Rank Constrained Temporal-Spatial Matrix Filters for CDMA Systems

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***Abstract*—Efficient interference suppression techniques are needed to maximally utilize the potential gains of code-division multiple-access systems. In this letter, a receiver structure which combines multiuser detection (temporal filtering) and receiver beamforming (spatial filtering) in a multipath environment is considered. Following previous work, we model the receiver as a linear matrix filter and use the minimum mean-squared error (MMSE) as the performance criterion. Motivated by the high complexity of the optimum receiver, we propose rank constrained temporal-spatial filters which are simpler and near-optimum. The MSE is minimized subject to a structural constraint, using an iterative algorithm based on alternating minimization. The constraint on the receiver matrix filter narrows down the solution space, which helps to solve the optimization problem more efficiently. The constraint can be set appropriately by the system designer to achieve the desired tradeoff between performance and complexity. Numerical results indicate that a performance close to that of the optimum filter can be achieved with a simple iterative structure, even in highly loaded systems. Adaptive implementation of the rank constrained filters is derived. A new adaptive scheme is proposed which is a combination of the alternating minimization and the least mean squares methods. The convergence properties are investigated along with the effect of the number paths.**

***Index Terms*—Minimum mean-squared error (MMSE), multiuser detection, temporal-spatial filtering.**

I. Introduction

FUTURE wireless systems are expected to be more reliable and provide higher data rate services. Code-division multiple-access (CDMA) has become a popular choice in wireless applications due to its potential to successfully accommodate these future demands. It is well known that CDMA systems suffer from multiaccess interference (MAI), which degrades performance. A number of techniques are proposed to combat MAI, both at the transmitter and the receiver. In this letter, we focus on two receiver signal-processing-based methods that combat MAI: multiuser detection (temporal filtering) and receiver beamforming (spatial filtering).

Multiuser detection [3] exploits the inherent structure of MAI to suppress it effectively. The high computational complexity of the joint maximum likelihood multiuser detector resulted in the investigation of a number of suboptimum detectors [4]. One of the near optimum detectors is the minimum mean-squared error (MMSE) detector, designed to minimize the MSE between the filter output and the information bit.

Antenna arrays have long been utilized to increase the capacity of wireless systems: beamforming separates the desired signal from interfering signals that originate from different locations [5]. It is shown in [6] that for flat-Rayleigh-fading channels, interferers can be nulled out with a cost of one degree of freedom per user. The capacity increase that is achieved with base-station antenna arrays in CDMA systems is shown in [7], where perfect instantaneous power control and matched filters are assumed for each user.

Combined temporal-spatial filtering was considered in [2] and the necessary statistics were derived along with several multiuser detectors. A recent paper examines several two-dimensional MMSE linear filter structures in a single path environment [8]. It is shown that the joint optimum temporal-spatial filter (OTSF) achieves the highest signal-to-interference ratio (SIR). The complexity of the OTSF led the authors to propose rank-1 constrained filters that employ the same temporal multiuser detector at each antenna element [8].

The rank-1 constrained filter's simplicity is appealing from an implementation point of view, but the performance of such filters may be far from that of the optimal filter due to the constrained solution space, especially under heavy loads. Motivated by the performance gap between the OTSF and the rank-1 constrained filter, in this letter, we search for filter structures whose performance lie between that of the OTSF and the rank-1 constrained filter and which can exploit the diversity provided by the multipath environment. We propose a general class of rank constrained filters, which are found subject to a structural constraint on the receiver filter. The strictness of the constraint determines the resulting filter's performance and complexity; the constraint can be relaxed in order to achieve a near optimum performance, at the expense of additional complexity.

Following the approach in [8], we express the temporal-spatial filter of a user as a receiver matrix filter, which performs joint multiuser detection, receiver beamforming and multipath combining. The MSE, with the appropriate rank constraint, is iteratively minimized using an alternating minimization algorithm. We first derive the deterministic iterations, which assume the knowledge of all users' parameters. Motivated by the fact that adaptive implementations require little information to track the constantly changing parameters in a wireless environment, we consider such implementations in Section IV. An adaptive algorithm which is a combination of the alternating minimization and the least mean squares (LMS) is formulated and the parameters that affect the convergence are explained.

