Optimal Power Allocation for Relay Assisted F/TDMA Ad Hoc Networks

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***Abstract*— In this paper, we study the power allocation problem at the relay nodes for two-hop F/TDMA networks with multiple sources and destinations. Considering the sum capacity as the performance metric, we solve the problem of optimally allocating the total power of each relay node between the transmissions it is assisting. We consider regenerative decode-and-forward (RDF), nonregenerative decode-and-forward (NDF), amplify-and-forward (AF) and compress-and-forward (CF) at the relay nodes. We observe that the optimum power allocation for the RDF and NDF cases are modified water-filling solutions. In RDF, the optimum power allocation considers both the direct links of the users and the relay to destination links, whereas the optimal power allocation for the NDF relaying considers only the relay to destination links. We also observe that relay nodes employing AF or CF may provide higher sum capacities than relay nodes employing DF techniques when sufficient power is available at the relay nodes. Motivated by the optimum power allocation identified for each case, we provide insights to relay selection strategies for relay assisted F/TDMA networks.**

I. INTRODUCTION

Increasing demand on wireless communications services continues to motivate innovative physical layer designs for next generation wireless systems. Recently, relay assisted multihop communications has become a prominent candidate to combat the impairments of the wireless channel by exploiting spatial diversity without needing to deploy physical antenna arrays [1]–[6]. Relay assistance also mitigates the effects of path loss, and provides the source nodes with extended battery life. Results on the capacity of the full duplex relay channel go back to [1]. Relay transmission schemes are derived in [2] using half duplex transmission. Recently, reference [3] showed that the uplink capacity of two-user systems can be increased by using cooperation, where each user also acts as a relay for the other.

Relay assisted transmission is expected to improve the performance of multiuser systems as well [7], [8]. Such networks, henceforth referred to as *multiuser relay networks* are ones where each relay node would serve multiple users, and the total transmission power budget for each relay node would be limited. When this is the case, each user's transmission should be relayed with a fraction of the power from its corresponding relay node. In such a scenario, the total relay power should be allocated between the transmissions of information from the sources that relay over this node, in order to obtain the best performance. Optimum power allocation for relay networks is studied up-to-date in [4]–[6] for several relay transmission schemes *with a single source-destination pair*. In contrast, in this paper, we will consider a relay assisted F/TDMA network with *multiple source-destination pairs*, and relay nodes each of which assists multiple sources. We address the optimum power allocation problem at the relay nodes that perform decode-and-forward, amplify-and-forward, and compress-and-forward relay transmission while considering the sum capacity as the performance metric.

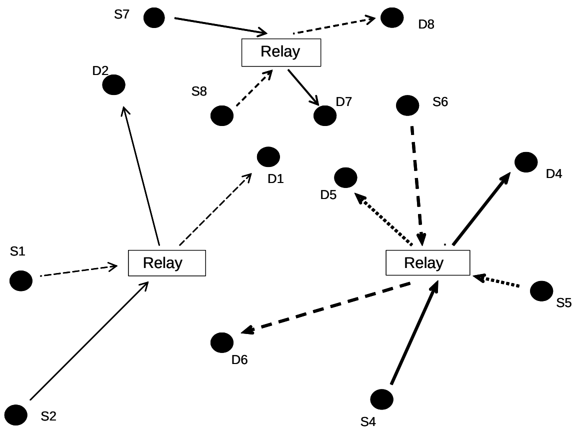

Fig. 1. System Model

II. SYSTEM MODEL

We consider a relay assisted F/TDMA ad hoc network with K users and L relay nodes (Figure 1). We assume that each user intends to transmit its signal to a different destination and has a pre-assigned relay node that will assist its transmission. The data transmission of each user occurs in *two* pre-assigned channels that can be either time slots or different frequencies. The user broadcasts its signal in the first channel, and the preassigned relay node transmits this user's information in the second channel. All channels of all users and relay nodes are distinct and nonoverlapping. The signal received by the destination in the ith user's first channel is

$$y_{di1} = \sqrt{P_{si}}\beta_i x_{si} + n_{di1} \tag{1}$$

where x_{si} is the symbol transmitted by user i, P_{si} is the transmit power of user i and β_i denotes the normalized channel gain from user i to the destination with n_{di1} as the zero mean

