Energy Harvesting Broadcast Channel with Inefficient Energy Storage

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***Abstract*—This paper considers the broadcast channel with an energy harvesting transmitter equipped with an inefficient energy storage device. For this setting, the optimal offline power policy that maximizes the average weighted sum rate of the system is derived. It is observed that this policy has a double threshold structure, with piecewise constant thresholds determined by the energy harvesting process. The convexity of the capacity region for the energy harvesting broadcast channel for a finite deadline is established, showing that the weighted maximum sum-rate traces the boundary of the region. Next, the optimal online policy is found using dynamic programming, and it is observed that the solution has a double threshold structure as well with state dependent thresholds. Lastly, a double threshold policy is proposed with fixed thresholds that performs near optimal with reduced complexity.**

I. Introduction

Energy harvesting for wireless communications has been a recent topic of interest due to its benefits about lifetime, maintenance costs and environmental impact. A wireless node with an energy harvester and a small battery for temporary storage can operate indefinitely in theory by perpetually replenishing its battery using the harvested energy. The energy available to these nodes are intermittent and varying; making power management, i.e., transmit power and energy storage control, a crucial matter for efficient operation.

Studies on energy harvesting wireless networks in the past few years considered different problems for various settings and models. An energy harvesting model with offline harvest information was first introduced in [1], where a transmission completion time minimization problem was formulated and solved for the optimal scheduling in a single transmitter-receiver pair with energy harvests and packet arrivals. This model was then extended to nodes with finite capacity battery, solving the problem of throughput maximization in [2]. Reference [3] introduced a directional water-filling algorithm for power allocation for throughput maximization in a fading channel. Multi user channels with energy harvesting transmitters were also considered, see for example [4]–[6]. In all these, it was observed that the solution to the offline problem consisted of piecewise constant transmit powers with the goal of transmitting with constant power for as long as possible while obeying the energy feasibility conditions.

For energy harvesting nodes, an energy storage device is desirable to provide such a power policy. In practice, however, this storage device would have storage losses, leakage and capacity fading. These in turn can affect the optimum transmission policy. Longer term losses of leakage and battery degradation were considered in [7]. A constant rate storage loss model in an offline energy harvesting setting was considered in [8] for a single user link, observing that in contrast to policies for ideal batteries, the optimal policy had a threshold structure and involved transmitting with harvested power without storage.

In this paper, we consider a two-user broadcast channel with an energy harvesting transmitter. The broadcast node is equipped with an inefficient battery which loses a constant fraction of the stored or retrieved energy, as in [8]. The offline weighted sum-rate maximization problem is solved and the solution is found to be a two-threshold policy as in its single user counterpart. Next, the convexity of the capacity region in this setup is shown, implying that each boundary point of the capacity region corresponds to the solution of the weighted sum rate maximization problem for some weight, and that the region can be traced using the problem solved. An online policy is formulated using dynamic programming, and is observed to have a two-threshold structure as well with state dependent thresholds. Finally, a computationally simpler online algorithm is proposed with fixed thresholds, whose performance is observed to be near-optimal. Details of the derivations in this paper can be found in [9].

II. System Model

We consider an energy harvesting broadcasting node powered by an inefficient battery communicating with two receivers. The node controls its instantaneous transmission power, which is supplied by the battery, energy harvesting device, or both. The harvested power can both be used to power the transmitter immediately, or to replenish the battery; however in the latter case, a fraction of the power is lost due to the inefficiency of the battery. The aim of the transmitter is to maximize its throughput in this setting.