II. System Model

We consider the uplink of a single cell direct-sequence (DS)-CDMA system in a multipath environment with K active users

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and the processing gain is N. An antenna array of M elements is employed at the receiver. The transmitted signal passes through a multipath channel with the impulse response given by

$$h_k(t) = \sum_{l=1}^{L} h_{k,l}\delta(t - \tau_{k,l}) \quad (1)$$

where $h_{k,l}$ and $\tau_{k,l}$ are the complex channel coefficient and the delay associated with path l of user k. Channel coefficients are modeled as independent zero-mean complex Gaussian random variables such that the amplitude and the phase have Rayleigh and uniform distributions, respectively. For simplicity, each user's channel is assumed to be composed of exactly L paths and the delays are assumed to be chip synchronous, i.e., $\tau_{k,l} = (l-1)T_c$. The received signal at the output of the antenna array is given by

$$\mathbf{r}(t) = \sum_{k=1}^{K}\sum_{l=1}^{L} h_{k,l}\sqrt{P_k}b_k s_k(t - \tau_{k,l} - \nu_k)\mathbf{a}_{k,l} + \mathbf{n}(t) \quad (2)$$

where P_k, b_k, and $s_k(t)$ represent the transmit power, information bit, and the signature waveform of user k; ν_k is the delay of user k; and $\mathbf{a}_{k,l}$ is the array response vector of path l of user k. The received signal $\mathbf{r}(t)$ is chip matched filtered and sampled over the entire observation interval of $(N+L-1)$ chips. In the sequel, we will concentrate on the desired user i, and assume that the receiver is synchronized to the first path of the desired user such that $\nu_i = 0$. The resulting data for the nth bit period of the desired user, can be arranged in a $(N+L-1) \times M$ dimensional matrix $\mathbf{R}$

$$\mathbf{R}(n) = \sum_{k=1}^{K}\sqrt{P_k}\left[b_k(n-1)\mathbf{S}_k^{(-1)} + b_k(n)\mathbf{S}_k + b_k(n+1)\mathbf{S}_k^{(+1)}\right]\mathbf{H}_k\mathbf{A}_k^T + \mathbf{N} \quad (3)$$

where $\mathbf{H}_k$ is the diagonal matrix of channel coefficients; $\mathbf{A}_k = [\mathbf{a}_{k,1}, \cdots, \mathbf{a}_{k,L}]$ is the matrix of array response vectors of user k; and $\mathbf{S}_k^{(-1)}$, $\mathbf{S}_k$, and $\mathbf{S}_k^{(+1)}$ are the $(N+L-1) \times L$-dimensional delayed signature sequence matrices. Despite being the most general model with asynchronous users and intersymbol interference (ISI), (3) adds little insight in terms of the performance and complicates the derivations. Therefore, we will assume synchronous users, i.e., $\nu_k = 0$ for all k, and that $\tau_{k,l} \ll T_s$ such that ISI can be ignored in the sequel. Hence, the received signal over the observation interval becomes

$$\mathbf{R} = \sum_{k=1}^{K}\sqrt{P_k}b_k\mathbf{S}_k\mathbf{H}_k\mathbf{A}_k^T + \mathbf{N} \quad (4)$$

where $\mathbf{S}_k$ is the $(N+L-1) \times L$ dimensional signature sequence matrix of user k, whose lth column is given by $[\mathbf{0}_{l-1}, s_k[1], \ldots, s_k[N], \mathbf{0}_{L-l}]^T$. The $\mathbf{N}$ matrix represents the spatially and temporally white noise $E[N_{kl}^* N_{mn}] = \sigma^2\delta_{km}\delta_{ln}$ where $(\cdot)^*$ denotes the conjugate operation. A linear matrix filter $\mathbf{X}_i$ is used to compute the decision statistic y_i and the bit decision of the desired user i is made by taking the sign of the real part of y_i

$$y_i = \sum_{n=1}^{N+L-1}\sum_{m=1}^{M}[X_i]_{nm}^* R_{nm} = tr\left(\mathbf{X}_i^H\mathbf{R}\right) \quad (5)$$

where $tr(\cdot)$ and $(\cdot)^H$ denote the trace and hermitian operations, respectively.