AWGN with unit variance. Similarly, the received signal at the relay node k to which user i is assigned, is

$$y_{ri} = \sqrt{P_{si}}\alpha_i x_{si} + n_{ri} \tag{2}$$

where α_i is the normalized channel gain from user i to the assigned relay node k, and n_{ri} is the zero mean AWGN with unit variance. In the second channel of the ith user, the kth relay node transmits x_{ri}, and the corresponding received signal at the destination is

$$y_{di2} = \sqrt{P_{ri}}\gamma_i x_{ri} + n_{di2} \tag{3}$$

where x_{ri}, P_{ri} and γ_i denote the signal transmitted for user i from the kth relay node, the transmit power of the kth relay node dedicated to user i and the normalized channel gain from the kth relay node to the destination of the ith user with a zero mean and unit variance AWGN n_{di2}, respectively. Note that the relay node should transmit after the source due to casuality constraints, and this constraint results in loss of one time slot when the channels represent different frequencies. We assume that each relay node has a total power constraint $\sum_{i \in A_k} P_{ri} \leq P_{Rk,total}$ where A_k denotes the set of users that relay their information through node k.

We consider four different relay transmission schemes at the relay nodes, and address the optimum power allocation in each case individually.

- **Regenerative Decode-and-Forward (RDF)**: When the transmission from the user is received reliably at the relay node, the relay node decodes the signal, re-encodes it with the same codebook used in the original user's transmission and transmits the signal in the second channel of the user [2], [5], [6].
- **Nonregenerative Decode-and-Forward (NDF)**: Similar to RDF, the relay decodes the signal, but re-encodes it with a codebook different than the original user and transmits it in the second channel of the user [4].
- **Amplify-and-Forward (AF)**: The signal received at the relay node is amplified and forwarded in the second channel of the user [2], [6].
- **Compress-and-Forward (CF)**: In this model, the relay node compresses and forwards the source's signal in the second channel of the user [8], [9].

III. OPTIMAL POWER ALLOCATION

In this work, we aim to optimally distribute the power of each relay node between the users' transmissions to be relayed by that node. Our goal is to maximize the sum capacity of the system. Clearly, the individual capacities of the users are a function of the relay transmission scheme used.

The optimum power allocation problem at the relay nodes is posed as

$$\max_{\{P_{ri}\}_{i=1,\cdots,K}} C_{sum} = \sum_{i=1}^{K} C_{i,*} \tag{4}$$

$$\text{s.t.} \sum_{i \in A_k} P_{ri} \leq P_{Rk,total}; \quad P_{ri} \geq 0 \quad \forall i,k \tag{5}$$

where $C_{i,*}$ is the individual capacity of user i and $*$ can be replaced with RDF, NDF, AF or CF according to the relay transmission scheme chosen. Since the power allocation at each relay node does not affect the individual capacities of the users that are served by other relay nodes, we focus on the sum capacity optimization problem at each relay node.

A. DF Type Relaying

For both RDF and NDF, the designated relay node must reliably decode the signal. Thus, the individual capacity of a relay assisted user cannot exceed the capacity of the user to relay link. This constraint leads to several important results in terms of optimum power allocation. When the direct link, β_i^2, is better than the relay link, α_i^2 for user i, the minimum of the capacity upper bounds of the direct link and the user to relay link is the latter. In this case, the capacity of the direct transmission is higher than that of the relay assisted transmission. Since by employing direct transmission for user i, the individual capacity of user i is maximized, *and* the relay has the potential to improve the sum capacity by investing its power in assisting the remaining users, the relay power allocated to user i should be

$$P_{ri} = 0 \quad \text{if} \quad \alpha_i^2 < \beta_i^2, \quad \forall i = 1, ..., K \tag{6}$$

For clarity of exposition, we denote the set of users that are served by the kth relay node, and have $\alpha_i^2 \geq \beta_i^2$ as A_k' in the sequel. In addition, observe that the maximum individual capacity of user i is upper bounded by

$$C_{i,RDF} \leq C_{i,NDF} \leq C_{upperDF} = \frac{1}{2}log(1 + P_{si}\alpha_i^2), \forall i \tag{7}$$

due to the decodability constraint at the relay. Thus, allocating more power of the relay node for the transmission of a user beyond a threshold will not increase the individual capacity of the user. These constraints should be taken into account for the power allocation problem in DF relay nodes.

