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Improved Achievable Rates for the Gaussian Interference Relay Channel

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***Abstract*—We consider the Gaussian interference channel with an intermediate relay. The known achievable schemes for this channel are based on decode-and-forward (DF) relaying, whose performance depends on the signal-to-noise (SNR) ratio of the received signal at the relay. Consequently, when the source-to-relay ($S-R$) links are weak, the resulting achievable rates have room for improvement. In this work, we design achievable schemes that provide this improvement. First, we consider compress-and-forward (CF) relaying to overcome the weakness of the DF schemes. Second, we consider employing structured codes and observe that when the signal strength from the direct link is subject to severe attenuation, using nested lattice codes yields higher rates than both the DF and CF schemes. Numerical results are presented to demonstrate the performance of different achievable schemes with the Potent Relay Outerbound derived in our previous work.**

I. Introduction

Wireless medium allows signals transmitted from one user to be overheard by surrounding users. This fact causes interference between different user pairs, but can also be utilized to facilitate cooperation between the nodes. The Gaussian interference relay channel (GIFRC) is a fundamental model that characterizes the case when interference and cooperation co-exist in the same network, and hence is an important building block for wireless ad hoc networks.

The GIFRC contains two source-destination pairs, and an intermediate relay which aims at helping both sources to communicate with the destinations. Relaying strategies have been extensively studied for the relay channel (RC), which consists of a single source, a receiver, and an intermediate relay node. It is well understood that with the help of a relay, the transmission rate of a point to point link can be increased using DF, or CF [1]. Recent efforts advocate employing lattice codes for relaying, i.e., compute-and-forward [2], which aims at decode a modulo-sum of a linear combination of the transmitted messages utilizing the linearity of the structured codes. This method is shown to have near optimal performance in many scenarios [2], [3].

Another feature of the GIFRC is the interference between two different source-destination pairs when both sources transmit through the same medium. The simplest model to characterize the interference between source-destination pairs is the interference channel, which consists of two senders with two corresponding receivers. The capacity region of the interference channel is known only for the case of strong, or very strong interference [4]. The best achievable rate region remains to be the Han-Kobayashi scheme [4], where sources use rate splitting to divide the messages into two parts, i.e., the private messages and common messages. At the decoder, the common message from the interference link is also decoded to facilitate decoding the message from the direct link.

For the GIFRC, we derived an outerbound in [5], where we considered the case when the power constraint at the relay is infinite. Based on this assumption, we derived sum rate upperbounds under both the weak and strong interference. We showed that these bounds is tighter than the cutset bound and close to the rate achieved by DF schemes when relay's power is large but finite. Other previous effort on GIFRC focused on identifying achievable rates. Reference [6] proposed an achievable scheme based on rate splitting at the sources and letting the relay decode the common and the private messages from both sources to cooperate. A modified model is also proposed in [7] and [8], and several achievable regions are obtained. Another achievability technique is the interference forwarding developed in [9], [10], which shows that forwarding interference can be beneficial. However, all of the known schemes focus on using DF schemes at the relay. The weakness of the DF scheme is that its performance is limited by the decoding capability of the relay. As a result, when the SNR of the received signal at the relay is low, the rates that can be achieved are small. In this work, we first design achievable schemes to overcome the weakness of the DF strategy. In section III, we use CF at the relay and rate splitting at sources to achieve higher rates than the DF relaying when $S-R$ links are weak. Secondly, we design a lattice encoding/decoding scheme to further improve the rates in some conditions. In section IV, we use Nested Lattice Codes with compute-and-forward relaying to achieve higher rates than both DF and CF relaying when the direct link is weak and the interference link is strong. We present numerical results to demonstrate the performance of different strategies in section V. It is shown that both the CF based scheme and the Lattice Code based scheme have performance close to the potent relay outerbound derived in [5] under some channel conditions. Section VI concludes the paper.

