Optimum Power Scheduling for CDMA Access Channels*

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Abstract: We consider the system access problem for CDMA networks where the access channels are non-orthogonal to the traffic channels. In such systems, the accessing user can degrade the performance of existing users by introducing extra interference. We study how transmit power should be varied with each successive access attempt so that the linear combination of average power expended and the average delay experienced by the accessing user is minimized. The *harm* caused by the accessing user to the existing communication links is also considered. We find that for all cost functions we consider, the absence of an access deadline implies that each access attempt should be made at some constant power. With an imposed access deadline we find that the optimum sequence of transmit powers can be determined recursively.

1 Introduction

For CDMA networks, access channels use the same carrier frequency as the traffic channels. This is in contrast to FDMA or TDMA systems where access channels occupy separate frequency or time bands. Non-orthogonal access channels introduce new challenges to the accessing user since care must be taken to avoid excessive interference with existing traffic while transmitting access messages with enough power to be heard. In a typical CDMA system, a user attempting to access the system sends a sequence of identical messages to the base station with increasing powers until access is achieved or a number of attempts are unsuccessful. After this timeout, the user must retry after waiting some random amount of time. Thus, there is some impetus to use large transmit powers to gain access quickly and reliably. For example, in the current IS-95 system [1], the power increment during the access attempts may be high, e.g. 3 dB, which may result in high transmit power requirements for the terminal and unacceptable interference to existing users. Therefore, careful planning of transmit power during access is important from the perspective of both existing and accessing users.

It should be noted that the performance of random access channels considering the effects of collisions or blocking on channel throughput and delay has been studied extensively in the literature (eg. [2,3]). Here we determine power schedules for an accessing user which minimize a linear combination of the average total power expended and the average access delay. We also consider *polite* access strategies where the user limits the interference to existing users.

2 General Assumptions

We consider an interference channel where a user tries to access the system by transmitting an access message at a certain power level p. If the access attempt is not successful, the same message is sent again. The probability of accessing the system at any step is a function $f(p)$ of the transmitter power level p. It must be noted here that we assume the channel is known and fixed during access and that access attempts are independent events. We denote the transmit power for the jth access attempt by p_j. The number of access attempts is denoted by the random variable K where $K = k$ if there are $k-1$ access failures followed by a success at the kth attempt. Hence,

$$P\{K=k\} = f(p_k)\prod_{j=1}^{k-1}(1-f(p_j)) \qquad (1)$$

We propose two distinct types of cost structure for the CDMA access channel power scheduling problem. The first is a weighted combination of the total transmitted power, the average access delay, and the disruptive effect of the accessing user on existing users. In this case, the user only stops signaling when access is achieved. The second cost function is simply the total transmitted power expended for access given an access deadline. With either cost function, our objective will be to minimize the expected cost.

It should be noted that we do not explicitly use the random retry time after a failure in expected delay calculations, but rather, use the expected number of access attempts $E[K]$ as a surrogate for the access delay.

2.1 Access without deadlines

We model the disruptive effect of the kth access attempt on the existing communication links by the *harm* function $g(p_k)$. This harm function may depend on the number and quality of existing communication links. Alternatively, we may choose to ignore the harm caused to the existing users by choosing $g(p) = 0$. The cost function is a weighted combination of

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the total transmitted power, the total harm, and the average access delay. With delay weight α, the expected cost is

$$C = \alpha E[K] + E\left[\sum_{j=1}^{K} (p_j + g(p_j))\right] \tag{2}$$

We seek a set of $\{p_k\}$ which minimize the expected cost.

Theorem 1 *The power schedule which minimizes C is a constant power schedule $p_k = p^*$. The optimal constant power p^* minimizes $\frac{p^*+g(p^*)+\alpha}{f(p^*)}$. The associated average access delay is given by $E[K] = 1/f(p^*)$.*

2.2 Access deadlines

In this section, we impose an access deadline. The deadline T is imposed by requiring the accessing user to transmit with very high power to *virtually* guarantee access if access has not been achieved before the deadline. That is, p_T is chosen so that $f(p_T) = 1 - \epsilon$ for a suitably small ϵ. This terminal power p_T is assumed given and included as a penalty in the cost function,

$$C_T = \sum_{k=1}^{T} P\{K = k\} \sum_{j=1}^{k} p_j \tag{3}$$

Given the above cost structure and the power level p_T to be used for the terminal access attempt at step T, we can recursively compute the optimal access schedule.