III. Previous Work

The optimum temporal-spatial filter $(\bar{\mathbf{X}}_i)$ minimizes the mean-squared error between the decision statistic and the information bit. Note that $\bar{\mathbf{X}}_i$ also achieves the highest SIR among all possible linear matrix filters [1]. $\bar{\mathbf{X}}_i$ is found as

$$\bar{\mathbf{X}}_i = \arg\min_{\mathbf{X}} E\left[\left|tr(\mathbf{X}^H\mathbf{R}) - b_i\right|^2\right]. \quad (6)$$

After reformulating the optimization problem with vector variables, the solution is given by [1], [4], [8]:

$$\bar{\mathbf{x}}_i = \sqrt{P_i}\left(\sum_{k=1}^{K} P_k\mathbf{q}_k\mathbf{q}_k^H + \sigma^2\mathbf{I}\right)^{-1}\mathbf{q}_i \quad (7)$$

where $\mathbf{q}_k$ is the long vector obtained by stacking up columns of $\mathbf{Q}_k = \mathbf{S}_k\mathbf{H}_k\mathbf{A}_k^T$. To reconstruct $\mathbf{X}_i$, we take every $(N+L-1)$ element of $\mathbf{x}_i$ and place as a column to $\mathbf{X}_i$. From (7), it is seen that finding the optimum filter requires the inversion of an $(N+L-1)M \times (N+L-1)M$-dimensional matrix which can be computationally costly. Motivated by this complexity of OTSF, the authors of [1] proposed a simpler receiver. In this case, the filter space, i.e., the solution space of the optimization problem, is constrained to contain filters of rank 1 only

$$\mathbf{X}_i = \arg\min_{\mathbf{X}} E\left[\left|tr(\mathbf{X}^H\mathbf{R}) - b_i\right|^2\right] \quad s.t. \quad \text{rank}(\mathbf{X}) = 1. \quad (8)$$

Note that any $\mathbf{X}_i$ of rank 1 can be decomposed as $\mathbf{X}_i = \mathbf{c}_i\mathbf{w}_i^T$, where $\mathbf{c}_i$ and $\mathbf{w}_i$ are $N+L-1$ and M-dimensional vectors, respectively. The optimization problem is expressed in terms of $\mathbf{c}_i$ and $\mathbf{w}_i$ as

$$[\bar{\mathbf{c}}_i, \bar{\mathbf{w}}_i] = \arg\min_{\mathbf{c}_i,\mathbf{w}_i} E\left[\left|\mathbf{c}_i^H\mathbf{R}\mathbf{w}_i^* - b_i\right|^2\right]. \quad (9)$$

A closed form expression for the minimizer of MSE does not exist and the MSE is not jointly convex in both vector variables. Fortunately, it is convex for a single variable when the other variable is fixed. An alternating minimization-based iterative algorithm was proposed in [1] and [9]. It was observed that with power control the performance of rank-1 constrained filters for single path channels were near optimum [8].

IV. Rank Constrained Temporal-Spatial Filters

Rank-1 constrained filter's simplicity is an advantage, but because of the tight constraint on the solution space, its performance is strictly suboptimal as compared to the OTSF's. This performance difference could be quite pronounced in heavily loaded systems: as the number of interfering users increase with respect to the dimensions provided by the temporal and spatial domains, the solutions found in the constrained space of rank-1 filters become more inadequate [10]. Under such conditions, filters whose performance lie between that of OTSF and the rank-1 constrained filter may be desired. We propose to achieve this performance increase by replacing the rank constraint with a looser version. By relaxing the constraint, the solution space will expand (including the matrices of rank 1) and the filters found in this new larger space will possibly perform better. Here, we will investigate the general class of rank $-r$[1] constrained filters, where $1 \leq r \leq \min\{N+L-1, M\}$. In other words, the solution space of the optimization problem will be the space of up to rank r matrices in $C^{N+L-1 \times M}$.

$$\mathbf{X}_i = \arg\min_{\mathbf{X}} E\left[\left|tr(\mathbf{X}^H\mathbf{R}) - b_i\right|^2\right]$$
$$\text{s.t. } \operatorname{rank}(\mathbf{X}) \leq r, \quad 1 \leq r \leq \min\{N+L-1, M\}. \quad (10)$$