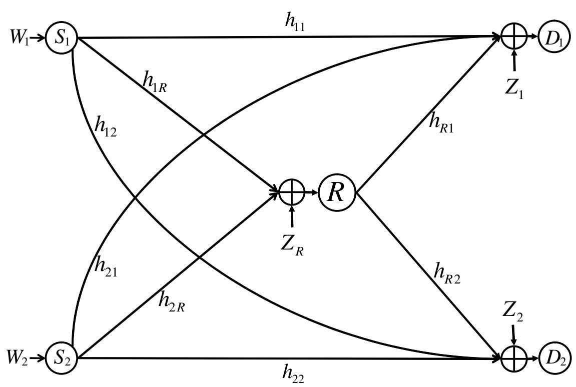

Fig. 1. GIFRC.

II. System Model

A. Discrete Memoryless Model

First we describe the discrete memoryless interference relay channel (DM-IFRC), since the CF scheme is derived for this model and then specialized to the Gaussian case. The system contains three finite input alphabets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_R$, three output alphabets $\mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_R$, and a probability distribution

$$p(y_1^n, y_2^n, y_R^n | x_1^n, x_2^n, x_R^n) = \prod_{i=1}^{n} p(y_{1,i}, y_{2,i}, y_{R,i} | x_{1,i}, x_{2,i}, x_{R,i})$$

which characterizes the channel. Each source S_i, $i = 1, 2$ wishes to communicate with a paired destination D_j, $j = 1, 2$. S_i chooses a message w_i from a message set $\mathcal{W}_i = \{1, 2, \ldots, 2^{nR_i}\}$, encodes this message into a length n codeword with an encoding function $f_i(w_i) = X_i^n$ and transmits the codeword through the channel. The relay employs an encoding function based on the information it received from previous transmissions, i.e., $X_{R,t} = f_R(Y_R^{t-1})$. Each destination uses a decoding function $g_i(Y_i^n) = \hat{W}_i$. A rate pair (R_1, R_2) is called achievable if there exists a message set, which is a set of encoding and decoding functions described above such that the error probability $Pr(\hat{W}_1 \neq W_1 \bigcup \hat{W}_2 \neq W_2)$ approaches zero as $n \to \infty$.

B. Gaussian Model

The GIFRC as shown in Fig. 1, in which there are two source-destination pairs, and a relay. The two sources and the relay transmit through the same medium, which causes interference at the receivers. The intermediate relay node is willing to help the transmission of information from both sources to both destinations. The signal X_R from the relay is based on its past received signals Y_R. The received signal at both destinations and relay can be modeled in the following way:

$$Y_1 = h_{11}X_1 + h_{21}X_2 + h_{R1}X_R + Z_1 \tag{1}$$

$$Y_2 = h_{12}X_1 + h_{22}X_2 + h_{R2}X_R + Z_2 \tag{2}$$

$$Y_R = h_{1R}X_1 + h_{1R}X_2 + Z_R \tag{3}$$

Here, $Z_i \sim \mathcal{N}(0, 1)$, $i = 1, 2, R$, denotes the additive Gaussian noise at each receiver. The power constraints are $\frac{1}{n}\sum_{k=1}^{n} X_{i,k}(w_i)^2 \leq P_i$, $i = 1, 2$, and $\frac{1}{n}\sum_{k=1}^{n} X_{R,k}^2 \leq P_R$.

Let $w_1 \in \{1, 2, \ldots, 2^{nR_1}\}$, $w_2 \in \{1, 2, \ldots, 2^{nR_2}\}$ denote the messages of the two sources. Each source employs an encoding function $X_i : w_i \to \mathbb{R}^n, i = 1, 2$, to obtain codewords $X_i^n(w_i)$ subject to the power constraints $\frac{1}{n}\sum_{k=1}^{n} X_{i,k}(w_i)^2 \leq P_i$. These codewords are to be transmitted in n channel uses to obtain a rate R_i per channel use. The relay uses an encoding function at the i th transmission $X_{R,i} : \{Y_R^{i-1}\} \to \mathbb{R}$ subject to the power constraint $\frac{1}{n}\sum_{k=1}^{n} X_{R,k}^2 \leq P_R$. At the receiver side, each decoder uses a decoding function $g_i : \mathbb{R}^n \to \{w_i\}, i = 1, 2$ to decode the messages, i.e., $g_i(Y_i^n) = w_i$. The corresponding probability of decoding error λ_i is defined as $P\{w_i \neq g_i(Y_i^n)\}$. A rate-tuple (R_1, R_2) is said to be achievable if there exists a sequence of codes X_1^n, X_2^n, X_R^n, satisfying the power constraint, such that the error probabilities $\lambda_1, \lambda_2 \to 0$ as $n \to \infty$.

