Optimal Power Policy for Energy Harvesting Transmitters with Inefficient Energy Storage

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***Abstract*—An energy harvesting transmitter with an inefficient energy storage device, i.e., battery or capacitor, is considered, where a fraction of the stored energy is lost in the process. An optimal transmission policy maximizing the average rate within a finite time horizon is found for a given energy harvesting scenario. In contrast to previous results for optimal power allocations with energy harvesting transmitters, it is observed that storage losses encourage instantaneous consumption of harvested energy without storage for moderate harvest rates. It is also shown that this policy converges to the known constant power transmission policies when storage efficiency goes to unity. Based on the optimal offline policy, an online policy based on constant charging and discharging thresholds is proposed. The performance of the optimal offline and proposed online algorithms are simulated along with naive transmission policies, and the behavior of the algorithms for various storage efficiencies is discussed.**

***Index Terms*—Energy harvesting, optimal scheduling, wireless networks, battery equipped nodes, inefficient energy storage.**

I. Introduction

Harvesting ambient energy in wireless networks is a crucial step in building next generation green and self-sufficient communication systems. This improves the environmental impact of wireless devices in the global scale, while extending the network lifetime indefinitely and making the devices truly mobile in the network. However energy harvesting comes with its own challenges; the harvested energy is variable and may be scarce, requiring tailored transmission policies to achieve the desired performance. For this reason, it is important to model and analyze energy harvesting wireless nodes and seek optimal energy management policies to maximize the utility of the node.

Network models with energy harvesting nodes has been the focus of green communication research in the past few years. The offline transmission completion time minimization problem for energy harvesting transmitters was solved in [1], introducing the energy causality constraint for harvester nodes. A similar model with the additional constraint of battery capacity was analyzed in [2], and it was shown that the transmission completion time minimization problem was closely related to the throughput maximization problem that was solved therein. The energy harvesting transmitter setting was further investigated in [3] for fading channels, in [4] for broadcast channels, in [5] for multiple access channels and in [6] for interference channels. These papers collectively used variations of the water-filling algorithm, by restricting the direction and amount of water flow, and by generalizing the water level formulation to more involved capacity expressions, to find the optimal offline power allocation. Similar water-filling results were obtained by [7] using an information theoretic approach. Building on [1], a two-hop communication model with energy harvesting transmitter and relay was considered in [8]. In common to all these papers is the fact that they utilized the concavity of the power-rate relationship to show that the optimal policies consist of epochs of constant power transmission. Recently in [9], a channel under time-varying amplitude constraints was studied, which could be interpreted as an energy harvesting transmitter without a battery whose instantaneous amplitude depends on the harvested energy, and a capacity based on Shannon's coding scheme with causal state information was given.

To implement an energy management policy, a node needs to be equipped with a battery to allow harvested energy to be stored for future use whenever needed. However, the behavior of a battery is far from an ideal energy storage device. A battery has many non-ideal characteristics such as rate-dependent capacity, temperature dependence, capacity fading and recovery effects. Therefore various analytical, electrical and stochastic models are used to accurately predict the behavior of a battery [10], [11]. The effect of some of these factors such as capacity fading or battery leakage were considered in [12] for an energy harvesting transmitter, giving rise to modified versions of the constant power policies in [1], [2]. Inefficiency of storage was modeled by assuming a constant loss of the storage energy, finding asymptotically optimal policies for sufficiently large batteries [13] and near-optimal duty-cycling techniques [14] with constant transmission rate under energy neutrality conditions.

In this paper, we focus on storage inefficiency, which leads to the loss of a certain fraction of the stored energy. We find the optimal offline power policies that maximize the bits departed by an energy harvesting transmitter with a finite battery. Contrary to the previous offline policies [1]–[5], the optimal policy in this case favors the consumption of harvested energy at the harvesting rate due to the inefficiency of storage. This observation also allows us to suggest an online policy with average thresholds for near-optimal performance in case of a non-predictable harvesting process. Our work differs from [13], where a similar energy loss model is considered, in allowing variable rate transmission, employing strict energy causality constraints, and assuming a finite battery of arbitrary capacity. In Section II, the transmission and harvesting model is presented along with the formulation of the problem. The optimal offline policy for an infinite battery is found in Section III, and is subsequently extended to the finite battery case in Section IV. The behavior of the optimal algorithm for the ideal storage case is compared to previous results in Section V, and a related online policy is proposed in Section VI. Finally the optimal offline and proposed online algorithms are simulated in Section VII and the paper is concluded in VIII.

