Low-Latency Communications over Zero-Battery Energy Harvesting Channels

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***Abstract*—We study the fundamental performance limits of energy harvesting channels with short-length channel codes that expend less processing energy and facilitate low-latency communications. In particular, we examine the zero-battery case, i.e, energy harvesting transmitters with no energy storage such as passive RFID tags, in which energy must be spent as it arrives or is lost. To analyze practical finite-length channel codes, we develop a second-order approximation to the communication rate for such channels with energy information causally known at the transmitter. We present two binary examples for which we explicitly calculate the channel capacity and channel dispersion and interestingly observe that a slight increase in the energy arrival probability can significantly boost the achievable rate, and further that the rate loss due to energy intermittency is more pronounced for channels with lower noise levels.**

I. Introduction

Energy harvesting enables green communications and extended wireless network lifetimes, as transmitters can continuously acquire the energy needed for their operation from external energy sources including nature, man-made sources, or even communicating devices [1]. Networking with energy harvesting nodes has been studied extensively in recent years identifying transmission (and reception) strategies, where it has been shown that energy arrivals as well as energy storage limitations impact system design, see for example [1]–[6] and many others.

More recently, energy arrivals at the channel use level has been considered in a variety of energy arrival and storage models highlighting these same issues. For infinite energy storage in a Gaussian channel, reference [7] shows that, the capacity of the energy harvesting channel is equal to the conventional Gaussian channel capacity. For a zero-battery Gaussian channel, the same authors observe discrete capacity achieving inputs in [8]. References [9], [10] study the binary energy harvesting channel with unit battery where the noiseless channel enables an equivalent timing channel representation and capacity. More recent information-theoretic capacity studies include Gaussian energy harvesting channels with finite battery models considering deterministic arrivals [11] and approximate capacity [12], and discrete memoryless energy harvesting channel with receiver side battery or energy arrival information [13], [14]. All of these studies consider the conventional capacity with blocklength tending to infinity.

Since practical channel codes always have finite and usually moderate lengths, the classical asymptotic analysis may fall short of describing the complete behavior of practical systems. In particular, the immediately available energy that can be used for transmission of a symbol is at a premium for energy harvesting nodes, and quantifying the performance with finite-length channel coding is a step of paramount importance towards future deployment of energy harvesting communication networks. Moreover, short-length codes are structurally less complex and require smaller processing energy, which is crucial to energy-limited applications. Such codes also enable faster information transfer and facilitate low-latency communications. These appealing practical advantages of finite-length codes, however, come at the cost of smaller communication rates.

The fundamental analysis of the back-off in coding rate from the conventional channel capacity as a result of using finite-length codes has emerged as a new trend in information theory. References [15], [16] have proved the following *second-order* or *normal* approximation to the channel coding rate for standard discrete and Gaussian channels,

$$R(n, \epsilon) \approx C - \sqrt{\frac{V}{n}} Q^{-1}(\epsilon), \tag{1}$$

for moderate values of blocklength n and any target block error probability $0 < \epsilon < 1$, where C is the channel capacity, V is the so-called *channel dispersion*, and $Q^{-1}(\cdot)$ is the functional inverse of the complementary Gaussian distribution function, $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-t^2/2) dt$. The approach has since been employed for more elaborate systems including multiple access [17] and fading [18] channels.

For energy harvesting channels, however, the only related work to date is reference [19] from 2014, where a binary *noiseless* energy harvesting channel with *infinite* energy storage is considered, and a finite blocklength achievable rate with $O(\log n/\sqrt{n})$ back-off from capacity is proved. The achievability strategy used therein from [7] relies on the ability to store infinite energy indefinitely.

In this work, we take the next step in the direction of finite-length channel codes for energy harvesting channels and consider the *noisy* energy harvesting channel. We focus on another extreme of energy storage as compared to [19], where the node intermittently acquires, but does not store energy. Such batteryless nodes are already used in near-field communications, e.g. passive RFID tags [20] that serve as our model motivation. For such a setup, we analyze the finite blocklength behaviour. In particular, we develop achievable rates with

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$O(1/\sqrt{n})$ back-off from capacity, similar to (1). The finite blocklength analysis uses the fact that the zero-battery energy harvesting channel is a channel with memoryless state that is causally known at the transmitter, as formalized in Section II. Thus, we state and prove our second-order achievability using Shannon strategies [21]. We present two binary examples in Section IV that provide insights into energy harvesting at finite blocklength. We conclude the paper with some remarks and future directions.

