The Energy Harvesting and Energy Cooperating Two-way Channel with Finite-Sized Batteries

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***Abstract*—In this paper, we consider the energy allocation problem for energy harvesting and energy cooperating nodes with finite-sized batteries. In particular, we solve the sum-throughput maximization problem in a two-way channel with energy harvesting nodes that can also transfer energy to one another. To do so, we non-trivially extend a class of policies which originally rely on an infinite-sized battery to be optimal, to the finite battery case. We observe that when we partition transferred energy into immediately used and stored components, an optimal policy has a non-zero stored component only when the battery of the transferring user is full. This enables the decomposition of the sum-throughput maximization problem into separate energy transfer and power allocation problems. Utilizing properties of this optimal class of policies, we solve the power allocation problem using a two dimensional directional water-filling algorithm with restricted transfers, where energy transfers only take place at full battery instances. Numerical results demonstrate that energy cooperation notably improves sum-throughput as one node gets energy deprived.**

***Index Terms*—Energy harvesting networks, energy cooperation, partially procrastinating power policies, two-way channels.**

I. Introduction

Recent findings on high efficiency medium range wireless energy transfer [1], [2] enable energy cooperation as a viable technology for future wireless networks. This option is particularly valuable in energy harvesting networks, where energy available to each node is non-uniform among the nodes and variable in time. Additionally, incorporating wireless energy transfer to energy harvesting networks brings a new dimension to the power allocation problem, calling for tailored energy policies in order to fully utilize the potential of the network [3]–[5].

Energy harvesting wireless networks are widely studied in the past few years, particularly from the perspective of transmission policies. In [6], the authors solve the transmission completion time minimization problem for a single energy harvesting transmitter with an infinite-sized battery. This is extended to a finite-sized battery and the throughput maximization problem in [7], and to fading channels in [8] using a directional water-filling algorithm. This is a variation of the conventional water-filling algorithm where water is allowed to flow only in the forward direction in time, in order to avoid consuming energy that is not yet harvested. Other channel models, e.g., multiple access [9], broadcast [10], [11], and relay channels [12], circuit power consumption models [13], [14], and energy harvesting transmitters [15] are subsequently considered. In common, these offline optimization problems are solved with variations of the directional water-filling algorithm.

Energy transfer adds a new dimension to wireless communications with energy harvesting nodes by enabling the possibility of energy cooperation, as proposed in [3]. Uni-directional energy transfer in energy harvesting channels is studied in [3] for various channel models. The authors develop a two dimensional directional water-filling algorithm, where water flow occurs both among the neighboring time slots of each transmitter, and also between transmitters. This work is extended to bi-directional energy transfers in [4], [5], which also shows the optimality of so-called procrastinating policies. With the help of procrastinating policies, the two dimensional directional water-filling algorithm is reduced to a single dimensional one. All of these references assume infinite-sized batteries at all nodes. Some other work on energy transfer in communications include inductively coupled models [16], energy token exchange models [17], and joint data and energy transfer over RF [18], [19].

In this work, we consider a two-way channel consisting of energy harvesting and cooperating nodes. Different than previous work which assumed infinite batteries at the nodes [3]–[5], we consider the practical case of finite-sized batteries. In particular, we show that procrastinating policies defined in [4], [5], which rely on infinite-sized storage for optimality, can be modified to apply to nodes with finite-sized batteries. Namely, we show that there exists an optimal *partially procrastinating policy*, where energy wirelessly received in a time slot cannot exceed energy consumed in that time slot unless the battery of the transferring node is full. This yields a decomposition of the sum-throughput maximization problem into energy transfer and power allocation problems. The former problem is over a single time slot, and is therefore solved directly. The latter problem is solved using the two dimensional directional water-filling (2D-DWF) algorithm in [3]. We further simplify the 2D-DWF algorithm using properties of partially procrastinating policies. Finally, we observe through simulations that energy cooperation significantly increases sum-throughput, particu-

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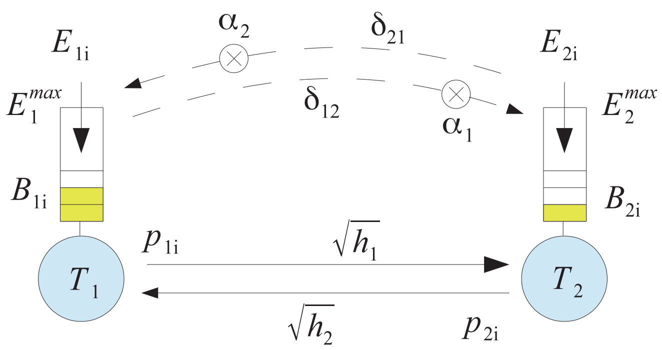

Fig. 1. Two-way channel with energy harvesting transmitters, energy cooperation, and finite-sized batteries.

larly when one of the nodes is energy-deprived.