Note that any matrix filter whose rank is less than or equal to r can be expressed in terms of at most r temporal-spatial filter pairs: $\mathbf{X}_i = \sum_{j=1}^{r} \mathbf{c}_{ij}\mathbf{w}_{ij}^T$. With this new representation, the MSE can be expressed as

$$\begin{aligned}\text{MSE} = &\sum_{l=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{K} P_k \mathbf{c}_{il}^H \mathbf{Q}_k \mathbf{w}_{il}^* \mathbf{w}_{ij}^T \mathbf{Q}_k^H \mathbf{c}_{ij} \\ &+ \sigma^2 \sum_{l=1}^{r}\sum_{j=1}^{r} \left(\mathbf{c}_{il}^H \mathbf{c}_{ij}\right)\left(\mathbf{w}_{il}^H \mathbf{w}_{ij}\right) \\ &- 2\sum_{j=1}^{r} \sqrt{P_i}\Re\left\{\mathbf{c}_{ij}^H \mathbf{Q}_i \mathbf{w}_{ij}^*\right\} + 1. \end{aligned} \quad (11)$$

As in the case with the rank-1 constrained filter, there is no closed form expression for the minimizer of (11) and MSE is not jointly convex in all vector variables. Note that in this case, we have $2r$ vector variables $\{\mathbf{c}_{i1}, \ldots, \mathbf{c}_{ir}, \mathbf{w}_{i1}, \ldots, \mathbf{w}_{ir}\}$. Fortunately, since MSE is convex for each variable given that the remaining $2r-1$ variables are fixed, alternating minimization approach will be used here as well to iteratively minimize the MSE. The expanded solution space causes the increase in the number of variables, and consequently, each step of the iterative algorithm will consist of $2r$ substeps. At each substep of the algorithm, a single variable is updated to minimize the MSE, while the remaining $2r-1$ variables are fixed. At the next substep a different variable is updated and this procedure continues in a round-robin fashion until convergence. With some abuse of notation let $\text{MMSE}(\{\mathbf{c}_{ij}\}_{j\neq x}, \{\mathbf{w}_{ij}\}_{j=1}^{r})$ and $\text{MMSE}(\{\mathbf{c}_{ij}\}_{j=1}^{r}, \{\mathbf{w}_{ij}\}_{j\neq x})$ denote values of $\mathbf{c}_{ix}$ and $\mathbf{w}_{ix}$ that minimize the MSE given that the remaining $2r-1$ variables are fixed. After setting the gradient equal to zero and solving for vector variables, the following equations are found:

$$\begin{aligned}\hat{\mathbf{c}}_{ix} &= \text{MMSE}\left(\{\mathbf{c}_{ij}\}_{j\neq x}, \{\mathbf{w}_{ij}\}_{j=1}^{r}\right) \\ &= \left(\sum_{k=1}^{K} P_k \mathbf{Q}_k \mathbf{w}_{ix}^* \mathbf{w}_{ix}^T \mathbf{Q}_k^H + \sigma^2 |\mathbf{w}_{ix}|^2 \mathbf{I}\right)^{-1} \\ &\times \left(\sqrt{P_i}\mathbf{Q}_i \mathbf{w}_{ix}^* - \sum_{j\neq x}^{r}\left(\sum_{k=1}^{K} P_k \mathbf{Q}_k \mathbf{w}_{ix}^* \mathbf{w}_{ij}^T \mathbf{Q}_k^H \right.\right. \\ &\qquad \left.\left. + \sigma^2 \mathbf{w}_{ix}^H \mathbf{w}_{ij} \mathbf{I}\right) \mathbf{c}_{ij}\right) \end{aligned} \quad (12)$$

$$\begin{aligned}\hat{\mathbf{w}}_{ix} &= \text{MMSE}\left(\{\mathbf{c}_{ij}\}_{j=1}^{r}, \{\mathbf{w}_{ij}\}_{j\neq x}\right) \\ &= \left(\sum_{k=1}^{K} P_k \mathbf{Q}_k^T \mathbf{c}_{ix}^* \mathbf{c}_{ix}^T \mathbf{Q}_k^* + \sigma^2 |\mathbf{c}_{ix}|^2 \mathbf{I}\right)^{-1} \\ &\times \left(\sqrt{P_i}\mathbf{Q}_i^T \mathbf{c}_{ix}^* - \sum_{j\neq x}^{r}\left(\sum_{k=1}^{K} P_k \mathbf{Q}_k^T \mathbf{c}_{ix}^* \mathbf{c}_{ij}^T \mathbf{Q}_k^* \right.\right. \\ &\qquad \left.\left. + \sigma^2 \mathbf{c}_{ix}^H \mathbf{c}_{ij} \mathbf{I}\right) \mathbf{w}_{ij}\right). \end{aligned} \quad (13)$$