Notation: We use lower case $x^n := (x_1, \cdots, x_n)$ and upper case $X^n := (X_1, \cdots, X_n)$ to denote n-length deterministic and random sequences, respectively. So, for a sequence s^n and some $t \in \{1, \cdots, n\}$, the t-th symbol is denoted by s_t and the first t-length subsequence $(s_1, \cdots, s_t)$ by s^t.

II. Problem Statement

The communication protocol over a discrete memoryless channel with zero-battery energy harvesting can be modeled as follows. At time $t = 1, 2, \cdots$, $E_t \in \mathcal{E} := \{0, \cdots, |\mathcal{E}|-1\}$ units of energy arrives at the transmitter according to an independent and identically distributed (i.i.d.) random process, i.e., we have $|\mathcal{E}|$ different possible values for the arrived energy value. The communication channel has a discrete input alphabet $\mathcal{X}$ and a discrete or continuous output alphabet $\mathcal{Y}$ but always satisfying $0 \in \mathcal{Y}$. For each energy level arrived at the transmitter, only a certain subset of the input alphabet becomes available for information transmission, and the remainder of input values simply correspond to a 0 in the output; the larger the energy level the more options available to the transmitter to choose the input from. In particular, inspired by *amplitude modulation* considerations and to indicate the energy significance of the input letters, we consider an input alphabet of the form $\mathcal{X} = \{0, \cdots, |\mathcal{E}|-1\}$ with the following transmission model: If no energy is harvested at time t, $E_t = 0$, no communication can take place and all input symbols $x \in \mathcal{X}$ lead to the output 0 with probability 1; if $E_t = e$ units of energy is available, for some $1 \leq e \leq |\mathcal{E}| - 1$, then a general transition probability law $P^{(e)}_{Y|X}(y|x)$ governs the channel from inputs $x \in \{0, \cdots, e\}$ to output $y \in \mathcal{Y}$, and for the remainder of inputs $e < x \leq |\mathcal{E}| - 1$ the output is set to 0 with probability 1, i.e., $P^{(e)}_{Y|X}(0|x) = 1$. In the binary case, for example, we have $|\mathcal{E}| = 2$ so that with $E_t = 1$, one bit can be transmitted over the binary-input memoryless channel $P^{(1)}_{Y|X}(y|x) := P_{Y|X}(y|x)$, $x \in \mathcal{X} = \{0, 1\}$. Although $P^{(e)}_{Y|X}(y|x)$ for different values of e are not related in general, they can belong for instance to the elements of a single underlying channel law. Finally, no battery is available so the harvested energy cannot be stored for future use, hence any unused energy is wasted.

One readily observes that, this energy harvesting setup corresponds to a memoryless channel $P_{Y|XS}(y|x, s)$ with i.i.d. memoryless state $s \in \{0, \cdots, |\mathcal{E}| - 1\}$ which is known *causally* at the transmitter. Here, the state represents the i.i.d. energy arrived at the transmitter, whose previous and current (but not future) levels are known prior to the transmission of each symbol, hence the term "causally". In particular, the state $s = 0$ corresponds to the no-energy case $E = 0$ so that $P_{Y|XS}(y|x, 0) = 1\{y = 0\}$ i.e., all input symbols x are mapped to $y = 0$ with probability 1. On the other hand, the state $s = e$ represents the case of $1 \leq e \leq |\mathcal{E}| - 1$ unit energy arrival for which $P_{Y|XS}(y|x, e) = P^{(e)}_{Y|X}(y|x)$.

An $(n, 2^{nR})$ code for this channel consists of a message set $\mathcal{M} = \{1, \cdots, 2^{nR}\}$, an encoder $x_t(m, s^t) : \mathcal{M} \times \mathcal{S}^t \to \mathcal{X}$ for all $m \in \mathcal{M}$ and $t = 1, \cdots, n$, and a decoder $\hat{m} : \mathcal{Y}^n \to \mathcal{M} \cup \{\mathcal{I}\}$ that, upon receiving y^n, assigns an estimate message $\hat{m} \in \mathcal{M}$ or declares an error $\mathcal{I}$. The average error probability of the code is defined as

$$\Pr[\mathcal{E}] := \frac{1}{2^{nR}} \sum_{m=1}^{2^{nR}} \Pr[\hat{M} \neq m | M = m]. \tag{2}$$

A rate R is called ϵ-achievable if there exists a sequence of $(n, 2^{nR})$ codes such that $\Pr[\mathcal{E}] \leq \epsilon$ as $n \to \infty$. The supremum of all ϵ-achievable rates is called the ϵ-capacity and denoted by C_ϵ, and capacity is defined as the limit of C_ϵ as $\epsilon \to 0$.