The system model is illustrated in Figure 1. The on-board energy harvester provides energy at a rate of $h(t)$, where $h(t)$ is non-negative and integrable. Out of this power, $s(t)$ is allocated by the transmitter to be stored in the battery of capacity E_{max}, $s(t) \leq h(t)$, and the remaining portion is used for transmission. The storage efficiency of the battery is denoted by η, $0 \leq \eta < 1$, and thus the battery is replenished at rate $\eta s(t)$. In practice, the loss may occur when storing or retrieving energy to and from the battery; these two losses can

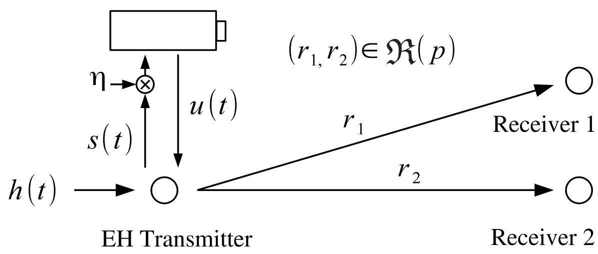

Fig. 1. Energy harvesting broadcast channel with inefficient storage.

be combined to one as in our model by scaling the battery capacity accordingly. The energy stored in the inefficient battery is retrieved at rate $u(t)$, determined by the transmitter, and is used for transmission as well. Thus, the instantaneous power available to the transmitter at time t is given by

$$p(t) = h(t) - s(t) + u(t) \tag{1}$$

which consists of the power retrieved from the battery and the portion of harvested power scheduled for transmission. It is assumed that this power is decided by the broadcast node through $s(t)$ and $u(t)$ in the presence of non-causal energy harvesting information $h(t)$, thus making the problem in consideration an offline optimization problem.

With allocated transmit power $p(t)$, the broadcaster can choose an instantaneous rate pair $(r_1(t), r_2(t)) \in \Re(p(t))$, where $\Re(p)$ is the capacity region of the broadcast channel for power p,

$$\Re_{AWGN}(p) = \Big\{(r_1, r_2) \big| r_1 \leq \frac{1}{2}\log_2\left(1 + \frac{\alpha p}{\sigma_1^2}\right), \\ r_2 \leq \frac{1}{2}\log_2\left(1 + \frac{(1-\alpha)p}{\alpha p + \sigma_2^2}\right),\ 0 \leq \alpha \leq 1\Big\} \tag{2}$$

for an additive white Gaussian noise (AWGN) broadcast channel with $\sigma_1 \leq \sigma_2$, and is achieved through superposition coding [10]. Here, σ_1^2 and σ_2^2 are noise variances experienced by the receivers after normalization of channel gains.

The energy supplied by the battery through $u(t)$ is limited by the energy available at the battery, which in turn is replenished by $s(t)$. The energy that can be stored in the battery is limited by the battery capacity E_{max}. Denoting the energy stored in the battery at time t as $E_{bat}(t)$, these two restrictions lead to the following constraints:

$$E_{bat}(t) = \int_0^t \eta s(\tau) - u(\tau)\ d\tau \geq 0, \tag{3}$$

$$\int_0^t \eta s(\tau) - u(\tau)\ d\tau \leq E_{max}, \quad 0 \leq t \leq T. \tag{4}$$

(3) implies that energy retrieved from the battery up to time t must not exceed the energy stored in the battery before time t, i.e., is the *energy causality* constraint. (4) ensures that $s(t)$ and $u(t)$ are chosen in such a way that the battery capacity is never exceeded, i.e., is the *battery capacity* constraint.

III. Weighted Sum-rate Maximization

Consider the problem of maximizing the average weighted sum-rate $r_{1,avg} + \alpha r_{2,avg}$, for $\alpha \geq 0$ within a given deadline T. We need to find optimal power allocations $s(t)$ and $u(t)$ that are energy feasible as in (3) and (4) and maximize the average weighted sum-rate for the broadcaster. This problem can be expressed as

$$\max_{s(t),u(t)} \frac{1}{T}\int_0^T r_1(t) + \alpha r_2(t)\ dt \tag{5a}$$

$$\text{s.t. } (r_1(t), r_2(t)) \in \Re(h(t) - s(t) + u(t)) \tag{5b}$$

$$0 \leq \int_0^t \eta s(\tau) - u(\tau) d\tau \leq E_{max} \tag{5c}$$

$$h(t) \geq s(t) \geq 0, \quad u(t) \geq 0, \quad \forall t \in [0, T] \tag{5d}$$

