The Ergodic Fading Interference Channel with an On-and-Off Relay

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***Abstract*—We consider the ergodic fading Gaussian interference relay channel (EF-GIFRC) with individual power constraints at the nodes. Aiming at design insights to emerge from the optimal power allocation, we focus on scenarios similar to that of the degraded/reversely degraded relay channels. In particular, we focus on models where the source-to-relay ($S-R$) links are either stronger than direct links, or completely blocked, i.e., with an "on-and-off" relay. To characterize the capacity of EF-GIFRC with an on-and-off relay, we first investigate the parallel interference relay channel. We propose an achievable scheme based on partial decode-and-forward (DF) strategy and show that the capacity for the parallel IFRC can be achieved under strong interference and degradedness conditions. Based on the achievable rate region for parallel IFRC, we propose an achievable rate region for EF-GIFRC, and present the properties of optimal power allocation. We also present a sum capacity result when the EF-GIFRC satisfies certain channel conditions.**

I. Introduction

Cooperation and interference are two attributes of a wireless network that have a fundamental impact on its design. The interference relay channel (IFRC), which consists of two senders with two corresponding receivers and an intermediate relay, is the simplest model that characterizes both interference and cooperation. Various achievable schemes for this channel based on decode-and-forward (DF) at the relay have been proposed in [1]–[3]. References [4], [5] considers a compress-and-forward based achievable scheme and improves the DF-based rates when the source-to-relay ($S-R$) links are weak. There have also been two outerbounds proposed for the GIFRC [5]–[7]. References [6] and [7] complement each other in the sense that the bound in [6] is tighter when the relay has large power, and the one in [7] is tighter when the relay has small power. We note that all the effort up to date consider the channel in a static environment.

One can easily argue the need for addressing fading channels particularly for a wireless environment. It is then a natural next step to consider a setting when the links between the nodes are subject to fading, and take advantage of the time varying nature of the channels. For example, in ergodic fading channels, when the channel side information (CSI) is available at the sources and destinations, the transmitter can opportunistically allocate its power to achieve higher rates. The capacity of fading channel is first considered in [8], where the authors showed that the capacity achieving power allocation has a water-filling structure. In [9], the authors showed that the optimal power allocation for the multiple access channel (MAC) is a generalization of the water-filling construction for single-user channels. In [10], the authors derived capacity region for the parallel relay channel under degradedness conditions. They also obtained the optimal power allocation for an asynchronous relay under ergodic fading by solving a max-min problem. In [11], the authors developed the capacity of ergodic fading interference channel (IC) under strong interference, and obtained the optimal power allocation based on the solution of the max-min problem. They also showed that the ergodic fading IC is in general not separable. In [12], the authors showed that for parallel Gaussian IC, under certain channel conditions, sum capacity can be achieved by independent encoding across all subchannels by treating interference as noise.

In this paper, we consider the GIFRC with a full-duplex relay under stationary and ergodic fading with individual power constraints at the nodes, where CSI is globally known. With the general aim of improving communication rates and finding capacity when possible, we investigate the optimal power allocation problem for the sources and the relay. In the relay channel, the decoding capability of the relay plays an important role on the performance of DF type of relaying strategies: When the $S-R$ link is weaker than the direct link, the relay is actually turned off. Motivated by this fact, to better characterize the influence of relay's decoding capability in the GIFRC with time varying links, we consider a model where the $S-R$ links are either stronger than the direct links, or completely blocked, which we term an "on-and-off" relay. This model simplifies the derivation, and is insightful in demonstrating the role the relay should play in communication, by inheriting the features of degraded/reversely degraded relay channel [13]. The physical reality this situation models is one where a relay is close to the sources and can occasionally encounter objects that block its signal.