Shannon has shown that the capacity of such channels is achieved by code-functions (instead of codewords) of the form $x(u, s)$ which map auxiliary codewords $u^n(m)$ and the state s^n to the channel input x^n, so following the definition above we have $x_t(m, s^t) = x(u_t(m), s_t)$ [21]. Consequently, such code-functions are known as the Shannon strategies [22]. In the following, we first prove a second-order achievable rate for such channels, and then use this result to shed some light on zero-battery energy harvesting with short-length codes.

III. Shannon Strategies at Finite Blocklength

In this section, we state a second-order achievable rate for discrete memoryless channels with causal state information at the transmitter via Shannon strategies. Our result will use the notation $i(u; y)$ for the *information density* defined as [23], [24]

$$i(u; y) := \log \frac{P_{Y|U}(y|u)}{P_Y(y)}, \tag{3}$$

where the transition probability $P_{Y|U}(y|u)$ is defined using the Shannon strategy

$$P_{Y|U}(y|u) = \sum_s P_S(s) P_{Y|XS}(y|x(u, s), s), \tag{4}$$

and $P_Y(y)$ is the corresponding marginal

$$P_Y(y) = \sum_u P_U(u) P_{Y|U}(y|u). \tag{5}$$

Once a pair of random variables (U, Y) is substituted as the arguments of the information density, $i(U; Y)$ becomes a random variable. In particular, the mean of the information density is the average mutual information, $\mathbb{E}[i(U; Y)] = \mathbb{I}(U; Y)$ [25].

We are now ready to state our result.

Theorem 1. *For a discrete memoryless channel $P_{Y|XS}(y|x, s)$ with causal state information (CSI) at the transmitter, all rates R satisfying*

$$R \leq C_{CSI} - \sqrt{\frac{V_{\epsilon,CSI}}{n}} Q^{-1}(\epsilon) + O\left(\frac{1}{n}\right), \tag{6}$$

are ϵ-achievable, where the corresponding capacity and dispersion are defined as

$$C_{CSI} := \max_{P_U(u),x(u,s)} \mathbb{E}[i(U;Y)], \tag{7}$$

$$V_{\epsilon,CSI} := \begin{cases} \min_{(P_U(u),x(u,s))\in\Pi} \mathrm{Var}[i(U;Y)] & \text{if } \epsilon < 1/2 \\ \max_{(P_U(u),x(u,s))\in\Pi} \mathrm{Var}[i(U;Y)] & \text{if } \epsilon > 1/2 \end{cases} \tag{8}$$

and Π is the set of capacity-achieving input pairs

$$\Pi := \{(P_U(u), x(u,s)) : \mathbb{E}[i(U;Y)] = C_{CSI}\}. \tag{9}$$

Remark 1. The expression (8) for channel dispersion ensures that, depending upon the sign of $Q^{-1}(\epsilon)$, the smallest possible penalty or the largest possible reward is considered for the finite blocklength performance, if multiple capacity-achieving input pairs $(P_U(u), x(u,s))$ are available. However, if all such pairs lead to the same channel dispersion value or if the capacity-achieving input pair is unique, we obtain a simplified expression

$$V_{\epsilon,\mathrm{CSI}} = \mathrm{Var}[i(U;Y)] \tag{10}$$

for all error probabilities $0 < \epsilon < 1$.

Remark 2. The cardinality of the auxiliary random variable in Theorem 1 can be bounded as $|\mathcal{U}| \le \min\{(|\mathcal{X}|-1)|\mathcal{S}| + 2, |\mathcal{Y}|+1\}$, using the convex cover method of [22, Appendix C]. In the energy harvesting model of this paper, however, the choice of $x(u, s=0)$ is irrelevant to the capacity and dispersion calculations, since the output for the case of $E = 0$ is deterministically set to 0. Therefore, one can strengthen this bound to $|\mathcal{U}| \le \min\{(|\mathcal{E}|-1)^2 + 2, |\mathcal{Y}|+1\}$.