In the case of RDF relay transmission, the individual capacity of user i is

$$C_{i,RDF} = min(\frac{1}{2}log(1 + P_{si}\beta_i^2 + P_{ri}\gamma_i^2), C_{upperDF}) \tag{8}$$

Similarly, for the case of NDF relay transmission, we have

$$C_{i,NDF} = min(\frac{1}{2}log(1+P_{si}\beta_i^2)+\frac{1}{2}log(1+P_{ri}\gamma_i^2), C_{upperDF}) \tag{9}$$

In the following analysis, we will use the following definitions for RDF and NDF networks:

Definition 1: *High potential users*: This is the set of users that are allocated nonzero power at their pre-assigned relay node and yet do not achieve the individual capacity upper bound (7). Thus, their individual capacities would be further increased, if more total power were available at the relay.

Definition 2: *Low potential users*: This is the set of users that achieve the maximum individual capacities indicated by (7), by the help of the relay node. For these users, even if more total relay power were available, the individual capacities would not increase.

Definition 3: *Nonrelayed users*: This is the set of users that are not assisted by the relay node. The users in this set have either high quality direct links, or low quality relay to destination links.

1) RDF Relaying: We are now ready to state our results for RDF relay networks.

***Theorem** 1:* The optimal power allocation for RDF relay networks results in three user sets, namely *high potential users*, *low potential users*, and *nonrelayed users* for each relay node.

1) The optimum relay power dedicated to high potential user i, and the achieved individual capacity of user i, are

$$P_{ri} = (\frac{1}{\mu_{k,RDF}} - \frac{1 + P_{si}\beta_i^2}{\gamma_i^2})^+ \tag{10}$$

$$C_{i,RDF} = \frac{1}{2} log(\gamma_i^2/\mu_{k,RDF}) \tag{11}$$

respectively, where $(.)^+ = max(.,0)$ and $\mu_{k,RDF}$ is the water level for the kth RDF relay node that satisfies $\sum_{i \in A_k} P_{ri} = P_{Rk,total}$.

2) The optimum relay power dedicated to low potential user i, and the achieved individual capacity of user i, are

$$P_{ri} = \frac{P_{si}(\alpha_i^2 - \beta_i^2)}{\gamma_i^2}; C_{i,RDF} = \frac{1}{2} log(1 + P_{si}\alpha_i^2) \tag{12}$$

3) The nonrelayed users set involves the users that either have better direct links than the source to relay links, i.e., $\alpha_i^2 < \beta_i^2$, or high quality direct links or low quality relay to destination links, i.e., $\frac{1+P_{si}\beta_i^2}{\gamma_i^2} > \frac{1}{\mu_{k,RDF}}$.

Proof: Using the fact that $P_{ri} = 0$ for the users that have $\beta_i^2 > \alpha_i^2$ the optimization problem at the kth relay node can be expressed as

$$\max_{\{P_{ri}\}_{i \in A'_k}} \sum_{i \in A'_k} \frac{1}{2} log(1 + P_{si}\beta_i^2 + P_{ri}\gamma_i^2) \tag{13}$$

$$\text{s.t.} \sum_{i \in A'_k} P_{ri} \leq P_{Rk,total}; \quad P_{ri} \geq 0, \quad \forall i \tag{14}$$

$$\frac{1}{2} log(1 + P_{si}\beta_i^2 + P_{ri}\gamma_i^2) \leq \frac{1}{2} log(1 + P_{si}\alpha_i^2), \quad \forall i \tag{15}$$

Constraint (15) is simply an upper bound for $\{P_{ri}\}$, and we have

$$0 \leq P_{ri} \leq \frac{P_{si}(\alpha_i^2 - \beta_i^2)}{\gamma_i^2} \tag{16}$$

Thus, the Lagrangian, $L(\{P_{ri}\}, \mu_{k,RDF}, \{\rho_{i,RDF}\})$, is

$$\sum_{i \in A'_k} \frac{1}{2} log(1 + P_{si}\beta_i^2 + P_{ri}\gamma_i^2) + \mu_{k,RDF}(\sum_{i \in A'_k} P_{ri} - P_{Rk,total})$$

$$+ \sum_{i \in A'_k} \rho_{i,RDF}(P_{ri} - \frac{P_{si}(\alpha_i^2 - \beta_i^2)}{\gamma_i^2})$$

where $\mu_{k,RDF}$ and $\rho_{i,RDF}$ are the Lagrange multipliers associated with the total transmit power constraint of the relay node k, and the upper bound for the relay power for user i, respectively. The cost function is a concave function and the $\{P_{ri}\}$ set is a convex set. Thus, simply using the KKT conditions, we arrive at the optimum relay power for user i as

$$P_{ri} = min((\frac{1}{\mu_{k,RDF}} - \frac{1 + P_{si}\beta_i^2}{\gamma_i^2})^+, \frac{P_{si}(\alpha_i^2 - \beta_i^2)}{\gamma_i^2}) \tag{17}$$