where $r_1(t)$ and $r_2(t)$ are instantaneous transmission rates to receivers 1 and 2 respectively, chosen from the capacity region $\Re(p(t))$. In order to simplify this problem, we first observe that given some $p(t)$, for any time instant t, the optimal choice of $(r_1(t), r_2(t))$ is the one in $\Re(p(t))$ that maximizes $r_1(t) + \alpha r_2(t)$. This is due to the linearity of the objective and independence of the constraints on $r_1(t)$ and $r_2(t)$ in t. Thus, for each p, one can find the rate pair in $\Re(p)$ that maximizes the instantaneous weighted sum-rate. Let the resulting weighted sum-rate be denoted by $r_\alpha^{BC}(p)$, i.e.,

$$r_\alpha^{BC}(p) = \max_{(r_1,r_2)\in\Re(p)} r_1 + \alpha r_2. \tag{6}$$

Then, the problem in (5) can be reduced to

$$\max_{s(t),u(t)} \frac{1}{T}\int_0^T r_\alpha^{BC}(h(t) - s(t) + u(t))\ dt \tag{7a}$$

$$\text{s.t. } 0 \leq \int_0^t \eta s(\tau) - u(\tau) d\tau \leq E_{max} \tag{7b}$$

$$h(t) \geq s(t) \geq 0, \quad u(t) \geq 0, \quad \forall t \in [0, T]. \tag{7c}$$

Lemma 1: The maximum weighted sum-rate $r_\alpha^{BC}(p)$ is non-decreasing, continuous and concave in p for any $\alpha \geq 0$.

Proof: The non-decreasing property is trivial, since the transmitter can discard excess energy to achieve any sum-rate with less power. Continuity follows from the fact that with power $p - \epsilon$, one can get arbitrarily close to any weighted sum-rate $r_\alpha^{BC}(p)$ by pausing for an arbitrarily short duration and transmitting with power p for the remaining time. This is in fact equivalent to time-sharing with an inactive state. Extending this to any two arbitrary powers p_1 and p_2, the transmitter can achieve any linear combination of $r_\alpha^{BC}(p_1)$ and $r_\alpha^{BC}(p_2)$ by time-sharing between these two powers, thus showing that the maximum weighted sum-rate $r_\alpha^{BC}(p)$ is concave in p by definition. ■

Lemma 1 implies that being a linear function of $r_\alpha^{BC}(p)$, the objective in (7a) is concave, and since the constraints are linear, the weighted sum-rate maximization problem (7) is convex. By taking the partial derivatives of the Lagrangian of (7), we obtain the KKT stationarity conditions [11]:

$$r'_\alpha(p(t)) = \eta \int_t^T (\lambda(\tau) - \beta(\tau))\ d\tau + \mu(t) - \sigma(t) \quad (8)$$

$$r'_\alpha(p(t)) = \int_t^T (\lambda(\tau) - \beta(\tau))\ d\tau - \nu(t),\ 0 \le t \le T \quad (9)$$

where $p(t) = h(t) - s(t) + u(t)$ and $r'_\alpha(p(t))$ stands for the derivative of r_α^{BC} with respect to p. Here, $\lambda(t)$, $\beta(t)$, $\mu(t)$, $\sigma(t)$ and $\nu(t)$ are the nonnegative Lagrangian multipliers corresponding to the energy causality, battery capacity, $s(t) \le h(t)$, and nonnegativity constraints for $s(t)$ and $u(t)$ respectively. The complementary slackness conditions are

$$\lambda(t) \left(\int_0^t \eta s(\tau) - u(\tau)\ d\tau \right) = 0, \quad (10a)$$

$$\beta(t) \left(E_{max} - \int_0^t \eta s(\tau) - u(\tau)\ d\tau \right) = 0, \quad (10b)$$

$$\mu(t)\,(h(t) - s(t)) = 0, \quad (10c)$$

$$\sigma(t)s(t) = 0, \quad \nu(t)u(t) = 0, \quad 0 \le t \le T. \quad (10d)$$

We provide the implications of these necessary conditions in five power allocation cases next.

