The Gaussian Interference Relay Channel with a Potent Relay

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***Abstract*—We consider the Gaussian interference channel with an intermediate relay. The relay is assumed to have abundant power and is named potent for that reason. A main reason to consider this model is to find good outerbounds for the Gaussian interference relay channel (GIFRC) with finite relay power. By setting the power of the relay constraint to infinity, we show that the capacity region is asymptotically equivalent to the case when the relay-destination links are noiseless and orthogonal to other links. The capacity region of the latter provides an outerbound for the GIFRC with finite relay power. We then show the capacity region of the former can be upper bounded by a single-input-multiple-output interference channel with an antenna common to both receivers. To establish the sum capacity of this channel, we study the strong and the weak interference regimes. For both regimes, we show that the upperbounds we find are achievable, thus establishing the sum capacity of GIFRC with the potent relay. Both results, in turn, serve as upperbounds for the sum capacity of the GIFRC with finite relay power. Numerical results show that the upperbounds are close to the known achievable rates for many scenarios of interest.**

I. Introduction

Wireless medium allows signals transmitted from one user to be overheard by surrounding users. This fact causes interference between different user pairs, but can also be utilized to facilitate cooperation between the nodes. The interference relay channel (IFRC) is a fundamental model that characterizes the case when interference and cooperation co-exist in the same network, and hence is an important building block for wireless ad hoc networks.

The relay channel (RC), which consists of a sender, a receiver, and an intermediate relay node, is the simplest model to characterize cooperation [1]. It is well understood that with the help of a relay, the transmission rate of a point to point link can be increased using Decode and Forward (DF), or Compress and Forward (CF) [1]. However, the capacity of the general relay channel remains open. Generalizations of relaying techniques to the multiple access relay channel (MARC) or the broadcast relay channel (BRC) are investigated in [2].

The interference channel, which consists of two senders with two corresponding receivers, is the simplest model to characterize interference. The capacity region of the interference channel also remains open in general, but is known only for the case of strong, or very strong interference [3], [4]. For other interference regimes, the best achievable rate region remains to be the Han-Kobayashi scheme. Various outerbounds for this channel have been derived to date, e.g. [5]. Notably, two important lines of work emerged recently. The authors in [6] proposed a new outerbound based on the extremal inequality [7], and showed that the sum capacity can be achieved by treating interference as noise under weak interference conditions. The same result is also obtained simultaneously by authors in [8] using a "smart and useful genie" approach.

It is a natural next step to consider a model where both cooperation and interference exist and examine their impact on wireless network design. This model, the interference relay channel (IFRC), which consists of two senders with two corresponding receivers, and an intermediate relay, has recently been considered. Sahin and Erkip in [9] have proposed an achievable scheme based on rate splitting at the sources and letting the relay decode both the common and the private messages to help both sources. A modified model is also proposed in [10] and [11], and several achievable regions are obtained. Another achievability technique is the interference forwarding developed in [12] and [13], which show that forwarding interference can be beneficial. Reference [14] provides a comprehensive view of a number of Gaussian IFRC (GIFRC) models, including in-band reception/transmission, in-band reception/out-of-band transmission, out-of-band reception/in-band transmission and out-of-band reception/transmission, and establishes capacity results for the out-of-band reception/in-band transmission and out-of-band reception/transmission scenarios.

In this paper, we advocate for a GIFRC model where the relay has very large (infinite) power. We call this the GIFRC *with a potent relay*. In reality, the GIFRC with potent relay can be thought of a system where the relay is a base station, whose power constraint is much larger compared to those of the users. From an information theoretic perspective, a main purpose of investigating this channel is to establish a good outerbound for the sum rate of the GIFRC (with finite relay power), since the capacity of GIFRC with infinite relay power is clearly an upperbound for the one with finite relay power. In section II, we propose the channel model and one equivalent model, which is the GIFRC with in-band reception/out-of-band

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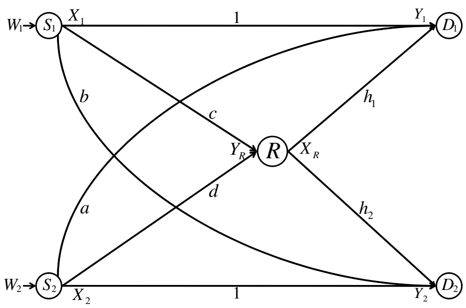

Fig. 1. GIFRC.