The users for which the upper bounds in (15) are inactive, and $P_{ri} = (\frac{1}{\mu_{k,RDF}} - \frac{1+P_{si}\beta_i^2}{\gamma_i^2}) > 0$, form the high potential users set. When the upper bound is active, $P_{ri} = \frac{P_{si}(\alpha_i^2 - \beta_i^2)}{\gamma_i^2}$, and the corresponding users are the low potential users. Finally, the users with $(\frac{1}{\mu_{k,RDF}} - \frac{1+P_{si}\beta_i^2}{\gamma_i^2}) < 0$, or $\alpha_i^2 < \beta_i^2$ form the set of nonrelayed users. □

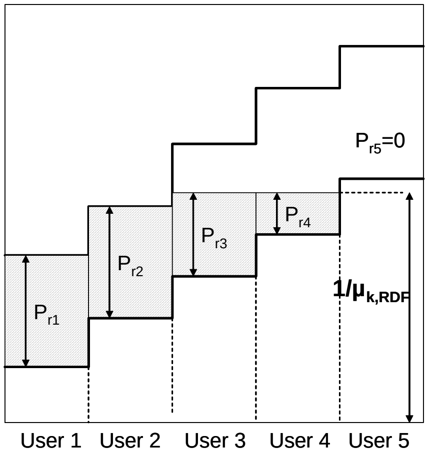

Fig. 2. Optimum power allocation for RDF relaying

The optimum power allocation for RDF networks is a modified water-filling solution where each user has both a base and an upper water level. The base level, $\frac{1+P_{si}\beta_i^2}{\gamma_i^2}$, is due to the direct link and the channel gain of the relay node to the destination for each user, whereas the upper level, $\frac{P_{si}(\alpha_i^2-\beta_i^2)}{\gamma_i^2} + \frac{1+P_{si}\beta_i^2}{\gamma_i^2}$, is due to the decodability constraints of the RDF relay nodes. Such a power allocation scheme is demonstrated in Figure 2 with five users and one relay. In this example, users 1 and 2 are the low potential users for which the relay node allocates enough power for each user to achieve their maximum individual capacities. Users 3 and 4 are high potential users since their individual capacities can still be improved by increasing the relay power. User 5 is a nonrelayed user and is not allocated any power because it has either a high-quality direct link or a low-quality relay-to-destination link. Observe that the relay node considers both the quality of the direct links of the users, and its own channel gain to the intended destinations, and will try to help the users with low quality direct links, and high quality relay to destination links.

In essence, the optimal power allocation tries to help the weak users that it can efficiently assist, providing fairness among the users. We note that, for low potential users, the benefit provided by the relay node does not increase with increased relay power. Thus, an appropriate relay selection strategy for RDF relay networks should be to select the relay nodes that will provide both high quality user to relay and relay to destination links. When the relay power is scarce, the relay node will help only one user that has the lowest $\frac{1+P_{si}\beta_i^2}{\gamma_i^2}$.

2) NDF Relaying: When the relays operate in the NDF mode, we have the following theorem for the optimal power allocation.

***Theorem** 2:* The optimal power allocation for NDF relay networks results in three user sets, namely *high potential users*, *low potential users* and *nonrelayed* users for each relay node.

1) The optimum relay power dedicated to high potential user i, and the achieved individual capacity of user i, are

$$P_{ri} = (\frac{1}{\mu_{k,NDF}} - \frac{1}{\gamma_i^2})^+ \tag{18}$$

$$C_{i,NDF} = \frac{1}{2}log(1 + P_{si}\beta_i^2) + \frac{1}{2}log(\frac{\gamma_i^2}{\mu_{k,NDF}}) \tag{19}$$

respectively, where $\mu_{k,NDF}$ is the water level for the kth NDF relay node that satisfies its power constraint.