Case 1: Simultaneous charge and discharge

This case corresponds to $s(t) > 0$ and $u(t) > 0$ simultaneously. Since utilizing the battery causes a net energy loss, the optimal policy must always avoid this.

Case 2: Discharging only

This case corresponds to $u(t) > 0$ and $s(t) = 0$. For this case, (10c) yields $\nu(t) = 0$, which is substituted in (9) to get

$$r'_\alpha(p(t)) = \int_t^T \lambda(\tau) - \beta\tau\ d\tau. \quad (11)$$

Since (10a) and (10b) imply that $\lambda(t)$ and $\beta(t)$ are only nonzero when $E_{bat} = 0$ and $E_{bat} = E_{max}$ respectively, the right hand side (RHS) of (11) remains constant for all values of $E_{bat} \in (0, E_{max})$. As a result, the transmit power $p(t)$ maintains a constant r'_α in between two extreme battery events. In general, the transmitter can choose any such p while satisfying the KKT conditions, and the optimal policy is not unique. For the sake of outlining only one optimal policy, we choose the smallest $p = p_u$ satisfying (11) and keep it constant until the next extreme battery event.

Another observation is that the RHS of (11) increases only when $\beta(t) > 0$, i.e., battery is full, and decreases only when $\lambda(t) > 0$, i.e., battery is empty. Therefore, the optimal transmit power is found to increase only when the battery is depleted and decrease only when it is full.

Case 3: Charging only with $s(t) < h(t)$

In this case, we focus on charging the battery only with charge rate $s(t) < h(t)$. Here, (10c) and (10d) along with $0 < s(t) < h(t)$ and $u(t) = 0$ imply that $\sigma(t)$ and $\mu(t)$ are zero. Substituting in (8), we get

$$r'_\alpha(p(t)) = \eta \int_t^T \lambda(\tau) - \beta(\tau)\ d\tau. \quad (12)$$

This expression is similar to (11), and thus we can say that an optimal policy has constant transmit power $p \triangleq p_s$ while charging the battery. The value of p_s changes only when the battery is at an extreme, increasing when it is depleted and decreasing when it is full.

Comparing (11) and (12), p_s and p_u are related by

$$\frac{r'_\alpha(p_s)}{r'_\alpha(p_u)} = \eta. \quad (13)$$

which implies that finding one of these powers is sufficient.

Case 4: Charging only with $s(t) = h(t)$

We now consider the case with $s(t) = h(t)$ and $u(t) = 0$, for which we have $\sigma(t) = 0$ due to (10d). Consequently, $\mu(t)$ remains in (8), and transmit power $p(t)$ is zero, yielding

$$r'_\alpha(0) = \eta \int_t^T \lambda(\tau)\ d\tau - \mu(t). \quad (14)$$

Since $\mu(t)$ is nonnegative by definition, the expression in (14) is not greater than $r'_\alpha(p_s)$. Since $r_\alpha(p)$ is concave, we have $r'_\alpha(0) \ge r'_\alpha(p_s)$ for any $p_s \ge 0$. Thus, the only feasible solution is $p_s = 0$, yielding a transmit power of $p(t) = 0$ as required by this case, as expected.