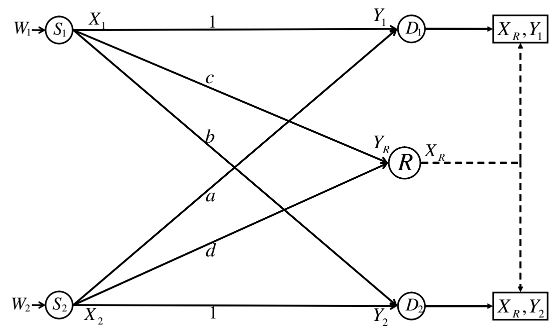

Fig. 2. GIFRC with in-band reception/out-of-band noiseless transmission.

noiseless transmission, with respect to the classification in [14]. We next observe that the SIMO interference channel with one antenna common to both receivers provides a capacity outerbound for this channel. To establish the sum capacity to this channel, we consider both strong and weak interference regimes. In section III, we utilize a "smart and useful genie" [8], to bound the sum rate for weak interference. In section IV, we use a genie argument as is also used for the Gaussian interference channel in strong interference. For both cases, we show that those upperbounds are achievable, thus establishing the sum capacity with the potent relay. Both results, in turn, serve as upperbounds for the sum capacity of the GIFRC. In section V, we demonstrate that with finite relay power, the sum-rate upperbounds are tighter than the cutset bound when the source-to-relay ($S-R$) gain is small, or when the power of the relay is larger than that of the users, but the difference between them is not significant. Also, the upperbounds are close to some recently proposed achievable schemes [9], [15] when the $S-R$ gain is strong and the power of the relay is large. Section VI concludes the paper.

II. System model

A. Gaussian interference relay channel (GIFRC)

The GIFRC is shown in Fig. 1, with channel outputs characterized by:

$$Y_1 = X_1 + aX_2 + h_1X_R + Z_1 \tag{1}$$

$$Y_2 = bX_1 + X_2 + h_2X_R + Z_2 \tag{2}$$

$$Y_R = cX_1 + dX_2 + Z_R \tag{3}$$

Here, $Z_i \sim \mathcal{N}(0,1),\ i = 1,2,R$, denotes the additive Gaussian noise at each receiver, and $a,b,c,d,h_1,h_2 \geq 0$ denote the channel gains. The power constraints are $\frac{1}{n}\sum_{k=1}^{n} X_{i,k}(w_i)^2 \leq P_i$, $i=1,2$, and $\frac{1}{n}\sum_{k=1}^{n} X_{R,k}^2 \leq P_R$. GIFRC with potent relay is the case when $P_R \to \infty$. Next, we will describe the model which we will show to be equivalent to GIFRC with a potent relay, and use in our capacity derivations.

B. GIFRC with in-band reception/out-of-band noiseless transmission

Several variations of GIFRC have been studied in [9]–[14]. Here, we shall investigate the channel as shown in Fig. 2. Following the notation in [14], this is the GIFRC with in-band reception/out-of-band noiseless transmission. In the sequel, we will show the equivalence between this model and the potent relay model, and derive the capacity results. The channel outputs are characterized by:

$$Y_1 = X_1 + aX_2 + Z_1 \tag{4}$$

$$Y_2 = bX_1 + X_2 + Z_2 \tag{5}$$

$$Y_R = cX_1 + dX_2 + Z_R \tag{6}$$

where $Z_i \sim \mathcal{N}(0,1),\ i = 1,2,R$ denotes the additive Gaussian noise at each receiver, and $a,b,c,d \geq 0$ denote the channel gains.

Let $w_1 \in \{1,2,\ldots,2^{nR_1}\}$, $w_2 \in \{1,2,\ldots,2^{nR_2}\}$ denote the messages of the two sources. Each source employs an encoding function $X_i : w_i \to \mathbb{R}^n, i=1,2$, to obtain codewords $X_i^n(w_i)$ subject to the power constraints $\frac{1}{n}\sum_{k=1}^{n} X_{i,k}(w_i)^2 \leq P_i$. These codewords are to be transmitted in n channel uses to obtain a rate R_i per channel use. The relay uses an encoding function at the i th transmission $X_{R,i} : \{Y_R^{i-1}\} \to \mathbb{R}$ subject to the power constraint $\frac{1}{n}\sum_{k=1}^{n} X_{R,k}^2 \leq P_R$. At the receiver side, each decoder uses a decoding function $g_i : \mathbb{R}^n \times \mathbb{R}^n \to \{w_i\}, i=1,2$ to decode the messages, i.e., $g_i(Y_i^n, X_R^n) = w_i$. The corresponding probability of decoding error λ_i is defined as $P\{w_i \neq g_i(Y_i^n, X_R^n)\}$. A rate-tuple $(R_1, R_2, \ldots, R_m)$ is said to be achievable if there exists a sequence of codes $X_1^n, X_2^n, \ldots, X_M^n$ such that the error probabilities $\lambda_1, \lambda_2, \ldots, \lambda_M \to 0$ as $n \to \infty$.