II. System Model and Problem Definition

We consider a two-way channel with two energy harvesting transmitters that can also bi-directionally transfer energy to one another. Unlike previous two-way models with energy cooperation [3], [4], we consider finite energy storage capability at both nodes. The channel model is shown in Fig. 1.

The time is slotted with unit slot length $\tau = 1$ for simplicity, while the extension to arbitrary slot lengths is straightforward. In time slot i, the transmitters T_1 and T_2 harvest E_{1i} and E_{2i} units of energy, and store the harvested energy in their batteries of size E_1^{max} and E_2^{max}, respectively. The energy available in the battery can either be used for transmission, or transferred to the other user. In particular, T_k, $k = 1, 2$, transfers $\delta_{ki} \geq 0$ units of energy to the other node in time slot i wirelessly. This transfer has efficiency $0 \leq \alpha_k < 1$, i.e., node T_ℓ, $\ell \neq k$, receives $\alpha_k \delta_{ki}$ units of energy as a result of the transfer. In addition, T_k transmits with power $p_{ki} \geq 0$ throughout time slot i, consuming $p_{ki}\tau = p_{ki}$ units of energy. Hence, the energy stored in the battery of T_k, $k = 1, 2$, at the end of time slot i evolves as

$$B_{ki} = \min\left\{E_k^{max}, B_{k(i-1)} + E_{ki} + \alpha_\ell \delta_{\ell i} - \delta_{ki} - p_{ki}\right\}, \tag{1}$$

where $\ell = 1, 2$, $\ell \neq k$.

We consider additive white Gaussian noise (AWGN) at both receivers, and noise powers are normalized through the channel coefficients $\sqrt{h_k}$ from T_k to T_ℓ. For full duplex operation, the channel output at T_k is given by

$$Y_k = X_k + \sqrt{h_\ell} X_\ell + Z_k, \quad \ell \neq k, \tag{2}$$

where $Z_k \sim \mathcal{N}(0, 1)$ is the received noise. Note that each receiver can cancel out the contribution from its own channel input, X_k, and treat the channel as a single transmitter AWGN channel. With the transmit powers p_{1i} and p_{2i} in time slot i, the channel capacity from T_k to T_ℓ is therefore given by [20]

$$C_k(p_{ki}) = \frac{1}{2}\log(1 + h_k p_{ki}). \tag{3}$$

In this work, we are interested in maximizing the sum-throughput of the system in a finite session length of N time slots. For clarity of exposition, we consider the case with $h_1 = h_2 = 1$, since the generalization to arbitrary h_1, h_2 is straightforward. Hence, we will use the sum-capacity C_S within each time slot, expressed as

$$C_S(p_{1i}, p_{2i}) = \frac{1}{2}\log(1 + p_{1i}) + \frac{1}{2}\log(1 + p_{2i}) \tag{4}$$

for transmit powers p_{1i} and p_{2i} in time slot i. However, the analysis in this paper can be extended to weighted sum-throughput functions by redefining $C_S(p_{1i}, p_{2i})$ accordingly.

In order to maximize the sum-throughput of the system, the transmitters choose the energy amounts allocated to transmission and transfer in each time slot, i.e., p_{ki} and δ_{ki}, $k = 1, 2$, $i = 1, \ldots, N$. We refer to the ensemble of these variables as the *power policy* of the system. In accordance with prior work [3]–[12], we consider the offline problem where both transmitters are aware of the energy arrivals E_{ki} for the duration of the communication session. Hence, the power policy is calculated from the entire energy arrival sequence, E_{ki}, $k = 1, 2$, $i = 1, \ldots, N$.

For this case, we first observe that a transmitter cannot consume more energy than it has available at any time. Namely, for T_k, the transmit and transfer energy in any time slot i is less than the stored, harvested, and received energy combined, i.e.,

$$\delta_{ki} + p_{ki} \leq B_{k(i-1)} + E_{ki} + \alpha_\ell \delta_{\ell i}, \tag{5}$$

where $\ell \neq k$. Note that this constraint can equivalently be expressed as $B_{ki} \geq 0$. We refer to this constraint as the *energy causality* constraint.