Proof: (Theorem 1) The proof combines Shannon strategies [21], [22] with the finite blocklength analysis of [15], [16] for discrete memoryless channels, as follows. Fix the pair $(P_U(u), x(u,s))$ that achieves the capacity C_{CSI} and dispersion $V_{\epsilon,\mathrm{CSI}}$, that is,

$$\mathbb{E}[i(U;Y)] = C_{\mathrm{CSI}}, \quad \mathrm{Var}[i(U;Y)] = V_{\epsilon,\mathrm{CSI}}. \tag{11}$$

Generate $M = 2^{nR}$ sequences $\{u^n(m)\}_{m=1}^M$ randomly and independently according to the i.i.d. distribution $P_U(u)$. To send message m, the encoder transmits $x_t = x(u_t(m), s_t)$ at time $t = 1, 2, \cdots$ i.e., only the current state s_t is used which is known at the transmitter. The decoder, upon observing the output y^n, finds the first $1 \le \hat{m} \le 2^{nR}$ such that

$$\frac{1}{n} i(u^n(\hat{m}); y^n) > R. \tag{12}$$

Note that, an error occurs if the true input $u^n(m)$ does not pass the threshold test in (12) or if another competing wrong input $u^n(\bar{m})$ with $\bar{m} \neq m$ passes the threshold test in (12). Due to the symmetry of the random code, the error probability of the random codebook can then be bounded as

$$\Pr[\mathcal{E}] \le \Pr\left[\frac{1}{n} i(U^n; Y^n) \le R\right] + 2^{nR} \Pr\left[\frac{1}{n} i(\bar{U}^n; Y^n) > R\right], \tag{13}$$

where the probability in the fist term is computed with respect to (w.r.t.) the joint distribution $P_{U^nY^n}(u^n, y^n) = P_{U^n}(u^n)P_{Y^n|U^n}(y^n|u^n)$, but the second term is computed w.r.t. the product distribution $P_{\bar{U}^nY^n}(u^n, y^n) = P_{U^n}(u^n)P_{Y^n}(y^n)$.

Since the input $P_{U^n}(u^n)$ is generated i.i.d. and the channel $P_{Y^n|U^n}(y^n|u^n)$ is memoryless, we can use the central limit theorem (CLT) to analyze the first term and the large deviations for the second term. In particular, using [15, Lemma 47], we find

$$2^{nR} \Pr\left[\frac{1}{n} i(\bar{U}^n; Y^n) > R\right] \le \frac{B_2}{\sqrt{n}}, \tag{14}$$

where B_2 is a positive constant. Moreover, from the Berry-Esseen approximation to CLT [15], we obtain

$$\begin{aligned} \Pr\left[\frac{1}{n} i(U^n; Y^n) \le R\right] &= \Pr\left[\frac{1}{n}\sum_{t=1}^n i(U_t; Y_t) \le R\right] \\ &\le Q\left(\frac{\mathbb{E}\left[\frac{1}{n}\sum_{t=1}^n i(U_t;Y_t)\right] - R}{\sqrt{\mathrm{Var}\left[\frac{1}{n}\sum_{t=1}^n i(U_t;Y_t)\right]}}\right) + \frac{B_1}{\sqrt{n}} \quad (15) \\ &= Q\left(\frac{\mathbb{E}\left[i(U;Y)\right] - R}{\sqrt{\mathrm{Var}\left[i(U;Y)\right]/n}}\right) + \frac{B_1}{\sqrt{n}}, \quad (16) \end{aligned}$$

where B_1 is another positive constant. Combining (13), (14), (16), and (11), we conclude that

$$\Pr[\mathcal{E}] \le Q\left(\frac{C_{\mathrm{CSI}} - R}{\sqrt{V_{\epsilon,\mathrm{CSI}}/n}}\right) + \frac{B_1 + B_2}{\sqrt{n}}. \tag{17}$$