II. System Model and Problem Definition

We consider a wireless transmitter powered with an energy harvester, supplying energy with a continuous non-negative time-varying rate. The harvested energy is to be used for transmission or is to be stored for future consumption, however a considerable amount is lost in the latter case due to the inefficiency of the on-board energy storage device (ESD). This encourages the system to consume energy as it is harvested, contrary to the previous cases where piecewise constant transmissions were favored.

The wireless system is modeled as follows: The transmitter harvests energy with rate $h(t)$ at time t throughout the duration of operation of length T. Within this time, the transmitter makes a transmission consuming power $p(t)$, and achieving an instantaneous rate of $r(p(t))$ where $r(.)$ is a concave non-decreasing non-negative function of instantaneous transmission power p. This is not necessarily the transmission power alone, but can be thought of as the overall consumed power as long as the above conditions for $r(p)$ are satisfied. The transmitter can choose to use the harvested energy for transmission or store some of it with efficiency $\eta \leq 1$, i.e., for every E units of energy stored, only $\eta \cdot E$ can be recovered from the ESD in future time instances.

Since the harvested energy can be partially stored and the previously stored energy can be consumed as desired, the problem becomes finding the energy storage and consumption rates that optimize a desired property of the system. In this paper, we consider maximizing the average transmission rate within the transmission duration $[0, T]$. We define the portion of harvested power intended to be stored in the battery as $s(t)$ and the rate at which battery energy is used up as $r(t)$. Assuming, without loss of generality, that the energy loss occurs at the storage stage, the energy stored in the battery at time t can be expressed as

$$E_{bat}(t) = \int_0^t \eta s(\tau) - r(\tau) \; d\tau. \tag{1}$$

Additionally, since of the energy drawn from the battery can not exceed the stored energy, this yields to the energy causality constraint

$$\int_0^t \eta s(\tau) - r(\tau) \; d\tau \geq 0 \qquad\qquad 0 \leq t \leq T. \tag{2}$$

In case of a finite battery where the maximum energy stored in the battery is limited to E_{max}, the additional battery capacity constraint arises as

$$\int_0^t \eta s(\tau) - r(\tau) \; d\tau \leq E_{max} \qquad\qquad 0 \leq t \leq T. \tag{3}$$

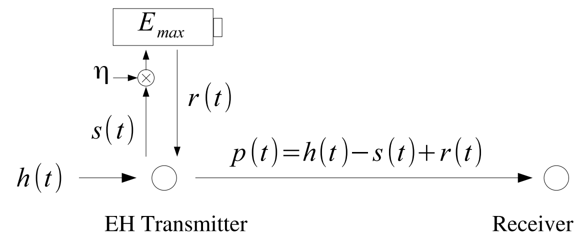

Fig. 1. Energy harvesting transmitter model with inefficient storage

Note that there is always the possibility to discard the excess energy that cannot be stored in the battery. However, since the achieved rate is nondecreasing in consumed power, any policy that discards energy cannot achieve a better average rate than a policy that uses the discarded energy for transmission instead. Therefore any average throughput achieved by discarding excess energy can be achieved, if not exceeded, without discarding it.

The transmit power $p(t)$ can be expressed in terms of the storage and consumption variables $s(t)$ and $r(t)$ as

$$p(t) = h(t) - s(t) + r(t). \tag{4}$$

The average rate maximization problem in this model becomes

$$\max_{s(t), r(t)} \frac{1}{T} \int_0^T r(h(t) - s(\tau) + r(\tau)) \; d\tau \tag{5}$$

$$\text{s.t. } 0 \leq \int_0^t \eta s(\tau) - r(\tau) \; d\tau, \tag{6}$$

$$\int_0^t \eta s(\tau) - r(\tau) \; d\tau \leq E_{max}, \tag{7}$$

$$s(t) \geq 0, \; r(t) \geq 0, \qquad\qquad 0 \leq t \leq T. \tag{8}$$

We first consider the problem without a battery capacity constraint, i.e., without the inequality in (7), in Section III, and extend the results to the battery limited case in Section IV.