C. Equivalence between channel models

Proposition 1: The capacity region ($\mathcal{C}_1$) of GIFRC with in-band reception/out-of-band noiseless transmission is asymptotically equivalent to the capacity region ($\mathcal{C}_2$) of GIFRC as the power of the relay $P_R \to \infty$.

Proof: $\mathcal{C}_1 \subseteq \mathcal{C}_2$: For any pair of rate $(R_1, R_2) \in \mathcal{C}_1$, we show that it can be achieved asymptotically in $\mathcal{C}_2$. Assume a TDMA scheme with two slots. In slot 1, the sources send their codewords to the relay and the destinations, whereas in slot 2, the relay send its codewords to the destinations. Decoding starts at the end of slot 2. Decoders first decode the relay's codewords. If the time assigned to slot 2 is ε, to successfully decode the codewords from relay at both decoders, we need $R_{relay} \leq \frac{\varepsilon}{2}\log(1+\min(h_1^2P_R, h_2^2P_R))$. Then equipped with the relay's codewords, we can achieve the rate $((1-\varepsilon)R_1, (1-$

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$\varepsilon)R_2)$. Since the relay has no power constraint, as $\varepsilon \to 0$, R_{relay} still can be infinite, which means $(1-\varepsilon)\mathcal{C}_1$ can be arbitrarily close to $\mathcal{C}_1$.

$\mathcal{C}_2 \subseteq \mathcal{C}_1$: Since destinations have the relay's signals, they can construct the same signals as in GIFRC with potent relay by adding relay's signal multiplied by infinite weight. ■

We have now established that the capacity region of GIFRC with in-band reception/out-of-band noiseless transmission is equivalent to that of the GIFRC with the potent relay. In the sequel, we will work with the former to establish the sum capacity results for the latter.

III. Sum capacity of GIFRC with potent relay in weak interference

In this section, we use a "smart and useful genie" [8] to upper bound the sum capacity of GIFRC with potent relay in weak interference. The main idea behind this is to give the minimum amount of information to receivers so that i.i.d. Gaussian inputs maximize the capacity.

Theorem 1: For each combination of channel gains (a,b,c,d), when there exists $\rho_1,\rho_2,\rho_3,\rho_4 \in [0,1]$ such that the following conditions hold

$$\frac{\rho_1^2}{(1+a^2P_2)^2}+\frac{c^2\rho_3^2}{(1+d^2P_2)^2} \geq \frac{b^2}{1-\rho_2^2}+\frac{c^2}{1-\rho_4^2} \quad (7)$$

$$\frac{\rho_2^2}{(1+b^2P_1)^2}+\frac{d^2\rho_4^2}{(1+c^2P_1)^2} \geq \frac{a^2}{1-\rho_1^2}+\frac{d^2}{1-\rho_3^2} \quad (8)$$

then the sum capacity of GIFRC with potent relay is

$$R_1+R_2 \leq C_\Sigma \quad (9)$$

with

$$C_\Sigma = \frac{1}{2}\log(1+\frac{(ac-d)^2P_1P_2+P_1+c^2P_1}{(a^2+d^2)P_2+1}) + \frac{1}{2}\log(1+\frac{(bd-c)^2P_1P_2+d^2P_2+P_2}{(c^2+b^2)P_1+1}) \quad (10)$$

Proof:

Converse:

$$\begin{aligned}
&n(R_1+R_2)\\
&\leq I(W_1;\hat{W}_1)+I(W_2;\hat{W}_2)\\
&\leq I(X_1^n;Y_1^n,X_R^n)+I(X_2^n;Y_2^n,X_R^n) && (11)\\
&\leq I(X_1^n;Y_1^n,X_R^n,Y_R^n)+I(X_2^n;Y_2^n,X_R^n,Y_R^n) && (12)\\
&\leq I(X_1^n;Y_1^n,Y_R^n)+I(X_1^n;X_R^n|Y_1^n,Y_R^n)\\
&\quad +I(X_2^n;Y_2^n,Y_R^n)+I(X_2^n;X_R^n|Y_2^n,Y_R^n) && (13)\\
&= I(X_1^n;Y_1^n,Y_R^n)+I(X_2^n;Y_2^n,Y_R^n) && (14)\\
&\leq I(X_1^n;Y_1^n,Y_R^n,S_1^n,S_R^n)+I(X_2^n;Y_2^n,Y_R^n,S_2^n,T_R^n) && (15)\\
&= h(S_1^n,S_R^n)-h(S_1^n,S_R^n|X_1^n)+h(Y_1^n,Y_R^n|S_1^n,S_R^n)\\
&\quad -h(Y_1^n,Y_R^n|S_1^n,S_R^n,X_1^n)+h(S_2^n,T_R^n)-h(S_2^n,T_R^n|X_2^n)\\
&\quad +h(Y_2^n,Y_R^n|S_2^n,T_R^n)-h(Y_2^n,Y_R^n|S_2^n,T_R^n,X_2^n) && (16)
\end{aligned}$$

here, (11) is due to the Markov chain $W_1 \to X_1^n \to Y_1^nX_R^n \to \hat{W}_1$, and (14) is due to the Markov chain $X_1^i,X_2^i \to Y_R^i \to X_{R,i+1}$. In inequality (15), we give genie information to both the receivers, where $S_1=bX_1+bN_1$, $S_2=aX_2+aN_2$, $S_R=cX_1+cN_3$, $T_R=dX_2+dN_4$, where $N_i \sim \mathcal{N}(0,\sigma_i^2)$, $E[N_iZ_i]=\rho_i\sigma_i$, $i=1,2$, $N_j \sim \mathcal{N}(0,\sigma_j^2)$, $E[N_jZ_R]=\rho_j\sigma_j$, $j=3,4$. Then to guarantee that i.i.d. Gaussian inputs maximize (16), we need the following terms to be maximized by Gaussian inputs, which is stated in *Lemma 1*.

$$h(bX_1^n+bN_1^n,cX_1^n+cN_3^n)-h(bX_1^n+Z_2^n,cX_1^n+Z_R^n|N_2^n,N_4^n) \quad (17)$$

$$h(aX_2^n+aN_2^n,dX_2^n+dN_4^n)-h(aX_2^n+Z_1^n,dX_2^n+Z_R^n|N_1^n,N_3^n) \quad (18)$$

Lemma 1: When there exist $\sigma_1^2,\sigma_2^2,\sigma_3^2,\sigma_4^2 \geq 0$ and $\rho_1,\rho_2,\rho_3,\rho_4 \in [0,1]$ such that the following condition holds

$$\frac{1}{\sigma_1^2}+\frac{1}{\sigma_3^2} \geq \frac{b^2}{1-\rho_2^2}+\frac{c^2}{1-\rho_4^2} \quad (19)$$

$$\frac{1}{\sigma_2^2}+\frac{1}{\sigma_4^2} \geq \frac{a^2}{1-\rho_1^2}+\frac{d^2}{1-\rho_3^2} \quad (20)$$

Then i.i.d. Gaussian inputs with variance P_1 and P_2 maximize (17) and (18).

For the proof of *Lemma 1*, see Appendix A.

It follows then that the expression (15) is equivalent to

$$nI(X_{1G};Y_1,Y_R,S_1,S_R)+nI(X_{2G};Y_2,Y_R,S_2,T_R)$$

where $X_{iG} \sim \mathcal{N}(0,P_i)$, $i=1,2$. Here, X_{iG} represents the i.i.d. Gaussian inputs. Next, we show how to make the genie that supplies S_1,S_2,S_R,T_R "smart".

Lemma 2: Under conditions below

$$\rho_1\sigma_1=1+a^2P_2 \qquad \rho_2\sigma_2=1+b^2P_1$$
$$c\rho_3\sigma_3=1+d^2P_2 \qquad d\rho_4\sigma_4=1+c^2P_1 \quad (21)$$

the genie is also smart in the sense that

$$\begin{aligned}
&I(X_{1G};Y_1,Y_R,S_1,S_R)+I(X_{2G};Y_2,Y_R,S_2,T_R) && (22)\\
&\quad =I(X_{1G};Y_1,Y_R)+I(X_{2G};Y_2,Y_R) && (23)
\end{aligned}$$

For the proof of *Lemma 2*, see Appendix B.

Then, using *Lemma 2* and *Lemma 1*, the sum rate can be bounded by

$$R_1+R_2 \leq I(X_{1G};Y_1,Y_R)+I(X_{2G};Y_2,Y_R) \quad (24)$$

which gives us the expression (9).