2) The optimum relay power dedicated to low potential user i, and the achieved individual capacity of user i, are

$$P_{ri} = \frac{P_{si}(\alpha_i^2 - \beta_i^2)}{\gamma_i^2(1 + P_{si}\beta_i^2)} \tag{20}$$

$$C_{i,NDF} = \frac{1}{2}log(1 + P_{si}\alpha_i^2) \tag{21}$$

3) The nonrelayed users are those users for which either their direct links are better than their source-to-relay links, i.e., $\alpha_i^2 < \beta_i^2$, or their relay-to-destination links have low quality, i.e., $\frac{1}{\mu_{k,NDF}} < \frac{1}{\gamma_i^2}$.

Proof: The power allocation problem at the kth NDF relay node can be expressed as

$$\max_{\{P_{ri}\}_{i\in A'_k}} \sum_{i\in A'_k} \frac{1}{2}log(1 + P_{si}\beta_i^2) + \frac{1}{2}log(1 + P_{ri}\gamma_i^2) \tag{22}$$

$$\text{s.t.} \sum_{i\in A'_k} P_{ri} \leq P_{Rk,total}; \qquad P_{ri} \geq 0, \quad \forall i \tag{23}$$

$$\frac{1}{2}[log(1 + P_{si}\beta_i^2) + log(1 + P_{ri}\gamma_i^2)] \leq \frac{1}{2}log(1 + P_{si}\alpha_i^2), \forall i \tag{24}$$

The decodability constraint in (24) yields the upper bound

$$0 \leq P_{ri} \leq \frac{P_{si}(\alpha_i^2 - \beta_i^2)}{\gamma_i^2(1 + P_{si}\beta_i^2)} \tag{25}$$

The Lagrangian, $L(\{P_{ri}\}, \mu_{k,NDF}, \{\rho_{i,NDF}\})$, is

$$\frac{1}{2}\sum_{i\in A'_k}[log(1 + P_{si}\beta_i^2) + log(1 + P_{ri}\gamma_i^2)] - \mu_{k,NDF}P_{Rk,total}$$
$$+ \sum_{i\in A'_k} \mu_{k,NDF}P_{ri} + \rho_{i,NDF}(P_{ri} - \frac{P_{si}(\alpha_i^2 - \beta_i^2)}{\gamma_i^2(1 + P_{si}\beta_i^2)})$$

where $\mu_{k,NDF}$ and $\rho_{i,NDF}$ are the Lagrange multipliers associated with the power constraint of the relay node k and the upper bound for the relay power used for user i, respectively. Once again, we have a convex program, and using KKT conditions, we arrive at the optimum relay power for user i as

$$P_{ri} = min((\frac{1}{\mu_{k,NDF}} - \frac{1}{\gamma_i^2})^+, \frac{P_{si}(\alpha_i^2 - \beta_i^2)}{\gamma_i^2(1 + P_{si}\beta_i^2)}) \tag{26}$$

The users for whom (24) is inactive, and $P_{ri} = \frac{1}{\mu_{k,NDF}} - \frac{1}{\gamma_i^2} > 0$ are high potential users. When (24) is active, $P_{ri} = \frac{P_{si}(\alpha_i^2 - \beta_i^2)}{\gamma_i^2(1+P_{si}\beta_i^2)}$, as the low potential users. The users that have $\frac{1}{\mu_{k,NDF}} < \frac{1}{\gamma_i^2}$ or $\alpha_i^2 < \beta_i^2$ are the nonrelayed users. □