III. Optimal Transmission Policy

We start by stating that the set of feasible $s(t)$ and $r(t)$ that satisfy the constraints (6,8) of this problem is convex. This can easily be shown by checking feasibility on a convex combination of two feasible $s(t)$ and $r(t)$ pairs. Together with the concavity of $r(p)$, this imposes that the problem is convex, allowing us to check KKT conditions for optimality due to Slater's condition for constraint qualification.

Ignoring the battery capacity constraint, the Lagrangian function of the maximum average rate problem in (5) can be written as

$$\begin{aligned}\mathfrak{L} = &-\int_0^T r(p(t))dt - \int_0^T \lambda(t) \left(\int_0^t \eta s(\tau) - r(\tau) d\tau \right) dt \\ &- \int_0^T \mu(t) s(t) dt - \int_0^T \delta(t) r(t) dt\end{aligned} \tag{9}$$

where $\lambda(t) \geq 0$, $\mu(t) \geq 0$ and $\delta(t) \geq 0$ are non-negative Lagrangian multipliers satisfying the complementary slackness conditions

$$\lambda(t)\cdot\int_0^t \eta s(\tau)-r(\tau)d\tau=0,\qquad 0\le t\le T \tag{10}$$

$$\mu(t)\cdot s(t)=0,\qquad 0\le t\le T \tag{11}$$

$$\delta(t)\cdot r(t)=0,\qquad 0\le t\le T \tag{12}$$

which indicate that λ is nonzero only when battery has no stored energy, and μ and δ are nonzero only when the system is not charging and not discharging respectively. The Karush-Kuhn-Tucker (KKT) stationarity conditions for this problem are found by differentiating $\mathfrak{L}$ w.r.t. $s(t)$ and $r(t)$ at time t as

$$r'(p(t))-\int_t^T\lambda(\tau)d\tau-\mu(t)=0\qquad 0\le t\le T \tag{13}$$

$$-\eta r'(p(t))+\int_t^T\lambda(\tau)d\tau-\delta(t)=0\quad 0\le t\le T \tag{14}$$

Remark 1: It is trivial that charging and discharging simultaneously is suboptimal, since an amount of energy is lost without any energy storage benefit. This is a condition that, similar to discarding energy being suboptimal, follows from the non-decreasing property of $r(p)$. Therefore it can be stated that μ and δ must not simultaneously be 0 for the policy to be optimal.

When the battery is charging, $\mu(t)=0$ yields

$$r'(p(t))=\int_t^T\lambda(\tau)d\tau \tag{15}$$

and when the battery is discharging, $\delta(t)=0$ yields

$$r'(p(t))=\frac{1}{\eta}\int_t^T\lambda(\tau)d\tau. \tag{16}$$

where $r'(.)$ stands for the derivative of $r(p)$ with respect to p. Since λ is nonnegative and nonzero only when battery is empty, the integral terms in (15) and (16) are constant between instances of an empty battery, and is nondecreasing. Therefore in between such instances, the transmission powers at which charging and discharging is performed, p_s and p_r respectively, are fixed and satisfy

$$\frac{r'(p_s)}{r'(p_r)}=\eta. \tag{17}$$

Consequently, transmission at any other power level requires energy to not be stored or taken from the battery, i.e., $p(t)=h(t)$. However, since μ and δ are non-negative, this is only possible when $p_r<h(t)<p_s$ which follows from (13) and (14). The remaining regions for $h(t)$ handled by storing or retrieving necessary amount of energy from the battery to achieve the corresponding transmission power p_s or p_r, i.e., storing with rate $h(t)-p_s$ when $h(t)>p_s$ or retrieving energy with $p_r-h(t)$ when $h(t)<p_r$. The optimal transmission policy for a fixed p_s and p_r is then

$$p(t)=\begin{cases}p_s, & h(t)\ge p_s\\ h(t), & p_s<h(t)<p_r\\ p_r, & h(t)\le p_r\end{cases} \tag{18}$$