Achievability: We only provide a sketch of the achievable scheme due to space limitations. The sum rate in (9) can be achieved by utilizing the noiseless relay-to-destination ($R-D$) links to form a virtual antenna array. Due to the causality constraint at the relay, we send a length n codeword in $n+1$ channel uses. The relay serves as the common antenna between two receivers, and the information it provides is delayed by one channel use. The receivers wait until they have all the information from the relay. Then the receivers treat interference as noise and perform maximum ratio combining with respect to Y_i and Y_R, $i=1,2$, which achieves the sum capacity. Note that the weight factors obtained at receiver $1(2)$ contain $P_2(P_1)$, and hence $P_2(P_1)$ is in the numerator of the first (second) term in the sum rate expression. ■

Remark 1: From (9), we can see that if $c = d = 0$, this is exactly the sum rate for interference channel with weak interference derived in [6] and [8]. The help from the relay provides the cooperative gain.

Remark 2: With some algebraic effort, we can see that, when $a = b$, $c = d$, $P_1 = P_2 = P$, $\rho_1 = \rho_2 = \rho$, $\rho_3 = \rho_4 = \rho_R$, i.e., the symmetric case, the conditions (7) and (8) are equivalent to

$$c^2 \leq \frac{1 - 2a(1 + a^2 P)}{1 + a^2 P} \tag{25}$$

This means that the interference links should be weak and the source-to-relay ($S - R$) links should not be strong for the sum capacity (9) of GIFRC with potent relay to hold.

IV. Capacity of GIFRC with potent relay under strong interference

Theorem 2: When $a \geq 1$ and $b \geq 1$, the capacity region of GIFRC with potent relay is

$$R_1 \leq \frac{1}{2}\log(1 + P_1 + c^2 P_1)$$
$$R_2 \leq \frac{1}{2}\log(1 + P_2 + d^2 P_2)$$
$$R_1 + R_2 \leq \frac{1}{2}\min\{\log(1 + (ac - d)^2 P_1 P_2 + (c^2 + 1)P_1 + (d^2 + a^2)P_2), \log(1 + (bd - c)^2 P_1 P_2 + (c^2 + d^2)P_1 + (d^2 + 1)P_2)\}$$

Proof: Following *Proposition 1*, we once again focus on the channel with in-band reception/out-of-band noiseless transmission. Based on the techniques bounding the strong interference channel in [3], [16], we first assume decoder 1 and decoder 2 can decode their own messages. For decoder 1, with $Y_1^n = X_1^n + aX_2^n + Z_1^n$ and X_R^n, by constructing $\frac{Y_1^n - X_1^n}{a} + bX_1^n + N_1^n$, where $N_1 \sim \mathcal{N}(0, \sqrt{1 - \frac{1}{a^2}})$, it can also decode w_2 if $a \geq 1$, . Similar result can be obtained for decoder 2 in the same way. Then any code for the GIFRC with potent relay is also a code for the compound SIMO MAC with an antenna common to both receivers. ■

Remark 3: Fig. 3 shows the sum capacity under different channel conditions in the weak interference and strong interference regimes. For the weak interference, as the gains of the interference links increase, the sum capacity decreases. This is because we are treating interference as noise. However, for the strong interference, increasing the gain of interfering links improves the sum capacity. For both cases, larger $S - R$ gains improve capacity because of the cooperation gain from relay.

V. Comparison

In the previous sections, we have established the sum capacity of GIRFC with a potent relay. It is clear that the model we described in Section II-B provides an outer bound for the GIFRC with finite relay power since it is tantamount to "giving" the relay signal to both destinations. This, in turn, means that our sum capacity results for the potent relay case are *sum rate* upperbounds for the GIFRC with finite relay power.

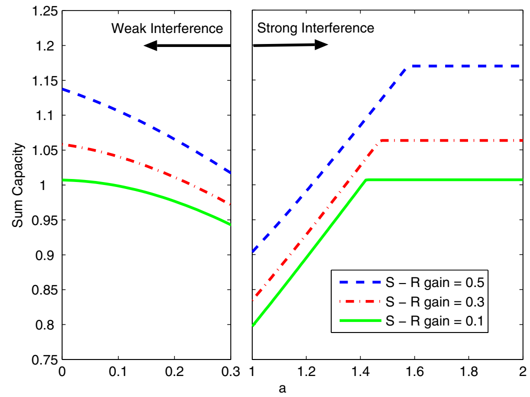

Fig. 3. Sum capacity of GIFRC with the potent relay under weak (left) and strong (right) interference when $P_1 = P_2 = 1$, $S_1(S_2) - D_1(D_2)$ gains are unity.

