Improving Soft Interference Cancellation for CDMA Systems

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***Abstract*– The optimum receiver to detect the bits of multiple CDMA users has exponential complexity in the number of active users in the system. Previous work showed that the successive and parallel soft interference cancellers correspond to *nonlinear programming relaxations* of the optimum multiuser detection problem. In this paper, we use this approximation method combined with the *slowest descent* approach to improve the performance of soft interference cancellers. The aim is to achieve a performance closer to the performance of the optimum receiver without significantly compromising the low complexity of the resulting receiver. We derive the resulting detectors and evaluate their performance. Results show that they can achieve near-optimum performance and outperform several previously proposed multiuser detectors.**

1 Introduction

It is well known that the capacity of a CDMA system is limited by the interference each user creates to other users in the system. Designing receivers that utilize the structure of the received signal to reduce the interference each user experiences is an effective method to enhance the system performance [1]. The optimum such receiver, which performs maximum likelihood detection of multiple users' bits has been shown to be exponentially complex in the number of users [2]. Following this development, many suboptimum linear and nonlinear receivers that outperform the matched filter receiver and that have reasonable complexity have been proposed [3–5].

In [6–8], it has been stressed that the linear decorrelator detector [3] quantizes the output of an unconstrained maximizer of the likelihood function. In [9, 10], it has been shown that several other well-known suboptimum multiuser detectors correspond to *approximate* solutions for the maximum likelihood multiuser detector. Among the suboptimum detectors, the multistage successive and parallel soft interference cancellers, proposed in [6, 11], are of particular interest due to their low bit error rate performance, and near-far resistant characteristics [6, 10]. It is thus worthwhile to consider modifications that would improve the performance of such a receiver without introducing a significant increase in computational complexity. To this end, we consider the application of the *slowest descent* method, proposed in [7, 8] for improving the performance of soft interference cancellers.

The slowest descent approach, introduced in [7], is applicable to detection techniques which quantize the relaxed maximum of an objective (e.g., likelihood) function. Specifically, the slowest descent method searches for a more likely bit vector by analyzing the slope of the likelihood function in a neighborhood of the relaxed maximum [8]. In the context of multiuser detection, the search involves a quantized relaxed maximizer bit vector estimate and a number of other estimates close to a line of least decrease in the relaxed likelihood function from its (relaxed) maximizer. In this work we investigate how the performance of soft interference cancellers being the quantized relaxed maximizers over a hypercube can be improved using the slowest descent method. We will offer evidence through numerical results that the resulting detectors perform better in terms of bit error rate compared with a number of other multiuser detectors including previously proposed interference cancellers.

2 System Model

We consider a synchronous CDMA system employing BPSK. Following references [10, 12], the received signal is given by

$$r(t) = \sum_{i=1}^{K} \sqrt{q_i} a_i s_i(t) + n(t) \tag{1}$$

where K is the number of users, q_i and a_i are received power and the transmitted bit (± 1 equiprobably) and $s_i(t)$ is the unit energy signature, i.e. $\int_0^T s_i(t)s_i(t)dt = 1$, of user i, and $n(t)$ is the additive white Gaussian noise (AWGN) process with power spectral density σ^2. The matched filter output of the ith user is given by $y_i = \int_0^T r(t)s_i(t)dt$. The received signal vector at the output of the matched filters, $\boldsymbol{y}$, is a sufficient statistic for the multiuser detection problem and is given by

$$\boldsymbol{y} = \boldsymbol{\Gamma\Lambda a} + \boldsymbol{n} \tag{2}$$

where $\boldsymbol{\Gamma}$ is the nonnegative definite cross correlation matrix with $\Gamma_{ij} = \int_0^T s_i(t)s_j(t)dt$, $\boldsymbol{\Lambda}$ is a diagonal matrix containing the users' received amplitudes $\Lambda_{ii} = \sqrt{q_i}$, $\boldsymbol{a}$ is the vector containing the information bits of the users and $\boldsymbol{n}$ is a zero mean Gaussian random vector with auto covariance matrix $E[\boldsymbol{n}\boldsymbol{n}^\top] = \sigma^2\boldsymbol{\Gamma}$.

