 1, pp. 253–259, January 1994.

[16] O. Simeone, O. Somekh, H. V. Poor, and S. Shamai, "Downlink multicell processing with limited-backhaul capacity," *EURASIP Journal on Advances in Signal Processing*, vol. 2009, pp. 1–10, 2009.