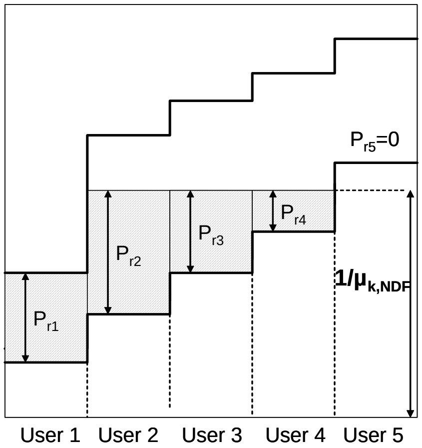

Fig. 3. Optimum power allocation for NDF relaying

Observe that the optimum power allocation for NDF relay networks tries to use the relay to destination channels as efficiently as it can without considering the direct links of the users. The optimum solution is a modified water-filling solution with base levels $\frac{1}{\gamma_i^2}$, and upper levels $\frac{1}{\gamma_i^2} + \frac{P_{si}(\alpha_i^2 - \beta_i^2)}{\gamma_i^2(1+P_{si}\beta_i^2)}$. The upper level is due to the decodability constraint of the NDF relay node. Such a power allocation scheme is demonstrated in Figure 3. In this example, user 1 is a low potential user and users 2, 3 and 4 are high potential users. User 5 is a nonrelayed user since the relay node has very low channel gain to its destination. Similar to the RDF case, even if the total transmit power of relay nodes are increased, the low potential users will not be able to achieve higher individual capacities. Thus, we can conclude that employing the appropriate relay selection strategy that provides high quality user to relay, and relay to destination links, improves the performance of the NDF relay networks.

B. AF Relaying

When AF relay transmission is used, the individual capacity of user i is

$$C_{i,AF} = \frac{1}{2}log(1 + P_{si}\beta_i^2 + \frac{P_{si}\alpha_i^2 P_{ri}\gamma_i^2}{P_{si}\alpha_i^2 + P_{ri}\gamma_i^2 + 1}) \tag{27}$$

For AF relay networks, we have the following theorem:

***Theorem** 3:* The optimal power allocation for AF relay networks results in nonzero power allocation for a subset of the users assigned to the relay node. The optimum power allocated to assist user i is

$$P_{ri} = (\frac{-(\frac{a_i}{b_i} + 2) + \sqrt{(\frac{a_i}{b_i})^2 + \frac{4a_i}{\mu_{k,AF}}(1 + \frac{a_i}{b_i})}}{2(a_i + b_i)})^+ \tag{28}$$

where $a_i = \dfrac{P_{si}\alpha_i^2/(P_{si}\alpha_i^2+1)}{(1+\beta_i^2 P_{si})/\gamma_i^2}$ and $b_i = \dfrac{\gamma_i^2}{P_{si}\alpha_i^2+1}$ while $\mu_{k,AF}$ is the water level for the kth AF relay node that satisfies $\sum_{i\in A_k} P_{ri} = P_{Rk,total}$.

Proof: The power allocation problem at the kth AF relay node can be expressed as

$$\max_{\{P_{ri}\}_{i\in A_k}} \sum_{i\in A_k} \frac{1}{2} log(1+P_{si}\beta_i^2 + \frac{P_{si}\alpha_i^2 P_{ri}\gamma_i^2}{P_{si}\alpha_i^2 + P_{ri}\gamma_i^2 + 1}) \qquad (29)$$

$$\text{s.t.} \sum_{i\in A_k} P_{ri} \le P_{Rk,total}; \qquad P_{ri} \ge 0, \quad \forall i \qquad (30)$$

which, again is a convex program. The Lagrangian is

$$L(\{P_{ri}\}, \mu_{k,AF}) = \frac{1}{2} log(1+P_{si}\beta_i^2 + \frac{P_{si}\alpha_i^2 P_{ri}\gamma_i^2}{P_{si}\alpha_i^2 + P_{ri}\gamma_i^2 + 1}) + \mu_{k,AF}(\sum_{i\in A_k} P_{ri} - P_{Rk,total})$$

where $\mu_{k,AF}$ is the Lagrange multiplier associated with the total transmit power constraint of the relay node k. Simply taking the derivative with respect to P_{ri} and equating it to zero, we arrive at the optimum relay power for user i in (28). □

Observe that the optimal power allocation for the AF relay nodes results in nonzero power allocation to the users that satisfy $\mu_{k,AF} < a_i$. When the relay node is very close to a user, then $a_i \approx \frac{\gamma_i^2}{P_{si}\beta_i^2+1}$ and $b_i \to 0$. This corresponds to the case when the users' received SNR at the relay node are very high. The optimal power allocation in this case is identical to the optimal power allocation in RDF as expected. It is important to note that in AF, the individual capacities of the users are not constrained by the capacity of the user to relay channel. The upper bound for the individual capacity of user i is

$$C_{i,AF} \le \frac{1}{2} log(1+P_{si}\beta_i^2 + P_{si}\alpha_i^2) \qquad \forall i \qquad (31)$$

Thus, AF relaying may perform better than the DF relaying.