Case 5: No charging or discharging

Finally, we consider $s(t) = u(t) = 0$, yielding a transmit power supplied entirely by the harvesting process, $p(t) = h(t)$. Without loss of generality, assume $h(t) > 0$, and we have $\mu(t) = 0$ from (10c). Substituting in (8) and (9), we get

$$r'_\alpha(p(t)) = \eta \int_t^T \lambda(\tau)\ d\tau + \sigma(t) = r'(p_s) + \sigma(t), \quad (15a)$$

$$r'_\alpha(p(t)) = \int_t^T \lambda(\tau)\ d\tau + \nu(t) = r'(p_u) - \nu(t) \quad (15b)$$

and thus conclude that for the transmitter to employ this policy, the transmitter power must satisfy $p_u \le p(t) \le p_s$ as $\sigma(t), \nu(t) \ge 0$. Since $p(t) = h(t)$, this is only possible when the harvested power itself lies within $[p_u, p_s]$.

The five cases outlined above restrict the power policy and the corresponding transmit power $p(t)$ of the broadcast node to the following three *modes*:

1) Charging only: $p(t) = p_s$, satisfying (12),
2) Discharging only: $p(t) = p_u$, satisfying (11),
3) Passive: $p(t) = h(t)$ with $p_u \le p(t) \le p_s$.

Hence, we have a double-threshold policy for $p(t)$, i.e., $p(t)$ is derived by employing two thresholds p_u and p_s on $h(t)$. The thresholds are constant between an empty (full) battery, and increase (decrease) together. When $h(t) > p_s$, the broadcaster uses a total power of p_s and redirects the remaining power to be stored in the battery, and when $h(t) < p_u$, the broadcaster uses a total power of p_u by retrieving the missing power from the battery. In between, no storage or retrieval is performed, and the broadcaster transmits with power $p(t) = h(t)$.

The set of thresholds can be calculated sequentially by observing the feasibility conditions. First, starting from $t = 0$, candidates for the (p_s, p_u) pair are found, which either deplete or fill the battery at some future instance $t_1 > 0$. Next, feasibility of these candidates are confirmed by checking what the next battery event would be if the same policy was employed past t_1. If the same battery event is repeated at some $t_2 > t_1$, the pair is not feasible, since changing the thresholds in the allowed direction would inevitably violate feasibility for $t > t_2$. Thus, the unique candidate pair that causes two different battery events at t_1 and t_2 and no events elsewhere in $[0, t_2]$ is chosen as the optimal set of thresholds for $[0, t_1]$. The process is then repeated starting from t_1 until deadline T is reached. This process is similar to the one in [2] for finite battery power allocation, and its details can be found in [9].

Remark 1: The optimal policy derived here can be shown to converge to the results of [4] as storage efficiency $\eta \to 1$, and $E_{max} \to \infty$. In this special case, a single threshold $p_s = p_u = p(t)$ emerges as the optimal policy. The power policy is therefore constant between empty battery instances and nondecreasing since no full battery event occurs, parallel to [4, Lemma 3].

IV. Capacity Region for the Energy Harvesting Broadcast Channel

With the solution to the sum-rate maximization problem, we next characterize the capacity region for the energy harvesting broadcast channel given an energy harvesting profile $h(t)$. For this purpose, it is necessary to find the maximum average rate region $\mathfrak{R}_{EH} = (r_{1,avg}, r_{2,avg})$ consisting of the union of average rate pairs that can be achieved under the energy harvesting constraints in (3) and (4).

It is trivial that the capacity region $\mathfrak{R}(p)$ for some fixed power p, as in (2), is convex. This follows from a simple time-sharing argument at the same broadcast power. Moreover, a similar time-sharing argument can be used to show that $\mathfrak{R}(p)$ is concave in p, i.e., for any rate pair in $\mathfrak{R}(p_1)$ and rate pair in $\mathfrak{R}(p_2)$, one can always achieve a convex combination of these rate pairs using a broadcast power equal to the convex combination of p_1 and p_2. Based on these properties, we state in Lemma 2 the convexity of the capacity region for the energy harvesting problem.

Lemma 2: The capacity region consisting of achievable average rates $\mathfrak{R}_{EH} = (r_{1,avg}, r_{2,avg})$ for an energy harvesting transmitter under power constraints (3) and (4) is convex.