What remains is determining p_s and p_r throughout transmission, which depend on the integral $\int_t^T\lambda(\tau)d\tau$. Therefore, one can equivalently look for the levels of $\int_t^T\lambda(\tau)d\tau$, which is decreasing due to λ being non-negative and constant between empty battery instances due to the complementary slackness condition in (10). This structure implies that one can calculate $\int_t^T\lambda(\tau)d\tau$ by finding its largest value resulting in an empty battery at $t_1\in(0,T]$ without violating any KKT conditions, fixing this value for $[0,t_1]$ and repeating this process recursively for $[t_1,T]$ until no further empty battery cases arise up to the end of transmission at T. Note that since the battery is empty at t_1, the policy can continue to transmit with harvested power $h(t)$ until its upper threshold p_s is reached, at which point the battery ceases to be empty and the lower threshold is also fixed.

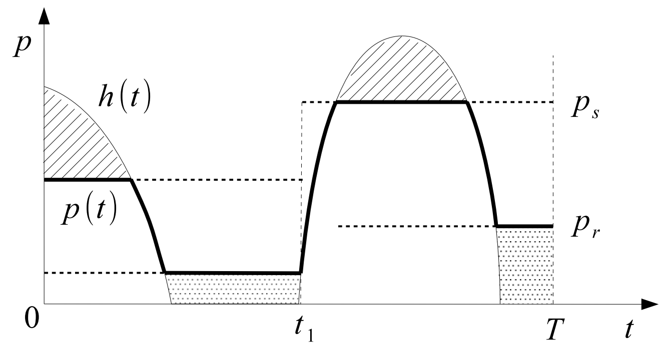

Fig. 2. Example optimal policy with transmission power thresholds p_s and p_r and a single empty battery instance at t_1

Remark 2: An interesting outcome of this approach is the battery being empty at T, since this is the requirement for termination of the above algorithm. This is another necessary condition for optimality, also suggested in [2, Lemma 5], since any remaining energy can be considered to be discarded, which is suboptimal.

The algorithm outlined above gives an increasing set of p_s and p_r values satisfying (15) and (16) respectively, that determine the optimal transmission policy according to (18). An example of the resulting policy is given in Figure 2. The thin solid line represents harvested energy $h(t)$ inspired from a solar panel, with harvest rate peaking during the day and vanishing at night. The first set of thresholds p_s and p_r is found by increasing the thresholds until the battery is empty at some time instance, shown with t_1. After t_1, the second set of thresholds is once again found by increasing the thresholds until the battery is empty at T. However the lower threshold p_r is not employed until the upper threshold p_s is reached for the first time after t_1. The resulting optimal transmission policy is shown with the bold line. The stored energy corresponds to the shaded regions, and is consumed at the dotted regions yielding an empty battery at t_1 and T.

In conclusion, the optimal transmission policy with an infinite battery consists of a transmitter power $p(t)$ mimicking harvested energy $h(t)$ within two thresholds p_s and p_r that are related by (17). The thresholds are non-decreasing, and increase whenever the battery is empty, just enough to yield another empty battery instance before or at the end of transmission T. The transmission is thus terminated at T with no stored energy.

IV. Extension to Finite Battery Models

In the previous section, it was assumed that the battery was sufficiently large, capable of storing as much energy as required. However, this is not always the case, particularly for small mobile nodes. In this section, we add the battery capacity constraint in (3) to the problem and analyze how the algorithm in Section III changes with this additional factor.