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3 Relaxations of the ML-MUD

The maximum likelihood multiuser detection problem (ML-MUD) [1,3] solves for $\boldsymbol{a}_{ML}$, the *best* estimate of the information bits of all users, given $\boldsymbol{y}$, by maximizing the likelihood function which is quadratic in $\boldsymbol{a}$. Specifically,

$$\boldsymbol{a}_{ML} = \arg \max_{\boldsymbol{a} \in \{-1,1\}^K} \underbrace{-\boldsymbol{a}^\top \boldsymbol{R} \boldsymbol{a} + 2\boldsymbol{a}^\top \boldsymbol{\Lambda} \boldsymbol{y}}_{l(\boldsymbol{a})}, \tag{3}$$

where $\boldsymbol{R} = \boldsymbol{\Lambda \Gamma \Lambda}$ with $R_{ij} = \sqrt{q_i}\sqrt{q_j}\Gamma_{ij}$. Henceforth, we will assume that the signatures of the users are independent and $\boldsymbol{\Gamma}$ and hence $\boldsymbol{R}$ are positive definite.

The ML multiuser detection problem for general correlation matrices was shown to be NP hard which means that one can find the optimum $\boldsymbol{a}$ only by exhaustive search of 2^K candidate vectors [2]. Recently, ML multiuser detection problem has been examined from a nonlinear programming perspective leading to relaxations of the original problem [10]. Specifically, with a positive definite $\boldsymbol{R}$, the likelihood function in (3) is strictly concave in $\boldsymbol{a}$ and it has a well defined unique maximizer over a convex set. Thus, one can find solutions by relaxing the constraint set –which in the original problem contains only the corners of the unit hypercube– such that the resulting "relaxed" constraint set, denoted in the following as Ω_c, is convex. The approach requires that the constraint set, Ω_c, contains the feasible set of the original problem. The solution, denoted as $\boldsymbol{a}^*$, can then be mapped to the feasible set of the original problem by taking the sign of each component of the relaxed solution vector (since bits are equiprobably ± 1). This is equivalent to mapping the maximizer to the closest (in Euclidean distance) bit vector. It has been shown that several well-known suboptimum detectors can be represented as relaxations of the ML multiuser detection problem [10].

4 The Slowest Descent Method

Consider an optimization problem where the feasible set consists of discrete values. If a closed form solution does not exist, one possible approach is to solve for a *continuous* optimizer of the problem and then exploit the assumed continuity of the likelihood function's second derivative in the neighborhood of this optimizer to generate a subset of the discrete feasible set which holds intuitively *more likely* values. The slowest descent method generates this subset by studying the slope of the likelihood function in the neighborhood of a maximizer and including all the feasible points closest (in Euclidean distance) to P lines of the least decrease in the likelihood function. The slowest descent estimate is the best (most likely) feasible point from this subset.

A slightly more general formulation of the slowest descent method than the one introduced in [8,13] follows. The slowest descent lines are the P mutually orthogonal lines $\{\boldsymbol{a}^* + \rho\hat{\Delta}^i_{min}, i = 1, \dots, P\}$ of the least local decrease in the likelihood function $l(\boldsymbol{a})$ from the maximizer $\boldsymbol{a}^*$ of $l(\boldsymbol{a})$ over the convex set Ω_c. Here, ρ is such that $\boldsymbol{a}^* + \rho\hat{\Delta}^i_{min} \in \Omega_c$. Each $\hat{\Delta}^i_{min}$ is a feasible direction at $\boldsymbol{a}^*$, that is, it belongs to the feasible direction set:

$$\Omega_\Delta(\boldsymbol{a}^*) = \text{closure}\{\Delta : \boldsymbol{a}^* + \epsilon\Delta \in \Omega_c,\ 0 < \epsilon < \epsilon_0\}, \tag{4}$$

where ϵ_0 is some positive number.