C. CF Relaying

In the case of CF relaying, when Gaussian codebooks are used, and the relay node compresses using Wyner-Ziv lossy source coding [10], the individual capacity of user i can be expressed as [9]

$$C_{i,CF} = \frac{1}{2} log(1+P_{si}\beta_i^2 + \frac{P_{si}\alpha_i^2}{1+\sigma_{Wi}^2}) \qquad (32)$$

with

$$\sigma_{Wi}^2 = \frac{P_{si}(\alpha_i^2+\beta_i^2)+1}{P_{ri}\gamma_i^2(P_{si}\beta_i^2+1)} \qquad (33)$$

For CF relay networks, we have the following theorem for optimum power allocation at each relay.

***Theorem** 4:* The CF relay with optimal power allocation assists a subset of the users that are assigned to the relay node. The optimum power allocation for user i is

$$P_{ri} = (\frac{-(\frac{X_i}{Y_i}+2) + \sqrt{(\frac{X_i}{Y_i})^2 + \frac{4X_i}{\mu_{k,CF}}(1+\frac{X_i}{Y_i})}}{2(X_i+Y_i)})^+ \qquad (34)$$

where $X_i = \frac{P_{si}\alpha_i^2\gamma_i^2}{(P_{si}\alpha_i^2+P_{si}\beta_i^2+1)}$ and $Y_i = \frac{\gamma_i^2(P_{si}\beta_i^2+1)}{(P_{si}\alpha_i^2+P_{si}\beta_i^2+1)}$ while $\mu_{k,CF}$ is the water level for the kth CF relay node that satisfies $\sum_{i\in A_k} P_{ri} = P_{Rk,total}$.

Proof: Proof follows identical steps to the proof of Theorem 3. □

Observe that, similar to AF case, the preassigned relay node k allocates nonzero power to user i if $\mu_{k,CF} < X_i$. When $P_{Rk,total} \to \infty$, $\sigma_{Wi}^2 \to 0$, which yields the same asymptotic upper bound for the individual capacity of user i as in the AF case:

$$C_{i,CF} \le \frac{1}{2} log(1+P_{si}\beta_i^2 + P_{si}\alpha_i^2) \qquad (35)$$

IV. NUMERICAL RESULTS

In this section, we present numerical results related to the performance of the F/TDMA multiuser relay network with optimum power allocation. For numerical results, we consider a F/TDMA multiuser relay network with 5 users and one relay node that serves all. The link SNRs of the users used throughout the simulations are $\{(P_{si}\beta_i^2, P_{si}\alpha_i^2, \gamma_i^2)\} = \{(1,4,3),(5,7,10),(9,11,6),(13,16,10),(15,22,2)\}$ dB. We investigate the individual capacities achieved by each relay transmission scheme with different values of power constraints for the relay node.

Figures 4, 5, 6 and 7 show the performance of the relay transmission for RDF, NDF, AF and CF with optimum power allocation, respectively. We observe that the individual capacities are improved as the relay power is increased up to a threshold for each user. In the RDF case, when the relay node has relatively low power, the relay node helps only the first user that has the highest $\frac{\gamma_i^2}{1+P_{s_i}\beta^2}$, since the rest of the users have higher direct links or the relay has low quality links to the destinations of these users. As the available power at the relay increases, the first user's potential is reached, and the relay node starts to help the rest of the users. We observe that individual capacities of the users 4 and 5 become approximately equal for larger relay power values. This is due to the fact that the terms $\frac{\gamma_i^2}{1+P_{s_i}\beta^2}$ and the channel gains from the relay node to the destinations become negligible with respect to $\mu_{k,RDF}$ when the relay has high power. We also observe that after a threshold, increasing the relay power does not help, since all users already achieve the maximum single-user capacities. In the NDF case, we again observe that the sum capacity is improved as the relay power is increased up to a threshold. Since NDF performs better than RDF, this threshold is much lower than the threshold in the RDF case. That is, for the maximum sum capacity, NDF requires less power at the relay node as compared to RDF. For NDF, we observe that the relay tries to use the relay to destination channels as efficiently as it can, without considering the performance of the direct links. However, the