To investigate the capacity of EF-IFRC with an on-and-off relay, we first consider the parallel IFRC with an on-and-off relay, since each fading realization for the ergodic fading channel can be considered as a subchannel for the parallel channel. This model is appropriate, for example, for wireless systems employing orthogonal frequency division

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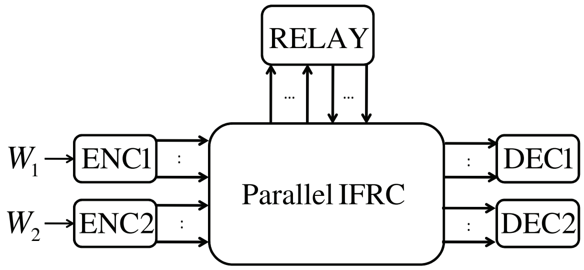

Fig. 1. Parallel Interference Relay Channel.

multiplexing (OFDM). We employ the partial DF scheme to obtain an achievable rate region which in general holds for any parallel IFRC. We further show that it yields the capacity region for our specific model when each subchannel satisfies certain strong interference and degradedness conditions. Based on the results for the parallel channel, we then obtain an achievable rate region for the EF-GIFRC with an on-and-off relay. In particular, we are interested in the case when the relay is asynchronous to the sources since this simplifies the transmitter design [10]. To find the optimal power allocation, we generalize the method in [10] to solve a max-min problem which contains multiple objective functions. We derive the resulting optimal power policy which offers design insights. The rate expressions imply that for both sources, the messages can be divided into two parts. One part is conveyed directly to the destinations, and the other part is conveyed through the relay, which is similar to the scheme in [14]. The rate splitting factor depends on the channel condition. The relay stores the messages to be conveyed through itself from both sources, and then forwards the messages to the destinations accordingly. We classify the resulting power allocation schemes as *(i) orthogonal waterfilling with nonselective fowarding*, *(ii) orthogonal waterfilling with selective forwarding*, *(iii) two-step waterfilling*, and *(iv) iterative waterfilling*. We further show that under certain fading distributions, the sum capacity of the EF-GIFRC can be achieved.

II. System model

A. The Parallel IFRC

In this section, we first provide the model for the discrete memoryless parallel IFRC, which is shown in Fig. 1. A parallel IFRC consists of K subchannels with channel transition probability distribution

$$\prod_{k=1}^{K} p_k(y_{1k}y_{2k}y_{Rk}|x_{1k}x_{2k}x_{Rk}) \tag{1}$$

The source encoder i $(i = 1, 2)$ maps a message into a codeword

$$x_i^n = (x_{i1}^n, \dots, x_{iK}^n) \tag{2}$$

The relay uses an encoding function f_m, $m = 1, \dots, n$, such that

$$x_{R,m} = (x_{R1,m}, \dots, x_{RK,m}) = f_m(y_{R1}^{m-1}, \dots, y_{RK}^{m-1}) \tag{3}$$

The decoder i maps the channel output $(y_{i1}^n, \dots, y_{iK}^n)$ into a message. Error occurs when the decoded message of either source is different from its transmitted message. The relay is allowed jointly encode and decode across all subchannels.

B. The Ergodic Fading Gaussian IFRC

For the Ergodic Fading Gaussian IFRC (EF-GIFRC), the received signals at the destinations and the relay are:

$$Y_1 = h_{11}X_1 + h_{21}X_2 + h_{R1}X_R + Z_1 \tag{4}$$
$$Y_2 = h_{12}X_1 + h_{22}X_2 + h_{R2}X_R + Z_2 \tag{5}$$
$$Y_R = h_{1R}X_1 + h_{2R}X_2 + Z_R \tag{6}$$

where the channel coefficients $h_{ij}, i, j = 1, 2, R$, are assumed to be independent random variables. We further assume that the fading processes $h_{ij}(n)$ are ergodic and stationary over time, where n is the time index. The noise Z_i is assumed to be a Gaussian random variable with unit variance. The power constraints for the input signals at each node are $\frac{1}{n}\sum_{k=1}^{n}\mathbb{E}[X_{i,k}^2] \le P_i$, $i = 1, 2$, and $\frac{1}{n}\sum_{k=1}^{n}\mathbb{E}[X_{R,k}^2] \le P_R$.

