Transmission Policies for Asymmetric Interference Channels with Energy Harvesting Nodes

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***Abstract*—Energy harvesting terminals are emerging as favorable alternatives to conventional terminals for wireless (sensor) networking due to environmental concerns and their potential to extend the network lifetime. Stochastic availability of energy for such networks calls for new network design insights on power control and scheduling, particularly in multi-user settings with interference. This paper considers an asymmetric interference channel with two transmitters and two receivers, and seeks optimal power policies to maximize sum capacity in an energy harvesting setting. It is shown that in the asymmetric interference case, the optimal sum capacity for the channel can be found by iteratively employing single-user optimizations, and the corresponding single-user problems are solved using modified water-filling algorithms such as directional water-filling and generalized water-filling. The performance of the proposed iterative algorithm is demonstrated through numerical results.**

I. Introduction

Environmental concerns and scarcity of energy resources are leading to the development of a rapidly growing industry on green technologies. In tune with this growth, with the increasing demand in data rates and number of users in wireless communications, design and implementation of green wireless alternatives is an important goal. Utilizing energy harvesters in wireless devices both serve this environmental purpose, and extend the lifetime of mobile terminals by eliminating its external energy dependence. In many practical wireless networks, nodes suffer from interference caused by other devices sharing the spectrum, which may severely affect the overall performance of the network. Thus the interference channel is the building block for the design of a multi-user network. For energy harvesting networks, interference becomes critical when the nodes rely on a common environmental source and therefore transmit simultaneously. Consequently, an intelligent power allocation in time could benefit the network as a whole in such settings. In this paper, we derive optimal power allocation policies for interference channels comprised of energy harvesting transmitters.

There has been a surge of interest in optimization for energy harvesting nodes using different approaches in the very recent past. These include queueing theoretic transmission policies to stabilize the data queue of a single-link communication system [1], modified back-pressure based algorithms with energy queues [2], or game theoretic sleep/wake-up strategies [3]. One approach, introduced by [4], aims to find an optimal scheduling through transmitter power control when the transmitter node harvests packets of energy, and thus available energy is strictly constrained. This work on a single transmission link was subsequently extended by [5] to battery-limited transmitters, and by [6] to fading channels. Multi-user settings were also considered for the broadcast channel [7] and the multiple access channel [8].

The Gaussian interference channel is a fundamental building block for multi-user wireless networks. As a result, its capacity analysis has been the focus of a number of previous studies. Although the strong interference case was thoroughly solved earlier [9], the sum-capacity for weaker interference settings was only recently characterized [10], with the exact capacity region still unknown. For the two user Gaussian interference channel the known sum capacities for various channel parameter regions are summarized in [10, Table 1]. These expressions indicate that the interaction of two transmitters play an important role on achieved sum-rate, making power allocation in interference networks an interesting and practical problem.

Although the basic energy harvester settings [4]–[8] are considered in previous work, no research has yet been done on a multi-transmitter multi-receiver setting. In this paper, we consider such a setting and focus on the asymmetric interference case where one receiver experiences higher level of interferences than the other. This is a plausible setting, for instance, in an ad hoc network with no central deployment or power control mechanism. In this setting, we identify the optimal transmission policies for two interfering transmitter-receiver pairs. Since transmitters harvest their own energy, but share the transmission medium, it becomes necessary for nodes to adapt to both their energy availabilities and interference from the other node. We develop iterative algorithms to find power policies for both transmitters maximizing sum capacity of the channel. Through simulations, we show that optimal transmission dictates a notable change on transmission powers of energy harvesting nodes compared to the single-user approach in previous work.

II. System Model

We consider two energy harvesting transmitter nodes $T1$ and $T2$ communicating with their respective receivers $R1$ and $R2$ in a shared channel as shown in Figure 1. The energy

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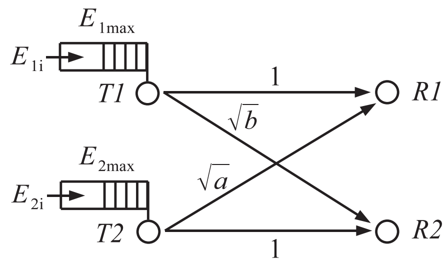

Fig. 1: System model.

