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Research Article

Resource Allocation for the Multiband Relay Channel: A Building Block for Hybrid Wireless Networks

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We investigate optimal resource allocation for the multiband relay channel. We find the optimal power and bandwidth allocation strategies that maximize the bounds on the capacity, by solving the corresponding max-min optimization problem. We provide sufficient conditions under which the associated max-min problem is equivalent to a supporting plane problem, which renders the solution for an arbitrary number of bands tractable. In addition, the sufficient conditions derived are general enough so that a class of utility functions can be accommodated with this formulation. As an example, we concentrate on the case where the source has two bands and the relay has a single band available and find the optimal resource allocation. We observe that joint power and bandwidth optimization always yields higher achievable rates than power optimization alone, establishing the merit of bandwidth sharing. Motivated by our analytical results, we examine a simple scenario where new channels become available for a transmitter to communicate; that is, new source to relay bands are added to a frequency division relay network. Given the channel conditions of the network, we establish the guidelines on how to allocate resources in order to achieve higher rates, depending on the relative quality of the available links.

1. Introduction

Future wireless networks are expected to enable nodes to communicate over multiple technologies and hops. Recent advances in the development of software defined radios support the vision where agile radios are employed at each node that utilize multiple standards and communicate seamlessly. Indeed, an intense research effort is directed towards having multiple communication standards coexist within one system, for example, the cellular network and IEEE 802.11 WLAN as in [1, 2]. We refer to a group of nodes capable of employing a number of communication technologies to find the best multihop route between the source-destination pairs, as a *hybrid wireless network*.

In this paper, we consider a simple hybrid wireless network with a source destination pair and aim at understanding its *performance limits*, that is, information theoretic rates with optimal resource allocation. In particular, we consider a scenario where a source node can communicate over multiple frequency bands to its destination, and a node that overhears the source transmission acts as a relay. We assume that the frequency bands that the source utilizes as well the ones used by the relay node are mutually orthogonal. The different bands are envisioned to represent links that operate with different wireless communication standards.

There has been considerable research effort up to date towards characterizing the information theoretic capacity of relay channels [3–7]. Most of the earlier work on relay channel capacity assumes that simultaneous transmission and reception at the relay is possible [4]. Since this is difficult to implement, recent work considers employing orthogonality at the relay via time-division [5, 8–10], frequency-division [11, 12], or code-division [13, 14]. To compensate the loss of spectral efficiency caused by this architecture and to increase the capacity, optimal resource allocation has been considered in [5, 8, 10, 11, 15, 16]. The optimal power and time slot duration allocation for the time-division relay channel has been considered in [5]. The work in [8] investigates three half-duplex time-division protocols that vary in the method of broadcasting they employ and the existence of receiver collision. The optimal power and time-slot allocation has been investigated for the protocol with the maximum degree of broadcasting and no receiver collision in [5].

We note that resource allocation in wireless relay networks is employed by utilizing the received SNR and the channel state information which are typically assumed to be available at the source and the relay node [5, 8, 10, 11, 16]. Notably, [16] studies optimal power and bandwidth allocation strategies for collaborative transmit diversity schemes for the situation when the source and the relay know only the magnitudes of the channel gains. The outage minimization and the corresponding optimal power control are considered when the network channel state is available at the source and the relay [10]. The model considered in this paper is in accordance with previous work and utilizes the received SNRs that are available at the source and the relay in order to find optimal resource allocation strategy.

In this paper, we investigate the optimal resource allocation strategies that maximize the capacity bounds for a simple *hybrid wireless relay network*. The channel model in this work can be traced back to a class of orthogonal relay networks first proposed in [11]. The three-node relay network in [11] is composed of two parts: a broadcast channel from the source node to the relay and destination node, and a separate orthogonal link from the relay node to the destination node. The parallel channel counterpart of [11] is later examined in [15]. A sum power constraint is imposed on the source node, and the relay node is restricted to perform a partial decode and forward operation. The sum rate from the source to the destination is then maximized by performing power allocation among different subchannels and the time sharing factor between the two parts of the network. A supporting plane technique is proposed in [15] to solve the associated max-min optimization problem. The results for the parallel network are then applied to the block fading model [15].

The model considered in this work is similar to the parallel relay network in [15]; yet, for the hybrid wireless network considered, the rate maximization leads to a different optimization problem than [15]: in a hybrid network, in addition to power allocation among different bands, it is conceivable to consider bandwidth allocation as well, and we find that the joint optimal power and bandwidth allocation yields higher rate than power optimization only. It is worth mentioning that dynamic bandwidth allocation is beneficial for a hybrid wireless network even in a scenario of a flat overall band. This is because different systems (standards) may exhibit different received SNR behavior even if the underlying channel gain and noise level are the same. This can be caused, for example, by different coding schemes or different requirements on feedback. Thus, one system will not, in general, be invariably better than all the others over all links.

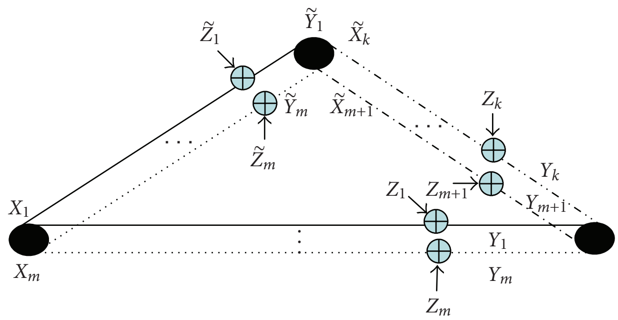

Figure 1: (k, m) Multiband Relay Channel.

At the outset, the joint power and bandwidth optimization appears challenging. Luckily, the resulting max-min optimization problem, we show, conforms to a set of sufficient conditions that render the solution manageable, even for an arbitrarily large number of bands. The technique that we can use under these sufficient conditions is the supporting plane technique used in [15]. We remark that the sufficient conditions are general enough that a class of utility functions can be optimized using the technique although our focus is on the information theoretic rates. This implies that the optimization technique used in this paper can be incorporated as a building block in a variety of resource allocation settings.

Lastly, in order to gain insight into the impact of optimal resource allocation on the construction of a hybrid wireless network, we examine a scenario where new wireless links can be added to the classical frequency division relay network to form a simple hybrid wireless network. Given the channel conditions between nodes, we study how to allocate resources to achieve the higher achievable rate. We observe that the source node is encouraged to communicate over the best network by dedicating all resources exclusively when condition of source-to-relay (SR) link and source-to-destination (SD) link of the new network is better (or worse) than that of SD link and SR link of the current network. Otherwise, it is beneficial to share resource between the current network and the new network to achieve a higher rate.

2. The Multiband Relay Channel

We consider the multiband relay channel (MBRC), which models a three-node hybrid wireless network where multiple frequency bands available from the source and the relay are mutually orthogonal. In particular, the situation where, among total k channels, there are m channels available for the source node and $k - m$ for the relay node, shown in Figure 1, is termed the (k, m)-MBRC.

The source node transmits information over m orthogonal channels to the relay and the destination node. The relay node uses a decode-and-forward scheme [4]. The (k, m)-MBRC input-output signal model is thus given by

$$\tilde{\mathbf{Y}}_{\mathbf{SR}} = \mathbf{X}_{\mathbf{S}} + \tilde{\mathbf{Z}}_{\mathbf{SR}}; \quad \mathbf{Y}_{\mathbf{RD}} = \tilde{\mathbf{X}}_{\mathbf{R}} + \mathbf{Z}_{\mathbf{RD}}; \quad \mathbf{Y}_{\mathbf{SD}} = \mathbf{X}_{\mathbf{S}} + \mathbf{Z}_{\mathbf{SD}}. \tag{1}$$

where $\mathbf{X}_{\mathbf{S}} = [X_1, X_2, \ldots, X_m]^T$ and $\tilde{\mathbf{X}}_{\mathbf{R}} = [\tilde{X}_{m+1}, \tilde{X}_{m+2}, \ldots, \tilde{X}_k]^T$ are the transmitted signal vectors from the source node and the relay node, respectively. $\mathbf{Y}_{\mathbf{SD}} = [Y_1, Y_2, \ldots, Y_m]^T$ and $\tilde{\mathbf{Y}}_{\mathbf{SR}} = [\tilde{Y}_1, \tilde{Y}_2, \ldots, \tilde{Y}_m]^T$ are the received signal vectors at the destination node and the relay node when the signal is transmitted from the source node. $\mathbf{Y}_{\mathbf{RD}} = [Y_{m+1}, Y_{m+2}, \ldots, Y_k]^T$ is the received signal vector at the destination from the relay. $\tilde{\mathbf{Z}}_{\mathbf{SR}} = [\tilde{Z}_1, \tilde{Z}_2, \ldots, \tilde{Z}_m]^T$ is the zero-mean independent additive white Gaussian noise (AWGN) vector with covariance matrix $E[\tilde{\mathbf{Z}}_{\mathbf{SR}}\tilde{\mathbf{Z}}_{\mathbf{SR}}^T] = \operatorname{diag}\{\tilde{N}_1/2, \tilde{N}_2/2, \ldots, \tilde{N}_m/2\}$ at the relay node. $\mathbf{Z}_{\mathbf{SD}} = [Z_1, Z_2, \ldots, Z_m]^T$ and $\mathbf{Z}_{\mathbf{RD}} = [Z_{m+1}, Z_{m+1}, \ldots, Z_k]^T$ are the zero-mean independent AWGN vectors with covariance matrices $E[\mathbf{Z}_{\mathbf{SD}}\mathbf{Z}_{\mathbf{SD}}^T] = \operatorname{diag}\{N_1/2, N_2/2, \ldots, N_m/2\}$, and $E[\mathbf{Z}_{\mathbf{RD}}\mathbf{Z}_{\mathbf{RD}}^T] = \operatorname{diag}\{N_{m+1}/2, N_{m+2}/2, \ldots, N_k/2\}$ at the destination node. $[\cdot]^T$ denotes the transpose operation, and $\operatorname{diag}\{a_1, \ldots, a_n\}$ is an $n \times n$ diagonal matrix. Since channels are independent, the channel transition probability mass function is given by

$$\begin{aligned} &P(y_1, y_2, \ldots, y_m, y_{m+1}, \ldots, y_k, \tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_m \mid \\ &\quad x_1, x_2, \ldots, x_m, \tilde{x}_{m+1}, \ldots, \tilde{x}_k) \\ &\quad = \prod_{i=1}^{m} P(y_i, \tilde{y}_i \mid x_i) \prod_{j=m+1}^{k} P\left(y_j \mid \tilde{x}_j\right), \end{aligned} \tag{2}$$

and we have the following theorem.