Next, we remark that a power policy yielding a battery overflow at any time slot cannot be sum-throughput optimal. This is the extension of [7, Lemma 2] to our two-way model with energy cooperation, and follows from the sum-capacity in (4) being increasing in p_{1i} and p_{2i}. In particular, if a power policy allows the battery of T_k to overflow in time slot i by an amount of ϵ, then increasing p_{ki} by ϵ avoids the overflow, increases the sum-throughput, and does not affect the rest of the transmission. Hence, without loss of generality, we restrict our attention to power policies that do not yield a battery overflow. With this constraint, we simplify (1) as

$$B_{ki} = B_{k(i-1)} + E_{ki} + \alpha_\ell \delta_{\ell i} - \delta_{ki} - p_{ki} \tag{6}$$

$$= \sum_{j=1}^{i} \left(E_{kj} + \alpha_\ell \delta_{\ell j} - \delta_{kj} - p_{kj}\right), \tag{7}$$

where (7) is obtained by telescoping (6) with initial battery state[1] $B_{k0} = 0$. With this definition, a power policy that does not yield a battery overflow satisfies $B_{ki} \leq E_k^{max}$. We refer to this constraint as the *battery capacity* constraint.

The sum-throughput maximization problem for the energy harvesting and energy cooperating two-way channel in a

[1] A non-zero initial battery state B_{k0} can be equivalently represented in this model by increasing E_{k1}, i.e., the first energy harvest, by B_{k0}.

transmission session of N time slots is expressed as

$$\max_{\{p_{ki},\delta_{ki}\}} \sum_{i=1}^{N} C_S(p_{1i},p_{2i}) \tag{8a}$$
$$\text{s.t. } 0 \le B_{ki} \le E_k^{max} \tag{8b}$$
$$p_{ki} \ge 0, \delta_{ki} \ge 0 \tag{8c}$$
$$k=1,2,\ i=1,\ldots,N, \tag{8d}$$

where (8b) are the energy causality and battery capacity constraints described above. In the next two sections, we propose a decomposition of (8) and find its solution.

III. Partially Procrastinating Policies for Finite-Sized Batteries

Reference [4] introduces procrastinating policies, which satisfy the condition

$$\alpha_k \delta_{ki} \le p_{\ell i}, \quad k,\ell=1,2,\ \ell \ne k \tag{9}$$

throughout the transmission. In the infinite-sized battery case in [4], for any power policy, a procrastinating policy with the same transmit powers can be found by postponing energy transfers which are not immediately needed, i.e., which do not satisfy (9). Hence, procrastinating policies are shown to contain an optimal policy, allowing the decomposition of the infinite-sized battery version of (8) into separate energy transfer and power allocation problems. This decomposition simplifies the analysis of the problem for two reasons: the energy transfer problem is defined over a single time slot only, and the power allocation problem is stripped from the energy transfer variables.

By their nature, procrastinating policies rely on being able to postpone energy transfers indefinitely without being limited by a finite-sized battery. Clearly, with finite-sized batteries, postponing transfers may yield a battery overflow which could have been avoided by transferring energy at the expense of violating (9). Hence, policies satisfying (9) do not necessarily contain an optimal solution to (8). In this section, we adapt procrastinating policies to the finite-battery case.

We decompose δ_{ki} into two components, namely

$$\delta_{ki} = \gamma_{ki} + \epsilon_{ki}, \tag{10}$$

where γ_{ki} and ϵ_{ki} are both non-negative. As will be made clear in the sequel, these components refer to transferred energy that is immediately consumed and transferred energy that is stored for future use, respectively. Note that by choosing γ_{ki} and ϵ_{ki}, any $\delta_{ki} \ge 0$ can be established. Hence, power policies defined as $\{p_{ki},\gamma_{ki},\epsilon_{ki}\}$, $k=1,2$, $i=1,\ldots,N$, include all feasible power policies for (8). We now define *partially procrastinating policies* for the finite-sized battery case as follows:

Definition 1 *A partially procrastinating policy consisting of* $\{p_{ki},\gamma_{ki},\epsilon_{ki}\}$, $k=1,2$, $i=1,\ldots,N$, *satisfies*

$$\alpha_k \gamma_{ki} \le p_{\ell i}, \quad k,\ell=1,2,\ \ell \ne k \tag{11}$$
$$\gamma_{1i}\gamma_{2i} = 0 \tag{12}$$
$$\epsilon_{ki}\left(B_{ki} - E_k^{max}\right) = 0, \quad k=1,2 \tag{13}$$

for $i=1,\ldots,N$.