III. The Parallel Interference Relay Channel

To investigate the capacity region for EF-GIFRC with an on-and-off relay, we start with the parallel IFRC with an on-and-off relay, where each subchannel can be categorized into four sets (A_1, A_2, A_3, A_4) based on the $S-R$ links. We denote A_1 as the set of subchannels where the relay can hear from both sources, $A_2 = \{k : p_k(y_R|x_1x_2x_R) = p_k(y_R|x_1x_R)\}$ as the set of subchannels where the relay can only hear from source 1, $A_3 = \{k : p_k(y_R|x_1x_2x_R) = p_k(y_R|x_2x_R)\}$ as the set of subchannels where the relay can only hear from source 2, $A_4 = \{k : p_k(y_R|x_1x_2x_R) = p_k(y_R|x_R)\}$ as the set of subchannels where the relay cannot hear from either source.

Since the quality of each subchannel is different from one another, DF relaying is not beneficial for some subchannels. Hence, we first propose an achievable rate region using the partial DF scheme for the IFRC. The proof of this proposition is omitted due to space limitations (see reference [15] for partial DF).

Proposition 1: For the IFRC, the following rate region is achievable for any rate pairs $R_1 = R_{1d}+R_{1r}, R_2 = R_{2d}+R_{2r}$ that satisfy $R_{1d}, R_{1r}, R_{2d}, R_{2r} \in \mathcal{R}_1 \bigcap \mathcal{R}_2 \bigcap \mathcal{R}_r$, where $\mathcal{R}_1$ includes rate pairs such that

$$R_{1d} \le I(X_1; Y_1|U_1V_1X_2X_R) \tag{7}$$
$$R_{1d} + R_{2d} \le I(X_1X_2; Y_1|U_1V_1U_2V_2X_R) \tag{8}$$
$$R_{1d} + R_{1r} \le I(X_1X_R; Y_1|U_2V_2X_2) \tag{9}$$
$$R_{1d} + R_{1r} + R_{2d} \le I(X_1X_2X_R; Y_1|U_2V_2) \tag{10}$$
$$R_{1d} + R_{2d} + R_{2r} \le I(X_1X_2X_R; Y_1|U_1V_1) \tag{11}$$
$$R_{1d} + R_{1r} + R_{2d} + R_{2r} \le I(X_1X_2X_R; Y_1) \tag{12}$$

and $\mathcal{R}_2$ is obtained by switching indices of 1 and 2 in $\mathcal{R}_1$, $\mathcal{R}_r$ includes rate pairs such that

$$R_{1r} \leq I(V_1; Y_R|X_R U_1 U_2 V_2) \tag{13}$$
$$R_{2r} \leq I(V_2; Y_R|X_R U_1 V_1 U_2) \tag{14}$$
$$R_{1r} + R_{2r} \leq I(V_1 V_2; Y_R|X_R U_1 U_2) \tag{15}$$

for any distribution

$$p(u_1)p(v_1|u_1)p(x_1|u_1v_1)p(u_2)p(v_2|u_2)p(x_2|u_2v_2)p(x_R|u_1u_2)$$

Remark 1: It is easy to see that this rate region includes the ones in [3] [7], by setting appropriate auxiliary random variables to $\emptyset$.

We are now ready to derive an achievable rate region for the parallel IFRC with an on-and-off relay.

Proposition 2: Rate pair $(R_1, R_2) \in \mathcal{R}_{p1} \bigcap \mathcal{R}_{p2}$ is achievable, where $\mathcal{R}_{p1}$ is the rate pairs that satisfy the following constraints. $\mathcal{R}_{p2}$ is obtained by switching the indices 1 and 2 in both the rate pairs and the random variables, and switching A_2 and A_3.