Theorem 1. *The upper and lower bounds for the capacity of (k, m)-MBRC are*

$$\begin{aligned} C_{low} = \max_{\substack{S \in \{1,\ldots,m\} \\ S^c = \{1,\ldots,m\}/S}} \Bigg[&\sup_{P(\cdot)} \min \Bigg\{ \sum_{i \in S} I(X_i; Y_i) \\ &+ \sum_{i=m+1}^{k} I\left(\tilde{X}_i; Y_i\right), \sum_{i \in S} I\left(X_i; \tilde{Y}_i\right) \Bigg\} \\ &+ \sup_{P(\cdot)} \sum_{i \in S^c} I(X_i; Y_i) \Bigg], \end{aligned} \tag{3}$$

$$\begin{aligned} C_{up} = \sup_{P(\cdot)} \min \Bigg\{ &\sum_{i=1}^{m} I(X_i; Y_i) \\ &+ \sum_{i=m+1}^{k} I\left(\tilde{X}_i; Y_i\right), \sum_{i=1}^{m} I\left(X_i; \tilde{Y}_i, Y_i\right) \Bigg\}, \end{aligned} \tag{4}$$

where $I(X; Y)$ is the mutual information between X and Y. The input distribution $P(\cdot)$ is

$$P(x_1, x_2, \ldots, x_m, \tilde{x}_{m+1}, \ldots, \tilde{x}_k) = P(x_1)P(x_2) \cdots P(\tilde{x}_k). \tag{5}$$

Proof. The lower bound is obtained by taking the maximum of all possible transmission rates given the total number of bands; that is, the lower bound includes all possible transmission schemes which depend on whether the transmission from the source band(s) is decoded at the relay.

We define S as the set of bands in which the transmission from the source is decoded at the relay. S^c is the complement of S and includes the set of bands for direct communication. For (k, m)-MBRC, the lower bound is given by

$$\begin{aligned} C_{\text{low}} = \max_{\substack{S \in \{1,\ldots,m\} \\ S^c = \{1,\ldots,m\}/S}} \Big\{ &C_{\text{DF}}\left(\mathbf{X}_S, \tilde{\mathbf{X}}_{\{m+1,\ldots,k\}}, \tilde{\mathbf{Y}}_S, \mathbf{Y}_{S \cup \{m+1,\ldots,k\}}\right) \\ &+ C_{\text{DT}}\left(\mathbf{X}_{S^c}, \mathbf{Y}_{S^c}\right) \Big\}, \end{aligned} \tag{6}$$

where $\mathbf{X}_S$ is the transmitted signal vector from the source and $\mathbf{X}_{S^c}$ is the transmitted signal vector from the source intended for direct transmission. Similarly, $\tilde{\mathbf{X}}_{\{m+1,\ldots,k\}}$ is the transmitted signal from the relay. $\tilde{\mathbf{Y}}_S$ is the received signal vector at the relay. $\mathbf{Y}_{S \cup \{m+1,\ldots,k\}}$ is the received signal vector at the destination. $\mathbf{Y}_{S^c}$ is the received signal vector at the destination as a result of direct transmission. $C_{\text{DF}}(\cdot)$ and $C_{\text{DT}}(\cdot)$ are given by

$$\begin{aligned} C_{\text{DF}}(\cdot) = \sup_{P\left(\mathbf{x}_S, \tilde{\mathbf{x}}_{\{m+1,\ldots,k\}}\right)} \min \Big\{ &I\left(\mathbf{X}_S, \tilde{\mathbf{X}}_{\{m+1,\ldots,k\}}; \mathbf{Y}_{S \cup \{m+1,\ldots,k\}}\right), \\ &I\left(\mathbf{X}_S; \tilde{\mathbf{Y}}_S \mid \tilde{\mathbf{X}}_{\{m+1,\ldots,k\}}\right) \Big\}, \end{aligned} \tag{7}$$

$$C_{\text{DT}}(\cdot) = \sup_{P(\mathbf{x}_{S^c})} I(\mathbf{X}_{S^c}; \mathbf{Y}_{S^c}), \tag{8}$$

where $P(\mathbf{x}_S, \tilde{\mathbf{x}}_{\{m+1,\ldots,k\}})$ is the input joint distribution with respect to S. Similarly, $P(\mathbf{x}_{S^c})$ is the input joint distribution with respect to S^c. We note that (7) can be readily obtained by using the results in [4] by taking $X = \mathbf{X}_S$, $\tilde{X} = \tilde{\mathbf{X}}_{\{m+1,\ldots,k\}}$, $\tilde{Y} = \tilde{\mathbf{Y}}_S$, and $Y = \mathbf{Y}_{S \cup \{m+1,\ldots,k\}}$. Applying the same approach, we obtain the following from the cut set bound [17]:

$$\begin{aligned} C_{\text{up}} = \sup_{P(\cdot)} \min \Big\{ &I\left(\mathbf{X}_S, \tilde{\mathbf{X}}_{\{m+1,\ldots,k\}}; \mathbf{Y}_{S \cup \{m+1,\ldots,k\}}\right), \\ &I\left(\mathbf{X}_S; \mathbf{Y}_{S \cup \{m+1,\ldots,k\}}, \tilde{\mathbf{Y}}_S \mid \tilde{\mathbf{X}}_{\{m+1,\ldots,k\}}\right) \Big\}, \end{aligned} \tag{9}$$

where $P(\cdot) = P(x_1, \dots, x_m, \tilde{x}_{m+1}, \dots, \tilde{x}_k)$. Following a similar approach to [11], (5) can be shown to maximize the mutual information in (7)–(9), and the optimization over (5) leads to (3)-(4). □

3. Capacity Bounds and Optimal Resource Allocation

In the remainder of the paper, we will consider optimal resource allocation on the bounds obtained for the MBRC, that is, for hybrid wireless networks where the source node has access to distinct bands (standards) and a second node that overhears the source information relays to the destination using additional orthogonal bands. We consider the Gaussian case, where all the transmitted signals are corrupted by additive white Gaussian noises.

We have the input-output signal model given by (1) under source and relay power constraints:

$$\begin{aligned} E[X_i^2] &\le \alpha_i P_s \quad i = 1, \dots, m; \\ E\left[\tilde{X}_i^2\right] &\le \zeta_i P_r \quad i = m+1, \dots, k, \end{aligned} \tag{10}$$

where P_s and P_r are the total available power at the source and relay node. α_i and ζ_i are the nonnegative power allocation parameters for each orthogonal band at the source and relay node, and $\sum_{i=1}^m \alpha_i = \sum_{i=m+1}^k \zeta_i = 1$. Unlike [5, 10], we do not have a total power constraint between the source and the relay and assume that each has its own battery.

We assume that the system has total bandwidth W. We define the received SNRs at the relay and the destination over channel $i = 1, \dots, k$ as

$$\begin{aligned} \chi_i &\triangleq \frac{P_s}{\tilde{N}_i W}, \quad \eta_i \triangleq \frac{P_s}{N_i W}, \quad i = 1, \dots, m, \\ \rho_i &\triangleq \frac{P_r}{N_i W}, \quad i = m+1, \dots, k. \end{aligned} \tag{11}$$

Note that the actual received SNR values are the scaled versions of (11) depending on the power and bandwidth allocation. For example, the actual received SNR at the relay from channel 1, which is allocated α_1 fraction of the source power and ϕ_1 fraction of the bandwidth, simply is $\alpha_1\chi_1/\phi_1$. Given the received SNRs which are available at the source and relay, our aim is to find the optimal resource allocation parameters that maximize capacity lower bound in terms of the transmitted power and the total bandwidth for (k, m)-MBRC, which leads to optimally allocating the source power among m source bands, the relay power among $k - m$ relay bands, and the total bandwidth among k bands. We can obtain the capacity lower and upper bounds of (k, m)-MBRC from Theorem 1 as follows.