The convergence of the overall algorithm is guaranteed by the fact that each substep guarantees to decrease the MSE function which is bounded below. Moreover, due to the nonconvexity of the MSE (11), the global minimum is not attained by a unique filter pair. This can be observed by noting that for any nonzero value of β, all filter pairs $[\beta\mathbf{c}_{ix}, \mathbf{w}_{ix}/\beta]$ will produce the same MSE. As a final remark, we note that since the MSE is possibly multimodal, there is no guarantee for the iterative algorithm to converge to the global optimum MSE value. To avoid getting stuck at local optima, we must refrain from starting our algorithm from obvious undesirable points such as filters that are orthogonal to the signal space. For our numerical results, we started the algorithm with randomly generated filters. We have observed that, experimentally our full-rank filter ($r = \min\{N+L-1, M\}$) always converged to the MMSE value that OTSF achieved. For the case of $1 \leq r < \min\{N+L-1, M\}$, the randomly initialized filter coefficients always converged to the same MSE value.

V. Adaptive Implementations

In this section, adaptive implementations of the rank constrained filters will be formulated. The adaptive implementation that we propose here will be a combination of the alternating minimization approach of the previous section and the least mean squares (LMS) algorithm. While keeping the main structure of the alternating minimization algorithm, each substep will be treated as an independent LMS problem. Because of the stochastic nature of LMS, in principle infinite iterations are required to reach the optimal point, whereas in the deterministic case, the same is accomplished with a single update [(12) and (13)]. Since it is not feasible to wait for such long periods, for each substep, we will only use B training bits. When the algorithm moves on to the next substep, the MSE function (error surface) will be changed and a new LMS algorithm will begin. Therefore, the same set of training bits could be reused, for a more efficient use of the resources, bandwidth, and time. The classic update rule of LMS is given by [11]

$$\mathbf{w}_i(n+1) = \mathbf{w}_i(n) + \mu\left[b_i(n) - y_i(n)\right]^* \mathbf{u}(n) \quad (14)$$

where $\mathbf{w}_i(n)$, μ, $b_i(n)$, $y_i(n)$, and $\mathbf{u}(n)$ represent the filter estimate, step size, the desired response, decision statistic, and the

[1] The term *rank*, which refers to the rank of the matrix filter here, should not be confused with the rank term in the *reduced-rank* algorithms, which is associated with the second order statistics of the filter input. The performance comparison of the two approaches can be found in [10].

received signal respectively. Applying this rule to our case results in the following equations:

$$\mathbf{c}_{ix}(n+1) = \mathbf{c}_{ix}(n) + \mu \left[b_i(n) - y_i(n)\right]^* \mathbf{R}(n)\mathbf{w}_{ix}^*(n) \tag{15}$$

$$\mathbf{w}_{ix}(n+1) = \mathbf{w}_{ix}(n) + \mu \left[b_i(n) - y_i(n)\right]^* \mathbf{R}^T(n)\mathbf{c}_{ix}^*(n). \tag{16}$$

Note that, for stability, we need $\mu < 2/\lambda_{\max}$, where $\lambda_{\max}$ is the maximum eigenvalue of the covariance matrix of the input vector [11]. In our case, μ can be appropriately selected by noting that $\mathbf{R}\mathbf{w}_{ix}^*$ and $\mathbf{R}^T\mathbf{c}_{ix}^*$ can be treated as the input vectors while updating $\mathbf{c}_{ix}$ and $\mathbf{w}_{ix}$, respectively.