In particular, for the finite-battery case, the procrastination condition in (9) applies only to the immediately consumed component of the transferred energy, γ_{ki}, as in (11). Hence, we refer to this as partial procrastination. The other component, namely ϵ_{ki}, can only be non-zero when the respective battery is full, i.e., $B_{ki} = E_k^{max}$, as dictated by (13). Next, we show that there exists a procrastinating policy which solves (8).

Lemma 1 *There exists at least one partially procrastinating policy, as defined in Definition 1, for which the transferred energy values* $\{\delta_{ki}\}$ *found from (10) and the transmit powers* $\{p_{ki}\}$ *solve the problem in (8).*

Proof: Let $\{p_{ki}^*,\delta_{ki}^*\}$, $k=1,2$, $i=1,\ldots,N$, be a solution to (8). To prove the lemma, we will construct a partially procrastinating policy $\{p_{ki}^*,\gamma_{ki}^*,\epsilon_{ki}^*\}$ which satisfies (10). Let $\delta_{k1} = \delta_{k1}^*$, $k=1,2$. Starting from $i=1$, calculate

$$\gamma_{ki} = \min\{\delta_{ki}, p_{\ell i}^*/\alpha_k\} \tag{14}$$
$$\delta_{k(i+1)} = \delta_{k(i+1)}^* + \delta_{ki}^* - \gamma_{ki} \tag{15}$$

for $k,\ell=1,2$ and $i=1,\ldots,N$. Note that by definition, $\{\gamma_{ki}\}$ in (14) satisfy (11). Next, for $i=1,\ldots,N$, let

$$\gamma_{ki}^* = \max\{0, \gamma_{ki} - \gamma_{\ell i}\}, \quad k,\ell=1,2, \quad \ell \ne k, \tag{16}$$

which ensures that $\{\gamma_{ki}^*\}$ satisfy both (11) and (12). Finally, let $\epsilon_{ki}^* = 0$ for $k=1,2$, $i=1,\ldots,N$. Starting from $i=1$, calculate δ_{ki} from (10) using $\{\gamma_{ki}^*,\epsilon_{ki}^*\}$, B_{ki} from (7) and $\{\delta_{ki}\}$, and let

$$\epsilon_{ki}^* = \max\{0, B_{ki} - E_k^{max}\}. \tag{17}$$

We remark that by definition, $\{\epsilon_{ki}^*\}$ satisfy (13). Note that the process in (14)-(15) postpones energy transfers to the next time slot when $\alpha_k\delta_{ki} \ge p_{\ell i}$, (16) eliminates cases where energy is transferred in both directions simultaneously, and (17) transfers any energy that is potentially overflowing to the other transmitter. As a result, both constraints in (8b) are satisfied for $k=1,2$ and $i=1,\ldots,N$ by construction. Hence, $\{p_{ki}^*,\gamma_{ki}^*,\epsilon_{ki}^*\}$ is feasible; and since the transmit powers are unchanged, this policy is also optimal. ■

In Lemma 1, we show that by procrastinating energy transfers γ_{ki}, and choosing ϵ_{ki} to avoid overflows, we can find a partially procrastinating policy for any feasible transmit power policy. This observation allows us to simplify the sum-throughput maximization problem, as we will present in the next section.

IV. Optimal Power Policy

We next apply the optimality of partially procrastinating policies to decompose and solve the problem in (8). We define *consumed powers*,

$$\bar{p}_{ki} = p_{ki} + \gamma_{ki} - \alpha_\ell \gamma_{\ell i}, \quad k, \ell = 1, 2, \ \ell \neq k \tag{18}$$

for the entire transmission duration, $i = 1, \ldots, N$. Substituting in (7), this yields the battery state

$$B_{ki} = \sum_{j=1}^{i} \left(E_{kj} + \alpha_\ell \epsilon_{\ell j} - \epsilon_{kj} - \bar{p}_{kj} \right). \tag{19}$$