Theorem 2. *The upper and lower bounds for the capacity of the Gaussian (k, m)-MBRC are*

$$\begin{aligned} C_{low}^{MBRC} &= \max_{\substack{S\in\{1,\dots,m\} \\ S^c\in\{1,\dots,m\}/S}} \max_{\substack{0\le\alpha_i,\phi_i,\zeta_i\le 1 \\ \sum_{i=1}^m \alpha_i=1 \\ \sum_{i=1}^k \phi_i=1 \\ \sum_{i=m+1}^k \zeta_i=1}} \min\Bigg\{\sum_{i\in S}\phi_i \log\left(1+\alpha_i\frac{\eta_i}{\phi_i}\right) \\ &\quad + \sum_{i=m+1}^{k}\phi_i \log\left(1+\zeta_i\frac{\rho_i}{\phi_i}\right) + \sum_{i\in S^c}\phi_i \log\left(1+\alpha_i\frac{\eta_i}{\phi_i}\right), \\ &\quad \sum_{i\in S}\phi_i \log\left(1+\alpha_i\frac{\chi_i}{\phi_i}\right) + \sum_{i\in S^c}\phi_i \log\left(1+\alpha_i\frac{\eta_i}{\phi_i}\right)\Bigg\}, \end{aligned} \tag{12}$$

$$\begin{aligned} C_{up}^{MBRC} &= \max_{\substack{0\le\alpha_i,\phi_i,\zeta_i\le 1 \\ \sum_{i=1}^m \alpha_i=1 \\ \sum_{i=1}^k \phi_i=1 \\ \sum_{i=m+1}^k \zeta_i=1}} \min\Bigg\{\sum_{i=1}^{2}\phi_i \log\left(1+\alpha_i\frac{\eta_i}{\phi_i}\right) + \sum_{i=m+1}^{k}\phi_i \log\left(1+\zeta_i\frac{\rho_i}{\phi_i}\right), \\ &\quad \sum_{i=1}^{2}\phi_i \log\left(1+\alpha_i\frac{\eta_i+\chi_i}{\phi_i}\right)\Bigg\}. \end{aligned} \tag{13}$$

We omit the proof for Theorem 2 since the derivation for each mutual information follows directly from [15]. For each broadcast channel, if the relay node sees a higher received SNR than the destination node, then a superposition coding scheme [17] is used to convey independent information to the relay node, which cannot be decoded by the destination directly. The relay node then collects this information from all the channels where superposition coding is used, and transmits it to the destination at the appropriate rate.

Based on whether the relay node is utilized by a certain channel (band), we note that there are 2^m possible schemes. We observe that these 2^m schemes are not exclusive to each other, since a superposition coding scheme may be reduced to a direct source-to-destination transmission if no band is allocated to the relay-to-destination link. We also note that which scheme yields the largest rate is completely decided by the SNR relationship, namely, the componentwise relationship between the received SNRs of the source-to-relay links, that is, $\chi_1, \dots, \chi_m$ and the received SNRs of the source-to-destination links, that is, $\eta_1, \dots, \eta_m$.

If $\chi_j \le \eta_j$, $j = 1, \dots, m$, then for any bandwidth allocation, the signal received by the relay over this broadcast channel can be viewed as a degraded version of the signal received by the destination. Therefore, direct link transmission should be used for this band, regardless of what scheme is used for the other bands. On the other hand, if $\eta_j < \chi_j$, then the relay node can always learn something more than the destination node over this band and uses the superposition code scheme, and although the superposition scheme may be reduced to a direct link transmission scheme, optimizing under this scheme does not incur any rate loss. Based on these observations, we conclude that there is no need to examine all the schemes to find the best rate and the corresponding resource allocation. That is, practically, the system checks the received SNRs and chooses one of 2^m schemes satisfying the relationship of the received SNRs to communicate and the rate with optimized resource allocation for the chosen scheme is the maximum achievable rate, and the corresponding resource allocation is the globally optimal solution.

Next, we maximize the capacity lower bound in (12). To achieve this goal, we introduce the following general max-min optimization problem. We define $G_1(\underline{R})$ and $G_2(\underline{R})$ as any utility function with any resource allocation vector $\underline{R}$ over the convex set $\mathbf{C}_0$:

$$\begin{aligned} &\max_{(c_1,c_2)\in\mathbf{B}_1} \quad \min\{c_1,c_2\} \\ &\text{where} \quad \mathbf{B}_1 = \{(G_1(\underline{R}),G_2(\underline{R})) : \underline{R}\in\mathbf{C}_0\}, \\ &\qquad\qquad \mathbf{C}_0 = \{\text{all feasible values of } \underline{R}\}. \end{aligned} \tag{14}$$

Proposition 1. *If $G_1(\underline{R})$ and $G_2(\underline{R})$ are nonnegative and concave over $\mathbf{C}_0$, there must exist $0 \le \beta \le 1$ such that maximizing the following equation with respect to $\underline{R}$ is equivalent to* (14)*:*

$$G(\beta,\underline{R}) = \beta G_1(\underline{R}) + (1-\beta)G_2(\underline{R}), \quad 0 \le \beta \le 1. \tag{15}$$

Proof. See Appendix A.

Note that the optimization problem in (14) corresponds to finding $\underline{R}$ and β maximizing the minimum of two end points in $G(\beta,\underline{R})$. One possible technique to solve the max-min optimization problem in (14) is given by the following proposition [15], which we will also utilize.

Proposition 2 ([15, Proposition 1]). *The relationship between optimal resource allocation parameters $\underline{R}^*$ and the corresponding optimal point β^* is given by the following.*

Case 1: *If $\beta^* = 1$, $G_1(\underline{R}^*) < G_2(\underline{R}^*)$.*

Case 2: *If $\beta^* = 0$, $G_1(\underline{R}^*) > G_2(\underline{R}^*)$.*

Case 3: *Neither case 1 nor 2 occurs; under this case, if $0 \le \beta^* \le 1$, $G_1(\underline{R}^*) = G_2(\underline{R}^*)$.*

Now, one can restate our max-min optimization problem given in Theorem 2 as follows:

$$\begin{aligned} &\max_{(c_1,c_2)\in\mathbf{B}_1} \min\{c_1,c_2\} \\ &\text{where } \mathbf{B}_1 = \{(C_1(\underline{R}),C_2(\underline{R})) : \underline{R}\in\mathbf{C}_0\}, \\ &\qquad \mathbf{C}_0 = \Big\{(\alpha_1,\dots,\alpha_m,\zeta_{m+1},\dots,\zeta_k,\phi_1,\dots,\phi_k) : \\ &\qquad\qquad 0 \le \alpha_i,\zeta_i,\phi_i \le 1, \\ &\qquad\qquad \sum_{i=1}^{m}\alpha_i = 1, \sum_{i=m+1}^{k}\zeta_i = 1, \sum_{i=1}^{k}\phi_i = 1\Big\} \subset \Re^{2k}, \end{aligned} \tag{16}$$

where $C_1(\underline{R})$ and $C_2(\underline{R})$ are the first and the second terms of max-min optimization problem in (12)*. Next, one needs to prove that $C_1(\underline{R})$ and $C_2(\underline{R})$ are concave over $\mathbf{C}_0$ in* (16)*. Define*

$$\begin{aligned} &F(x_1,\dots,x_n,y_1,\dots,y_n) \\ &\quad = \sum_{i\in D} x_i \log\left(1 + y_i\frac{t_i}{x_i}\right), \\ &\qquad t_i > 0,\ x_i \ge 0,\ y_i \ge 0, \quad i = 1,\dots,n, \\ &D = \{i : x_i > 0, i = 1,\dots,n\}. \end{aligned} \tag{17}$$

It is easy to see that $F(x_1,\dots,x_n,y_1,\dots,y_n)$ is continuous over $\{x_i \ge 0, y_i \ge 0, i = 1,\dots,n\}$. Then, one has the following proposition.

Proposition 3. *$F(x_1,\dots,x_n,y_1,\dots,y_n)$ is concave over $x_i \ge 0$ and $y_i \ge 0, i = 1,\dots,n$.*

Proof. First, note that due to the continuity of $F(\cdot)$, we only need to prove that $F(\cdot)$ is concave over the *interior* of the region, that is, $x_i > 0$, $y_i > 0, i = 1,\dots,n$. This is done by examining the Hessian, $\mathbf{H}$, of (17). The second-order derivatives of (17) with respect to x_i and y_i are

$$\frac{\partial^2 F(\cdot)}{\partial x_i^2} = \frac{-t_i^2 y_i^2}{x_i(t_i y_i + x_i)^2}; \quad \frac{\partial^2 F(\cdot)}{\partial y_i^2} = \frac{-t_i^2 x_i}{(t_i y_i + x_i)^2}, \tag{18}$$

$$\frac{\partial^2 F(\cdot)}{\partial x_i \partial y_i} = \frac{t_i^2 y_i}{(t_i y_i + x_i)^2}. \tag{19}$$

We note that $\partial^2 F(\cdot)/\partial x_i\partial x_j = \partial^2 F(\cdot)/\partial y_i\partial y_j = \partial^2 F(\cdot)/\partial x_i\partial y_j = 0$, for all $i \ne j$.