In this section, we consider the symmetric case of GIFRC to evaluate the performance of our results as outerbounds. We first show how our results behave as sum rate upperbounds for the GIFRC with finite relay power constraint. This is done by comparing the sum rate with the known achievable schemes from [9], [15] and the cutset bound. The difference between the two achievable schemes is that [15] does not use rate splitting while [9] does. Since these schemes are based on decode-and-forward, the performance can be improved when $S - R$ gains are weak. Thus, in this regime, we also provide a comparison with the TDMA scheme with amplify-and-forward relay, which is similar to that in [17], stated below:

Proposition 2: For the symmetric case where $a = b, c = d, h_1 = h_2 = h, P_1 = P_2 = P$, sum rate $R_{sum} = \frac{1}{2}\log\det(I + PAA^T(B\Sigma_Z B^T)^{-1})$ can be achieved using a TDMA scheme with amplify-and-forward relaying (TDMA-AF), where $A = [1 \;\; h\gamma c]^T$, $\gamma = \sqrt{\frac{P_R}{1 + d^2 P_2 + c^2 P_1}}$ is the amplification factor at the relay,

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & h\gamma \end{bmatrix}, \Sigma_Z = \begin{bmatrix} 1 + b^2 P & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 + d^2 P \end{bmatrix}.$$

Note also that the cutset bound for the GIFRC is:

$$R_1 \leq \min\{I(X_1 X_R; Y_1|X_2), I(X_1; Y_1 Y_R|X_2 X_R)\} \tag{26}$$
$$R_2 \leq \min\{I(X_2 X_R; Y_2|X_1), I(X_2; Y_2 Y_R|X_1 X_R)\} \tag{27}$$
$$R_1 + R_2 \leq \min\{I(X_1 X_2; Y_1 Y_2 Y_R|X_R), I(X_1 X_2 X_R; Y_1 Y_2)\} \tag{28}$$

where Y_1, Y_2, Y_R are from the equations (1)-(3).

Fig. 4 compares our outerbound to the cutset bound under weak interference. We observe that our sum rate upperbound is tighter than the cutset bound even when the power of the relay is of the same level as that of the users.

For the strong interference case, Fig. 5 shows that our sum rate upperbound is tighter the cutset bound under some channel conditions. Moreover, for cases when $S - R$ gain is weak, our bound is better even when the power of the relay is small. This

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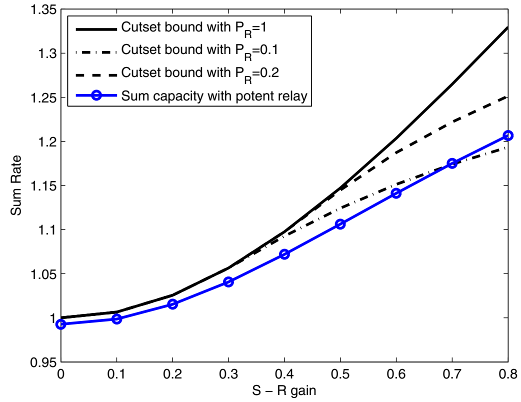

Fig. 4. Comparison of the cutset bound and our bound under weak interference when $P_1 = P_2 = 1, a = 0.1$, $S_{1(2)} - D_{1(2)}$ gain and $R - D$ gains are unity.

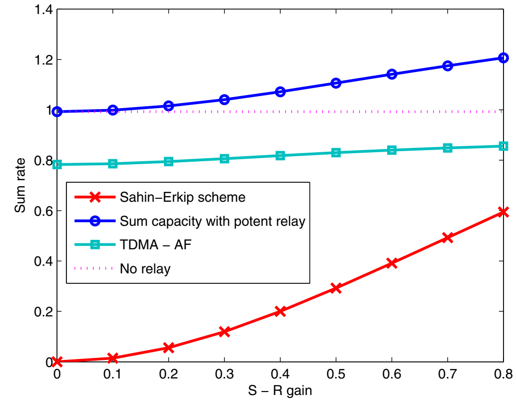

Fig. 6. Comparison of our result and several achievable schemes under weak interference when $P_1 = P_2 = P_R = 1, a = 0.1$, $S_{1(2)} - D_{1(2)}$ gain and $R - D$ gains are unity.