For a partially procrastinating policy, we have $\bar{p}_{ki} \geq 0$ from (11). In addition, due to (11)-(12), the constraint $p_{ki} \geq 0$ implies $\bar{p}_{ki} \geq \gamma_{ki}$. Given these two constraints on $\bar{p}_{ki}$, all power policies $\{p_{ki}, \gamma_{ki}, \epsilon_{ki}\}$ can be represented with a consumed power policy $\{\bar{p}_{ki}, \gamma_{ki}, \epsilon_{ki}\}$. Therefore, without loss of generality, we add (11)-(12) to the problem in (8) and rewrite the sum-throughput maximization problem as

$$\max_{\{\bar{p}_{ki}, \gamma_{ki}, \epsilon_{ki}\}} \sum_{i=1}^{N} C_S \left(\{\bar{p}_{ki} - \gamma_{ki} + \alpha_\ell \gamma_{\ell i}\}_{k=1}^{2} \right) \tag{20a}$$

$$\text{s.t. } 0 \leq B_{ki} \leq E_k^{max} \tag{20b}$$

$$\bar{p}_{ki} \geq \gamma_{ki} \geq 0, \quad \gamma_{1i}\gamma_{2i} = 0 \tag{20c}$$

$$\bar{p}_{ki} \geq 0, \ \epsilon_{ki} \geq 0 \tag{20d}$$

$$k, \ell = 1, 2, \quad \ell \neq k, \quad i = 1, \ldots, N. \tag{20e}$$

Here, $\left(\{p_k\}_{k=1}^2\right)$ in (20a) represents (p_1, p_2) with $\ell \neq k$, and B_{ki} is given in (19). In this problem, the objective in (20a) and the constraints in (20c) are independent of ϵ_{ki}, while the remaining constraints are independent of γ_{ki}. Furthermore, the summation terms in the objective and the constraints in (20c) are separable in i. Hence, the problem in (20) can be decomposed into two parts as

$$f(\bar{p}_1, \bar{p}_2) = \max_{\gamma_1, \gamma_2} C_S \left(\{\bar{p}_k - \gamma_k + \alpha_\ell \gamma_\ell\}_{k=1}^{2} \right) \tag{21a}$$

$$\text{s.t. } \bar{p}_k \geq \gamma_k \geq 0 \tag{21b}$$

$$\gamma_1 \gamma_2 = 0 \tag{21c}$$

$$k, \ell = 1, 2, \quad \ell \neq k, \tag{21d}$$

and

$$\max_{\{\bar{p}_{ki}, \epsilon_{ki}\}} \sum_{i=1}^{N} f(\bar{p}_{1i}, \bar{p}_{2i}) \tag{22a}$$

$$\text{s.t. } 0 \leq B_{ki} \leq E_k^{max} \tag{22b}$$

$$\bar{p}_{ki} \geq 0, \ \epsilon_{ki} \geq 0 \tag{22c}$$

$$k = 1, 2, \quad i = 1, \ldots, N. \tag{22d}$$

We refer to (21) as the *energy transfer problem* and to (22) as the *power allocation problem*, since the former optimizes energy transfers γ_{ki} within each time slot, while the latter considers the problem of allocating consumed power $\bar{p}_{ki}$ among time slots.

A. Solving the Energy Transfer Problem

Since (21) is uncoupled from the battery dynamics in (22b), its solution is identical to the solution of the energy transfer problem in [4]. In particular, it is observed in [4, Lemma 1] that the solution to (21) without the constraint (21c) satisfies (21c). Thus, this is also the solution to (21). Namely, the optimal transferred energy values are

$$\gamma_{ki}^* = \max \left\{ \frac{1}{2} \left(1 + \bar{p}_{ki} - \frac{1 + \bar{p}_{\ell i}}{\alpha_k} \right), 0 \right\} \tag{23}$$

for $k, \ell = 1, 2$, $\ell \neq k$, yielding the solution

$$f(\bar{p}_{1i}, \bar{p}_{2i}) = \begin{cases} C_S(\bar{p}_{1i}, \bar{p}_{2i}), & \gamma_{1i}^* = \gamma_{2i}^* = 0, \\ \log\left(\frac{\sqrt{\alpha_1}}{2} \left(1 + \bar{p}_{1i} + \frac{1+\bar{p}_{2i}}{\alpha_1} \right) \right), & \gamma_{1i}^* > 0, \\ \log\left(\frac{\sqrt{\alpha_2}}{2} \left(1 + \bar{p}_{2i} + \frac{1+\bar{p}_{1i}}{\alpha_2} \right) \right), & \gamma_{2i}^* > 0. \end{cases} \tag{24}$$

B. Solving the Power Allocation Problem

The power allocation problem in (22) is a generalized version of (8). In particular, only the function $f(\bar{p}_1, \bar{p}_2)$ in the objective is different. The solution to (8) without the battery capacity constraints and with uni-directional energy transfer is found using a *two dimensional directional water-filling (2D-DWF) algorithm* in [3]. However, for (22), we also know that an optimal policy satisfies (13). This follows from the optimality of partially procrastinating policies shown in Lemma 1, and can be used to develop a simpler power allocation algorithm for this model.