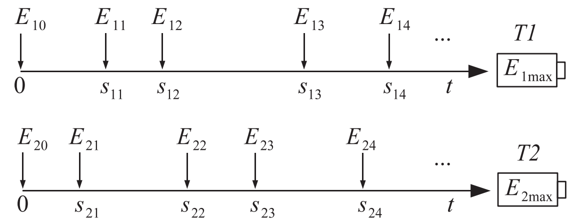

Fig. 2: Energy harvesting model.

harvested by the transmitters is normalized to the respective receiver's Gaussian noise power and channel gain, so that the direct channel coefficients are 1. With this normalization, the cross-channel coefficients are $\sqrt{a}$ and $\sqrt{b}$ and the channel outputs at the two receivers can be expressed as:

$$\begin{aligned} Y_1 &= X_1 + \sqrt{a}X_2 + Z_1 \\ Y_2 &= \sqrt{b}X_1 + X_2 + Z_2 \end{aligned} \tag{1}$$

where Z_1 and Z_2 are the Gaussian noise with unit variance. For this channel, the sum capacity for certain ranges of a and b are known [10, Table 1]. In this paper, we focus on *asymmetric* interference, which is the case with $a < 1$ and $b > 1$ ($a > 1$ and $b < 1$). This is the case when one of the receivers experiences strong interference, while the other weak interference. The sum capacity in this regime is expressed for two regions depending on whether $ab > 1$ or not, and is given by:

$$C_s^I = \frac{1}{2}log\left(1 + \frac{p_1}{1+ap_2}\right) + \frac{1}{2}log\left(1+p_2\right), \qquad ab \geq 1 \tag{2}$$

$$C_s^{II} = \min\left\{ \begin{matrix} \frac{1}{2}log\left(1+\frac{p_1}{1+ap_2}\right) + \frac{1}{2}log\left(1+p_2\right) \\ \frac{1}{2}log\left(1+bp_1+p_2\right) \end{matrix} \right\}, \quad ab \leq 1 \tag{3}$$

for the case with stronger interference at $R2$, i.e., $a < 1$ and $b > 1$.

Each transmitter node uses only the harvested energy for transmission, and it is assumed that transmission energy is prominent in the system, i.e., processing power and base power consumption are ignored. The nodes harvest packets of energies E_{1i} and E_{2i} at times s_{1i} and s_{2i} respectively, and stored in the batteries of the nodes with capacities E_{1max} and E_{2max}, as shown in Figure 2. The energy overflowing the battery is lost, and other factors such as battery leakage or inefficiency are ignored. Recall that E_i and E_{max} values are normalized to have unit channel coefficients and unit noise at the receiver, and note that E_i's are truncated at E_{max} since the remaining portion cannot be utilized. We consider the offline power allocation problem, so it is assumed that all information about energy harvests are known to both transmitters, or a centralized decision mechanism, before start of transmission. The offline formulation is intended for benchmark purposes for online problems, as well as some cases where energy harvest is highly predictable such as solar energy.

The model suggests that the users shall choose transmission powers that they can supply the energy for. Therefore the power allocations for users are strictly constrained to not exceed the amount of harvested energy at any time of transmission. Also, since capacity expressions (2,3) are monotonically increasing in p_1 and p_2 within the parameter range, allowing energy to overflow at any node is strictly suboptimal, as can be observed by comparing to spending the overflowing amount of energy just before the overflow. This implies the optimal power allocation should also satisfy a minimum transmission power requirement in order to prevent any overflows. In parallel to the single user notation in [5], both constraints for user $j \in \{1,2\}$ described above will be expressed as the set $\mathfrak{P}_j$ of feasible power allocations $p_j(t)$,