III. Compress-and-Forward Relaying Strategy

In this section, we first derive the achievable rate region using CF scheme in DM-IFRC, and then extend the result to the GIFRC. We use block Markov encoding, where the sources use rate splitting, and the relay transmits a quantized version of its received signal to the destinations to facilitate decoding.

Theorem 1: The following rate tuples are achievable for the general interference relay channel:

$$R_{11} \leq I(X_1; \hat{Y}_R, Y_1 | U_1, U_2, X_R) \tag{4}$$

$$R_{10} + R_{11} \leq I(X_1; \hat{Y}_R, Y_1 | U_2, X_R) \tag{5}$$

$$R_{11} + R_{20} \leq I(U_2, X_1; \hat{Y}_R, Y_1 | U_1, X_R) \tag{6}$$

$$R_{10} + R_{11} + R_{20} \leq I(U_2, X_1; \hat{Y}_R, Y_1 | X_R) \tag{7}$$

$$R_{22} \leq I(X_2; \hat{Y}_R, Y_2 | U_1, U_2, X_R) \tag{8}$$

$$R_{20} + R_{22} \leq I(X_2; \hat{Y}_R, Y_2 | U_1, X_R) \tag{9}$$

$$R_{22} + R_{10} \leq I(U_1, X_2; \hat{Y}_R, Y_2 | U_2, X_R) \tag{10}$$

$$R_{20} + R_{22} + R_{10} \leq I(U_1, X_2; \hat{Y}_R, Y_2 | X_R) \tag{11}$$

subject to

$$I(X_R; Y_1) \geq I(Y_R; \hat{Y}_R | X_R, Y_1) \tag{12}$$

$$I(X_R; Y_2) \geq I(Y_R; \hat{Y}_R | X_R, Y_2) \tag{13}$$

for all joint probability distribution

$$p(u_1)p(u_2)p(x_1|u_1)p(x_2|u_2)p(x_R)p(y_1 y_2 y_R | x_1 x_2 x_R)p(\hat{y}_R | y_R x_R)$$

For the proof of *Theorem 1*, see Appendix A.

Remark 1: This result can be extended to the Gaussian case by setting $U_1 \sim \mathcal{N}(0, \alpha P_1)$, $U_2 \sim \mathcal{N}(0, \beta P_2)$, $V_1 \sim \mathcal{N}(0, (1-\alpha)P_1)$, $V_2 \sim \mathcal{N}(0, (1-\beta)P_2)$, $X_R \sim \mathcal{N}(0, P_R)$, which are all independent from each other, and $X_1 = U_1 + V_1$, $X_2 = U_2 + V_2$, $\hat{Y}_R = Y_R + \hat{Z}_R$, where $\hat{Z}_R \sim \mathcal{N}(0, \sigma_R^2)$. Here, U_1, U_2 represent the common messages to be decoded at both receivers, whereas V_1, V_2 represent the private messages to be decoded at the intended receivers only. $\hat{Y}_R$ represents the compressed version of the received signal at the relay, which is to be forwarded to the receivers. Hence we have:

Proposition 1: For the GIFRC, the following rate tuples are achievable for $\forall \alpha, \beta \in [0,1]$

$$R_{11} \leq C\Big(\frac{h_1^2 \bar{\alpha}\bar{\beta} P_1 P_2 + \bar{\alpha} c' P_1}{\bar{\beta} d' P_2 + 1 + \sigma_R^2}\Big) \quad (14)$$

$$R_{10} + R_{11} \leq C\Big(\frac{h_1^2 \bar{\beta} P_1 P_2 + c' P_1}{\bar{\beta} d' P_2 + 1 + \sigma_R^2}\Big) \quad (15)$$

$$R_{11} + R_{20} \leq C\Big(\frac{h_1^2 \bar{\alpha} P_1 P_2 + \bar{\alpha} c' P_1 + \beta d' P_2}{\bar{\beta} d' P_2 + 1 + \sigma_R^2}\Big) \quad (16)$$

$$R_{10} + R_{11} + R_{20} \leq C\Big(\frac{h_1^2 P_1 P_2 + c' P_1 + \beta d' P_2}{\bar{\beta} d' P_2 + 1 + \sigma_R^2}\Big) \quad (17)$$