$$R_1 \leq \sum_k I(X_{1k} X_{Rk}; Y_{1k}|U_{2k} X_{2k}) \tag{16}$$

$$\begin{aligned} R_1 \leq &\sum_{k\in A_1} I(X_{1k}; Y_{Rk}|X_{Rk} X_{2k} U_{1k}) \\ &+ \sum_{k\in A_2} I(X_{1k}; Y_{Rk}|X_{Rk} U_{1k}) \\ &+ \sum_{k\in A_3, A_4} I(X_{1k}; Y_{1k}|X_{Rk} X_{2k} U_{1k}) \end{aligned} \tag{17}$$

$$R_1 + R_2 \leq \sum_k I(X_{1k} X_{2k} X_{Rk}; Y_{1k}) \tag{18}$$

$$\begin{aligned} R_1 + R_2 \leq &\sum_{k\in A_1} I(X_{1k} X_{2k}; Y_{Rk}|X_{Rk} U_{1k} U_{2k}) \\ &+ \sum_{k\in A_2} (I(X_{1k}; Y_{Rk}|X_{Rk} U_{1k}) + I(X_{2k}; Y_{1k}|X_{1k} X_{Rk} U_{2k})) \\ &+ \sum_{k\in A_3} (I(X_{2k}; Y_{Rk}|X_{Rk} U_{2k}) + I(X_{1k}; Y_{1k}|X_{2k} X_{Rk} U_{1k})) \\ &+ \sum_{k\in A_4} I(X_{1k} X_{2k}; Y_{1k}|X_{Rk} U_{1k} U_{2k}) \end{aligned} \tag{19}$$

$$\begin{aligned} &R_1 + R_2 \\ &\leq \sum_{k\in A_1} (I(X_{2k}; Y_{Rk}|X_{Rk} X_{1k} U_{2k}) + I(X_{1k} X_{Rk}; Y_{1k}|U_{2k} X_{2k})) \\ &\quad + \sum_{k\in A_3} (I(X_{2k}; Y_{Rk}|X_{Rk} U_{2k}) + I(X_{1k} X_{Rk}; Y_{1k}|U_{2k} X_{2k})) \\ &\quad + \sum_{k\in A_2, A_4} I(X_{1k} X_{2k} X_{Rk}; Y_{1k}|U_{2k}) \end{aligned} \tag{20}$$

$$\begin{aligned} &R_1 + R_2 \\ &\leq \sum_{k\in A_1} (I(X_{1k}; Y_{Rk}|X_{Rk} X_{2k} U_{1k}) + I(X_{2k} X_{Rk}; Y_{1k}|U_{1k} X_{1k})) \\ &\quad + \sum_{k\in A_2} (I(X_{1k}; Y_{Rk}|X_{Rk} U_{1k}) + I(X_{2k} X_{Rk}; Y_{1k}|U_{1k} X_{1k})) \\ &\quad + \sum_{k\in A_3, A_4} I(X_{1k} X_{2k} X_{Rk}; Y_{1k}|U_{1k}) \end{aligned} \tag{21}$$

Proof: This can be shown by replacing

$$X_i, Y_i, U_j, V_j \quad (i \in \{1, 2, R\}, j \in \{1, 2\})$$

with $(X_{i1}, .., X_{iK}), (Y_{i1}, .., Y_{iK}), (U_{j1}, .., U_{jK}), (V_{j1}, .., V_{jK})$ where $(X_{1k}, X_{2k}, X_{Rk}, Y_{1k}, Y_{2k}, Y_{Rk}, U_{1k}, U_{2k}, V_{1k}, V_{2k})$ are independent for each different k. In addition, we need to set $V_{1k} = X_{1k}, V_{2k} = X_{2k}$ when $k \in A_1$, $V_{1k} = X_{1k}, V_{2k} = \emptyset$ when $k \in A_2$, $V_{1k} = \emptyset, V_{2k} = X_{2k}$ when $k \in A_3$, $V_{1k} = \emptyset, V_{2k} = \emptyset$ when $k \in A_4$,and and apply Fourier-Motzkin elimination. ■

Remark 2: The achievable scheme is, in general, valid for *any* parallel IFRC. Above, we have made the particular choices on V_{1k}, V_{2k} in different subchannels in order to specialize the rate to the relevant scenario at hand.