With the battery capacity constraint, the Lagrangian function in (9) becomes

$$\begin{aligned}\mathfrak{L} = &-\int_0^T r(p(t))dt - \int_0^T \lambda(t)\left(\int_0^t \eta s(\tau) - r(\tau)d\tau\right)dt \\ &-\int_0^T \beta(t)\left(E_{max} - \int_0^t \eta s(\tau) - r(\tau)d\tau\right)dt \\ &-\int_0^T \mu(t)s(t)dt - \int_0^T \delta(t)r(t)dt \end{aligned} \tag{19}$$

where $\beta(t) > 0$ is the corresponding multiplier with the complementary slackness condition

$$\beta(t)\cdot\left(E_{max} - \int_0^t \eta s(\tau) - r(\tau)d\tau\right) = 0, \qquad 0 \le t \le T. \tag{20}$$

Similar to the multiplier for energy causality, the complementary slackness condition only allows $\beta(t)$ to be nonzero whenever the battery is full, i.e., energy stored is equal to the battery capacity. This term reflects to the charging and discharging conditions in (15) and (16) as

$$r'(p(t)) = \int_t^T \lambda(\tau) - \beta(\tau)d\tau \tag{21}$$

$$r'(p(t)) = \frac{1}{\eta}\int_t^T \lambda(\tau) - \beta(\tau)d\tau. \tag{22}$$

Consequently, the ratio of threshold transmission powers in (17) still holds; however the thresholds are instead determined by the integral $\int_t^T \lambda(\tau) - \beta(\tau)d\tau$. Due to the restrictions on λ and β, we know that this integral is constant whenever stored energy is within $(0, E_{max})$, decreasing when battery is empty, and increasing when battery is full. Therefore the approach is once again finding each constant level by searching the feasible values yielding an empty or full battery at some time instance $t_1 \in [0, T]$.

Remark 3: If at time t_1 the battery is empty, the integral term is bound to decrease, and the threshold powers p_s and p_r are bound to increase after t_1. Thus, if harvest rate just after t_1 satisfies $h(t_1^+) < p_r$, these thresholds and corresponding t_1 cannot be feasible. Analogously, if at time t_1 the battery is full, the integral term is bound to increase, and the threshold powers p_s and p_r are bound to decrease. In this case, $h(t_1^+) > p_s$ indicates infeasibility for these thresholds and t_1. These values are not considered as candidates for optimal policy.

Once policy candidates that satisfy feasibility outlined in Remark 3 are found, the optimal transmission policy is chosen as the candidate with the largest t_1 that, if allowed to run beyond t_1, violates the other constraint first. In other words, if the candidate policy depletes the battery at t_1, the next critical battery state changing the value of the integral term at time $t' > t_1$ should be a full battery, and vice versa. This condition ensures that there exists a policy extending feasibly beyond t' after the change in the integral term at t_1, and is parallel to the decision algorithm in [2]. Similar to the infinite battery case, at t_1 the battery is either empty or full, indicating that either λ or β is free to be non-negative until energy is stored or retrieved respectively. Therefore the other threshold, p_r or p_s remains equal to its value before t_1 until $h(t)$ reaches the next p_s or p_r candidate.

When employed recursively for the remaining portion of the problem $[t_1, T]$, the defined procedure gives the optimal set of p_s and p_r values increasing when battery is empty and decreasing when battery is full, satisfying (21) and (22) respectively. Along with the policy definition in (18), the power allocation maximizing average rate in $[0, T]$ is reached.

V. Ideal Energy Storage

The structure of the optimal policy in (18) suggests transmitting with the harvested rate $h(t)$ within the harvesting rate $[p_r, p_s]$ and not storing or using up battery energy. This differs from the case with an ideal battery in [1] and [2], where the optimal policy aims to transmit with constant power as much as the energy constraints allow. As we can see below, the optimal policy we suggest in fact converges to the policies in these papers for $\eta = 1$.