First, we remark that (22) is a convex program. We write the Karush-Kuhn-Tucker (KKT) conditions

$$-\frac{df(\bar{p}_{1i}, \bar{p}_{2i})}{d\bar{p}_{ki}} + \sum_{j=i}^{N} (\lambda_{kj} - \beta_{kj}) - \sigma_{ki} = 0, \tag{25}$$

$$\sum_{j=i}^{N} (\lambda_{kj} - \beta_{kj}) - \alpha_k \sum_{j=i}^{N} (\lambda_{\ell j} - \beta_{\ell j}) - \mu_{ki} = 0, \tag{26}$$

for $k = 1, 2$, $i = 1, \ldots, N$, where $\lambda_{ki} \geq 0$ and $\beta_{ki} \geq 0$ are the Lagrange multipliers for the constraints in (22b), and $\sigma_{ki} \geq 0$ and $\mu_{ki} \geq 0$ are those for the constraints in (22c), respectively. The complementary slackness conditions are

$$\lambda_{ki} B_{ki} = 0, \quad \beta_{ki}(B_{ki} - E_k^{max}) = 0, \tag{27}$$

$$\sigma_{ki} \bar{p}_{ki} = 0, \quad \mu_{ki} \epsilon_{ki} = 0. \tag{28}$$

We define water-levels v_{ki}, $k = 1, 2$, $i = 1, \ldots, N$, as

$$v_{ki} = \left(\frac{df(\bar{p}_{1i}, \bar{p}_{2i})}{d\bar{p}_{ki}} \right)^{-1}, \tag{29}$$

which are found from (24) as

$$v_{1i} = \begin{cases} 2(1 + \bar{p}_{1i}), & \gamma_{1i}^* = \gamma_{2i}^* = 0, \\ 1 + \bar{p}_{1i} + \frac{1+\bar{p}_{2i}}{\alpha_1}, & \gamma_{1i}^* > 0, \\ \alpha_2 (1 + \bar{p}_{2i} + \frac{1+\bar{p}_{1i}}{\alpha_2}), & \gamma_{2i}^* > 0, \end{cases} \tag{30}$$

for $k = 1$ and

$$v_{2i} = \begin{cases} 2(1 + \bar{p}_{2i}), & \gamma_{1i}^* = \gamma_{2i}^* = 0, \\ \alpha_1 (1 + \bar{p}_{1i} + \frac{1+\bar{p}_{2i}}{\alpha_1}), & \gamma_{1i}^* > 0, \\ 1 + \bar{p}_{2i} + \frac{1+\bar{p}_{1i}}{\alpha_2}, & \gamma_{2i}^* > 0, \end{cases} \tag{31}$$