Theorem 2 *Let the optimal power schedule for deadline $T-1$ be $\{p_k^*(T-1)\}$ with associated mean cost C_{T-1}^*. The optimal schedule for T steps is*

$$p_k^*(T) = \begin{cases} p_{k-1}^*(T-1) & k = 2, 3, \ldots, T \\ p^* & k = 1 \end{cases} \tag{4}$$

where p^* minimizes $p^* + [1 - f(p^*)]C_{T-1}^*$.

The proofs of the theorems are omitted here due to space limitations and can be found in [4].

3 System Model

We assume a DS/CDMA system with processing gain G where there are N users already communicating with a base station using the available traffic channels. The existing users have perfect power control and their transmit power levels will not change during a new user's access attempt. We assume that during the course of access for one user, there will be no other new user access attempts, i.e. the access channel load is low. We also assume that during access the user sends a sequence of 1s to the base station. Lastly, we assume that the access code used is known to both the accessing user and the base station, and that the user is tuned to a downlink paging channel where it can receive information about access parameters.

The propagation delay between the accessing user and the base station is not known and must be determined during access. We assume a coarse acquisition where determining the propagation delay within a chip is sufficient for access purposes. After the access stage, a fine delay tracking algorithm will be implemented when the user starts transmitting actual information bits. Thus, we assume the propagation delay is an integer multiple of the chip duration T_c and is equally likely to be any integer between 0 and $M-1$ chips. We use a parallel acquisition receiver which consists of a bank of M correlators where the ith correlator is matched to the access signature sequence with offset i chips [5]. The coarse acquisition is achieved by applying a threshold test to the output of each matched filter. If the base station determines a user present at a certain delay as a result of the test, an ACK declaring a user's presence is broadcasted. If the base station sends an ACK even though no user is present, a *false alarm* occurs.

We model this detection problem as a binary hypothesis testing problem where the hypotheses are:

- H_1 : The user is trying to access the system
- H_0 : The user is not present

To determine which hypothesis is acting we use a Neyman-Pearson test where a threshold for the decision statistic is set based on a fixed test false alarm probability α_F [6]. Given α_F, one can determine the threshold value Λ against which each of the decision statistics (matched filter outputs) must be compared. In what follows, we determine the detection (successful access) probabilities for two system models that we will consider. In doing so, we use the simplifying approximation that the interference seen due to existing users at the output of each correlator is a Gaussian random variable [7].

The N existing users transmit with power values $\{q_i\}$ and have path gains $\{h_i\}$ to the base station. We assume the existing users are all at 0 chip offset, the accessing user transmits with power level p and has path gain h_a to the base station. The received signal at the base station is:

$$r(t) = I_a\sqrt{ph_a}\, c_a(t - DT_c) + \sum_{i=1}^{N} \sqrt{q_i h_i} b_i(t) c_i(t) + n(t) \tag{5}$$

where I_a is the indicator of user's presence, $c_a(t)$ is the signature sequence of the accessing user, D is the propagation delay in chips, $c_i(t)$ and $b_i(t)$ are the signature sequence and the information bit of the existing user i at time t and $n(t)$ is the Gaussian noise process with power spectral density σ^2.

Given that the access message is L bits long, each correlator the base station uses to compute the decision statistics is an L-bit (LG-chip) correlator. Using the assumption that all existing users transmit random bits with random signature sequences, i.e. all chips generated and the information bits that the existing users transmit are independent and all equally likely to be 1's or -1's, we find that each correlator output, r_j, $j = 0, ..., M-1$, given the delay of the accessing user $D = d$, is a Gaussian random variable with mean and variance

$$E[r_j \mid D=d] = \begin{cases} \sqrt{ph_a}\Gamma_{j,d} & \text{if } H_1 \text{ is acting} \\ 0 & \text{if } H_0 \text{ is acting} \end{cases} \quad (6)$$

$$\sigma^2_{r_j|D=d} = \sigma^2_{\hat{r}} = (L/G)\sum_{i=1}^{N} q_i h_i + L\sigma^2 \quad (7)$$

where

$$\Gamma_{j,d} = \int_{jT_c}^{(LG+j)T_c} c_a(t-jT_c)\; c_a(t-dT_c)dt \quad (8)$$

is the L-bit autocorrelation of the access signature sequence.