Proof: The proof is by time-sharing. Consider two feasible power allocation policies $p(t)$ and $p'(t)$ with respective variables $s(t)$, $u(t)$, $s'(t)$ and $u'(t)$, achieving average rate pairs $(r_{1,avg}, r_{2,avg})$ and $(r'_{1,avg}, r'_{2,avg})$. It is sufficient to show that the convex combination of these rate pairs with $0 \le \gamma \le 1$ is also achievable with a feasible policy. Employing $u_\gamma(t) = \gamma u(t) + (1-\gamma)u'(t)$ and $s_\gamma(t) = \gamma s(t) + (1-\gamma)s'(t)$, one can achieve an average rate pair at least as good as the convex combination of the rate pairs due to the concavity of the capacity region $\mathfrak{R}(p)$ in p. The proposed policies are feasible since the constraints (3) and (4) are linear. ■

Since the energy harvesting capacity region is convex, for any point on its boundary, one can find a separating hyperplane. The slope of this hyperplane defines an α for which the point is the maximizer of the weighted sum-rate maximization problem with parameter α. Consequently, the boundary of the capacity region can be outlined by tracing $0 \le \alpha \le 1$ and solving the weighted sum-rate problem in Section III. This is employed in Section VI to evaluate and compare the rate regions of policies in simulations.

V. Online Transmission Policies

So far, we have considered the offline problem where the broadcast node has non-causal information about the energy harvesting process $h(t)$. In practice, this may not be possible and online power policies that only have causal harvesting information may be desired. For this purpose, we first formulate a dynamic program for the online problem, and together with the insights from the offline problem, propose a simpler policy.

A. Optimal Online Policy

For the power decision the broadcaster needs to make in an online fashion, what the transmitter knows is limited to the state of its battery and the current energy harvesting rate. The optimal action for such a node can be formulated as a dynamic program, which can be solved recursively to yield an optimal policy. To be able to evaluate the optimal action with a recursion, we quantize time with a sufficiently small δ and assume that the realization of $h(t)$ is constant within an interval of length δ while Markovian or i.i.d. among intervals. Expressing the value, i.e., expected average weighted sum-rate as $V(E_{bat}, h)$ at the state with battery energy E_{bat} and harvest rate h, we can express the discounted dynamic program with discount factor $\Delta \le 1$ as

$$V(E_{bat}, h) = \max_{\phi} r_\alpha^{BC}(\phi(E_{bat}, h))\delta + \Delta \mathbb{E}\left[V(E_{bat}^\delta, h^\delta)\right]$$

where E_{bat}^δ and h^δ are the battery and channel states after a time δ has elapsed. Here, the value of a state is the weighted sum-rate r_α^{BC} obtained from the immediate action $\phi(E_{bat}, h)$ and the discounted expected value of the next state. Solving this recursion for $E_{max} = 100mJ$ and h distributed uniformly in $[0, 20]mW$, we observe that the optimal action $\phi(E_{bat}, h)$ has the structure shown in Figure 2. Here, it can be observed that the online optimal power allocation also conforms to a double threshold policy for any fixed E_{bat}, with the thresholds changing as a function of E_{bat}. In region I, the transmit power $p(t)$ equals the harvest rate $h(t)$, while in regions II and III the broadcast node is charging and discharging its battery respectively. The boundaries of these regions can also be seen to conform to the relation in (13).

B. Proposed Online Policy

Observing that both the optimal offline and optimal online policies have a two-threshold structure, we propose that a simpler policy composed of a constant pair of thresholds can be used when the dynamic program is computationally costly. In an attempt to balance the energy input and output of the battery, we choose the thresholds for the proposed online policy to satisfy

$$\int_{p_s}^{\infty}(p-p_s)f_h(p)dp - \int_0^{p_u}(p_u-p)f_h(p)dp = 0, \quad \frac{r'_\alpha(p_s)}{r'_\alpha(p_u)} = \eta. \tag{16}$$

where $f_h(p)$ stands for the distribution of the energy harvesting rate h. This choice aims to prevent the node from employing a policy that yields battery overflows or outages systematically. As $\eta \to 1$, the thresholds approach average harvesting rate, performing similar to the best-effort transmission scheme of