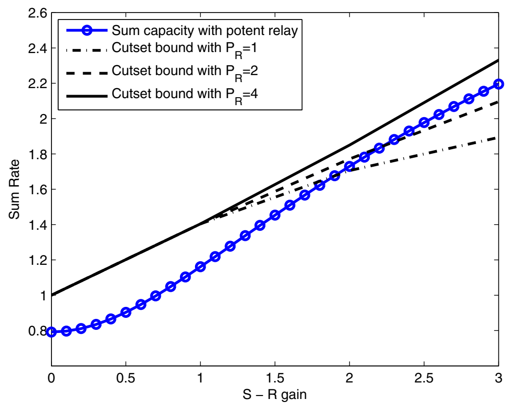

Fig. 5. Comparison of the cutset bound and our bound under strong interference when $P_1 = P_2 = 1, a = 1$, $S_{1(2)} - D_{1(2)}$ gain and $R - D$ gains are unity.

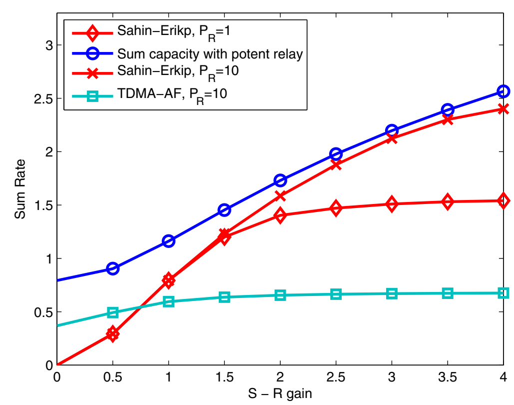

Fig. 7. Comparison of our result and several achievable schemes under strong interference when $P_1 = P_2 = 1, a = 1$, $S_{1(2)} - D_{1(2)}$ gain and $R - D$ gains are unity.

is because if $S - R$ gain is weak, the information available at the relay is limited, thus a powerful relay cannot help too much.

Next, we plot our sum rate bound and compare with the achievable schemes in Fig. 6. Sahin-Erkip achievability scheme relies on the decoding capability of relay, so it is outperformed by TDMA-AF in this regime. One important observation is that when the $S - R$ links are weak, not using the relay performs better than employing DF at the relay. For this and the remaining figures, the achievable scheme from Maric-Dabora-Goldsmith has similar performance with Sahin-Erkip scheme and thus is not shown to preserve clarity.

In contrast, in the strong interference regime, Sahin-Erkip achievable scheme performs better than TDMA-AF scheme. Fig. 7 shows the gap between our outerbound and the Sahin-Erkip achievable scheme. When the power of the relay is 10dB more than the transmitters, the gap becomes considerably small. Fig. 8 shows when the power of the relay is 19dB more than the transmitters, both Sahin-Erkip and Maric-Dabora-Goldsmith achievable schemes match our outerbound when $S - R$ links are strong, but the former scheme performs better than the latter one since it uses rate splitting.

VI. Conclusion

In this paper, we have studied the Gaussian interference channel with an intermediate relay. In particular, we considered the case when the relay has very large power. We proposed an equivalent channel model which is the GIFRC with in-band reception/out-of-band noiseless transmission. We have found the sum capacity of this channel under both weak and strong interference conditions. Both results serve, in turn, as a sum rate upperbound for GIFRC with finite relay power constraint. Simulations show that our results as bounds in this case are better than the cutset bound and close to the known achievable schemes under channel conditions described in Section V. The capacity region of GIFRC in general remains open. Future directions include improving achievable rates and accommodating rate constrained links.

Appendix A
Proof of Lemma 1

Proof: We first rewrite (17) and (18) as

$$h(bX_1^n + Z_2^n, cX_1^n + Z_R^n | N_2^n, N_4^n) = h(bX_1^n + W_2^n, cX_1^n + W_4^n)$$

$$h(aX_2^n + Z_1^n, dX_2^n + Z_R^n | N_1^n, N_3^n) = h(aX_2^n + W_1^n, dX_2^n + W_3^n)$$

where $W_i \sim \mathcal{N}(0, 1-\rho_i^2),\ i = 1, 2, 3, 4$. W_1, W_2, W_3, W_4 are independent of N_1, N_2, N_3, N_4, respectively.