The Hessian is the $2n\times 2n$ block diagonal matrix with the following matrix in its ith diagonal:

$$A_i = \begin{bmatrix} \dfrac{\partial^2 F(\cdot)}{\partial x_i^2} & \dfrac{\partial^2 F(\cdot)}{\partial x_i \partial y_i} \\ \dfrac{\partial^2 F(\cdot)}{\partial y_i \partial x_i} & \dfrac{\partial^2 F(\cdot)}{\partial y_i^2} \end{bmatrix} \quad i = 1,\dots,n. \tag{20}$$

It is readily seen that A_i is singular. Since $\partial^2 F(\cdot)/\partial x_1^2 < 0$ for $y_1 > 0$ from (18), $\mathbf{H}$ is the negative semidefinite. Thus, $F(\cdot)$ is concave over $x_i > 0$ and $y_i > 0$, $i = 1, \ldots, n$. Since $F(\cdot)$ is continuous over $x_i \geq 0, y_i \geq 0, i = 1, \ldots, n$, $F(\cdot)$ is concave over $x_i \geq 0$ and $y_i \geq 0, i = 1, \ldots, n$. □

We note that for any choice of set $S \in \{1, \ldots, m\}$, $C_1(\underline{R})$ corresponds to $F(\cdot)$ in (17) with $x_i = \phi_i$, $i = 1, \ldots, k$, $y_i = \alpha_i$, $i = 1, \ldots, m$, $y_i = \zeta_i$, $i = m+1, \ldots, k$, $t_i = \eta_i$, $i = 1, \ldots, m$, and $t_i = \rho_i$, $i = m+1, \ldots, k$. For $C_1(\underline{R})$, the Hessian is a $2k \times 2k$ block diagonal matrix. Similarly, $C_2(\underline{R})$ corresponds to $F(\cdot)$ with $x_i = \phi_i$, $i = 1, \ldots, m$, $y_i = \alpha_i$, $i = 1, \ldots, m$, $t_i = \chi_i$, $i \in S$, and $t_i = \eta_i$, $i \in S^c$. For $C_2(\underline{R})$, the Hessian is a $2m \times 2m$ block diagonal matrix.

Remark 1. Since $F(\cdot)$ is concave over the set $x_i \geq 0$ and $y_i \geq 0, i = 1, \ldots, n$, it is also concave over any convex subset of it. Thus, $C_1(\underline{R})$ and $C_2(\underline{R})$ are concave over $\underline{R} \in \mathbf{C}_0$. (It is readily seen that the sum constraints define a convex set.) This establishes that the local optimal for (16) is also the global optimal [18, Theorem 3.4.2, page 125-126].

Remark 2. We further find that $F(\cdot)$ is strictly concave over any convex subset of $x_i > 0 (y_i > 0)$, $i = 1, \ldots, n$, jointly when $y_i > 0$ $(x_i > 0)$, $i = 1, \ldots, n$, are held constant. Note that when all $y_i > 0$, $i = 1, \ldots, n$, are held constant, that is, $y_i = c_i$, we have $F(\cdot)$ as a function of x_i, $i = 1, \ldots, n$. In this case, it is easily seen that the Hessian is the $n \times n$*diagonal* matrix in which ith diagonal term is given by $\partial^2 F(\cdot)/\partial x_i^2 = -t_i^2 c_i^2/(x_i(t_i c_i + x_i)^2)$, $c_i > 0$, $i = 1, \ldots, n$. Since now all of the diagonal terms are strictly negative when $x_i > 0$, $i = 1, \ldots, n$, $F(\cdot)$ is strictly concave over all $x_i > 0$, $i = 1, \ldots, n$, jointly when all $y_i > 0$, $i = 1, \ldots, n$, are held constant. Similarly, $F(\cdot)$ is strictly concave over $y_i > 0$, $i = 1, \ldots, n$, jointly when all $x_i > 0$, $i = 1, \ldots, n$, are held constant. Since if a function is strictly concave over a set, it is also strictly concave over any convex subset of that set, the preceding argument implies that $F(\cdot)$ is strictly concave over any convex subset of $x_i > 0$ $(y_i > 0), i = 1, \ldots, n$, when all $y_i > 0$ $(x_i > 0)$, $i = 1, \ldots, n$, are held constant. This fact will be useful in the sequel.

Based on Proposition 1 and Proposition 3, the methodology given in Proposition 2 can be applied to our max-min optimization problem in (16) for an arbitrary (k, m). That said, in the remainder of the paper, we will examine the optimal resource allocation for $(3, 2)$-MBRC where the source has two bands and the relay has a single band available to communicate and uses its own full power P_r. We find this network model representative and meaningful because of the following two observations. First, if there is more than one band available for the link between the relay and the destination, then only the best band among them will be used. This can be seen by fixing the overall band for this link and performing joint power and bandwidth optimization. Therefore, as long as the relay-to-destination SNRs are different, which is usually the case in practice, (k, m)-MBRC will have the same resource allocation parameters as those of $(m+1, m)$-MBRC. Secondly, the case with $m > 2$ is similar to the case with $m = 2$ except that there are more schemes to choose from. Therefore, we focus on the $(3, 2)$-MBRC in the sequel.

3.1. Maximization of Capacity Bounds for the Gaussian $(3, 2)$-*MBRC.*

For $(3, 2)$-MBRC, there are four schemes to choose from. Let us label them Schemes I through IV. From Theorem 2, upper and lower bounds for the capacity of the Gaussian $(3, 2)$-MBRC are

$$C_{\text{low}}^{\text{MBRC}} = \max_{\substack{0 \leq \alpha_i, \phi_i \leq 1 \\ \sum_{i=1}^{2} \alpha_i = 1 \\ \sum_{i=1}^{3} \phi_i = 1}} \min\left\{ \sum_{i=1}^{2} \phi_i \log\left(1 + \alpha_i \frac{\eta_i}{\phi_i}\right) + \phi_3 \log\left(1 + \frac{\rho_3}{\phi_3}\right), \phi_1 \log\left(1 + \alpha_1 \frac{\kappa}{\phi_1}\right) + \phi_2 \log\left(1 + \alpha_2 \frac{\nu}{\phi_2}\right) \right\}, \tag{21}$$

$$C_{\text{up}}^{\text{MBRC}} = \max_{\substack{0 \leq \alpha_i, \phi_i \leq 1 \\ \sum_{i=1}^{2} \alpha_i = 1 \\ \sum_{i=1}^{3} \phi_i = 1}} \min\left\{ \sum_{i=1}^{2} \phi_i \log\left(1 + \alpha_i \frac{\eta_i}{\phi_i}\right) + \phi_3 \log\left(1 + \frac{\rho_3}{\phi_3}\right), \sum_{i=1}^{2} \phi_i \log\left(1 + \alpha_i \frac{\eta_i + \chi_i}{\phi_i}\right) \right\}, \tag{22}$$

where $(\kappa, \nu) = (\chi_1, \chi_2)$, (χ_1, η_2), (η_1, χ_2), and (η_1, η_2) for schemes I to IV, respectively. Each scheme materializes as a function of the received SNRs as follows.

Scheme I: $S = \{1, 2\}$, the scenario where transmission from the source node over both links is decoded at the relay node. This scheme is chosen if $\eta_1 \leq \chi_1$ and $\eta_2 \leq \chi_2$.

Scheme II: $S = \{1\}$, the scenario where transmission from the source node over band 1 is decoded at the relay node while band 2 is used for direct transmission only. This scheme is chosen if $\eta_1 \leq \chi_1$ and $\eta_2 \geq \chi_2$.

Scheme III: $S = \{2\}$, the scenario where transmission from the source node over band 2 is decoded at the relay node while band 1 is used for direct transmission only. This scheme is chosen if $\eta_1 \geq \chi_1$ and $\eta_2 \leq \chi_2$.

Scheme IV: $S = \{\phi\}$, the scenario where transmissions from the source node from both bands are used only for direct transmission. This scheme is chosen if $\eta_1 \geq \chi_1$ and $\eta_2 \geq \chi_2$.

We define $\underline{R}^* = (\alpha_1, \alpha_2, \phi_1, \phi_2, \phi_3)$ as the optimal resource allocation parameters for (21). $C_1(\underline{R})$ and $C_2(\underline{R})$ are the first and second terms in (21). From (21), we note that the capacity for scheme IV is given by

$$C_{\text{direct}}^{\text{MBRC}} = \max_{\substack{0\le\alpha_i,\phi_i\le 1\\ \sum_{i=1}^{2}\alpha_i=1\\ \sum_{i=1}^{2}\phi_i=1}} \left\{\phi_1 \log\left(1+\alpha_1\frac{\eta_1}{\phi_1}\right)+\phi_2\log\left(1+\alpha_2\frac{\eta_2}{\phi_2}\right)\right\}. \tag{23}$$

In this case, the max-min optimization problem reduces to a maximization problem and it is readily shown that the optimal resource allocation for the rate of scheme IV is given by

$$\underline{R}^* = \begin{cases}(1,0,1,0,0) & \text{if } \eta_1 > \eta_2,\\ (0,1,0,1,0) & \text{if } \eta_1 < \eta_2.\end{cases} \tag{24}$$

For schemes I, II, III, once the appropriate scheme is decided upon, parameters (κ, ν) can be substituted accordingly and we can examine $\underline{R}^*$ for each of the cases in Proposition 2.

Case 1. $\beta^* = 1$, and $\underline{R}^*$ maximizes $C_1(\underline{R})$.