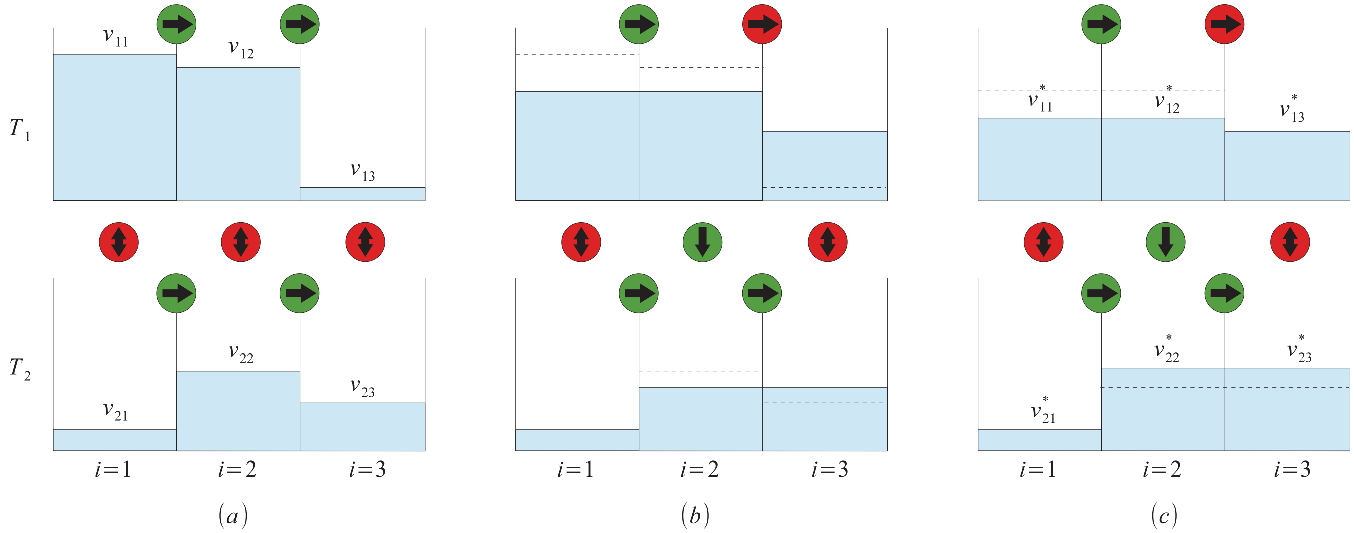

Fig. 2. Two dimensional directional water-filling with restricted transfers, with (a) initial water-levels, (b) water-levels after flow within each node, and (c) water-levels after flow between the two nodes. The flow from T_k to T_ℓ is not allowed unless the battery of T_k is full, as seen at $i = 2$ in (b).

for $k = 2$. We make the following observations: From (27)-(28), the Lagrange multiplier σ_{ki} is zero if $\bar{p}_{ki}$ is non-zero, and λ_{ki} and β_{ki} are non-zero only if the battery of T_k is empty or full at time slot i, respectively. Hence, assuming non-zero transmit powers, from (25), the water-level is found as

$$v_{ki} = \left(\sum_{j=i}^{N} (\lambda_{kj} - \beta_{kj}) \right)^{-1}, \tag{32}$$

which remains constant unless the battery is full or depleted, increasing when depleted and decreasing when full. Finally, from (26), whenever $\epsilon_{ki} > 0$, i.e., energy is transferred from T_k to T_ℓ, then the two water-levels must satisfy $v_{\ell i} = \alpha_k v_{ki}$. Otherwise, we must have $v_{\ell i} > \alpha_k v_{ki}$ and $\epsilon_{ki} = 0$, since $v_{\ell i} < \alpha_k v_{ki}$ cannot satisfy (26).

In order to find an optimal policy, we need to find water-levels $\{v_{ki}\}$ that satisfy the conditions outlined above, from which we can calculate the Lagrange multipliers using (25)-(28). To this end, we use the two dimensional directional water-filling algorithm in [3]. In this algorithm, harvested energy E_{ki} is initially allocated to the respective time slot, and water-levels are calculated from (30)-(31). Energy then flows from time slot i to time slot $i+1$ for T_k if $v_{ki} > v_{k(i+1)}$, or from T_k to T_ℓ in time slot i if $\alpha_k v_{ki} > v_{\ell i}$. This is repeated iteratively until water-levels are stabilized. We refer the reader to [3, Alg. 1] for the implementation of this algorithm without battery capacity constraints and bi-directional energy cooperation. In addition to [3, Alg. 1], to conform to the battery capacity conditions, if the energy flowing into time slot $i+1$ exceeds E_k^{max} for that transmitter, then energy flow into time slot $i+1$ from time slot i stops. With this algorithm, we need to iteratively update water-levels for all neighboring slots, and regulate water-flow in both dimensions.

The contribution of partially procrastinating policies to this algorithm emerges when we utilize Lemma 1. In particular, for ϵ_{ki}, (13) holds for at least one optimal policy, i.e., energy is transferred from T_k to T_ℓ only when the battery of T_k is full. In the 2D-DWF algorithm described above, this provides an important simplification by restricting the energy flow between T_1 and T_2 to a small number of time slots where one of the batteries is full. Therefore, we refer to the resulting algorithm as the 2D-DWF algorithm with restricted transfers. This restriction also eliminates the ambiguity about whether the energy in T_k at time slot i should flow to the next time slot or to T_ℓ if both water-levels are lower. In the 2D-DWF algorithm with restricted transfers, water flow to T_ℓ does not take place unless water flow to time slot $i+1$ is blocked due to a full battery.