$\hat{\Delta}^i_{min}$ are mutually orthogonal solutions of

$$\hat{\Delta}^i_{min} = \arg \min_{\|\Delta\|=1,\, \Delta \in \Omega_\Delta(\boldsymbol{a}^*)} \{l(\boldsymbol{a}^*) - l(\boldsymbol{a}^* + \epsilon\Delta)\} \tag{5}$$

for a sufficiently small (positive) $\epsilon < \epsilon_0$.

Efficient recursive algorithms for computing bit vectors closest to the line intervals of the least decrease in $l(\boldsymbol{a})$ and their respective likelihoods are given in [8].

It is easily seen that the unconstrained maximizer of the ML multiuser detection problem given in (3) results in the non-quantized (and normalized) decorrelating solution [1], i.e., for $\Omega_c = \mathcal{R}^K$, $\boldsymbol{a}^* = \hat{\boldsymbol{a}}_c = \Lambda^{-1}\Gamma^{-1}\mathbf{y}$. It can also be seen that $\hat{\Delta}^i_{min}$ are the eigenvectors of the Hessian of $l(\boldsymbol{a})$, $\mathbf{R}$ [7]. In Section 6 we derive the slowest descent solution based on the convex set $\Omega_c = [-1, 1]^K$.

5 Soft Interference Cancellation

The constraint set of the ML multiuser detection problem (3) consists of the corner points of the unit hypercube. An effective approximation method is to relax the constraint set to cover the whole hypercube and use nonlinear programming algorithms to find the solution of the new convex programming problem [14]. The relaxed problem is

$$\boldsymbol{a}^* = \arg \max_{\boldsymbol{a} \in [-1,1]^K} -\boldsymbol{a}^\top \boldsymbol{R} \boldsymbol{a} + 2\boldsymbol{a}^\top \boldsymbol{\Lambda} \boldsymbol{y}. \tag{6}$$

The cost function in (6) is concave and has a unique maximum over the convex constraint set $[-1, 1]^K$. However, the optimum point does not have a closed form representation and one should use iterative methods to obtain a solution. The class of iterative methods that can be used include the nonlinear Gauss-Seidel and nonlinear Jacobi methods [15]. Reference [10] showed that the resulting iterative (multi stage) algorithms using these two methods yield the successive and the parallel *soft interference cancellers* respectively. The algorithms have been shown to converge to $\boldsymbol{a}^*$ in (6) under mild conditions [10]. For each user i, the first step of the Gauss-Seidel iteration is

$$\hat{x}(t+1) = \frac{1}{\sqrt{q_i}}\left(y_i - \sum_{j=1}^{i-1}\sqrt{q_j}\Gamma_{ji}a_j(t+1) - \sum_{j=i+1}^{N}\sqrt{q_j}\Gamma_{ji}a_j(t)\right) \tag{7}$$

and the first step for the Jacobi iteration is

$$\hat{x}(t+1) = \frac{1}{\sqrt{q_i}}\left(y_i - \sum_{j=1, j\neq i}^{K}\sqrt{q_j}\Gamma_{ji}a_j(t)\right) \tag{8}$$

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The second step for both algorithms is

$$a_i(t+1) = \begin{cases} -1, & \hat{x}(t+1) < -1 \\ \hat{x}(t+1), & -1 \le \hat{x}(t+1) \le 1 \\ 1, & \hat{x}(t+1) > 1 \end{cases} \quad (9)$$

At each stage, to get the estimate of each user's bit, both receivers use soft estimates of the bits to reconstruct the interference and subtract this estimate from the user's matched filter output, scale the result by the amplitude of the user and project onto $[-1, 1]$. The difference between the two is that while the Gauss-Seidel algorithm uses the available current stage estimates of the users (successive), i.e., feedback from a group of users whose bit estimates are already computed, the Jacobi algorithm uses only bit estimates from the previous stage (parallel).