This case holds if the following condition is satisfied:

$$\sum_{i=1}^{2}\phi_i^* \log\left(1+\alpha_i^*\frac{\eta_i}{\phi_i^*}\right)+\phi_3^*\log\left(1+\frac{\rho_3}{\phi_3^*}\right) < \phi_1^*\log\left(1+\alpha_1^*\frac{\kappa}{\phi_1^*}\right)+\phi_2^*\log\left(1+\alpha_2^*\frac{\nu}{\phi_2^*}\right), \tag{25}$$

and we obtain

$$\underline{R}^* = \begin{cases}\left(1,0,1-\dfrac{\rho_3}{\rho_3+\eta_1},0,\dfrac{\rho_3}{\rho_3+\eta_1}\right) & \text{if } \eta_1 > \eta_2,\\ \left(0,1,0,1-\dfrac{\rho_3}{\rho_3+\eta_2},\dfrac{\rho_3}{\rho_3+\eta_2}\right) & \text{if } \eta_1 < \eta_2.\end{cases} \tag{26}$$

The received SNRs must satisfy

$$\frac{\eta_1}{\rho_3+\eta_1}\log\left(1+\frac{\kappa(\rho_3+\eta_1)}{\eta_1}\right) > \log(1+\rho_3+\eta_1) \quad \text{for } \eta_1 > \eta_2, \tag{27}$$

$$\frac{\eta_2}{\rho_3+\eta_2}\log\left(1+\frac{\nu(\rho_3+\eta_2)}{\eta_2}\right) < \log(1+\rho_3+\eta_2) \quad \text{for } \eta_1 < \eta_2. \tag{28}$$

Proof. See Appendix B. □

Case 2. $\beta^* = 0$, and $\underline{R}^*$ maximizes $C_2(\underline{R})$.

This case holds if the following condition is satisfied:

$$\sum_{i=1}^{2}\phi_i^* \log\left(1+\alpha_i^*\frac{\eta_i}{\phi_i^*}\right)+\phi_3^*\log\left(1+\frac{\rho_3}{\phi_3^*}\right) > \phi_1^*\log\left(1+\alpha_1^*\frac{\kappa}{\phi_1^*}\right)+\phi_2^*\log\left(1+\alpha_2^*\frac{\nu}{\phi_2^*}\right), \tag{29}$$

and we obtain

$$\underline{R}^* = \begin{cases}(1,0,1,0,0) & \text{if } \kappa > \nu, \eta_1 > \kappa,\\ (0,1,0,1,0) & \text{if } \kappa < \nu, \eta_2 > \nu.\end{cases} \tag{30}$$

Proof. See Appendix B. □

Remark 3. By substituting the appropriate parameters for (κ, ν) for each scheme into (30), we observe that Case 2 does not ever materialize for schemes I, II, III.

Case 3. $0 \le \beta^* \le 1$, and $\underline{R}^*$ maximizes $\beta^* C_1(\underline{R}) + (1-\beta^*)C_2(\underline{R})$ for a fixed β^*.

This case occurs when (25) or (29) doES not hold. The closed form solution for this optimization problem does not exist. Thus, we have to rely on an iterative algorithm. We propose to use alternating maximization algorithm that calls for optimizing $\boldsymbol{\alpha} = \{\alpha_1, \alpha_2\}$ in one stage, followed by optimizing $\boldsymbol{\phi} = \{\phi_1, \phi_2, \phi_3\}$ in the next stage. The iterations are obtained by finding KKT points of the corresponding optimization problem with the variable vector $\boldsymbol{\alpha}$ or $\boldsymbol{\phi}$. We note that the objective function is not differentiable at the boundary of the feasible region, that is, for $\phi_i = 0$, $i = 1, 2, 3$ and the corresponding KKT points are not defined. Thus, we need to introduce a small positive value, ε, and define a modified feasible region as illustrated in (B.3) and (B.4) that excludes the boundary point. Every time an iteration reaches the boundary of the new feasible region, we expand the feasible region by successively reducing ε so that we can continue with the iterations until convergence. The detailed description of the following proposed iterative algorithm and proof of its convergence to the global optimal solution is given in Appendix C.

Step 1. (i) Initialization: for initial values of $\beta^*, \mu, \lambda, \omega_i$, $i = 1, 2, 3$, ψ_i, $i = 1, 2$, and assign values to ϕ_1, ϕ_2, ϕ_3, such that $\phi_1 + \phi_2 + \phi_3 = 1$.

(ii) Iteration n: update $\alpha_i(n), i = 1, 2$ by finding the solution of KKT condition of (C.2) with respect to $\alpha_i, i = 1, 2$; find $\mu(n)$ and $\psi_i(n)$, $i = 1, 2$ such that $\alpha_1(n) + \alpha_2(n) = 1$ and $\alpha_i(n) \ge \varepsilon$, $i = 1, 2$.

(iii) Iteration $n+1$: update $\phi_1(n+1)$, $\phi_2(n+1)$, and $\phi_3(n+1)$ by finding the solution of KKT condition of (C.2) with respect to ϕ_i, $i = 1, 2, 3$; find $\lambda(n+1)$ and $\omega_i(n+1)$, $i = 1, 2, 3$ such that $\phi_1(n+1)+\phi_2(n+1)+\phi_3(n+1) = 1$, and $\phi_i(n) \ge \varepsilon$, $i = 1, 2, 3$.

(iv) Repeat step (ii) until the optimal β^* is found by $C_1(\underline{R}^*) = C_2(\underline{R}^*)$ in (C.3).

Step 2. If the iteration does not reach the boundary of the feasible region of (B.3) and (B.4), the algorithm terminates.

Step 3. Otherwise, set $\varepsilon = \varepsilon/d$, $d > 1$ in (B.3) and (B.4) and repeat Steps (1) to (2) by using the KKT points from the previous iteration as the initial points. (For numerical results, we use $d = 2$.)

We reiterate that based on the scheme at hand, we would substitute the correct parameters for $(\kappa, \nu) \in$

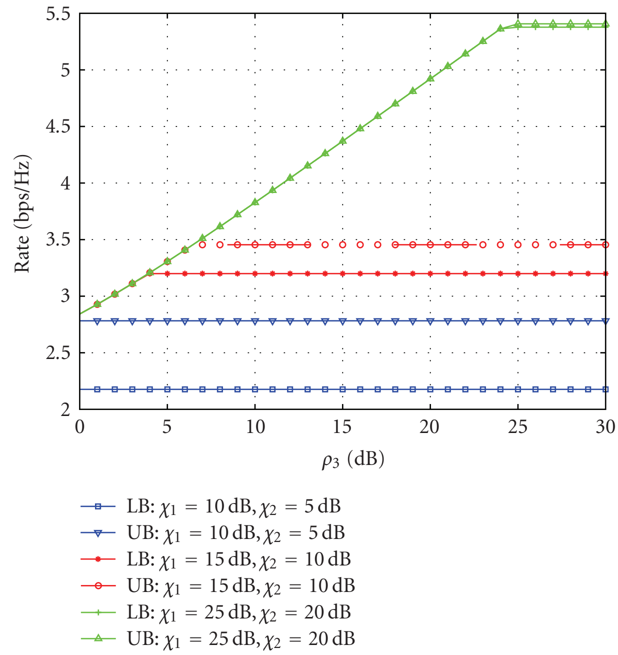

Figure 2: Upper and lower bounds of (3, 2)-MBRC with power optimization only: SNRs at SD, η_1 = 10 dB and η_2 = 5 dB.

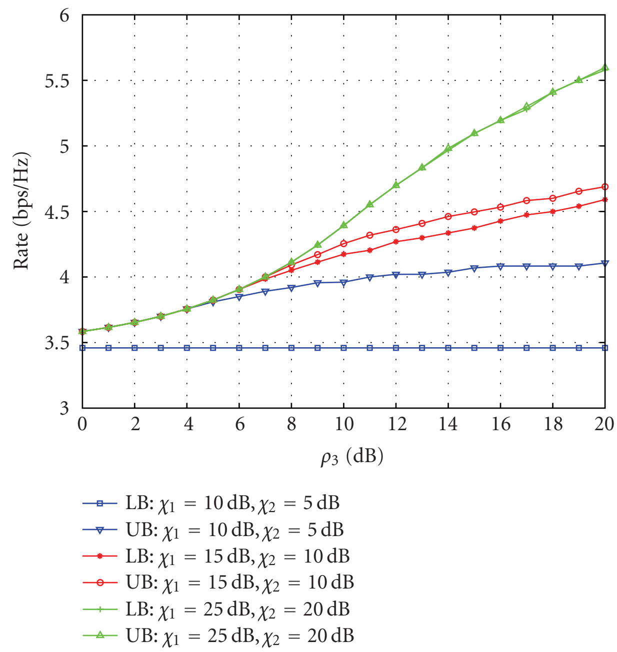

Figure 3: Upper and lower bounds of (3, 2)-MBRC with joint power and bandwidth optimization: SNRs at SD, η_1 = 10 dB and η_2 = 5 dB.

$\{(\chi_1, \chi_2), (\chi_1, \eta_2), (\eta_1, \chi_2)\}$ to find the optimal resource allocation strategy.

3.2. Upper Bound on Capacity. Recall that the upper bound given by (13) is obtained by the max-flow min-cut theorem. The maximization for the upper bound follows same steps to that of the lower bound, details of which we will omit here. In general, the upper bound is not tight. One exception is that for (3, 2)-MBRC, since Case 2 for schemes I, II, and III is not possible, the optimal resource allocation parameters $\underline{R}^*$ maximize $C_1(\underline{R})$ (Case 1) or $C_1(\underline{R}) = C_2(\underline{R})$ (Case 3). There exists a ρ_3' such that $C_1(\underline{R}^*) < C_2(\underline{R}^*)$ if $\rho_3 < \rho_3'$, otherwise $C_1(\underline{R}^*) = C_2(\underline{R}^*)$. Since the first term of the upper bound in (22) is the same as $C_1(\underline{R})$, we know that for (3, 2)-MBRC, $\underline{R}^*$ maximizes $C_{\text{low}}^{\text{MBRC}} = C_{\text{up}}^{\text{MBRC}}$ for $\rho_3 < \rho_3'$ and the resulting optimized rate is the capacity of (3, 2)-MBRC. A similar observation was made for the frequency division relay network, that is, when one band exists from the source in [11]. It is interesting to observe that the same observation extends to the multiband case.