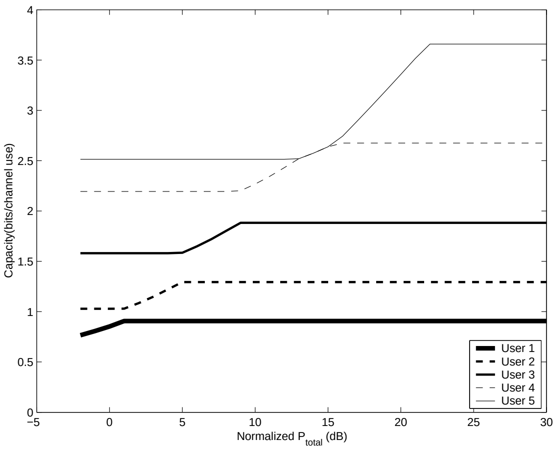

Fig. 4. RDF relay networks

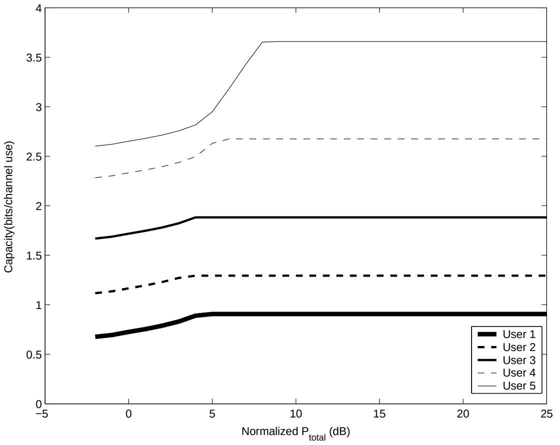

Fig. 5. NDF relay networks

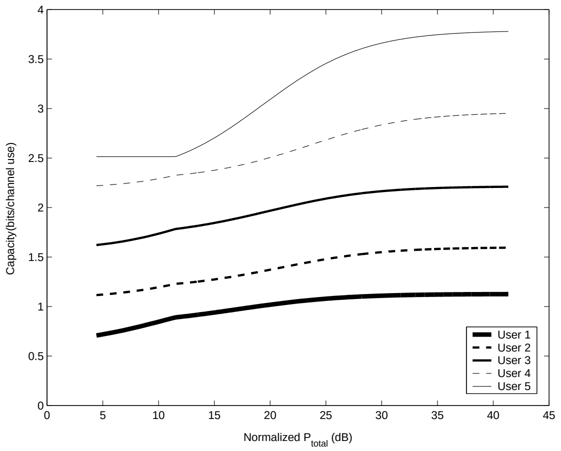

Fig. 6. AF relay networks

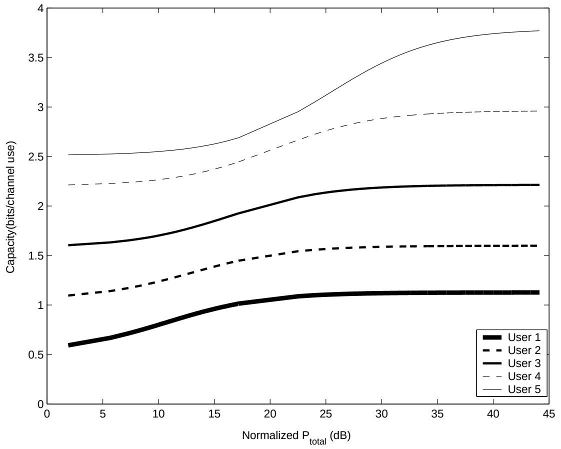

Fig. 7. CF relay networks

benefit that can be provided by the relay node is limited by the quality of the user to relay link. In Figure 6, we observe that the benefit obtained by the AF relay nodes converges to its maximum point gradually for each user. Similar behavior is observed for the CF relay transmission in Figure 7. We also observe that in the AF and CF mode, both individual capacities and the resulting sum capacities may be higher than the capacities that result from operating in a DF mode. This is due to the fact that DF relaying has the decodability constraints in the user to relay links whereas the AF and CF do not.

V. CONCLUSION

In this work, we have considered a two-hop multiple source-destination F/TDMA wireless network where intermediate nodes relay the information of source nodes. We have solved the problem of optimally allocating the power of each relay node between the users' transmissions it is assisting. We have observed that the optimum power allocation for RDF relay nodes helps the users that have low quality direct links and have destinations near to the relay first, and tries to improve the individual capacities of the weak users. The optimum power allocation in NDF relay networks tries to use the relay to destination channels as efficiently as it can. We also observe that the AF and CF relay nodes provide higher sum capacities than the DF relay nodes with high relay powers due to the decodability constraints of DF relaying. Finally, we note that the performance can be further improved by optimizing the fraction of time the relay dedicates to assisting each user in addition to its power, a topic of current interest.

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