VI. Simulation Results and Discussion

We consider a single-cell synchronous CDMA system that employs a base station antenna array. Signature sequences, direction of arrivals (DOA), and channel coefficients are randomly generated for each run. The results are time averages of 100 runs. We assume a uniform linear array of antennas equispaced at half a wavelength. We assume that both the signature sequences and array responses are of unit energy. For all experiments, the SNR level of the desired user is set to 10 dB. Channel coefficients are zero mean complex Gaussian variables, normalized such that $E[|h_{k,l}|^2] = 1$. For different experiments, the SIR of the desired user is plotted to investigate the behaviors of the algorithms. Note that with appropriate scaling, the MSE and the SIR of every linear filter $(\mathbf{X})$ can be related [1], [8].

We first consider a system with processing gain $N = 16$ and $M = 8$ array elements. There are $K = 40$ users, each with $L = 3$ paths and to simulate a near-far environment, interfering users' powers are set to 10 dB higher than the desired user. The output SIR is plotted in Fig. 1 using (12) and (13). It is seen that, for each rank constrained filter, each iteration increases the SIR monotonically, which indicates that the alternating minimization algorithm is working as expected. It is also clear that, filters with higher maximum allowable rank perform better: rank-4 and rank-2 filters converge faster and achieve a higher SIR compared to the rank-1 filter. However, this performance improvement comes with a price of complexity increase. In the rank-1 case, at each iteration, two variables are updated, whereas in the rank-4 case, this number goes up to eight. It can also be observed that near optimum performance can be achieved with rank-4 filters.

The effect of the number of paths on the performance is shown in Fig. 2, where maximum SIRs achieved at the convergence point of the algorithm are plotted versus the number of paths. It is seen that the proposed filters perform better with increasing multipath diversity: increasing the number of paths has the effect of increasing the "effective" temporal signature length. In a way, the effective temporal signature length is $N + L - 1$, which improves the performance.

Fig. 3 compares the adaptive implementations of different rank constrained filters with the LMS implementation of OTSF. Blocks of $B = 10$ training bits are used at each substep of the alternating minimization algorithm such that each block of ten bits corresponds to one step (iteration index) of the deterministic case. An appropriate step size (μ) has been chosen for the implementation of the rank constrained filters. Equations (15) and (16) are used to obtain the results. Similar to their deterministic counterparts, filters with higher maximum allowable rank perform better: the max SIR achieved increases with respect to the maximum allowable rank of $\mathbf{X}$. We note that, for this system, our experiments showed that, for small step sizes, i.e., at the expense of convergence speed, the adaptive OTSF can achieve an SIR that is as much as 3 dB higher than the rank-4 filter.

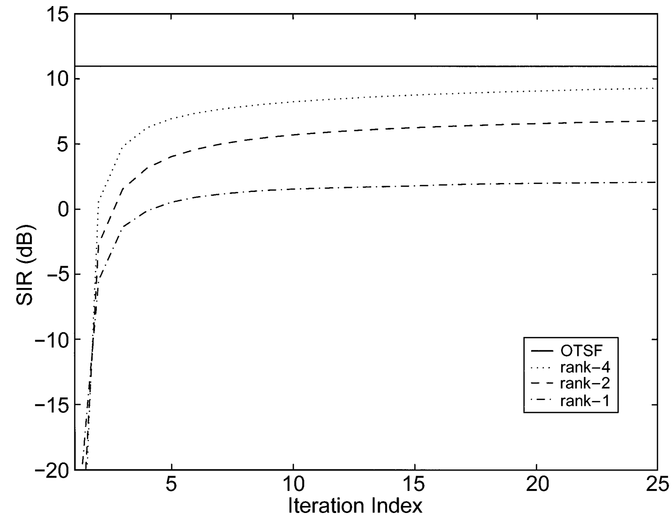

Fig. 1. $K = 40, N = 16, M = 8$, and $L = 3$.