4. Numerical Results and Discussion

4.1. Capacity Bounds. In this section, we present numerical results to support our analysis described in Section 3. Specifically, for (3, 2)-MBRC, we plot the capacity lower bound (LB) obtained by optimal resource allocation as well as the capacity upper bound (UB) with the same resource allocation parameters. For comparison purposes, we also consider the case where overall bandwidth W is equally divided between the three bands and only optimal power allocation is done.

Figure 2 shows the capacity UB and LB for (3, 2)-MBRC with optimal power allocation only. When the source-to-relay (SR) SNRs χ_1 and χ_2 are smaller than or equal to the source-to-destination (SD) SNRs η_1 and η_2, respectively, the lower bound does not increase and saturate even if the relay-to-destination (RD) SNR ρ_3 increases. This is expected, since using the relay is not beneficial when the source-to-relay channel is worse than the source-to-destination channel.

In contrast, when χ_1 and χ_2 are larger than η_1 and η_2, respectively, the lower bound increases as ρ_3 increases and saturates after a certain threshold of ρ_3. This threshold becomes larger as the quality of the SR links improves as compared to the SD links, that is, as χ_1 and χ_2 get larger compared to η_1 and η_2. Indeed, the fact that we can achieve higher rates when the SR channel is better than the SD channel is intuitively pleasing as the power allocation becomes more effective when we have a better SR channel. It is noticeable that the upper and lower bounds approach each other as the SR link quality improves as compared to that of SD.

Figure 3 shows the capacity UB and LB for (3, 2)-MBRC with joint optimal power and bandwidth allocation. We observe that the lower bound does not saturate when the SR links are better than the SD links. This additional improvement is thanks to the dynamic bandwidth allocation. By comparing Figure 2 and 3, we observe that the achievable rate of MBRC with joint optimal power and bandwidth is always larger than that of power optimization only, sometimes by a significant margin. This points to the advantage of joint

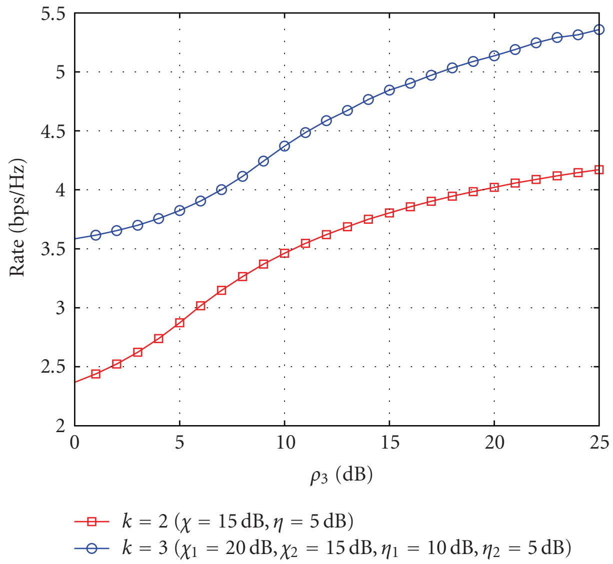

Figure 4: Comparison of achievable rates: the new SR ink is better than the current SR link, and the new SD link is better than the current SD link.

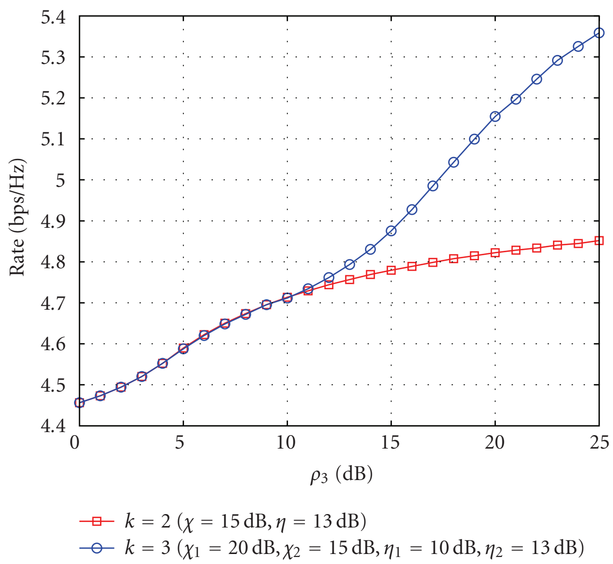

Figure 5: Comparison of achievable rates: the new SR link is better than the current SR link, and the new SD link is worse than the current SD link.

power and bandwidth optimization, promoting the idea of different wireless technologies lending each other frequency resources to improve capacity.

4.2. Guidelines for Hybrid Network Design. When a new wireless link becomes available at the source in addition to the existing single band relay network, a hybrid wireless network can be formed. In this case, a meaningful question is how to allocate resources between links in order to maximize the data rate. It is evident that the resource allocation strategy is a function of the channel quality of the available

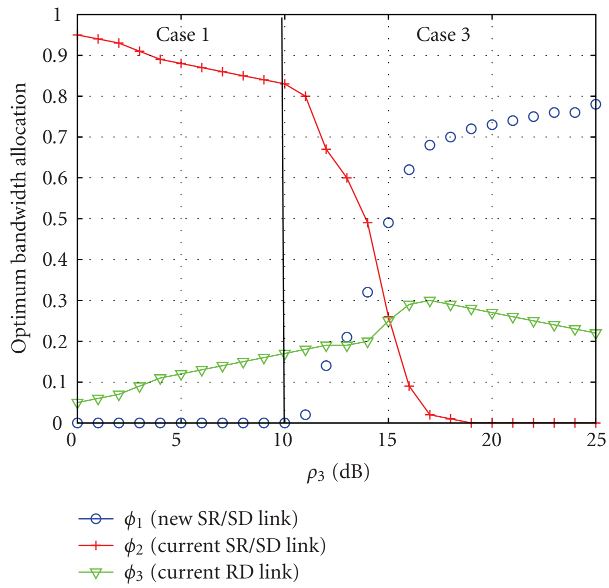

Figure 6: Optimal bandwidth allocation: the new SR link is better than the current SR link, and the new SD link is worse than the current SD link.

links (SD/SR/RD). To answer this question, we compare the achievable rates with optimal resource allocation for $k = 2$ and 3 and observe the effect of adding a new link on the maximum achievable rate.

Figure 4 shows the achievable rates when the new SR ink is better than the current SR link, and the new SD link is better than the current SD link. Comparing $k = 3$ and $k = 2$, we observe that the achievable rate of $k = 3$ is better than that of $k = 2$. This is because quality of the new link is better than that of the current link, and all resources are allocated to the new link. If the new links were worse, the maximum achievable rates would stay the same since all resources would be allocated to the current link.

Figure 5 shows the achievable rates when the new SR link is better than the current SR link, and the new SD link is worse than the current SD link. We observe that the achievable rates for $k = 2$ and 3 are almost same for low RD SNR. This is because when the RD link is poor, the relay becomes less useful, and most of bandwidth and power are allocated into channel with the best direct link. As the RD SNR increases, we observe that the achievable rate for $k = 3$ is larger than that of $k = 2$. This is because it is optimal resource allocation that we allocate more bandwidth and power to the new link with the best SR link. The observation is justified by examining bandwidth allocation (the power allocation follows a similar pattern) for $k = 3$ shown in Figure 6. We see that more bandwidth is allocated to the current link (ϕ_2 for $k = 3$) for low received RD SNR. More bandwidth is allocated to the new link (ϕ_1 for $k = 3$) when the RD link becomes better. We also observe that Case 1 and Case 3 of our proposed optimal resource allocation occur depending on the RD SNR: with both SR SNRs better than both SD

SNRs, Case 1 occurs at low RD SNR (from 0 dB to 10 dB); otherwise, the optimal resource allocation corresponds to Case 3. We note that the optimal resource allocation scheme would be reversed if the new SR link were worse than the current SR link, and the new SD link were better than the current SD link.

We note that the given received SNRs in the numerical results correspond to scheme I (i.e., $\eta_i \leq \chi_i$, $i = 1, 2$). Similarly, we can examine the effect of adding a new link under different received SNR relationship between η_i and χ_i which corresponds to scheme II or scheme III, and we could readily apply the optimal resource allocation solution found in Section 3.1.

5. Conclusions

In this paper, we have investigated the optimal resource allocation for a hybrid three-node relay network where the source, with the help of a relay node, communicates to the destination via multiple orthogonal channels (MBRCs). In particular, we have studied joint optimal power and bandwidth allocation strategies that maximize the bounds on the capacity, which results in a max-min optimization problem. We have solved this problem using a supporting plane technique [15]. In particular, we have provided sufficient conditions for when this max-min optimization problem can be solved using this technique. It is worthwhile to mention that these sufficient conditions are general enough so that other utility functions that rely on SNR can be considered as well as the information theoretic rates considered in this paper.