In the following, we describe how to use the slowest descent approach (SD) to improve upon the performance of the soft interference cancellers (SIC).

6 Improving on SIC Using SD Search

We have observed in Section 5 that the maximizer $\boldsymbol{a}^*$ of $l(\boldsymbol{a})$ over $\Omega_c = [-1, 1]^K$ is the convergence point of the iterative algorithms given by (7)-(9), i.e., the non-quantized output of the soft interference canceller after several stages (iterations) are performed [10]. One can take the sign of $\boldsymbol{a}^*$ as is conventionally done and arrive at a users' joint bit vector estimate. Instead of selecting this quantized relaxed maximizer as our estimate, we will choose the most likely of bit vectors which are closest (in terms of the Euclidean distance) to the slowest descent lines determined by direction vectors (5) and $\boldsymbol{a}^*$. A solution to the problem on the right hand side of (5) for $\Omega_c = [-1, 1]^K$ is given below.

First, we notice that the difference on the right hand side of (4) is non-negative for any feasible direction defined in (5). The difference can be expanded as

$$l(\boldsymbol{a}^*) - l(\boldsymbol{a}^* + \epsilon\Delta) = \epsilon^2 \Delta^T \mathbf{R} \Delta - 2\epsilon \Delta^T \mathbf{g}_l(\boldsymbol{a}^*), \quad (10)$$

where

$$\mathbf{g}_l(\boldsymbol{a}^*) = -\mathbf{R}\boldsymbol{a}^* + \Lambda \mathbf{y}$$

is the gradient of $l(\boldsymbol{a})$ at the maximizer $\boldsymbol{a}^*$.

Since $\boldsymbol{a}^*$ is the maximizer of $l(\boldsymbol{a})$ on Ω_c we have

$$\Delta^T \mathbf{g}_l(\boldsymbol{a}^*) \le 0 \quad (11)$$

for all feasible directions $\Delta \in \Omega_\Delta(\boldsymbol{a}^*)$, i.e., the function has to be locally non-increasing in Ω_c.

From (10), we observe that, for a sufficiently small ϵ, the dominating term is the linear term. Thus to minimize this difference, we first need to seek those feasible Δ which are orthogonal to $\mathbf{g}_l(\boldsymbol{a}^*)$;

$$[\hat{\Delta}^i_{min}]^T \mathbf{g}_l(\boldsymbol{a}^*) = 0 \quad (12)$$

for $i = 1, \ldots, P$. We select $P \le K$ in such a manner that all solutions to (5) are orthogonal to $\mathbf{g}_l(\boldsymbol{a}^*)$. The feasible directions non-orthogonal to $\mathbf{g}_l(\boldsymbol{a}^*)$ are deemed to decrease $l(\boldsymbol{a})$ excessively and are neglected.

For $\Omega_c = [-1, 1]^K$, it can be seen that

$$\Omega_\Delta(\boldsymbol{a}^*) = \{\Delta\colon \text{sign}\{\delta_i\} = \text{sign}\{-a_i^*\} \text{ when } |a_i^*| = 1\} \quad (13)$$

and

$$[\mathbf{g}_l(\boldsymbol{a}^*)]_i = 0 \text{ iff } |a_i^*| < 1.$$

Thus, for (12) to hold, we need

$$\hat{\Delta}^i_{min} \in \Omega^*_\Delta(\boldsymbol{a}^*) = \{\Delta : \delta_i = 0 \text{ when } |a_i^*| = 1\}. \quad (14)$$