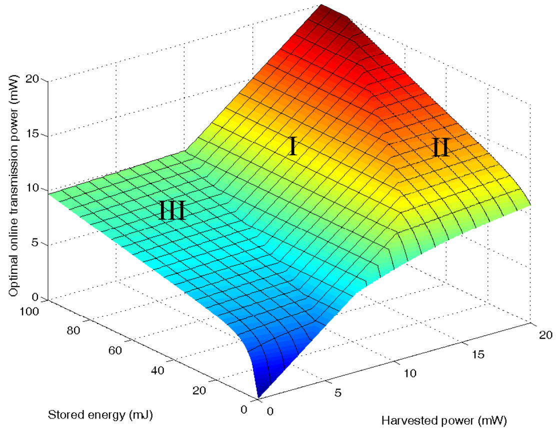

Fig. 2. Optimal online transmission power as a function of node states.

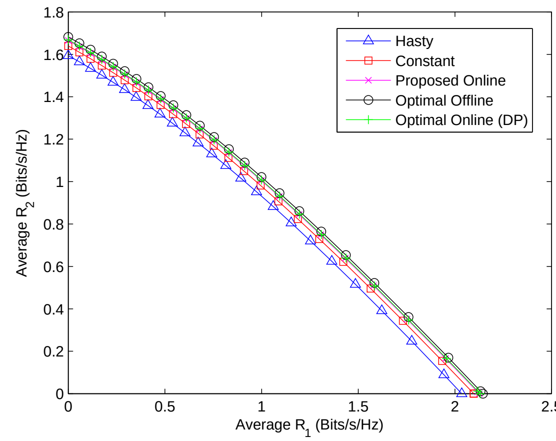

Fig. 3. Average transmission rate regions in an energy harvesting broadcast setting with $\eta = 0.6$.

[12] which is optimal for an infinite length transmission. Conversely as $\eta \to 0$, the thresholds diverge and the transmitter always chooses $p(t) = h(t)$, which is trivially optimal for zero storage efficiency. Thus, the proposed policy can be shown to be asymptotically optimal for the extreme values of η.

VI. Numerical Results

In this section, we present the results of simulations of the offline and online policies compared with naive policies. Namely, we find the achievable rate regions for all policies by solving the weighted sum-rate maximization problem for different values of α, as suggested in Section IV. The resulting average rate regions are presented in Figure 3. In these simulations, a broadcast node with battery capacity $E_{max} = 100mJ$ with efficiency $\eta = 0.6$ given a bandwidth of $1MHz$ in an AWGN channel is considered. The noise spectral density is taken as $N_0 = 10^{-19}W/Hz$ and the path loss coefficients to receivers 1 and 2 are set to be $-100dB$ and $-103dB$ respectively. For a simulation duration of $T = 10000$ seconds, the simulation is performed in 1 second slots, within which the harvesting process is assumed to yield a constant power distributed uniformly in $[0, 40]mW$. The hasty and constant algorithms transmit with harvested power and a pre-determined constant power respectively, and are simulated for comparison. It can be observed that the proposed online algorithm performs close to the optimal online algorithm, while both outperform naive algorithms such as hasty and constant.

VII. Conclusion

In this paper, the weighted sum rate optimal power allocation policy for an energy harvesting broadcaster node with inefficient energy storage is found and shown to trace the boundary of the capacity region for this channel. The optimal offline policy is shown to have a double threshold structure, where the two thresholds are related through the efficiency of the battery. The thresholds are shown to remain constant unless the battery is depleted or full. It is further shown that the optimal online policy established with dynamic programming also yields a two-threshold policy, and a computationally simple near-optimal online policy with fixed thresholds is proposed.

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