$$R_{22} \leq C\Big(\frac{h_2^2 \bar{\alpha}\bar{\beta} P_1 P_2 + \bar{\beta} d'' P_2}{\bar{\alpha} c'' P_1 + 1 + \sigma_R^2}\Big) \quad (18)$$

$$R_{20} + R_{22} \leq C\Big(\frac{h_2^2 \bar{\alpha} P_1 P_2 + d'' P_2}{\bar{\alpha} c'' P_1 + 1 + \sigma_R^2}\Big) \quad (19)$$

$$R_{22} + R_{10} \leq C\Big(\frac{h_2^2 \bar{\beta} P_1 P_2 + \bar{\beta} d'' P_2 + \alpha c'' P_1}{\bar{\alpha} c'' P_1 + 1 + \sigma_R^2}\Big) \quad (20)$$

$$R_{20} + R_{22} + R_{10} \leq C\Big(\frac{h_2^2 P_1 P_2 + d'' P_2 + \alpha c'' P_2}{\bar{\alpha} c'' P_1 + 1 + \sigma_R^2}\Big) \quad (21)$$

where $C(x) = \frac{1}{2}\log(1+x)$,

$$\sigma_R^2 = \max\Big(\frac{h_1^2 P_1 P_2 + (h_{11}^2 + h_{1R}^2) P_1 + (h_{21}^2 + h_{2R}^2) P_2 + 1}{h_{R1}^2 P_R},$$
$$\frac{h_2^2 P_1 P_2 + (h_{22}^2 + h_{2R}^2) P_2 + (h_{12}^2 + h_{1R}^2) P_1 + 1}{h_{R2}^2 P_R}\Big),$$

$h_1 = h_{21}h_{1R} - h_{11}h_{2R}$, $h_2 = h_{12}h_{2R} - h_{22}h_{1R}$, $c' = (h_{11}^2(1+\sigma_R^2) + h_{1R}^2)$, $d' = (h_{21}^2(1+\sigma_R^2) + h_{2R}^2)$, $c'' = (h_{12}^2(1+\sigma_R^2) + h_{1R}^2)$, $d'' = (h_{22}^2(1+\sigma_R^2) + h_{2R}^2)$.

IV. Nested Lattice Codes For GIFRC

Structured codes have been shown to outperform random codes in several cases [2]. Specifically, relay nodes can decode the modulo-sum of transmitted messages and forward the sum to the destinations. The linear structure of the codes can be exploited by both the relay and the destinations to achieve higher rates. For the GIFRC, as well it is natural to consider lattice codes as an option when designing the relaying schemes. In this section, we consider the symmetric case for clarity of exposition. We present an achievable rate using Nested Lattice Codes based on the result from [3].

Theorem 2: For the symmetric GIFRC, where $h_{11} = h_{22}, h_{12} = h_{21}, h_{1R} = h_{2R}, h_{R1} = h_{R2}, P_1 = P_2 = P$, the following symmetric rate is achievable using lattice code

$$R \leq \min\Big\{C(\frac{h_{12}^2 P}{1 + h_{11}^2 P}), C(\frac{h_{R1}^2 P_R}{1 + h_{11}^2 P}),$$
$$\frac{1}{2}C(\frac{h_{12}^2 P + h_{R1}^2 P_R}{1 + h_{11}^2 P}), C(h_{1R}^2 P - \frac{1}{2})\Big\} \quad (22)$$