$$\begin{aligned} & h(bX_1^n + bN_1^n, cX_1^n + cN_3^n) - h(bX_1^n + W_2^n, cX_1^n + W_4^n) \\ &= h(cX_1^n + cN_3^n) + h(bX_1^n + bN_1^n | cX_1^n + cN_3^n) \\ &\quad - h(cX_1^n + W_4^n) - h(bX_1^n + W_2^n | cX_1^n + W_4^n) \qquad (29) \\ &= h(cX_1^n + cN_3^n) + h(bN_1^n - bN_3^n | cX_1^n + cN_3^n) \\ &\quad - h(cX_1^n + W_4^n) - h(W_2^n - \frac{b}{c} W_4^n | cX_1^n + W_4^n) \\ &= h(cX_1^n + cN_3^n | bN_1^n - bN_3^n) + h(bN_1^n - bN_3^n) \\ &\quad - h(cX_1^n + W_4^n | W_2^n - \frac{b}{c} W_4^n) - h(W_2^n - \frac{b}{c} W_4^n) \\ &= h(cX_1^n + V_{13}^n) + h(bN_1^n - bN_3^n) - h(cX_1^n + U_{24}^n) \\ &\quad - h(W_2^n - \frac{b}{c} W_4^n) \qquad (30) \end{aligned}$$

where

$$V_{13} \sim \mathcal{N}(0, \frac{c^2\sigma_1^2\sigma_3^2}{\sigma_1^2 + \sigma_3^2}),\ U_{24} \sim \mathcal{N}(0, \frac{c^2(1-\rho_2^2)(1-\rho_4^2)}{c^2(1-\rho_2^2) + b^2(1-\rho_4^2)})$$

From the worst case noise lemma in [7], we have

$$\begin{aligned} & h(cX_1^n + V_{13}^n) - h(cX_1^n + U_{24}^n) \leq \\ & \quad nh(cX_{1G} + V_{13}) - nh(cX_{1G} + U_{24}) \end{aligned}$$

if

$$\frac{c^2\sigma_1^2\sigma_3^2}{\sigma_1^2 + \sigma_3^2} \leq \frac{c^2(1-\rho_2^2)(1-\rho_4^2)}{c^2(1-\rho_2^2) + b^2(1-\rho_4^2)}$$

where $X_{1G} \sim \mathcal{N}(0, P_1)$, which gives us the condition (19). Using similar method we can obtain the condition (20). ∎

APPENDIX B
PROOF OF LEMMA 2

Proof:

$$\begin{aligned} & I(X_{1G}; S_1, S_R | Y_1, Y_R) \\ &= h(S_1, S_R | Y_1, Y_R) - h(S_1, S_R | Y_1, Y_R, X_{1G}) \\ &= h(S_1 | Y_1, Y_R) + h(S_R | S_1, Y_1, Y_R) \\ &\quad - h(S_1 | Y_1, Y_R, X_{1G}) - h(S_R | S_1, Y_1, Y_R, X_{1G}) \\ &\leq h(S_1 | Y_1) + h(S_R | Y_R) - h(S_1 | Y_1, Y_R, X_{1G}) \\ &\quad - h(S_R | S_1, Y_1, Y_R, X_{1G}) \\ &= h(X_{1G} + N_1 | X_{1G} + aX_{2G} + Z_1) - h(N_1 | aX_{2G} + Z_1) \\ &\quad + h(cX_{1G} + cN_3 | cX_{1G} + dX_{2G} + Z_R) \\ &\quad - h(cN_3 | dX_{2G} + Z_R) \qquad (31) \end{aligned}$$

As long as (31) is 0, the genie S_1, S_R is smart. We can perform similar operation for the other term $I(X_{2G}; S_2, T_R | Y_2, Y_R)$, and conditions (21) can be obtained. ∎

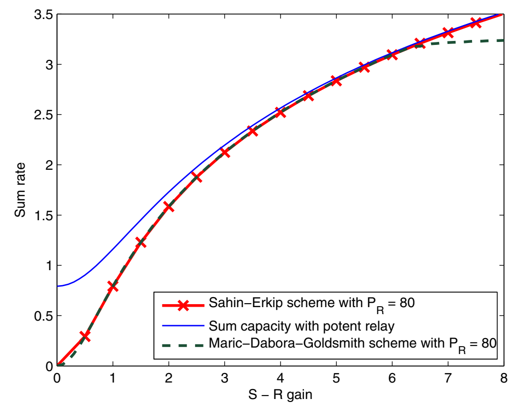

Fig. 8. Comparison of our outerbound between achievable scheme with powerful relay in strong interference regime when $P_1 = P_2 = 1, P_R = 80, a = 1$, $S_{1(2)} - D_{1(2)}$ gain and $R - D$ gains are unity.

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