For $(3, 2)$-MBRC, we have found the joint power and bandwidth allocation. We have observed that the upper and lower bounds approach each other as the source-to-relay channel condition improves as compared to the source-to-destination channel condition, and joint power and bandwidth optimization always yields better performance than power optimization only.

Our numerical results have also investigated the scenario where a new link at the source becomes available for an existing frequency division relay network, and the power and bandwidth resources are to be reallocated. We have observed that the source node is encouraged to communicate over the best link by dedicating all resources when the new SR link and SD link are better (or worse) than the current SD link and SR link. Otherwise, it is beneficial to share resources between the current link and the new link to achieve the higher rate.

The simple MBRC investigated in this paper can be considered as a building block for more complex hybrid wireless networks. From the system design point of view, we conclude that, for this two-hop, simple network, higher achievable rates can be obtained by optimally allocating resources between multiple standards. It would be of interest to gain an understanding of the set of conditions under which using multiple communication links (standards) and optimal sharing of resources would be beneficial for multihop hybrid wireless networks.

Appendices

A. Proof of Proposition 1

Proof. Suppose that both $G_1(\underline{R})$ and $G_2(\underline{R})$ are nonnegative and concave over convex set C_0. Then, we claim that the optimization problem (14) can be relaxed as follows:

$$\max_{(c_1,c_2)\in\mathbf{B}} \min\{c_1, c_2\}, \tag{A.1}$$

$$\text{where } \mathbf{B} = \text{dominance closure}\{\text{convex closure}\{\mathbf{B}_1\}\}, \tag{A.2}$$

$$\mathbf{B}_1 = \{(G_1(\underline{R}), G_2(\underline{R})) : \underline{R} \in \mathbf{C}_0\}, \tag{A.3}$$

$$\mathbf{C}_0 = \{\text{all feasible values of } \underline{R}\}, \tag{A.4}$$

$$\text{dominance closure}\{\mathbf{A}\} := \text{closure}\left\{\bigcup_{(x,y)\in\mathbf{A}} \{\text{rectangular}(x,y)\}\right\}, \tag{A.5}$$

$$\text{where rectangular}(x, y) = \{(a, b) : 0 \leq a \leq x, 0 \leq b \leq y\}. \tag{A.6}$$

To see that, we devise the following notion of dominance: pair (a, b) is said to be dominated by (c, d) if $a \leq c$ and $b \leq d$. We say that a set $\mathbf{A}_1$ is dominated by the other set $\mathbf{A}_2$, or $\mathbf{A}_1 \lhd \mathbf{A}_2$ if every point in $\mathbf{A}_1$ is dominated by some point in $\mathbf{A}_2$. Since $G_1(\underline{R})$ and $G_2(\underline{R})$ are concave over $\mathbf{C}_0$, we realize that $\mathbf{B}_1$ dominates its convex closure $\overline{\mathbf{B}}_1$. Furthermore, from the definition of dominance closure in (A.5), it is easy to see $\mathbf{B} \lhd \overline{\mathbf{B}}_1$. Since $\mathbf{B} \lhd \overline{\mathbf{B}}_1$ and $\overline{\mathbf{B}}_1 \lhd \mathbf{B}_1$, we have $\mathbf{B} \lhd \mathbf{B}_1$. We note that adding dominated points to $\mathbf{B}_1$ does not change the value of optimization problem (14), which allows us to consider problem (A.1)–(A.6) instead.

Set $\mathbf{B}$ has the following properties. (1) It is a closed convex set. To see that, consider two points in $\mathbf{B}$: $(a_1, b_1) \in$ rectangular (x_1, y_1) and $(a_2, b_2) \in$ rectangular(x_2, y_2). Then we must have $(\lambda a_1 + (1-\lambda)a_2, \lambda b_1 + (1-\lambda)b_2) \in$ rectangular $(\lambda x_1 + (1-\lambda)x_2, \lambda y_1 + (1-\lambda)y_2)$. (2) Consider any supporting plane of this set, which is a line in $\Re^2$ in this case. The slope of this line cannot be both positive and finite. Otherwise, suppose the supporting plane passes through point (x, y) in $\mathbf{B}$, then rectangular (x, y), defined by (A.6), will not be in $\mathbf{B}$.

We then observe that (A.1)–(A.6) must be solved when $c_1 = c_2 \neq 0$ and (c_1, c_2) is at the boundary of $\mathbf{B}$. The maximum of (A.1)–(A.6) should be attained at the boundary of $\mathbf{B}$ since every interior point of $\mathbf{B}$ must be dominated by some point at its boundary. Also, there must be such a point on the boundary with $c_1 = c_2$. We then show that the point with $c_1 = c_2 \neq 0$ must be a local maximal point. This is because any improvement over this point would require increasing c_1, c_2 simultaneously. Suppose such improved point exists. Then it will be strictly separated from set $\mathbf{B}$ by the support plane passing through (c_1, c_2), since no supporting plane has finite positive slope. Since $\mathbf{B}$ is a closed convex set and $\min\{x, y\}$ is a concave function over $\Re^2$, any local maximum must be globally optimal [18, Theorem 3.4.2, page 125-126]. This completes our claim that optimality must be attained at the diagonal line. We observe that this point may not be in $\mathbf{B}_1$. Therefore, it may not be parameterizable by $\underline{R}$. This necessitates consideration of the three cases as we will show later.

Since optimality must be attained at the boundary of the convex set $\mathbf{B}$, we observe that (A.1)–(A.6) is equivalent to (A.7) for a certain β:

$$\max_{(c_1,c_2)\in\mathbf{B}} \beta c_1 + (1-\beta)c_2, \quad 0 \le \beta \le 1. \tag{A.7}$$

(A.7) can be solved by examining three cases: $\beta = 0$, $\beta = 1$, $0 < \beta < 1$.

For $\beta = 1$ and $\beta = 0$, we have

$$\max_{\underline{R}\in\mathbf{C}_0} G_1(\underline{R}), \quad \beta = 1; \qquad \max_{\underline{R}\in\mathbf{C}_0} G_2(\underline{R}), \quad \beta = 0. \tag{A.8}$$

For $0 < \beta < 1$, we prove below that (A.7) is equivalent to (A.9). Let the optimal solution of (A.7) be (c_1^*, c_2^*). Since $0 < \beta < 1$, (c_1^*, c_2^*) cannot be dominated by any point in $\mathbf{B}$ other than itself. (If such a point exists, it would be separated from $\mathbf{B}$ by the supporting plane passing through (c_1^*, c_2^*).) Since $\mathbf{B} \lhd \mathbf{B}_1$ and $\mathbf{B} \supset \mathbf{B}_1$, we must have $(c_1^*, c_2^*) \in \mathbf{B}_1$. This means that we can solve (A.9) instead:

$$\max_{(c_1,c_2)\in\mathbf{B}_1} \beta c_1 + (1-\beta)c_2, \quad 0 < \beta < 1. \tag{A.9}$$

Since all points in $\mathbf{B}_1$ can be parameterized by $\underline{R}$, problem (A.9) is equivalent to (A.10). β is adjusted until the solution to (A.10) yields $G_1(\underline{R}) = G_2(\underline{R})$:

$$\max_{\underline{R}\in\mathbf{C}_0} \beta G_1(\underline{R}) + (1-\beta)G_2(\underline{R}), \quad 0 < \beta < 1. \tag{A.10}$$

In summary, we proved that there must exist a $\beta \in [0,1]$ such that (14) is equivalent to (A.7) if $G_1(\underline{R})$ and $G_2(\underline{R})$ are nonnegative and concave over convex set $\mathbf{C}_0$. Depending on the value of β, we find that there are three cases for the max-min optimization (A.7) as given in Proposition 2.

B. Optimal Resource Allocation

Proof. We provide the proof for optimal resource allocation for $(3,2)$-MBRC. To do so, we utilize the following theorem which we restate here with our notation for convenience. □

Theorem 3 ([19, Proposition 3.9, pages 219–221]). *Suppose that $F : \mathfrak{R}^{\mathrm{n}} \mapsto \mathfrak{R}$ is continuously differentiable and concave on the set X, a Cartesian product of sets X_i, where each X_i is a closed convex subset of $\mathfrak{R}^{\mathrm{n_i}}$ $(n_1 + \cdots + n_m = n)$. Furthermore, suppose that for $\mathbf{x} = \{\mathbf{x_1}, \ldots, \mathbf{x_m}\}$, $\mathbf{x_i} \in \mathfrak{R}^{\mathrm{n_i}}$, $F(\mathbf{x})$ is a strictly concave function of each $\mathbf{x_i}$, when the other components of $\mathbf{x}$ are held constant. Let $\{\mathbf{x}(t)\}$ be the sequence generated by the iterative algorithm obtained by optimizing $F(\mathbf{x})$ over one vector variable at a time. Then, every limit point of $\{\mathbf{x}(t)\}$ maximizes $F(\mathbf{x})$ over X.*

For a fixed $0 \le \beta^* \le 1$, the objective function of our max-min problem corresponds to

$$F(\mathbf{x}) = \beta^* C_1(\mathbf{x}) + (1-\beta^*)C_2(\mathbf{x}), \tag{B.1}$$

where $\mathbf{x}$ corresponds to $\underline{R} = (\alpha_1, \alpha_2, \phi_1, \phi_2, \phi_3)$.