Consequently, (5) becomes

$$\begin{aligned} \hat{\Delta}^i_{min} &= \arg \min_{\|\Delta\|=1,\, \Delta \in \Omega^*_\Delta(\boldsymbol{a}^*)} \epsilon^2 \Delta^T \mathbf{R} \Delta \\ &= \arg \min_{\|\tilde{\Delta}\|=1} \epsilon^2 \tilde{\Delta}^T \tilde{\mathbf{R}} \tilde{\Delta}. \end{aligned} \quad (15)$$

where $\tilde{\Delta}$ is a reduced vector obtained by eliminating the zero elements of Δ, and $\tilde{\mathbf{R}}$ is the corresponding reduced submatrix of $\mathbf{R}$ whose corresponding rows/columns have been eliminated. Formally, we can express $\tilde{\mathbf{R}}$ as:

$$\tilde{\mathbf{R}} = \tilde{\mathbf{P}} \Lambda \mathbf{S}^T \mathbf{S} \Lambda \tilde{\mathbf{P}},$$

where $\tilde{\mathbf{P}}$ is the projection matrix

$$\tilde{\mathbf{P}} = \text{diag}\{\delta(|a_i^*| < 1) \text{ for } i = 1, \ldots, K\},$$

where $\delta(x) = 1$ for $x = 1$ and 0 otherwise. Note, that $\mathbf{S}\Lambda\tilde{\mathbf{P}}$ results in eliminating those columns (user sequences and corresponding amplitudes) of $\mathbf{S}\Lambda$ which correspond to the index i for which $|a_i^*| = 1$. Thus, from (15) we see that $\hat{\tilde{\Delta}}^i_{min}$ are left eigen-vectors of the reduced matrix $\tilde{\mathbf{S}}$ obtained by eliminating sequences of all users for which the corresponding element of $\boldsymbol{a}^*$ is in $\{\pm 1\}$.

Thus, the proposed improved soft interference canceller can be summarized as follows:

- Compute $\boldsymbol{a}^*$ from (4) using a soft interference canceller.
- Compute the index set $\tilde{\mathcal{I}} = \{i : |a_i^*| < 1\}$ and the corresponding projection matrix

$$\tilde{\mathbf{P}} = \text{diag}\{1 \text{ for } i \in \tilde{\mathcal{I}} \text{ and } 0 \text{ otherwise}\}$$

- Find feasible directions $\{\hat{\Delta}^i_{min}\}$ orthogonal to $\mathbf{g}_l(\boldsymbol{a}^*)$ which correspond to the P smallest eigen-values of $\tilde{\mathbf{S}} = \mathbf{S}\Lambda\tilde{\mathbf{P}}$.
- Employ the recursive algorithms from [8] to compute bit vectors closest to the lines of the least decrease in $l(\boldsymbol{a})$, $\{\boldsymbol{a}^* + \rho\hat{\Delta}^i_{min} : \rho \in \mathcal{R}\}$ for $i = 1, \ldots, P$, and their respective likelihoods, and select the best one.

Note that, in the general case, the eigen-vectors $\{\hat{\Delta}^i_{min}\}$ are a function of $\mathbf{y}$ and, thus, pre-computing all possible eigen-vector sets has prohibitive complexity. On the other hand, if

$\boldsymbol{a}^*$ is in the interior of the hypercube $[-1,1]^K$ then $\boldsymbol{a}^* = \hat{\boldsymbol{a}}_c$ is the (normalized) non-quantized decorrelator output. Furthermore, $\{\hat{\Delta}^i_{min}\}$ are the left eigen-vectors of the matrix $\mathbf{S}$, and, consequently, can be pre-computed.

In the following section, we present the numerical results reporting the bit error performance of the proposed detectors. We also present, for comparison purposes, the simplified improved detectors that searches the lines corresponding to eigen-vectors of $\mathbf{S}$ instead of the eigen-vectors of $\tilde{\mathbf{S}}$.