Proof: Due to limited space, we only describe the encoding/decoding schemes. A more detailed treatment is given in [3]. Also, for preliminaries of lattice codes, see [11]. We choose a pair of nested lattice codes $\Lambda \subset \Lambda_c \subset \mathbb{R}^n$ with nesting ratio R, such that the coarse lattice Λ is Rogers-good and Poltyrev-good [12], and the fine lattice Λ_c is Poltyrev-good. Moreover, we choose the coarse lattice such that $\sigma^2(\Lambda) = P$. The codewords are the fine lattice points that are within the fundamental Voronoi region of the coarse lattice. We use the block Markov coding to transmit b messages in $b+1$ blocks. Source i, $i = 1, 2$ maps its message $w_i(k)$ in block k into a lattice point $t_i^n(k) \in \Lambda_c \bigcap \mathcal{V}(\Lambda)$, and transmits $X_i^n(k) = (t_i^n(k) + U_i^n(k)) \mod \Lambda$, where $U_i^n(k) \sim \text{Unif}(\mathcal{V}(\Lambda))$ is the dither. It can be shown that $X_i^n(k)$ satisfies the power constraint and is independent of $t_i^n(k)$ [11]. At the end of each block, the relay decodes $t^n(k) = (t_1^n(k) + t_2^n(k)) \mod \Lambda$, and uses rate splitting to transmit the index of this modulo-sum message to the destinations with power P_R in next block. At the destination, each decoder treats the the signal from direct link, which is X_i for receiver i, $i = 1, 2$, as noise. It then uses successive decoding to decode the modulo-sum of the two source messages and the interference message, and thus it can recover its intended message. Error probability decrease exponentially as $n \to \infty$ if the nesting ratio R satisfies (22). ∎

Remark 2: When the direct link is strong, DF and CF schemes outperform this scheme, since the signal from the direct link, which contains significant amount of information about the intended message, is treated as noise. However, when direct link is weak but the interference link is strong, the information contained in the direct link is limited. Thus treating it as noise does not incur much rate loss. Instead, we can use the interference link and compute-and-forward relaying scheme to recover the message transmitted in the direct link. This scheme has better performance than DF and CF schemes in scenarios noted in the next section.

V. Numerical Results

In this section, we consider the symmetric channel and compare the new individual achievable sum rates derived in this paper with the rates achieved using DF [6], [10]. Note that the figures are plotted according to the rate expressions in *Theorem 1* and *Theorem 2*. In [6], the authors proposed one achievable scheme based on rate splitting for the GIFRC, which is denoted as Sahin-Erkip scheme in the figures. In [10], the authors proposed another achievable scheme which is also based on DF at the relay, but no rate splitting at the sources is used. Note that for all figures, this achievable scheme has similar performance with Sahin-Erkip scheme and thus is not shown to preserve clarity. In addition, we compare the rates achieved by these various schemes with the Potent Relay Outerbound from our previous work [5].

Fig. 2 compares the sum rates achieved by the new CF based scheme, Nested Lattice Code based scheme and Sahin-Erkip scheme with the Potent Relay Outerbound. Since the Sahin-Erkip scheme requires the relay to decode the signals from both sources perfectly, the rate is very low when the $S-R$ link is weak. In this scenario, CF scheme performs

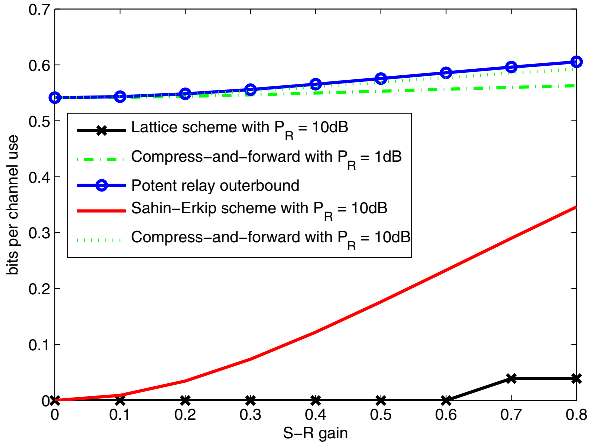

Fig. 2. Comparison of several achievable schemes and our outerbound under weak interference when $P = 1\text{dB}, h_d = 1, h_c = \sqrt{0.1}, h_R = 1$.