For an ideal battery with no storage loss, the power thresholds satisfy (17) with $\eta = 1$, i.e.,

$$r'(r_s) = r'(r_p) \tag{23}$$

and the two thresholds are thus equal, forcing the transmission power to be a constant, $p_s = p_r$, until the value of the integral $\int_t^T \lambda(t) - \beta(t)dt$ changes. In Section III, the procedure on finding the value of these integrals give an increasing sequence with empty batteries at the end of each step, which is identical to the optimal policy in [1]. In Section IV, the procedure at each recursion seeks the longest constant power transmission that allows a feasible and optimal continuation of the algorithm, which matches with the policy outlined in [2]. From this perspective, the threshold policy can be considered as relaxing the constant power requirement of the previous work to an interval of powers, the width of which depends on how inefficient the energy storage device is. This allows sufficiently small variations in transmission power whenever storing and retrieving energy is not beneficial due to inefficiency.

VI. Online Transmission Policy

The procedure outlined in Sections III and IV requires non-causal knowledge of the harvesting process $h(t)$ in order to calculate the optimal offline policy. Although assuming this knowledge may be justified for highly predictable harvesting applications, it is desirable to perform reasonably well in the absence of such knowledge as well. In this section, we suggest an online policy based on its optimal offline counterpart that only requires causal knowledge of harvested energy.

The optimal offline policies we have presented consist of upper and lower thresholds that change throughout the transmission based on the local and future behavior of the harvesting process, i.e., the thresholds go up (down) when the average harvesting rate is increasing (decreasing). However in the online formulation, the trend of the harvesting process is not known, except possibly the overall statistics of the process. Thus, a straightforward adaptation of the offline policy would be to fix the thresholds based on the statistical information about the harvesting process, and transmit with $p(t) = h(t)$ whenever the battery state does not allow further charging or discharging above or below p_s or p_r respectively. For a harvesting process with probability density function $f_h(p)$, the proposed online transmission power $p_o(t)$ is given by

$$p_o(t) = \begin{cases} p_{os}, & h(t) \geq p_{os} \text{ and } E_{bat} < E_{max} \\ p_{or}, & h(t) \leq p_{or} \text{ and } E_{bat} > 0 \\ h(t), & otherwise \end{cases} \quad (24)$$

where the fixed thresholds p_{os} and p_{or} are found by solving (17) against an average storage/consumption equality condition,

$$\eta \int_{p_{os}}^{\infty} (p - p_{os}) f_h(p) \, dp \;=\; \int_0^{p_{or}} (p_{or} - p) f_h(p) \, dp. \quad (25)$$

This formulation is similar to that in [13], which considers a general sense energy neutrality instead of strict energy constraints for a constant rate transmitter with fading.

The proposed online policy converges to the optimal offline policy if the battery is sufficiently large, and starts from a sufficiently charged state, that the optimal policy does not require to change its thresholds throughout the transmission. This is the case where all fluctuations in harvesting energy can be accommodated by the battery without reaching an empty or full state. A similar statement also holds when the energy harvesting rate is less varying. Furthermore, as battery efficiency decreases, the difference between p_{os} and p_{or} grows, eventually transmitting without storage for all harvest rates at $\eta = 0$, as expected. On the other hand, as efficiency approaches the ideal storage case, the thresholds converge, favoring a constant power transmission at $\eta = 0$ to best utilize the concavity of the rate function.

VII. Simulations

In this section, we present simulation results for the optimal offline policy as well as the proposed online policy against some naive alternative transmission policies. We consider an energy harvesting transmitter equipped with a battery of capacity $100mJ$ in an AWGN channel of bandwidth $1MHz$. The path loss between the transmitter and its intended destination is taken to be $-100dB$ and the receiver noise spectral density as $N_0 = 10^{-19}W/Hz$.

Energy harvesters utilize ambient events such as vibrations or wind. For simulation purposes, we assume that the energy is harvested from random environmental events in time, each contributing a random amount of energy, to best resemble the nature of ambient events. The energy arrival scenario is generated using a set of discrete packets of energy, the arrivals of which is adapted to the continuous model using scaled versions of the tan^{-1} function. In particular, packets of energy distributed uniformly in $[0, 5]mJ$ are generated with a rate of 0.2 packets per second uniformly over $[0, T]$ where T is the simulation duration of 10000 seconds. A packet of size E_i arriving at time t_i is then represented in the continuous model with the following cumulative energy component

$$E_i \left(\frac{1}{\pi} tan^{-1} \left(\lambda_i (t - t_i) \right) + \frac{1}{2} \right) \quad (26)$$

where λ_i is an exponentially distributed parameter for the i^{th} arrival, representing the variance of the arrival duration for each packet. All components from each energy arrival event is then added to get the cumulative energy harvested by the node.