Upper bounding the RHS of (17) by ϵ, we have proved the existence of a code with $\Pr[\mathcal{E}] \le \epsilon$ and rate R satisfying

$$R \le C_{\mathrm{CSI}} - \sqrt{\frac{V_{\epsilon,\mathrm{CSI}}}{n}} Q^{-1}\left(\epsilon - \frac{B_1 + B_2}{\sqrt{n}}\right), \tag{18}$$

which simplifies to (6) via Taylor's theorem and completes the proof of Theorem 1. ■

IV. Binary Energy Harvesting Examples

In this section, we present two binary examples to better illustrate our models and results. In the following examples, for numbers $0 \le p, q \le 1$, we use the binary complement notation $\bar{p} := 1 - p$, the binary entropy function $H_b(p) := -p\log p - \bar{p}\log\bar{p}$, and the binary convolution operation $p * q := p\bar{q} + \bar{p}q$. Note that, all $\log$ and $\exp$ operations are to the base 2.

Example 1. Consider a zero-battery energy harvesting setup as in Section II with a binary noiseless channel, i.e., $\mathcal{X} = \mathcal{Y} = \{0, 1\}$ and $P_{Y|X}(y|x) = 1\{y = x\}$ when $E = 1$, and with energy arrival according to an i.i.d. Bernoulli process with parameter $p := \Pr[E_t = 1]$ which captures the level of energy intermittency in the communication system at hand. For this example, the cardinality bound of Remark 2 gives $|\mathcal{U}| \le 3$, but it turns out that a binary auxiliary $|\mathcal{U}| = 2$ is sufficient; in particular, any optimal ternary auxiliary can be reduced to a binary auxiliary with the same capacity and dispersion by combining two of the symbols. The analysis then reduces to that for a Z channel. It is straightforward to show that the capacity is

$$C_{\mathrm{CSI}} = H_b(pq^*) - q^* H_b(p), \tag{19}$$

where

$$q^* := \left(p\left[1 + \exp\left(\frac{H_b(p)}{p}\right)\right]\right)^{-1}, \tag{20}$$

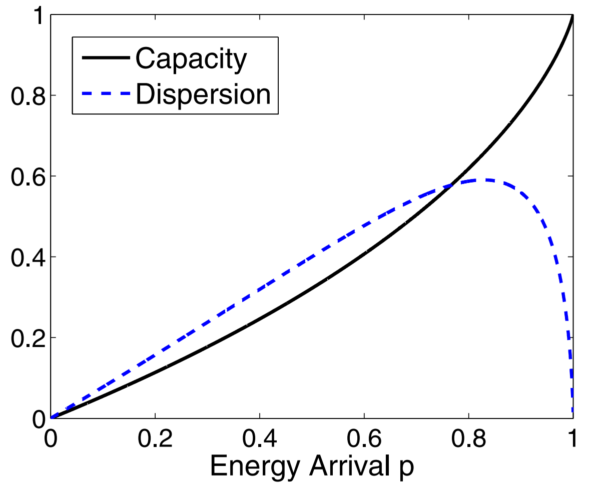

Fig. 1. Capacity and dispersion of Example 1 as a function of the energy arrival probability p.

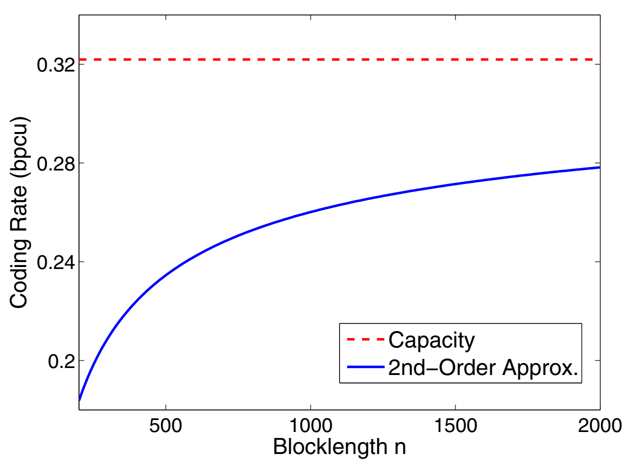

Fig. 2. Comparison of capacity and second-order approximation of the coding rate for Example 1 with energy arrival probability $p = 0.5$ and target error probability $\epsilon = 10^{-3}$.

and is achieved by either

$$x(u=0, s=1) = 0,\ x(u=1, s=1) = 1,\ \Pr[U=1] = q^*, \tag{21}$$

or

$$x(u=0, s=1) = 1,\ x(u=1, s=1) = 0,\ \Pr[U=0] = q^*. \tag{22}$$

Both capacity-achieving selections attain the same dispersion, so that

$$V_{\text{CSI}} = q^* p\bar{p}\ \log^2\left(\frac{1-pq^*}{\bar{p}q^*}\right). \tag{23}$$

Figure 1 illustrates the capacity and dispersion of this model as a function of the energy arrival probability p. A comparison of the asymptotic capacity with the second-order approximation of the coding rate per Theorem 1 is depicted in Figure 2.