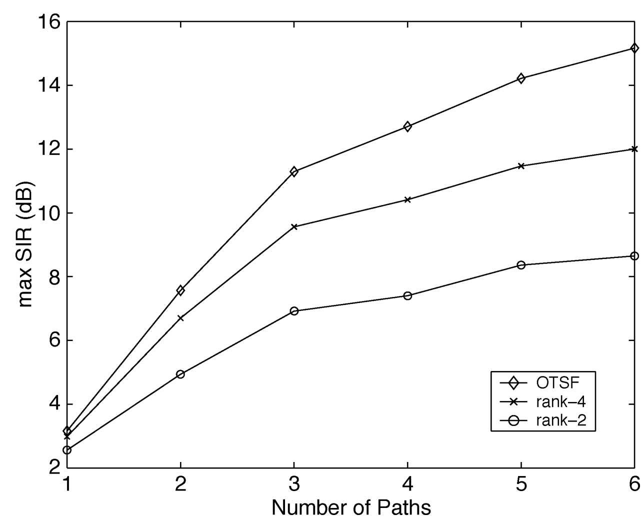

Fig. 2. $K = 40, N = 16$, and $M = 8$.

The effect of μ on the convergence speed of the algorithm is clearly observed in Fig. 4, where the curve corresponding to a higher step size converge faster, but the residual error at the convergence point is higher. On the contrary, the curve with a smaller μ converge slower, but at the convergence point it achieves a smaller MSE.

In Fig. 5, the SIR performance of the rank-4 filter is plotted in different multipath environments. It is seen that an increase in the multipath diversity causes an improvement in performance.

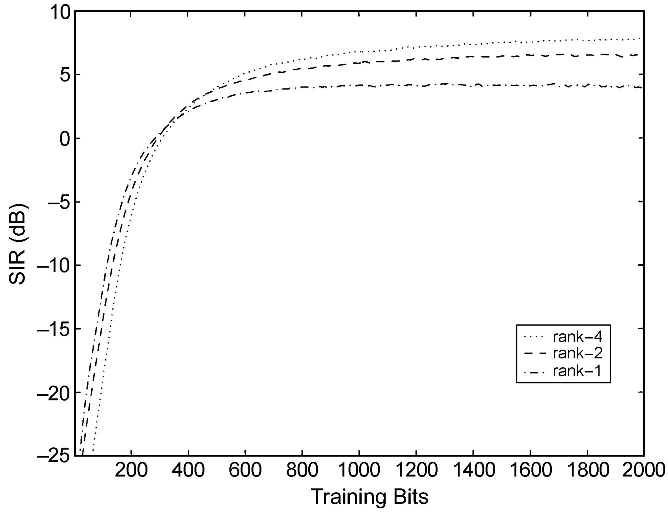

Fig. 3. $K = 40, N = 16, M = 8, L = 3$, and $\mu = 0.01$.

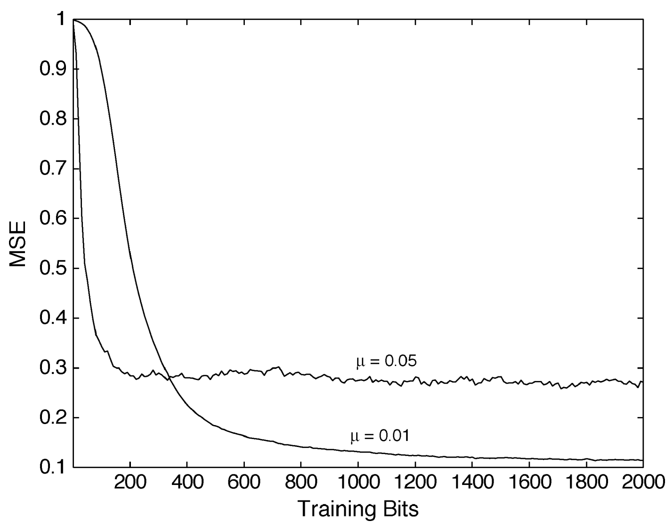

Fig. 4. $K = 40, N = 16, M = 8, L = 3$, rank($\mathbf{X}$) $= 2$, and $B = 10$.