Considering the case when each subchannel is in one of the sets A_1, A_2, A_3 or A_4, and satisfies the following degradedness conditions:

$$D_1 : p_k(y_1 y_2|y_R x_R x_1 x_2) = p_k(y_1 y_2|y_R x_R), k \in A_1$$
$$D_2 : p_k(y_1 y_2|y_R x_R x_1 x_2) = p_k(y_1|y_R x_R) p_k(y_2|x_R), k \in A_2$$
$$D_3 : p_k(y_1 y_2|y_R x_R x_1 x_2) = p_k(y_2|y_R x_R) p_k(y_1|x_R), k \in A_3$$
$$D_4 : p_k(y_1 y_2|y_R x_R x_1 x_2) = p_k(y_1 y_2|x_1 x_2 x_R), k \in A_4$$

we can characterize the capacity region of the parallel IFRC with an on-and-off relay as follows:

Theorem 1: Under the above degradedness conditions $D_1 - D_4$, the rate region in *Proposition* 2 is in fact the capacity region for parallel IFRC with an on-and-off relay when all subchannels satisfy the following strong interference condition

$$I(\underline{X_{1k}}, \underline{X_{Rk}}; \underline{Y_{1k}}|\underline{X_{2k}}) \leq I(\underline{X_{1k}}, \underline{X_{Rk}}; \underline{Y_{2k}}|\underline{X_{2k}}) \tag{22}$$

where $\underline{X_{ik}} = \{X_{i1}, \ldots, X_{iK}\}$, the following "average very strong interference" conditions

$$\sum_{k\in A_3 A_4} I(X_{1k}; Y_{1k}|X_{2k} X_{Rk} U_{1k}) \leq \sum_{k\in A_3 A_4} I(X_{1k}; Y_{2k}|U_{1k}) \tag{23}$$

$$\sum_{k\in A_2 A_4} I(X_{1k} X_{Rk}; Y_{1k}|X_{2k} U_{2k}) \leq \sum_{k\in A_2 A_4} I(X_{1k} X_{Rk}; Y_{2k}|U_{2k}) \tag{24}$$

$$\begin{aligned} &\sum_{k\in A_1 A_4} I(X_{1k}; Y_{1k} Y_{Rk}|X_{2k} X_{Rk} U_{1k}) \\ &\qquad \leq \sum_{k\in A_1 A_4} I(X_{1k}; Y_{2k} Y_{Rk}|X_{Rk} U_{1k}) \end{aligned} \tag{25}$$

and the counterpart of (22)(23)(24)(25) obtained by switching the indices of sets A_2 and A_3 and the indices of the random variables 1 and 2.

Sketch of the Proof: Due to the space limitation, we only provide a sketch of the proof. The sum rate bounds (20)-(21) and the corresponding bounds in $\mathcal{R}_{p2}$ become non-binding under the average very strong interference condition (23)-(24). The bounds (16)-(18) and the corresponding bounds in $\mathcal{R}_{p2}$ can be bounded by applying (22) as in [7] using vector inputs along with the fact that each subchannel is independent. The rest can be obtained from the cut set bound and by applying the condition (25) and the degradedness conditions $D_1 - D_4$.

IV. Power Allocation for the Ergodic Fading Gaussian Interference Relay Channel

Equipped with the results for the parallel IFRC, we are now ready to study the EF-GIFRC with an on-and-off relay with individual power constraint at each node. Reference [10] considered the power allocation problem in a fading relay channel, where they assumed an asynchronous relay which simplifies the transmitter design by resulting in a convex problem. This leads to an assumption we adopt here as well, namely, the signals sent from the relay and the sources being independent. Consistent with the notation in the previous section, we set $A_1 = \{h_{ij} : h_{1r} \geq h_{11}, h_{2r} \geq h_{22}\}$, $A_2 = \{h_{ij} : h_{1r} \geq h_{11}, h_{2r} = 0\}$, $A_3 = \{h_{ij} : h_{1r} = 0, h_{2r} \geq h_{22}\}$, $A_4 = \{h_{ij} : h_{1r} = 0, h_{2r} = 0\}$. We further denote $H = \{h_{ij}\}$. All fading realizations are constrained to belong to one of these sets. We first derive an achievable rate region for the EF-IFRC.