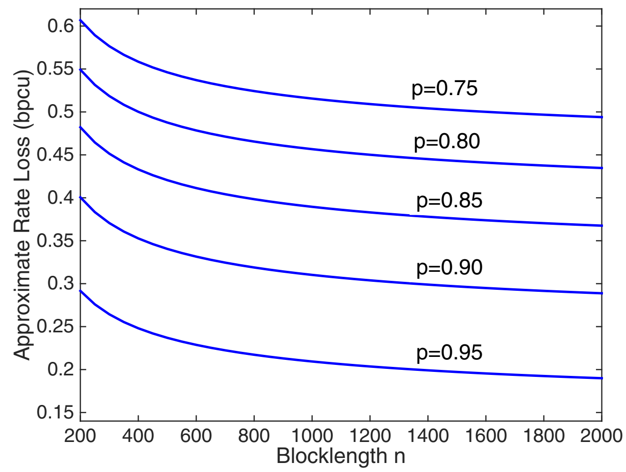

Fig. 3. Approximate rate loss due to energy intermittency for Example 1 with target error probability $\epsilon = 10^{-3}$.

A further comparison can be made between the achievable communication rate over this binary noiseless energy harvesting channel with that of a standard noiseless channel when energy is always available ($p = 1$). Obviously, in the latter case, the transmitter can always send 1 bit of information per channel use for any blocklength n, since no channel coding is needed over such a noiseless channel. This also matches with our expressions (19) and (23) that indicate $C_{\text{CSI}} \to 1$ and $V_{\text{CSI}} \to 0$ when $p \to 1$. Hence, for all energy arrival probabilities $0 \le p < 1$, one can consider

$$1 - \left(C_{\text{CSI}} - \sqrt{\frac{V_{\text{CSI}}}{n}}Q^{-1}(\epsilon)\right) \tag{24}$$

as the approximate rate loss due to the energy intermittency. An approximation of this rate loss via Theorem 1 is illustrated in Figure 3 for several values of p and suggests that, as the energy arrival probability approaches 1, the rate loss diminishes very fast. Furthermore, although the increase of blocklength decreases this rate loss, such an improvement for a greater arrival probability in comparison with smaller arrival probability appears to be almost independent of the operating blocklength.

Example 2. Consider a zero-battery binary energy harvesting channel as in Section II with $\mathcal{X} = \mathcal{Y} = \{0, 1\}$ and a binary symmetric channel (BSC) with crossover probability $0 \le c \le 1/2$ when $E = 1$, and again with energy arrival according to an i.i.d. Bernoulli process with parameter $p := \Pr[E_t = 1]$. The cardinality bound of Remark 2 gives $|\mathcal{U}| \le 3$, but a binary auxiliary $|\mathcal{U}| = 2$ is again sufficient, since any optimal ternary auxiliary can be reduced to a binary auxiliary with the same capacity and dispersion by combining two of the symbols. It is again straightforward to show that the capacity is

$$C_{\text{CSI}} = H_b(pA^*) - q^* H_b(p\bar{c}) - \overline{q^*} H_b(pc), \tag{25}$$

where q^* and A^* satisfy

$$q^* * c = A^* := \left(p\left[1 + \exp\left(\frac{H_b(p\bar{c}) - H_b(pc)}{p\bar{c} - pc}\right)\right]\right)^{-1}. \tag{26}$$