An example of the 2D-DWF algorithm with restricted transfers is presented in Fig. 2. The system is initialized by allocating $p_{ki} = E_{ki}$, and calculating v_{ki} from (30)-(31), as in Fig. 2a. At this point, directional water flow for each user is allowed, as indicated by green arrows, while energy transfer between users is not allowed, as indicated by red arrows. After the water flows, we observe that the battery of T_1 is full at the end of $i = 2$, as in Fig. 2b, preventing further water flow into $i = 3$. This enables energy transfer at $i = 2$, leading to the optimal water-levels in Fig. 2c. We remark that this does not imply no energy transfer takes place in $i = 1$ or $i = 3$, but those transfers are represented by γ_{ki} obtained from (23), while the vertical water flow represents ϵ_{ki} only.

V. Numerical Results

We consider transfer efficiency values $\alpha_1 = \alpha_2 = 0.5$, channel coefficients $h_1 = h_2 = -100$dB, receiver noise density $N_1 = N_2 = 10^{-19}$W/Hz, and a bandwidth of 1MHz for $N = 100$ time slots of length $\tau = 1$sec. Energy arrivals E_{2i} are distributed independently and uniformly in $[0, 10]$mJ,

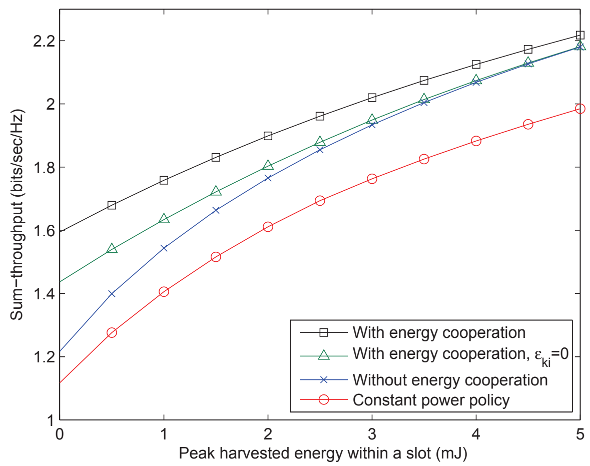

Fig. 3. Sum-throughput versus E_h for a TWC with and without energy transfer, compared to constant power scheme.

while E_{1i} are distributed independently and uniformly in $[0, E_h]$mJ. The energy storage capacity, i.e., battery size, for both transmitters are $E_1^{max} = E_2^{max} = 10$mJ.

The sum-throughput of the two-way channel is plotted in Fig. 3 versus E_h, for policies found with and without energy cooperation. We also present the optimal policy when ϵ_{ki}, $k = 1, 2$, $i = 1, \ldots, N$ is restricted to be zero. Therefore, the top three plots correspond to a) $\gamma_{ki}, \epsilon_{ki} \geq 0$, b) $\gamma_{ki} \geq 0$, $\epsilon_{ki} = 0$, and c) $\gamma_{ki} = \epsilon_{ki} = 0$ cases, respectively. Furthermore, we compare the optimal policy to the *constant power* policy, in which all nodes transmit with power equal to the expected harvested power, i.e., $p_{ki} = \mathbb{E}[E_{ki}]$, whenever possible. We observe that energy cooperation has a notable effect on sum-throughput, particularly when T_1 is energy deprived, i.e., for small E_h. This is due to energy transfers from T_2 to T_1 playing an important role in increasing the sum-throughput of the system. The difference between the energy cooperation plot and the cooperation with $\epsilon_{ki} = 0$ plot demonstrates the impact of transferring overflows, i.e., ϵ_{ki}. Similarly, the difference between the cooperation with $\epsilon_{ki} = 0$ plot and the without cooperation plot demonstrates the impact of immediately consumed energy transfers, i.e., γ_{ki}. Although the impact of γ_{ki} diminishes with increasing peak harvested energy E_h, the impact ϵ_{ki} remains notable. Finally, we remark that the optimal policies perform significantly better than the constant power policy for all E_h.

VI. Conclusion

In this work, we have solved the problem of sum-throughput maximization for two energy harvesting and cooperating transmitters in a two-way communication scenario. We have proven the optimality of partially procrastinating policies by partitioning transferred energy into immediately consumed and stored components, and showing that the stored component in the optimal policy is non-zero only when the battery of the energy-transferring user is full. We have shown the decomposition of the sum-throughput maximization problem into energy transfer and power allocation problems. We have presented the two dimensional directional water-filling algorithm with restricted transfers to solve the power allocation problem. With this storage-limited model, we have shown the benefits of partial procrastinating policies in practical scenarios. Future work includes extensions to other multiterminal models, peak power constrained transmitters, and online policies.

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