The correlator outputs, $\{r_j\}$ are generally correlated and thus not independent. This fact necessitates evaluation of M-fold integrals to calculate the false alarm probability (the test threshold) and the probability of access. However, if the accessing user's codeword is also generated such that each chip is independent and equally likely, i.e. the access codeword is pseudorandom, it can be shown using law of large numbers [8] that the correlations between the correlator outputs approach 0 as the processing gain $G \to \infty$. So, for large processing gains it can be argued that the $\{r_j\}$ are approximately independent. We will use this approximation in calculating the access probabilities in the next section.

3.1 Access Probabilities

We adopt two models for declaring user access. The models use the same parallel acquisition receiver to test user's presence, but differ in physical resources available that are used to verify potential user's propagation delay.

Model 1: This model assumes that an ACK by the base station will be transmitted if one of the decision statistics (r_k) exceeds the threshold and all others ($r_j, j = 0, \ldots k-1, k+1, \ldots, M-1$) are below. In this case, the base station determines the incoming signal's propagation delay as k. If no user is present, then we say a false alarm occurred and we fix the probability of this event to be α_F. If indeed a user is present, a verification[1] process will determine whether k is the correct delay of the accessing user or will reject the value of k. If the verification is not affirmative, further information needed for the user to establish communication with the system is not sent by the base station making the overall access attempt unsuccessful. Thus, access is successful only if correct delay value k is acquired at the base station. This model assumes only one delay value can be verified at a time.

In this case, the false alarm probability is the probability that one of the M matched filter outputs exceeds the threshold and all others are below given no user is present and can be written as

$$\alpha_F = Q(\Lambda_1/\sigma_{\hat{r}})[\overline{Q}(\Lambda_1/\sigma_{\hat{r}})]^{M-1} \quad (9)$$

where $Q(x)$ is the standard normal complementary CDF and $\overline{Q}(x) = 1 - Q(x)$. Since α_F is given, the threshold value Λ_1 can be calculated using Equation 9. Thus, the probability of correct detection, i.e. probability of successfully accessing the system, for our first model is given as

$$f(p) = \frac{1}{M}\sum_{k=0}^{M-1} Q\left[\frac{\Lambda_1 - \sqrt{ph_a}L}{\sigma_{\hat{r}}}\right] \prod_{\substack{i=0 \\ i\neq k}}^{M-1} \overline{Q}\left[\frac{\Lambda_1 - \sqrt{ph_a}\Gamma_{i,k}}{\sigma_{\hat{r}}}\right] \quad (10)$$

Model 2: This model assumes that an ACK by the base station will be transmitted whenever a decision statistic (r_i) exceeds the threshold. In this case, the base station determines the candidates for the incoming signal's propagation delay as all delays whose corresponding decision statistics exceed the threshold. Again, if no user is present, we say a false alarm occurred and we fix the probability of this event to be α_F. If indeed a user is present, a verification process will determine the correct delay k is among the candidates, and reject the access if it is not. Thus, access attempt is successful only if correct delay value k is among the delays acquired at the base station. This model assumes that it may be necessary to verify all M delays simultaneously.

In this case, a false alarm occurs if any of the M matched filter outputs exceed the threshold which has probability

$$\alpha_F = 1 - [\overline{Q}(\Lambda_2/\sigma_{\hat{r}})]^M \quad (11)$$

so that, $\Lambda_2 = \sigma_{\hat{r}}\, Q^{-1}\left(1 - (1-\alpha_F)^{\frac{1}{M}}\right)$. Given the threshold value Λ_2, probability of access in a single access attempt is the probability that the decision statistic that corresponds to the user's actual delay exceeds the threshold which equals

$$f(p) = Q[(\Lambda_2 - \sqrt{ph_a}L)/\sigma_{\hat{r}}] \quad (12)$$

3.2 The Harm Function

The harm function should be representative of the service degradation imposed on existing users by the accessing user. Many formulations of such harm functions are possible. Since the signal to interference ratio (SIR) of an existing user decreases when a new user tries to access the system, we choose the harm function $g(p_j)$ to be the average of the reciprocal of the SIRs $\gamma_i(j)$ of the existing connections i during access attempt j. Given that the existing users transmit information using random signature sequences, the average harm function $g(p_j)$ per access attempt can be written as:

$$g(p_j) = \frac{L}{N}\sum_{i=1}^{N} E[\frac{1}{\gamma_i(j)}] = g_1 + g_2 p_j \quad (13)$$

where

$$g_1 = \frac{L}{N}\sum_{i=1}^{N}\frac{\sum_{k\neq i} q_k h_k \frac{1}{G} + \sigma^2}{q_i h_i}, \quad g_2 = \frac{L}{N}\sum_{i=1}^{N}\frac{\frac{h_a}{G}}{q_i h_i} \quad (14)$$