$$\mathfrak{P}_j = \left\{ p(t) | \sum_{k=0}^{n} E_{jk} - E_{jmax} \leq \int_0^{s_{jn}} p(t)dt \leq \sum_{k=0}^{n-1} E_{jk},\ \forall n \right\} \tag{4}$$

where $p(t)$ is a positive bounded integrable power allocation function defined on $[0,T]$ with T being the deadline for transmission. Here we used the fact that the two energy constraints are flat between arrivals, and thus it suffices to check feasibility only at arrival instances. The truncation of E_{ji} introduced above is crucial for constructing this feasibility set, since $E_{ji} > E_{jmax}$ yields an empty feasible set. Finally note that the set $\mathfrak{P}_j$ is convex, i.e., the convex combination of any two feasible power allocation functions is also feasible.

With this definition, we now state the optimal transmission problem. Given the energy harvesting scenario and a deadline T, the goal is to maximize sum capacity of the interference network by optimal allocation of transmission powers of the users, where each user is constrained with the feasible power allocation set $\mathfrak{P}_j$. The problem is formulated as

$$\begin{aligned} \max \quad & \frac{1}{T}\int_0^T C_s(p_1(t),p_2(t))dt \\ \text{s.t.} \quad & p_1(t) \in \mathfrak{P}_1,\ p_2(t) \in \mathfrak{P}_2 \end{aligned} \tag{5}$$

where $C_s(p_1(t),p_2(t))$ is the sum capacity defined for the corresponding region in (2,3).

III. Sum Capacity Maximization for Region I

Region I corresponds to when the channel parameters satisfy $ab \geq 1$. The sum capacity for this region with $a \leq 1$ is given in (2), which translates to a decoding scheme where $R1$ treats interference as noise and $R2$ decodes and removes the interference. In this section, we present an iterative algorithm that converges to the optimal power allocations for both users that maximize sum capacity. This solution involving the sum capacity expression for $a \leq 1$ also applies to its symmetry with $b \leq 1$ when the two users are swapped.

We first argue that C_s^I is strictly concave in (p_1, p_2) for $a < 1$. This is achieved by showing the Hessian matrix of $-C_s^I$, H_I, is positive definite. Since H_I is real and symmetric, it is Hermitian. Therefore, it is sufficient to check Sylvester's criterion for positive definiteness. Determinant of H_I reveals that the condition for positive definiteness is $p_2 \geq (a^2-1)/(2a-2a^2)$, which holds for $p_2 \geq 0$ and $a < 1$. Therefore C_s^I is strictly concave for $a < 1$.

The strict concavity of C_s^I and the convexity of constraint sets $\mathfrak{P}_1$ and $\mathfrak{P}_2$ imply that problem defined in (5) for Region I is convex and has a unique solution. Therefore the solution can be found iteratively by fixing the power of one user and performing a single variable maximization for the other user, changing the fixed user at each iteration. When p_2 is fixed, the expression $C_s^I(p_1)$ is identical to a single user directional water-filling in [6] with a constant term and effective base level $\frac{1}{h'(t)} = 1+ap_2(t)$. This is a variation of the conventional water-filling policy with two extra constraints: water initially placed based on harvests is allowed to flow in only forward direction to preserve the causality of energy, and no more than E_{1max} amount of water can flow past an instant to comply with the battery constraint. To solve the single-user problem with fixed p_2, $T1$ only performs directional water-filling at each iteration with base levels updated as needed.