7 Numerical Results

We consider two systems, one with a processing gain $G = 7$, and $K = 6$ users and another one with $G = 7$ and $K = 7$. In all experiments, we have simulated the bit error rate of the user of interest which is taken to be user 3. Two types of graphs are presented. One set depicts the error rate of user 3 versus the equal SNR of all users in the system. The second set analyzes the near-far resilience of provided algorithms by depicting the error-rate versus the SNR of the interfering users. The SNR of the user of interest is kept fixed and is depicted on the graph as well.

The following list relates the abbreviations for the employed algorithms to either their description or their more complete name:

- ML: the maximum likelihood detector
- DEC: the decorrelating detector
- SIC: soft interference canceller
- DEC-EV*: the slowest descent method from [7] based on $\Omega_c = \mathcal{R}^K$; the non-quantized decorrelator and the singular value decomposition (svd) of $\mathbf{S}$, which searches
  - DEC-EV1: one slowest descent direction
  - DEC-EVA: all slowest descent directions
- SIC-EV*: the soft interference canceller described in Section 6 based on $\Omega_c = [-1,1]^K$; the non-quantized SIC and the singular value decomposition (svd) of the reduced matrix $\tilde{\mathbf{S}}$, which searches
  - SIC-EV1: one slowest descent direction
  - SIC-EVA: all slowest descent directions
- EV*-SIC: the improved soft interference canceller described in Section 6 based on $\Omega_c = [-1,1]^K$; the non-quantized SIC and the singular value decomposition (svd) of the non-reduced matrix $\mathbf{S}$, which searches
  - EV1-SIC: one slowest descent direction
  - EVA-SIC: all slowest descent directions
- PIC: The two-stage parallel interference canceler (PIC), initialized with a decorrelator. Further iterations did not provide a significant improvement.

The cross correlation matrix for the $K = 7$ system is the symmetric three-valued Gram matrix, $\bar{\boldsymbol{\Gamma}}$, with $\bar{\Gamma}_{ii} = 1, \bar{\Gamma}_{ij} = -1/7$, for all $i \neq j$, except $\bar{\Gamma}_{17} = \bar{\Gamma}_{71} = -3/7$. The cross correlation matrix for the $K = 6$ system is [16]:

$$\boldsymbol{\Gamma} = \frac{1}{7}\begin{bmatrix} 7 & 3 & 1 & -1 & 1 & -1 \\ 3 & 7 & -3 & 3 & 1 & -1 \\ 1 & -3 & 7 & -3 & 3 & -3 \\ -1 & 3 & -3 & 7 & -3 & -1 \\ 1 & 1 & 3 & -3 & 7 & -3 \\ -1 & -1 & -3 & -1 & -3 & 7 \end{bmatrix} \tag{16}$$

Figures 1 through 4 show that the SIC-EV* detectors perform very close to the ML detector in both the equal power and the near-far situations, with SIC-EVA performing slightly better. In most cases, the EV*-SIC algorithms contribute very little degradation relative to the SIC-EV* detectors. The slowest descent based methods significantly improve on their "initialization" detectors: decorrelator and the soft interference canceller and frequently outperform both of these regardless of the "initialization" method used. In the near-far situation, the slowest descent based techniques behave in a consistent manner with ML detector, while the SIC and the decorrelator's performance is insensitive to near-far ratios [1, 6]. The PIC detector has poorer performance consistently in the equal-power case, and works better in the near-far situations.

8 Conclusion

In this work, we have introduced a detector which improves on the performance of the soft interference canceller for multiuser detection in CDMA systems based on both the idea of constraint optimization and the slowest descent approach. Simulation examples show that the derived improved soft interference canceller and its simplified version perform very well and can achieve near-optimal performance for two situations of interest in a CDMA system: the case of equal-power and the case of unequal-power interferers. It is also observed that they outperform several previously proposed detectors. Finally, the simplified version of the improved soft interference cancellers, EV*-SIC, has almost identical performance as the non-simplified improved interference cancellers, SIC-EV*. The significant reduction in the implementation complexity, via the pre-computation of necessary eigen directions, favors the use of the simplified version of the improved soft interference cancellers in practical systems.