[1]We assume the verification is done quickly and reliably

Inserting this harm function into Equation (2), the polite-access cost function with this model becomes

$$C = (\alpha + g_1)E[K] + (1 + g_2)E\left[\sum_{j=1}^{K} p_j\right] \qquad (15)$$

4 Results and Conclusions

For numerical results, we construct a system with $N = 10$ existing users and 1 accessing user. Processing gain is $G = 100$ and all traffic channel users have a common SIR target $\gamma^* = 5$ which is achieved with minimum total transmit power [9] by all the existing users before the access. Given these conditions and normalized ambient noise power, $\sigma^2 = 1$, the total interference plus the noise power the access codeword sees per bit is calculated to be $\hat{\sigma} = \frac{1}{L}\sigma_{\hat{r}}^2 = 1.9091$. We assume the receiver false alarm probability to be $\alpha_F = 10^{-5}$ and that the accessing user can have a propagation delay between 0 and 4 chips. Also, for the following numerical examples, we have defined the received power of the accessing user as the product of its transmit power and uplink gain and searched for the optimum *received* power schedules. Thus, the mobile should determine its transmit power by dividing the received power levels (obtained and broadcasted by the base station) by its uplink gain. We first address the case where no access deadlines are given. Consider first the cost function defined by Equation (2) with $g(p) = 0$. The delay factor $\alpha = 0$ implies a minimum at $p^* = 0$. However, we note that this is a degenerate case since receiving an ACK in this case will correspond to a false alarm event and access will never be verified. In Figure 1, we have plotted the optimum power level that should be used to access the system versus the delay weight factor α for both models. We observe that sending longer access messages results in smaller optimum power values. The rationale behind this is it can be shown that as the total energy of the access message increases, the probability of correct detection increases for both models leading to a smaller optimum power value for access. We also observe that the power levels required for Model 1 are lower than those of Model 2. The false alarm event for the first model is defined as the event where one of the decision statistic exceeds the threshold when all others are below where as for the second model it is defined as the event where any number of threshold crossings occur. If the same threshold were used for both models, the second model's false alarm probability would be higher. So, to keep the *same* false alarm performance, one must choose a *higher* threshold for the second test which results in higher power requirements for the second model. The corresponding average delay in both access attempts and in bits are given in Figure 2. Longer access messages, due to the expenditure of higher total energy, result in smaller average number of access attempts for both models. We have also plotted the optimum power values versus the length of the access message in Figure 3 for the cost given by Equation (15). It was observed again that the optimum power is lowered by using longer access messages and the optimum power values required by the first model are lower than that of the second model. Also, same effect on delay as in the previous case is observed (Figure 4). Next we consider the access deadline scheme where the cost to be minimized is given by Equation (3). It is observed that the structure of the optimum schedules as the deadline for access gets larger suggests the use of nearly zero power values for the initial steps, followed by constant power, and lastly high power levels toward the deadline to avoid the termination cost. Thus, the amount of power to be spent as a function of the maximum number of attempts allowed (T) first decreases dramatically, then more slowly as the delay tolerance increases (Figure 5). Similarly, the delay grows slowly first and almost linearly after a while with an increasing deadline (Figure 6). It is observed that different values of ϵ, probability of access failure at T, although result in different p_T values, suggest similar structures and costs. This is due to the fact that for a reasonable number of access attempts allowed, the probability of reaching step T is small and thus the action at this step does not change the policy dramatically. The ϵ used in results shown is 10^{-4}.

To summarize, the numerical results for our models show that the received power to be used during access can be reduced by using longer access messages at the expense of increasing delay. However, since the optimum schedules result in a fairly small number of access attempts, using longer access messages does not introduce unacceptable access delay. We also observe that if an access deadline is present and is lenient enough, the optimum power schedule requires very low energy expenditure for initial steps, resembles a constant schedule for some steps and uses high power levels near the deadline. The theoretical results stated in the paper are valid for any interference channel access power scheduling problem where access attempts are independent. Notably, we have ignored the collisions with other accessing users and assumed that the channel conditions during access are known. What actions must be taken in the absence of these assumptions is also of interest.