When p_1 is fixed, $C_s^I(p_2)$ has two variable terms that do not simplify to a conventional water-filling expression. Therefore a more generalized approach is necessary. This single user problem becomes

$$\max \int_0^T C_s(p_2(t))dt \text{ s.t. } p_2(t) \in \mathfrak{P}_2$$

for which the Karush-Kuhn-Tucker(KKT) conditions for optimality at a time instant t that falls in the k^{th} energy arrival epoch can be written as

$$\frac{d}{dp_2(t)} C_s^I(p_2(t)) - \sum_{j=k}^{N} \lambda_j - \sum_{j=k}^{N} \mu_j + \eta_k = 0 \qquad (6)$$

where λ_j, μ_j and η_j are the Lagrange multipliers for the upper and lower bounds in (4) and $p(t)$ respectively, and N is the number of arrivals within deadline T. The Lagrange multipliers also satisfy the complementary slackness conditions:

$$\lambda_n \cdot \left(\int_0^{s_{2n}} p_2(t)dt - \sum_{i=0}^{n-1} E_{2i} \right) = 0 \quad \forall n \qquad (7)$$

$$\mu_n \cdot \left(\sum_{i=0}^{n} E_{2i} - \int_0^{s_{2n}} p_2(t)dt - E_{2max} \right) = 0 \quad \forall n \qquad (8)$$

$$\eta_n \cdot p_2(t) = 0 \quad \forall n, t \in [s_{n-1}, s_n] \qquad (9)$$

which dictate that λ_j and μ_j are only nonzero when the optimal policy reaches the boundary for one of the constraints in $\mathfrak{P}_2$, and η_j is nonzero only when transmission power is zero. Together with (6), this implies that the derivative

$$\frac{d}{dp_2} C_s^I(p_2) = \frac{-ap_1}{2(1+p_1+ap_2)(1+ap_2)} + \frac{1}{2(1+p_2)} \qquad (10)$$

only changes at arrivals at which a constraint in $\mathfrak{P}_2$ is satisfied with equality, while η_j ensures that (6) is met when $p_2 = 0$. This observation leads to a constrained version of the *generalized iterative water-filling* approach in [11]. For optimal transmission, $T2$ aims to keep the expression in (10) constant, allowing it to change only when a constraint is tight. This can be performed similar to directional water-filling, but by comparing the value of the expression in (10) instead of the water levels. A similar approach is adopted in [8] for multiple access channels.

With the two single user problems individually solved, the unique optimal transmission policy can be obtained by iteratively solving the directional water-filling problem for $T1$ and the directional generalized water-filling problem for $T2$.

IV. Sum Capacity Maximization for Region II

In this section, we extend the results of Section III to Region II with channel parameters $ab \leq 1$. The sum capacity given in (3) for $b \geq 1$ is used throughout the section, but the results can be reflected to the symmetry with $a \geq 1$ in a similar manner to Region I by swapping the two users.

Once again we start by stating the concavity of C_s^{II}. Notice that the first term of the minima in (3) is identical to C_s^I, which was shown to be strictly concave in Section III. The joint concavity of the second term is trivial, yet it is not strictly concave. Consequently the minimum of these two concave terms is also concave, implying that a similar iterative method will converge to the solution of (5) for this region. However the uniqueness of the solution in this case is not certain since concavity is not strict.

It can be observed that the value of p_2 is sufficient to decide which of the two terms in (3) dominates the minimum. The condition for the first term being smaller can be simplified to a threshold p_c,

$$p_2 \leq \frac{b-1}{1-ab} \triangleq p_c. \qquad (11)$$

This fact significantly simplifies the single-user problem for $T1$, since p_2 values are fixed. The solution has a directional water-filling interpretation, but with the effective base levels expressed conditioned to the dominating term of (3) as

$$\frac{1}{h'(t)} = \begin{cases} 1+ap_2(t) & p_2(t) < p_c \\ \frac{1+p_2(t)}{b} & p_2(t) \geq p_c \end{cases} \qquad (12)$$

For the single user problem of $T2$, the directional generalized water-filling approach still holds, but with the following conditional water level expression:

$$\frac{d}{dp_2} C_s^{II}(p_2) = \begin{cases} -\frac{ap_1}{2(1+p_1+ap_2)(1+ap_2)} + \frac{1}{2(1+p_2)} & p_2(t) < p_c \\ \frac{1}{2(1+bp_1+p_2)} & p_2(t) \geq p_c \end{cases} \qquad (13)$$