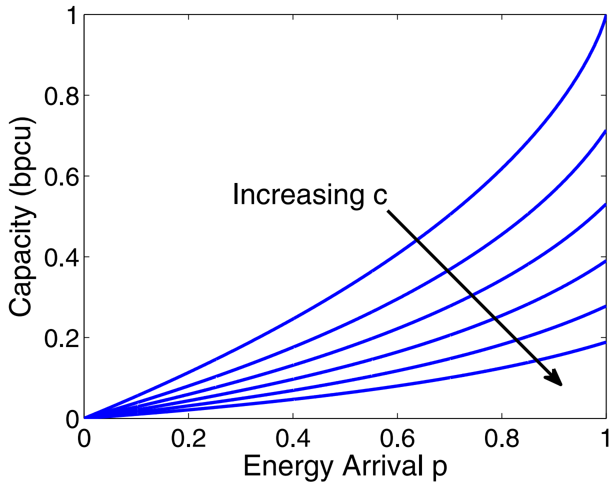

Fig. 4. Capacity of Example 2 as a function of the energy arrival probability p for varying values of the channel noise c.

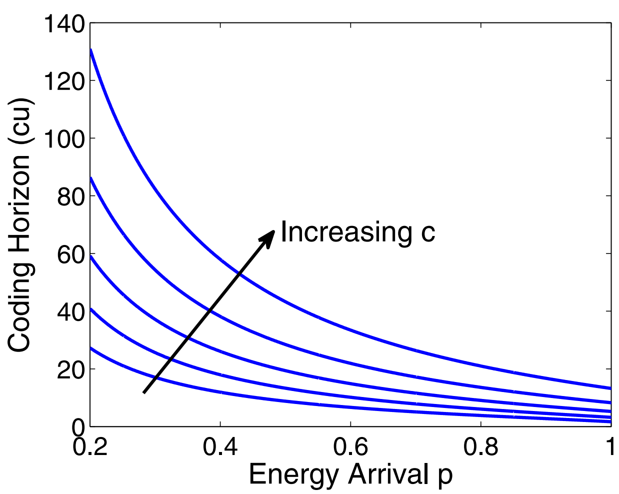

Fig. 5. Coding horizon V/C^2 of Example 2 as a function of the energy arrival probability p for varying values of the channel noise c.

The capacity is achieved by either

$$x(u=0,s=1)=0,\ x(u=1,s=1)=1,\ \Pr[U=1]=q^*, \tag{27}$$

or

$$x(u=0,s=1)=1,\ x(u=1,s=1)=0,\ \Pr[U=0]=q^*. \tag{28}$$

Both capacity-achieving selections attain the same dispersion, so that

$$\begin{aligned} V_{\text{CSI}} = q^*(p\bar{c})(1-p\bar{c})\ \log^2\left(\frac{p\bar{c}}{1-p\bar{c}}\frac{1-pA^*}{pA^*}\right) \\ +\overline{q^*}(pc)(1-pc)\ \log^2\left(\frac{pc}{1-pc}\frac{1-pA^*}{pA^*}\right). \end{aligned} \tag{29}$$

As expected, the capacity and dispersion expressions for crossover probability $c=0$ reduce to the results of the binary noiseless channel of Example 1. One can verify that, as the channel noise c increases, the channel capacity decreases, as illustrated in Figure 4. However, the channel noise also reduces the dispersion (except for a small range of p values close to 1 and c values close to 0). To make a fair judgment of the system behavior, we use the "coding horizon" V/C^2 metric defined by [15], which is related to the blocklength required for achieving a certain fraction η of the capacity:

$$n \approx \frac{V}{C^2}\left(\frac{Q^{-1}(\epsilon)}{1-\eta}\right)^2. \tag{30}$$

Figure 5 shows that the coding horizon for this example indeed increases, as the channel noise increases. For further reference, a comparison of the corresponding asymptotic capacity with the second-order approximation of the coding rate per Theorem 1 is depicted in Figure 6. Comparing Figures 2 and 6 shows that the channel noise has a significant impact on the channel coding rate, both in the asymptotic and finite

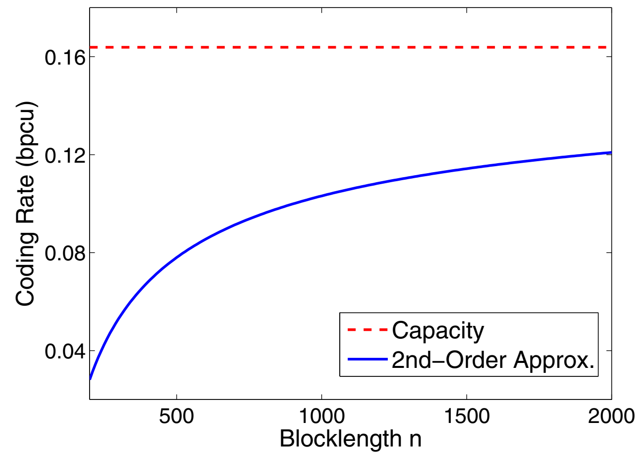

Fig. 6. Comparison of capacity and second-order approximation of the coding rate for Example 2 with energy arrival probability $p=0.5$, crossover probability $c=0.11$, and target error probability $\epsilon=10^{-3}$.