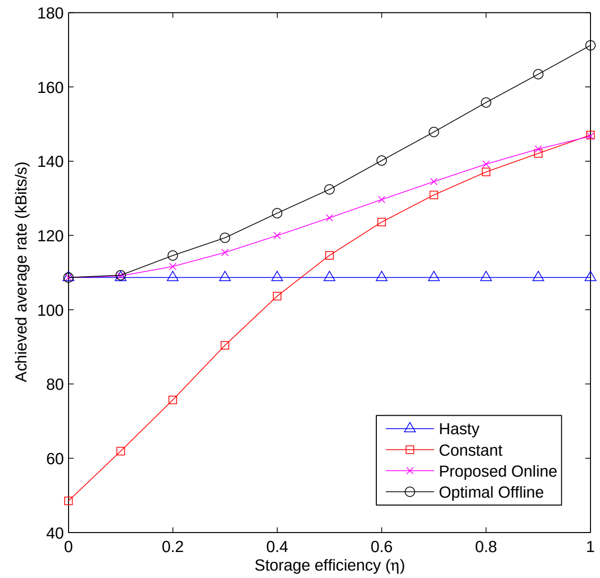

Fig. 3. Average transmission rates with varying battery efficiency for the optimal and online algorithms.

In Figure 3, we compare the performance of the optimal offline algorithm and the online algorithm proposed in Section VI with the following two naive alternatives. The first one, labeled *hasty*, uses up the energy as it is harvested, not storing any energy in the battery. This policy yields the optimal average rate when battery efficiency η is zero, and all stored energy is lost. Furthermore, the performance of this policy is invariant with changing η since it does not utilize the battery at all. The second naive algorithm, labeled *constant*, chooses a constant power and attempts to transmit with this constant power whenever the harvested or stored energy allows, and stores the excess energy if harvesting rate is in fact larger than the chosen power. The constant power in this case is calculated by finding the transmission power for which the expected rates of storing excess energy and using battery energy are equal, also considering η. This is similar to a special case of the online algorithm for $p_{os} = p_{or}$, and thus achieves the same performance for $\eta = 1$, i.e., the ideal energy storage case.

In comparison, the hasty and constant algorithms arguably perform better for smaller and larger values η respectively. Inspired by the optimal offline policy structure, the suggested online policy consequently mimics the hasty algorithm for small values of η by moving p_{os} and p_{or} apart, and mimics the constant algorithm by moving p_{os} and p_{or} closer as $\eta \to 1$. As seen in Figure 3, the proposed online algorithm performs at least as good as both naive algorithms for all values of η. On the other hand, the optimal offline algorithm changes its threshold values according to the harvesting process to best utilize the harvested energy and the battery, performing notably better than the online algorithms with increasing battery efficiency.

VIII. Conclusion

Considering inefficiencies of energy storage can significantly change the structure of an optimal power policy in an energy harvesting network. In this paper, we find the average rate maximizing policy for an energy harvesting transmitter with an inefficient finite battery that loses a constant fraction of the stored energy. Contrary to the piecewise constant power policies suggested for similar settings with ideal energy storage, the optimal policy in this problem involves transmitting with the energy harvesting rate within a range of transmission powers bounded by an upper and lower threshold, which are subject to change throughout the transmission depending on the trend of the harvesting process. We also point out that this policy converges to the previous constant power policies as storage efficiency approaches the ideal case. Finally, we propose an online policy that is inspired by the offline optimal structure, and simulate both algorithms in comparison to naive algorithms that are expected to perform well for ideal and no storage cases. We observe that the online algorithm combines the characteristics of both of these naive algorithms, outperforming both of them for any storage efficiency. Future work on this topic may include extensions to more involved energy storage models, network topologies, or channels with fading or outage.

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