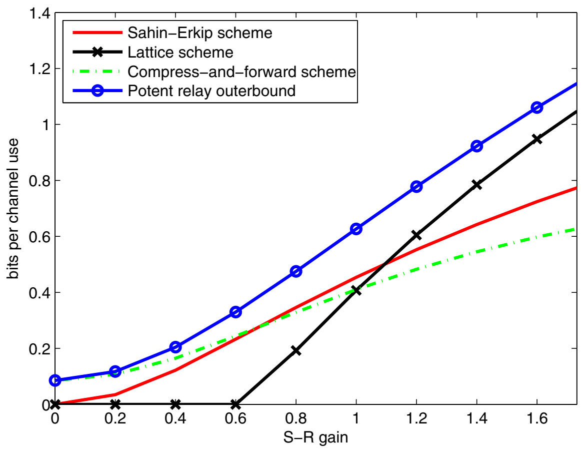

Fig. 3. Comparison of several achievable schemes and our outerbound under strong interference when $P = 1\text{dB}$, $P_R = 10\text{dB}$, $h_d = \sqrt{0.1}, h_c = \sqrt{10}, h_R = \sqrt{3}$.

better. Also, the rate achieved by CF scheme is close to the outerbound as the power of the relay increases. The lattice code performs poorly in this setting since the interference is weak and direct link is strong.

Next, we compare the sum rates achieved by various achievable schemes with the outerbound under the condition that the direct link is weak but the interference link is strong. For this case, Fig. 3 shows that the Nested Lattice Code based scheme outperforms the DF and CF relaying schemes for a range of the $S-R$ link values. In this setting, the relay is assumed to be more powerful than the source node to show the goodness of our potent relay outerbound. CF scheme is close to the outerbound when $S-R$ link is weak, but it is outperformed by DF scheme when $S-R$ gain is large. Fig. 4 shows sum rates achieved by different schemes as the power increases in the scenario where all nodes are assumed to have equal power. Lattice codes perform close to optimal for some range of the power level. However, CF scheme in this case is worse than the DF scheme. On the one hand, $S-R$ link is not weak in this case, so decoding the source messages is helpful. On the other hand, CF scheme needs to use the received signal at the destination as side information to help decode the intended

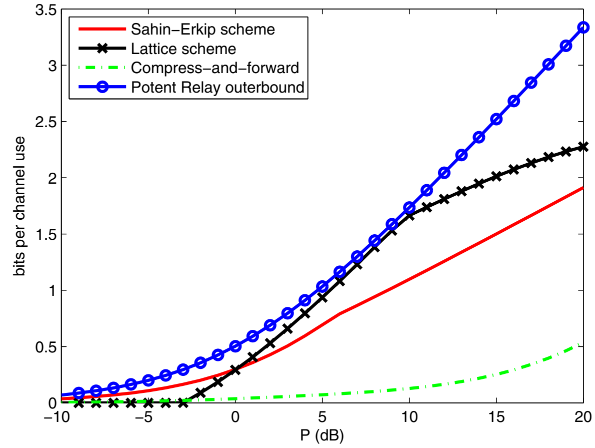

Fig. 4. Comparison of several achievable schemes and our outerbound under strong interference as the power varies and $h_d = 0.1, h_c = \sqrt{10}, h_R = 1, h_s = 1,$.

message. When the direct link is weak, the received signal does not contain much information about the source message, and thus it limits the performance.

VI. Conclusion

In this paper, we first proposed a new achievable scheme based on compress-and-forward at the relay. The performance of this scheme is better than the decode-and-forward assisted relaying schemes when the source-to-relay links are weak. In addition, we proposed a scheme utilizing the structured codes, which is shown to have better performance when the direct link is weak and interference link is strong. We also compared the various achievable schemes with our outerbound. It is shown that the achievable rates are close to our outerbound in the scenarios described in the previous section. The investigation in this paper shows that it can be beneficial to employ structured codes when designing transmission schemes for the GIFRC. Future work includes developing lattice schemes that can be used in more general channel settings.

Appendix A
Proof of Theorem 1

Proof: We use the block Markov encoding [1]. The main idea is to let the relay send a quantized version of its received signal to destinations to help decode the intended message. Also, rate splitting at the sources is used to improve the rates.

Codebook for the sources: Choose a joint distribution

$$p(u_1)p(u_2)p(x_1|u_1)p(x_2|u_2)p(x_R)p(y_1y_2y_R|x_1x_2x_R)p(\hat{y}_R|y_Rx_R)$$

Split message w_i into w_{i0}, w_{ii} where $w_{i0} \in \{1, \dots, 2^{nR_{i0}}\}$, $w_{ii} \in \{1, \dots, 2^{nR_{ii}}\}$, and $R_{i0} + R_{ii} = R_i$, $i = 1, 2$. For each w_{i0}, generate the codeword $u_i^n(w_{i0})$ according to $p(u_i^n(w_{i0})) = \prod_{k=1}^n p(u_{i,k})$. For each $u_i^n(w_{i0})$, generate $2^{nR_{ii}}$ codewords $x_i^n(w_{ii}|w_{i0})$ for each message w_{ii} according to $p(x_i^n(w_{ii}|w_{i0})) = \prod_{k=1}^n p(x_{i,k}|u_{i,k}(w_{i0}))$.