The iterative execution of the single-user water-filling solution for $T1$ with conditional base levels, and single-user

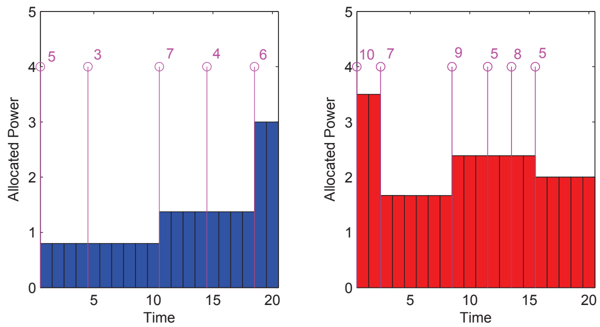

Fig. 3: Energy arrivals and power policies with single link directional water-filling [6] for $T1$ (left) and $T2$ (right).

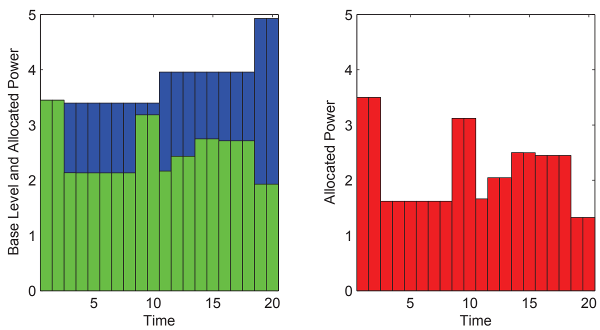

Fig. 4: Optimal power policies with iterative directional water-filling for $T1$ (left) and $T2$ (right).

generalized directional water-filling solution for $T2$ with the modified water level expression in (13) converges to an optimal transmission policy for the two users in this region.

V. Simulations

In this section, we present simulation results to demonstrate how the optimal iterative solution yields different transmission policies than single-user solutions. We simulate a system with deadline $T = 20$, $E_{1max} = E_{2max} = 10$ and channel coefficients $a = 0.9$ and $b = 2$ which falls within Region I. The harvested energies E_{ji} and their times s_{ji} are marked on Figure 3 for $T1$ (left) and $T2$ (right).

Figure 3 shows the optimal single-user power allocation functions for $T1$ and $T2$, calculated using directional water-filling of [6]. In this case, each user calculates its optimal single-user power allocation by ignoring the interference of the other user. Therefore, nodes only adapt to their own energy arrivals. The power allocation functions of the iterative algorithms suggested in this paper are presented in Figure 4 for the same energy harvesting scenario. Note that for $T1$, the base level is shown in green, and the transmission policy is represented by the difference between water level and the base level, shown in blue. It can be observed that in this case users adapt both to their own arrivals, as seen at the end of $p_1(t)$ and the beginning of $p_2(t)$; and to the interference caused by the other user, notable around $t = 9$. The overall change in transmission powers compared to Figure 3 is due to the interaction of the two transmitters, and is significant even for a two user case.

VI. Conclusion

In this paper, we derived and demonstrated iterative algorithms to find the optimal transmission power policies for a Gaussian asymmetric interference channel with cross-channel coefficients $\sqrt{a}$ and $\sqrt{b}$ satisfying $a < 1$ and $b > 1$ or its symmetry. We showed that an iteration of single-user optimizations converge to the sum capacity maximizing power-allocation, and solved the single-user problems using variants of water-filling. We observed that the interaction of users in such networks notably affect the optimal power allocation.

The regions considered in this paper incorporate various approaches to interference such as treating interference as noise, which is commonly used in practice for its simplicity. Therefore the implications of this paper point toward practical interest in identifying and solving optimality problems in such systems. Current research in this direction includes analysis for general regions of operations, and distributed or online versions of the proposed allocation schemes [12].

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