Proposition 3: For the EF-IFRC with an on-and-off asynchronous relay, the following rate region is achievable.

$$\bigcup_{\underline{P}(H) \in \mathcal{P}(H)} \mathcal{R}(\underline{P}(H)) \tag{26}$$

The union is for all the power allocation $\underline{P}(H) = [P_1(H)\ P_2(H)\ P_{R1}(H)\ P_{R2}(H)] \in \mathcal{P}(H)$ satisfying

$$\mathbb{E}_H[P_1(H)] = P_1, \quad \mathbb{E}_H[P_2(H)] = P_2$$
$$\mathbb{E}_H[P_{R1}(H)] + \mathbb{E}_H[P_{R2}(H)] = P_R \tag{27}$$

$\mathcal{R}(P(H))$ is the set of rate pairs (R_1, R_2) that satisfy the following rate constraints under power policy $P(H)$.

$$R_1 \leq \min\{T_1, T_2\} \tag{28}$$
$$R_2 \leq \min\{T_3, T_4\} \tag{29}$$
$$R_1 + R_2 \leq \min\{T_5, T_6, T_7, T_8\} \tag{30}$$
$$R_1 + R_2 \leq \min\{T_9, T_{10}, T_{11}, T_{12}\} \tag{31}$$

where $C(x) = \frac{1}{2}\log(1+x)$ and

$$T_1 = \mathbb{E}_H[C(h_{11}^2 P_1(H) + h_{R1}^2 P_{R1}(H))]$$
$$T_2 = \mathbb{E}_{A_1A_2}[C(h_{1R}^2 P_1(H))] + \mathbb{E}_{A_3A_4}[C(h_{11}^2 P_1(H))]$$
$$T_5 = \mathbb{E}_H[C(h_{11}^2 P_1(H) + h_{21}^2 P_2(H) + h_{R1}^2 P_{R1}(H) + h_{R1}^2 P_{R2}(H))]$$
$$T_6 = \mathbb{E}_{A_1}[C(h_{1R}^2 P_1(H) + h_{2R}^2 P_2(H))] + \mathbb{E}_{A_2}[C(h_{1R}^2 P_1(H)) + C(h_{21}^2 P_2(H))] + \mathbb{E}_{A_3}[C(h_{2R}^2 P_2(H)) + C(h_{11}^2 P_1(H))] + \mathbb{E}_{A_4}[C(h_{11}^2 P_1(H)) + C(h_{21}^2 P_2(H))]$$
$$T_7 = \mathbb{E}_{A_1A_3}[C(h_{2R}^2 P_2(H)) + C(h_{11}^2 P_1(H) + h_{R1}^2 P_{R1}(H))] + \mathbb{E}_{A_2A_4}[C(h_{11}^2 P_1(H) + h_{21}^2 P_2(H) + h_{R1}^2 P_{R1}(H))]$$
$$T_8 = \mathbb{E}_{A_1A_2}[C(h_{1R}^2 P_1(H)) + C(h_{21}^2 P_2(H) + h_{R1}^2 P_{R2}(H))] + \mathbb{E}_{A_3A_4}[C(h_{11}^2 P_1(H) + h_{21}^2 P_2(H) + h_{R1}^2 P_{R2}(H))]$$

$T_3, T_4, T_9, T_{10}, T_{11}, T_{12}$ are obtained by switching indices 1 and 2, and switching A_2 and A_3 in $T_1, T_2, T_5, T_6, T_7, T_8$ respectively.