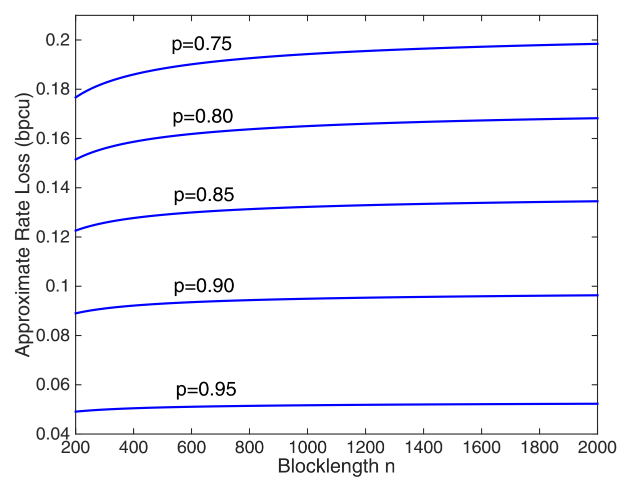

Fig. 7. Approximate rate loss due to energy intermittency for Example 2 with crossover probability $c=0.11$ and target error probability $\epsilon=10^{-3}$.

blocklength regime. In particular, increasing the noise level from $c = 0$ to only $c = 0.11$ almost halves the coding rate.

Analogous to the noiseless case, one can also study the rate loss due to energy intermittency by comparing the achievable communication rate of the binary symmetric energy harvesting channel above with that of the standard BSC when energy is always available ($p = 1$). It is known from [15] that $C_{\text{BSC}} = 1 - H_b(c)$ and $V_{\text{BSC}} = c(1-c)\log^2\left(\frac{1-c}{c}\right)$ which matches with our expressions (25) and (29) as $p \to 1$. Hence, for all energy arrival probabilities $0 \le p < 1$, one can define the approximate rate loss due to energy intermittency as

$$\left(C_{\text{BSC}} - \sqrt{\frac{V_{\text{BSC}}}{n}}Q^{-1}(\epsilon)\right) - \left(C_{\text{CSI}} - \sqrt{\frac{V_{\text{CSI}}}{n}}Q^{-1}(\epsilon)\right). \tag{31}$$

This approximate rate loss is illustrated in Figure 7 for the noise level $c = 0.11$ whose pattern resembles to that for other values of noise level c as well. This figure suggests that, similar to the noiseless case of Example 1, a slight increase in the energy arrival probability leads to a significant improvement in the rate loss. Moreover, in comparison with the noiseless case of Example 1, the rate loss values are much smaller and they further decrease as the noise level increases, since the communication rate over the noisier baseline BSC decreases and makes the effect of energy intermittency less pronounced. We would also like to remark that, the unexpectedly low values of rate loss in Figure 7 for very short blocklengths is an artifact of neglecting third- and lower-order terms in the achievable rate expansion.

V. Concluding Remarks

We have considered the binary energy harvesting channel with zero-battery and short-length channel codes. To facilitate the analysis, we have proved a second-order achievable coding rate for discrete memoryless channels with causal state information, i.e., the energy arrival information, at the transmitter. We have observed, through two binary examples, that the noise level can have a significant impact on the communication rate with short-length codes, and that energy intermittency mostly impacts on channels with smaller noise levels.

A critical remaining issue is to prove a converse for Theorem 1 showing that it is indeed the second-order performance. This is part of our ongoing research, where we are building on the strong converse arguments of [26], [27]. Upon introducing appropriate code functions (resembling Shannon strategies), the channel essentially becomes a regular discrete memoryless channel for which the converse techniques of [15], [16] should apply. However, proving such a tight second-order converse is challenging, similar to many other illusive second-order converse problems in finite blocklength information theory involving auxiliary random variables.

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