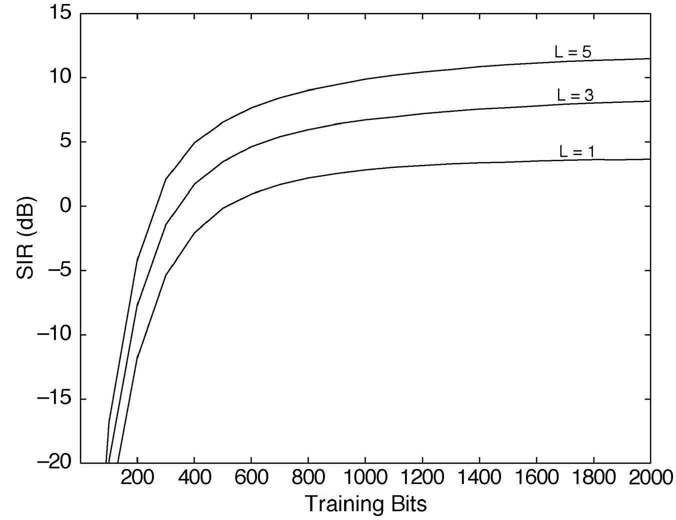

Fig. 5. $K = 40, N = 16, M = 8, \mu = 0.01$, rank($\mathbf{X}$) $= 4$, and $B = 100$.

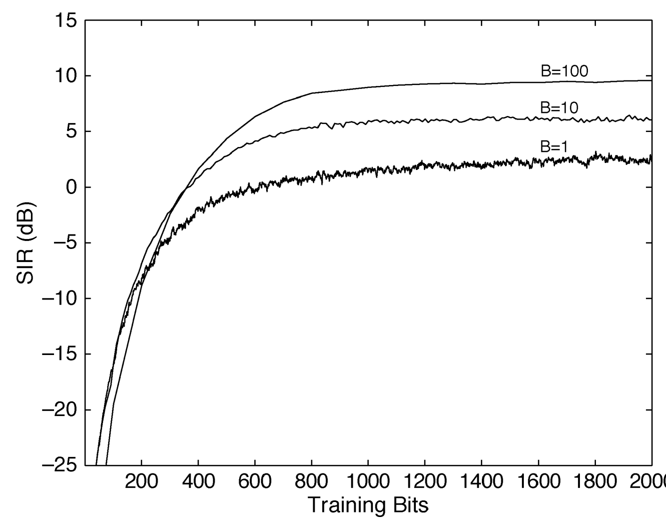

Fig. 6. $K = 20, N = 16, M = 8, L = 3$, rank($\mathbf{X}$) $= 4$, and $\mu = 0.01$.

This result can be explained be the fact that the "effective" temporal signature length increases as a result of the increase in the number of paths.

The effect of B on the convergence is investigated in Fig. 6. In this scenario, we consider a system with $K = 20$ users, $N = 16$ processing gain, $M = 8$ antenna elements, and the interfering users are 10 dB stronger than the desired user. Note that for a duration of 1000 b, the curve with $B = 100$ corresponds to an alternating minimization algorithm with ten steps where each step uses 100 b, $B = 1$ case, on the other hand, is an alternating minimization algorithm of 1000 steps with only 1 b per step. Even though the latter case has more steps of alternating minimization, because each step performed poorly as compared to the former, it converges to a lower SIR value.

VII. Conclusion

In this letter, we have proposed the rank constrained filters and derived their adaptive counterparts in multipath channels. The main motivation of the work is the suboptimal performance of rank-1 constrained filters in heavily loaded systems and the need for adaptive algorithms that could be implemented in a wireless scenario. It is shown that with a looser rank constraint, better performance can be achieved at the expense of additional complexity. Even in heavily loaded systems, where there is a significant performance gap between OTSF and rank-1 filter, near full-rank performance can be achieved with a mild increase in complexity with respect to the rank-1 constrained filter. Adaptive implementation of the proposed filters based on a combination of the LMS and the alternating minimization is formulated and its convergence properties are investigated. The resulting adaptive algorithm is able to suppress interference and perform multipath combining without the need for any knowledge of interferers' parameters or the knowledge of the channel of the desired user. Thus, the need for channel estimation for the desired user (other than the timing of the first arrival path) is alleviated. The new adaptive algorithm exhibits the classic convergence properties with respect to the step size, μ. Also, a new and equally important convergence parameter B, the number of training bits to be used in each subiteration, is introduced. In both the deterministic and the adaptive cases, the existence of multiple paths provides diversity and by appropriate combining included in our filter structures, we are able to improve performance. We conclude by noting that interference management capability of the system can be further enhanced by combining the rank constrained filters with power control algorithms [1].

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