Proof: To show this, we first fix a power policy $\mathcal{P}(H)$ satisfying the power constraint. For each channel state realization, we have one subchannel. We replace each input variable in *Proposition 2* by Gaussian inputs according to the power policy. Specifically, we set $X_R = U_1 + U_2$, where U_1, U_2 are Gaussian inputs with power $P_{R1}(H), P_{R2}(H)$. The region is obtained by replacing the sum over all the subchannels by averaging over all fading realizations, and taking the union over all power policies. ∎

Remark 3: From the rate constraints, R_1 can be written as

$$\mathbb{E}_{A_3A_4}[C(h_{11}^2 P_1(H))] + \min\{\mathbb{E}_{A_1A_2}[C(h_{1R}^2 P_1(H))], \mathbb{E}_H\left[C\left(\frac{h_{R1}^2 P_{R1}(H)}{h_{11}^2 P_1(H)+1}\right)\right] + \mathbb{E}_{A_1A_2}[C(h_{11}^2 P_1(H))]\} \tag{32}$$

R_2 can also be written in a similar form. We can see that part of the message is transmitted directly to the destinations, while the other part is transmitted through the relay, as in [14]. When the $S-R$ links are present, the relay decodes the messages to flow through itself, and transmits these messages to the destinations at all channel realizations.

It is easy to see that the rate region is convex. Next, we aim to find the optimal power allocation to maximize the sum rate. We denote $S_1(\underline{P}(H)) = T_1 + T_3$ as the rate under power policy $\underline{P}(H)$, and $S_2(\underline{P}(H)) = T_1 + T_4$, $S_3(\underline{P}(H)) = T_2 + T_3$, $S_4(\underline{P}(H)) = T_2 + T_4$, $S_i(\underline{P}(H)) = T_i$, for $i = 5, 6, \ldots, 12$. The problem is formulated as

$$\max_{\underline{P}(H) \in \mathcal{P}(H)} \min S_1(\underline{P}(H)), S_2(\underline{P}(H)), \ldots, S_{12}(\underline{P}(H)) \tag{33}$$

The authors in [10] proposed a method to solve the max-min problem with two objective functions. In the same spirit, we now generalize this method to N objective functions to solve our problem (where for our case $N = 12$). First, construct a function

$$V(\underline{\alpha}) = \max_{\underline{P}(H) \in \mathcal{P}(H)} \underline{\alpha}^T \underline{S}(\underline{P}(H)) \tag{34}$$

where $\underline{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_N]^T$, $\sum_{i=1}^N \alpha_i = 1$, $\underline{S}(\underline{P}(H)) = [S_1(\underline{P}(H)), S_2(\underline{P}(H)), \ldots, S_N(\underline{P}(H))]^T$. The solution of this optimization problem depends on the shape of $V(\underline{\alpha})$. The form of the function $V(\underline{\alpha})$ is not known in advance. Hence, the procedure in [10] is needed to find the optimal solution. In essence, this procedure is to determine which subset of the objective functions are "active" for the max-min problem. Note that since the problem is convex, Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for optimality. We let $\mathcal{S} \subset \{1, 2, \ldots, 12\}$, and $M = |\mathcal{S}|$ is the cardinality of $\mathcal{S}$. We further denote $k_1, \ldots, k_M$ as the elements in $\mathcal{S}$. If for $\sum_{i=1}^M \alpha_{k_M} = 1$, the power policy $\underline{P}_{\mathcal{S}}^*(H)$ which maximizes $\sum_{i=1}^N \alpha_{k_M} S_{k_i}(\underline{P}(H))$ satisfies $S_{i\in\mathcal{S}}(\underline{P}_{\mathcal{S}}^*(H)) = S_{j\in\mathcal{S}, j\neq i}(\underline{P}_{\mathcal{S}}^*(H))$ and

$$S_{i\in\mathcal{S}}(\underline{P}_{\mathcal{S}}^*(H)) < S_{i\in\mathcal{S}^c}(\underline{P}_{\mathcal{S}}^*(H)) \tag{35}$$

then $\underline{P}_{\mathcal{S}}^*(H)$ is the optimal solution for the max-min problem. To find the optimal solution, we need to search over all possible subsets $\mathcal{S}$. This yields the following different types of power allocation policies.