Based on Proposition 3, we know that the objective function (B.1) of our max-min optimization problem given in (21) is concave over the constraint set X, which is a Cartesian product of the following two sets, X_1 and X_2, which are closed convex subset of $\mathfrak{R}^{\mathrm{n_i}}$ $(n_1 = 2, n_2 = 3)$:

$$X_1 = \left\{\boldsymbol{\alpha} = (\alpha_1, \alpha_2) : 0 \le \alpha_1, \alpha_2 \le 1, \sum_{i=1}^{2} \alpha_i = 1\right\},$$
$$X_2 = \left\{\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3) : 0 \le \phi_1, \phi_2, \phi_3 \le 1, \sum_{i=1}^{3} \phi_i = 1\right\}. \tag{B.2}$$

Also, for the interior points of the feasible region of $\underline{R}$, we note that (B.1) is strictly concave over $\boldsymbol{\alpha}(\boldsymbol{\phi})$ when $\phi(\boldsymbol{\alpha})$ are fixed; see Remark 2. We note that the objective function (B.1) is not differentiable at the boundary of the feasible region, that is, for $\phi_i = 0$, $i = 1, 2, 3$. Therefore, we cannot directly apply Theorem 3. We define a modified feasible region as follows:

$$X_{1_{\text{new}}} = \left\{\boldsymbol{\alpha} = (\alpha_1, \alpha_2) : \varepsilon \le \alpha_1, \alpha_2 \le 1, \sum_{i=1}^{2} \alpha_i = 1\right\}, \tag{B.3}$$

$$X_{2_{\text{new}}} = \left\{\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3) : \varepsilon \le \phi_1, \phi_2, \phi_3 \le 1, \sum_{i=1}^{3} \phi_i = 1\right\} \tag{B.4}$$

with a small $\varepsilon > 0$. Since (B.1) is continuously differentiable over the new feasible region given by (B.3) and (B.4), we can now apply Theorem 3. Thus, we can devise an iterative algorithm to maximize the function, by maximizing over $\boldsymbol{\alpha}$ for fixed $\boldsymbol{\phi}$ ($\boldsymbol{\phi}$ for fixed $\boldsymbol{\alpha}$) and then maximizing $\boldsymbol{\phi}$ for fixed $\boldsymbol{\alpha}$ ($\boldsymbol{\alpha}$ for fixed $\boldsymbol{\phi}$). The iterative method, by Theorem 3, will converge to the maximizer of $F(\mathbf{x})$ over (B.3) and (B.4), which is the global optimal. If the iterative algorithm converges to a point that is at the boundary of the new feasible region given by (B.3) and (B.4), we need to reduce ε further and repeat the iteration using the KKT points from the previous iteration as the initial point. By repeating this procedure, the iterative algorithm converges to the global optimal solution.

For Case 1 ($\beta^* = 1$ in (B.1)) and Case 2 ($\beta^* = 0$ in (B.1)), we provide the closed form solution below. For Case 3 ($0 \le \beta^* \le 1$ in (B.1)), we provide the iterative algorithm in Appendix C.

Case 1. $\underline{R}^*$ maximizes $C_1(\underline{R})$. Using $\alpha_1 = 1 - \alpha_2 = \alpha$ and $\phi_2 = 1 - \phi_1 - \phi_3$, $C_1(\underline{R})$ in (21) can be rewritten as

$$\begin{aligned} C_1(\alpha, \phi_1, \phi_3) &= \phi_1 \log\left(1 + \frac{\alpha\eta_1}{\phi_1}\right) \\ &\quad + (1 - \phi_1 - \phi_3) \log\left(1 + \frac{(1-\alpha)\eta_2}{(1 - \phi_1 - \phi_3)}\right) \\ &\quad + \phi_3 \log\left(1 + \frac{\rho_3}{\phi_3}\right). \end{aligned} \tag{B.5}$$

Differentiating (B.5) with respect to ϕ_1, we obtain $\phi_1 = \alpha\eta_1(1-\phi_3)/(\alpha\eta_1 + (1-\alpha)\eta_2)$. Observe that $0 \le \phi_1 \le 1$ for $0 \le \alpha \le 1$ and $0 \le \phi_3 \le 1$. Substituting this expression for ϕ_1 into (B.5), we obtain

$$\begin{aligned} C_1(\alpha, \phi_3) = (1 - \phi_3) \log\left(1 + \frac{\alpha\eta_1 + (1-\alpha)\eta_2}{(1-\phi_3)}\right) \\ + \phi_3 \log\left(1 + \frac{\rho_3}{\phi_3}\right). \end{aligned} \tag{B.6}$$

For fixed ϕ_3, we can maximize (B.6) in terms of α by

$$\alpha^* = \begin{cases} 1 & \text{if } \eta_1 > \eta_2, \\ 0 & \text{if } \eta_1 < \eta_2. \end{cases} \tag{B.7}$$

Substituting (B.7) into (B.6), we have

$$C_1(\phi_3) = (1-\phi_3) \log\left(1 + \frac{\max(\eta_1, \eta_2)}{(1-\phi_3)}\right) + \phi_3 \log\left(1 + \frac{\rho_3}{\phi_3}\right). \tag{B.8}$$

Maximizing (B.8) with respect to ϕ_3 leads to $\rho_3/(\rho_3 + \max(\eta_1, \eta_2))$. The optimal power allocation parameter in (B.7) indicates that one of channels whose received SNR at the destination is smaller is not used. Thus, for the case where $\eta_1 > \eta_2$, $\phi_2^* = 0$. On the other hand, for the case where $\eta_1 < \eta_2$, $\phi_1^* = 0$. Thus, using $\sum_{i=1}^{3} \phi_i = 1$, the optimal resource allocation parameter for Case 1 is given by (26). By Remark 1, this is the global optimal solution. The condition of the received SNRs given by (27) and (28) for Case 1 to occur can be readily found by substituting (26) into (25).

Case 2. $\underline{R}^*$ maximizes $C_2(\underline{R})$. By applying the same technique as in Case 1 , we find the optimal resource allocation parameters given in (30). By Remark 1, this is the global optimal solution.

C. Iterative Algorithm for Case 3

Proof. For Case 3, $\underline{R}^*$ maximizes the following.

$$C(\beta^*, \underline{R}) = \beta^* C_1(\underline{R}) + (1 - \beta^*) C_2(\underline{R}), \quad 0 \le \beta^* \le 1 \tag{C.1}$$

Since the closed form solution does not exist for case 3, we rely on the iterative algorithm given in Theorem 3. As we noted in Appendix B, (C.1) is not differentiable at the boundary of the feasible region. Thus, we start with the new feasible region given in (B.3) and (B.4). Then, the Lagrangian is

$$\begin{aligned} \mathcal{L} = \beta^* &\left(\sum_{i=1}^{2} \phi_i \log\left(1 + \alpha_i \frac{\eta_i}{\phi_i}\right) + \phi_3 \log\left(1 + \frac{\rho_3}{\phi_3}\right)\right) \\ &+ (1 - \beta^*) \left(\phi_1 \log\left(1 + \alpha_1 \frac{\kappa}{\phi_1}\right) + \phi_2 \log\left(1 + \alpha_2 \frac{\nu}{\phi_2}\right)\right) \\ &- \lambda\left(\sum_{i=1}^{3} \phi_i - 1\right) - \mu\left(\sum_{i=1}^{2} \alpha_i - 1\right) + \sum_{i=1}^{3} \omega_i \phi_i + \sum_{i=1}^{2} \psi_i \alpha_i, \end{aligned} \tag{C.2}$$

where λ and μ are Lagrange multipliers corresponding to sum constraints for bandwidth and power allocation, respectively. ω_i and ψ_i are inequality constraints for bandwidth and power allocation, respectively, that is, $\alpha_i \ge \varepsilon$, $i = 1, 2$, and $\phi_i \ge \varepsilon$, $i = 1, 2, 3$. For a fixed β^*, we start with values of ϕ_1, ϕ_2, and ϕ_3 such that $\sum_{i=1}^{3} \phi_i = 1$ and find the optimal $\boldsymbol{\alpha}$ such that $\alpha_1 + \alpha_2 = 1$. In iteration n, we update $\boldsymbol{\alpha}(n)$ by optimizing the objective function over $\boldsymbol{\alpha}$ while keeping the total power constraints satisfied, and fixing $\boldsymbol{\phi} = \boldsymbol{\phi}(n-1)$. In iteration $n+1$, $\boldsymbol{\phi}(n+1)$ is found by optimizing the objective function with respect to $\boldsymbol{\phi}$ while keeping the bandwidth constraints satisfied, and fixing $\boldsymbol{\alpha} = \boldsymbol{\alpha}(n)$. By Remark 1 and 2, and Theorem 3, this algorithm converges to the global optimal solution. Note that the optimal solution $(\boldsymbol{\alpha}^*, \boldsymbol{\phi}^*)$ satisfies (see Proposition 2)

$$\begin{aligned} \sum_{i=1}^{2} \phi_i^* \log\left(1 + \alpha_i^* \frac{\eta_i}{\phi_i^*}\right) + \phi_3^* \log\left(1 + \frac{\rho_3}{\phi_3^*}\right) \\ = \phi_1^* \log\left(1 + \alpha_1^* \frac{\kappa}{\phi_1^*}\right) + \phi_2^* \log\left(1 + \alpha_2^* \frac{\nu}{\phi_2^*}\right). \end{aligned} \tag{C.3}$$

□

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