Codebook for the relay: Choose 2^{nR_0} codewords $x_R^n(w_0)$ for $w_0 \in \{1, \dots, 2^{nR_0}\}$ according to $p(x_R^n(w_0)) =$

$\prod_{k=1}^{n} p(x_{R,k}(w_0))$. Then, for each $x_R^n(w_0)$, choose $2^{n\hat{R}}$ codewords $\hat{y}_R^n(z|w_0)$for each $z \in \{1,\ldots,2^{n\hat{R}}\}$ according to $p(\hat{y}_R^n(z|w_0)) = \prod_{k=1}^{n} p(\hat{y}_{R,k}|x_{R,k}(w_0))$, where we define

$$p(\hat{y}_R|x_R) = \sum_{\mathcal{P}} p(u_1)p(u_2)p(x_1|u_1)p(x_2|u_2) \\ p(y_1 y_2 y_R|x_1 x_2 x_R)p(\hat{y}_R|y_R x_R) \quad (23)$$

where $\mathcal{P} = \{u_1, u_2, x_1, x_2, y_1, y_2, y_R\}$. Randomly partition the set $\{1,\ldots,2^{n\hat{R}}\}$ into 2^{nR_0} cells $\{S(w_0)\}$.

Encoding: In block k, let $w_{10}(k), w_{11}(k), w_{20}(k), w_{22}(k)$ and $w_0(k), z(k)$ be the messages to be sent from the sources and the relay, respectively. For the sources, choose the corresponding codewords $u_1^n(w_{10}(k)), x_1^n(w_{11}(k)|w_{10}(k))$, $u_2^n(w_{20}(k)), x_2^n(w_{22}(k)|w_{10}(k))$ to be sent in this block. For the relay, assume that $(\hat{y}_R^n(z(k-1)|w_0(k-1)), y_R^n(k-1), x_R^n(w_0(k-1))) \in T_\epsilon$, where T_ϵ stands for the jointly ϵ-typical set, and $z(k-1) \in S(w_0(k))$, then $x_R^n(w_0(k))$ is transmitted in block k.

Decoding: At the end of block k, two receivers decode $w_0(k)$ to obtain an estimate $\hat{w}_0(k)$ independently. To successfully decode $w_0(k)$, we need $R_0 \le \min\{I(X_R;Y_1), I(X_R;Y_2)\}$. Then both receivers try to find $\hat{z}(k-1)$ such that $(\hat{y}_R^n(\hat{z}(k-1)|\hat{w}_0(k-1)), y_i^n(k-1), x_R^n(\hat{w}_0(k-1))) \in T_\epsilon$ and $\hat{z}(k-1) \in S(w_0(k))$. To successfully decode this, we need $\hat{R} \le \min\{I(\hat{Y}_R;Y_1|X_R) + I(X_R;Y_1), I(\hat{Y}_R;Y_2|X_R) + I(X_R;Y_2)\}$.

After the relay correctly decodes the quantized version of the signal it received in block $k-1$, decoder 1 tries to find $(w_{10}(k-1), w_{11}(k-1), w_{20}(k-1))$ such that

$$\Big(u_1^n(w_{10}(k-1)), x_1^n(w_{11}(k-1)|w_{10}(k-1)), u_2^n(w_{20}(k-1)), \\ \hat{y}_R^n(\hat{z}(k-1)|\hat{w}_0(k-1)), y_1^n(k-1), x_R^n(\hat{w}_0(k-1))\Big) \in T_\epsilon$$

Decoder 2 uses the same method to decode $(w_{20}(k-1), w_{22}(k-1), w_{10}(k-1))$.