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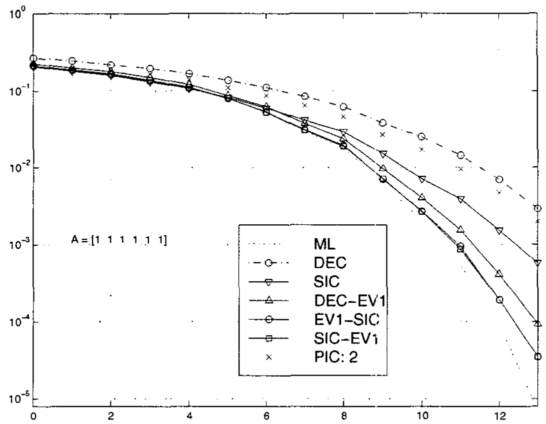

Figure 1: Equal power users, detectors based on one search direction, Gram matrix $\mathbf{\Gamma}$

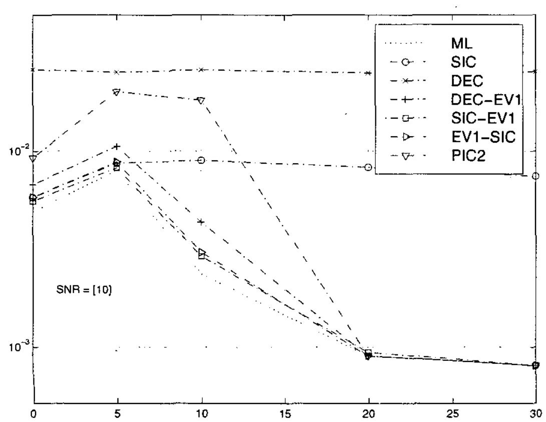

Figure 2: Unequal power users, detectors based on one search direction, Gram matrix $\mathbf{\Gamma}$

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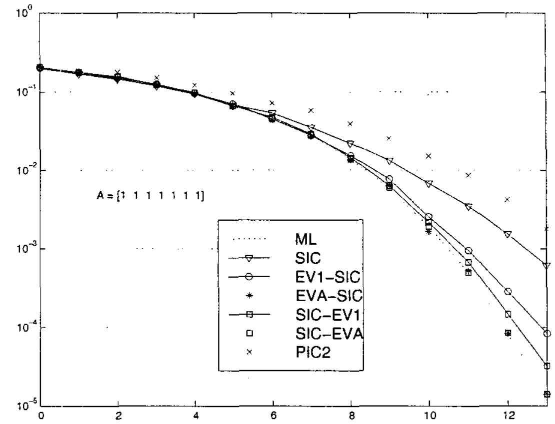

Figure 3: Equal power users, ML, decorrelator and SIC based detectors, Gram matrix $\bar{\mathbf{\Gamma}}$

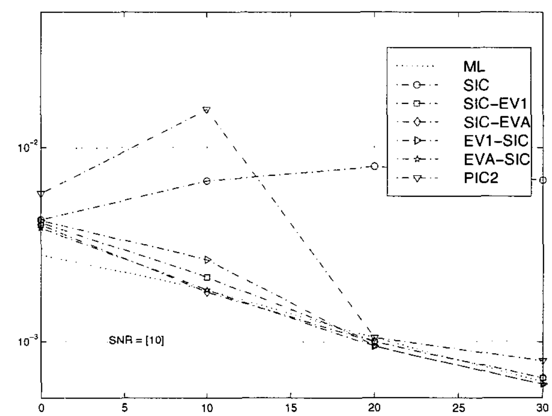

Figure 4: Unequal power users, ML, decorrelator and SIC based detectors, Gram matrix $\bar{\mathbf{\Gamma}}$

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