i) Orthogonal Waterfilling with Non-selective Forwarding: First, we consider the case when $\alpha_1 = 1$. The optimal power allocation $\underline{P}_1^*(H)$ has an orthogonal waterfilling format [10]. The relay decodes the messages from both sources, and allocates power to be used for helping each source according to the strength of $R - D$ links. The relay keeps forwarding messages from both sources irrespective of whether the $S-R$ links are present or not. When $\alpha_5 = 1$ or $\alpha_9 = 1$, the power policies also have orthogonal waterfilling format.

ii) Orthogonal Waterfilling with Selective Forwarding: For the cases when $\alpha_7 = 1$ or $\alpha_8 = 1$ or $\alpha_{11} = 1$ or $\alpha_{12} = 1$, the optimal power allocation strategies also have orthogonal waterfilling format. However, even if the relay can decode the messages to be flowed through itself from both sources, it only selectively forwards the messages from one source for all channel realizations. Thus, from the perspective of the other pair of users, the relay performs *interference forwarding*. This occurs when one of the $S-R$ link is in average much stronger than the other.

Note that for the special case when $\alpha_2 = 1$, the rate expression requires to allocate all of the relay's power to help source 1, i.e., $P_{R2}(H) = 0$, which yields $S_1(\underline{P}_2^*(H)) < S_2(\underline{P}_2^*(H))$, due to the definition of the sets A_1, A_3. Hence $\underline{P}_2^*(H)$ is not a solution to the max min problem. Similar arguments hold for the case when $\alpha_3 = 1$ and $P_{R1}(H) = 0$.

iii) Two-Step Waterfilling: For the cases when $\alpha_4 = 1$ or $\alpha_6 = 1$ or $\alpha_{10} = 1$, the optimal power allocation for the sources in these cases is to waterfill on the corresponding links for different channel realizations. However, to further obtain the condition under which these cases are active, we need to solve the power allocation problem for the relay and then arrive at a condition that depends on the average power constraint at the relay. The relaying strategy depends on the solution of the power allocation problem for the relay, which also has waterfilling format. Hence, we term this policy *two-step waterfilling*.

iv) Iterative Waterfilling: The above schemes characterize the boundary cases for the optimization problem, i.e., $\alpha_i = 1$, and $\alpha_j = 0$ for all $j \neq i$. For all other cases, the optimal power allocation does not have a simple format or a closed form, and can be found by an iterative waterfilling strategy as in [10]. Due to limited space, the details are omitted here.

Lastly, we present a capacity result for the EF-GIFRC.

Theorem 2: For the EF-GIFRC with an on-and-off asynchronous relay where all fading realizations are subject to strong interference, if the fading distribution dictates power policy $\underline{P}_5^*(\underline{P}_9^*)$ for the case $\alpha_5 = 1 (\alpha_9 = 1)$ the solution for the max-min problem, or equivalently $S_5(\underline{P}_5^*) < S_i(\underline{P}_5^*), i \neq 5$ $(S_9(\underline{P}_9^*) < S_i(\underline{P}_9^*), i \neq 9)$, the sum capacity is $S_5(\underline{P}_5^*)$ $(S_9(\underline{P}_9^*))$, and power policy $\underline{P}_5^*(\underline{P}_9^*)$ is optimal.

Sketch of Proof: Note that for the static channel, under strong interference, the following rates are always upperbounds for the sum capacity [7]:

$$R_1 + R_2 \le I(X_1 X_2 X_R; Y_1) \quad R_1 + R_2 \le I(X_1 X_2 X_R; Y_2)$$

These bounds are maximized by Gaussian inputs, and the end result is obtained by extending them to fading case as in [8].

V. Conclusion

In this paper, we investigated the capacity region for the ergodic fading GIFRC (EF-GIFRC). We first presented a partial DF scheme for the IFRC and generalized this scheme to parallel IFRC. Under the strong interference and degradedness conditions, the capacity region of parallel IFRC is characterized. The achievable scheme is used in the EF-GIFRC to study the optimal power allocation for the DF relaying scheme. It is shown that to maximize the sum rate, the relay decodes the messages from both sources, and selectively or nonselectively forwards the messages to the destinations according to the channel realizations. Sum capacity of the EF-GIFRC is obtained under certain channel conditions.

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