Error Probability Analysis: Suppose $(w_{10}, w_{11}, w_{20}) = (1,1,1)$ is sent. Then the error events at decoder 1 are $E_1 = \{w_{10} \ne 1, w_{11} = 1, w_{20} = 1\}$, $E_2 = \{w_{10} = 1, w_{11} \ne 1, w_{20} = 1\}$, $E_3 = \{w_{10} \ne 1, w_{11} \ne 1, w_{20} = 1\}$, $E_4 = \{w_{10} = 1, w_{11} \ne 1, w_{20} \ne 1\}$, $E_5 = \{w_{10} \ne 1, w_{11} = 1, w_{20} \ne 1\}$, $E_6 = \{w_{10} \ne 1, w_{11} \ne 1, w_{20} \ne 1\}$. The probability of each event is

$$P(E_1) = \sum_{w_{10}=2}^{2^{nR_{10}}} P[(u_1^n(w_{10}), x_1^n(1|w_{10}), u_2^n(1), \hat{y}_R^n(\hat{z}|\hat{w}_0), \\ y_1^n, x_R^n(\hat{w}_0) \in T_\epsilon] \\ \le 2^{-n(I(X_1;\hat{Y}_R,Y_1|U_2,X_R)-R_{10}-3\epsilon)} \quad (24)$$

$$P(E_2) = \sum_{w_{11}=2}^{2^{nR_{11}}} P[(u_1^n(1), x_1^n(w_{11}|1), u_2^n(1), \hat{y}_R^n(\hat{z}|\hat{w}_0), \\ y_1^n, x_R^n(\hat{w}_0) \in T_\epsilon] \\ \le 2^{-n(I(X_1;\hat{Y}_R,Y_1|U_1,U_2,X_R)-R_{11}-3\epsilon)} \quad (25)$$

$$P(E_3) = \sum_{w_{10}=2}^{2^{nR_{10}}} \sum_{w_{11}=2}^{2^{nR_{11}}} P[(u_1^n(w_{10}), x_1^n(w_{11}|w_{10}), u_2^n(1), \\ \hat{y}_R^n(\hat{z}|\hat{w}_0), y_1^n, x_R^n(\hat{w}_0) \in T_\epsilon] \\ \le 2^{-n(I(X_1;\hat{Y}_R,Y_1|U_2,X_R)-(R_{10}+R_{11})-3\epsilon)} \quad (26)$$

$$P(E_4) = \sum_{w_{20}=2}^{2^{nR_{20}}} \sum_{w_{11}=2}^{2^{nR_{11}}} P[(u_1^n(1), x_1^n(w_{11}|1), u_2^n(w_{20}), \\ \hat{y}_R^n(\hat{z}|\hat{w}_0), y_1^n, x_R^n(\hat{w}_0) \in T_\epsilon] \\ \le 2^{-n(I(U_2,X_1;\hat{Y}_R,Y_1|U_1,X_R)-(R_{11}+R_{20})-3\epsilon)} \quad (27)$$

$$P(E_5) = \sum_{w_{20}=2}^{2^{nR_{20}}} \sum_{w_{10}=2}^{2^{nR_{10}}} P[(u_1^n(w_{10}), x_1^n(1|w_{10}), u_2^n(w_{20}), \\ \hat{y}_R^n(\hat{z}|\hat{w}_0), y_1^n, x_R^n(\hat{w}_0) \in T_\epsilon] \\ \le 2^{-n(I(U_2,X_1;\hat{Y}_R,Y_1|X_R)-(R_{10}+R_{20})-3\epsilon)} \quad (28)$$

$$P(E_6) = \sum_{w_{10}=2}^{2^{nR_{10}}} \sum_{w_{11}=2}^{2^{nR_{11}}} \sum_{w_{20}=2}^{2^{nR_{20}}} P[(u_1^n(w_{10}), x_1^n(w_{11}|w_{10}), u_2^n(w_{20}), \\ \hat{y}_R^n(\hat{z}|\hat{w}_0, y_1^n), x_R^n(\hat{w}_0) \in T_\epsilon] \\ \le 2^{-n(I(U_2,X_1;\hat{Y}_R,Y_1|X_R)-(R_{10}+R_{11}+R_{20})-3\epsilon)} \quad (29)$$

To guarantee that the probability of every error event decreases exponentially, we need the rates (4)-(7) after eliminating the redundant terms. Similarly we can obtain (8)-(11). ■

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