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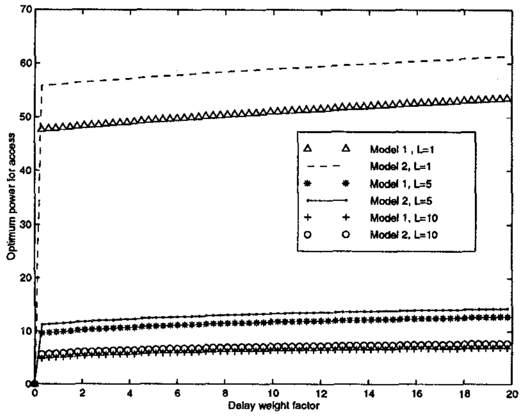

Figure 1: Optimum received power vs. the delay weight factor α.

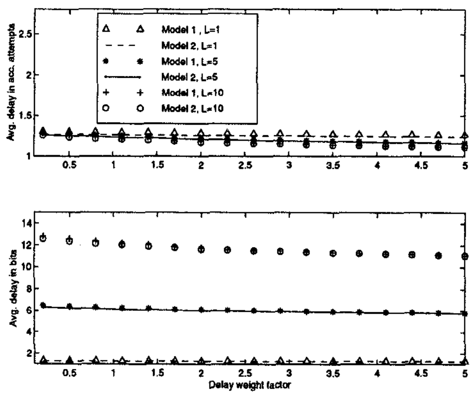

Figure 2: Average delay in access attempts and in bits vs. the delay weight factor α

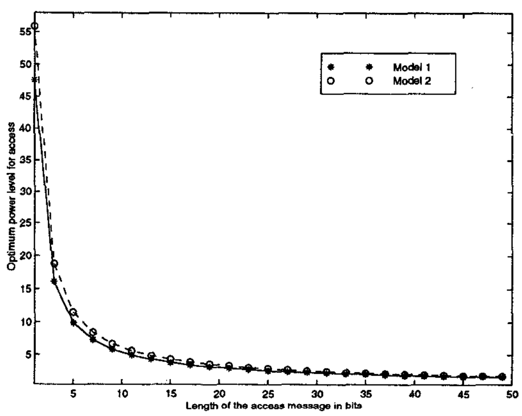

Figure 3: Optimum received power in the presence of the harm function versus L. $\alpha = 0$, $g_1 = 0.2\ L$, $g_2 = 0.0011\ L$.

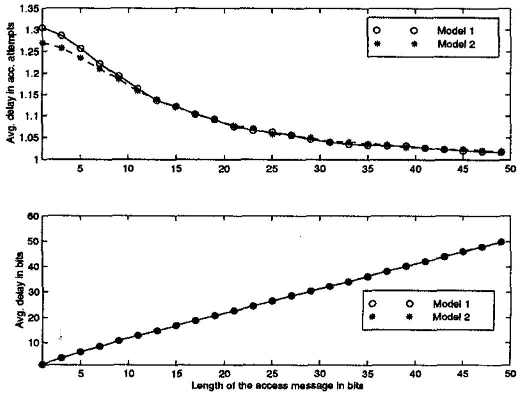

Figure 4: Average delay performance in access attempts and in bits vs L.

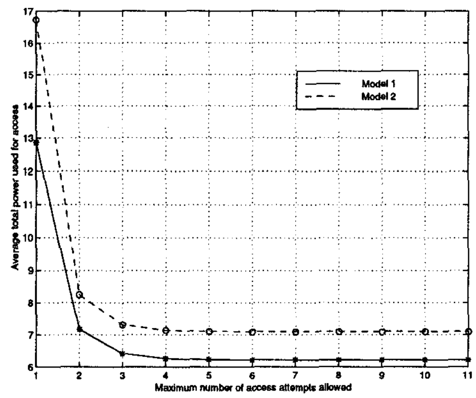

Figure 5: Average total transmit power vs. T.

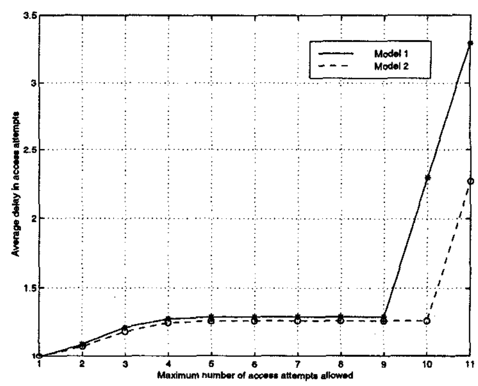

Figure 6: Average access delay vs T.