Optimal Power Allocation for Relay Nodes in Time Division Multiuser Relay Networks

Semih Serbetli and Aylin Yener
Electrical Engineering Department
The Pennsylvania State University
University Park, PA 16802
serbetli@psu.edu yener@ee.psu.edu

Abstract — **This paper studies the power allocation problem at the relay nodes for the uplink of a relay assisted TDMA system considering the sum capacity as the performance metric. We consider three different relay transmission schemes namely, regenerative decode-and-forward (RDF), nonregenerative decode-and-forward (NDF) and amplify-and-forward (AF) at the relay nodes, and address the optimum power allocation in each case. We observe that the optimum power allocation for RDF considers the direct links of the users and tries to equalize the rates of the users providing fairness to the system. In the NDF case, the optimum power allocation tries to provide equal improvement in the individual capacities of the users by the aid of the relay nodes. Motivated by the optimum power allocation identified for each case, we provide insights to relay selection strategies for relay assisted TDMA networks and resource allocation for cooperative communications. We observe that cooperation among the users provides fairness to the system through sharing of the resources.**

I. Introduction

Relay assisted wireless communication systems are attractive due to their potential of combating the impairment of the wireless channel using space diversity without the need of physical antenna arrays [1–8]. Relay assistance also mitigates the effects of path loss, and provides the source nodes with extended battery life. The relay channel is studied in [1], and upper and lower bounds for the capacity of the relay channel is identified in [2]. Simple relay transmission schemes are derived in [3] using half duplex transmission. Recently, reference [4] showed that the uplink capacity of two user systems can be increased by using cooperation, where each user also acts as a relay for the other.

In wireless networks, transmission power of the nodes is limited. Hence, power efficiency is a critical concern when designing relay transmission strategies. It has been shown that significant performance improvement is achieved by the optimum power allocation for various relay assisted networks [5–8].

Relay assisted transmission is expected to improve the performance of multiuser systems as well. Such networks, henceforth referred to as *multiuser relay networks* are ones where, each relay node would serve for multiple users, and the total transmission power budget for each relay node would be limited. When this is the case, a fraction of the power should be devoted to relay each user' transmission by the assisting relay node. In such a scenario, the total relay power should be allocated between the transmissions of information from the sources that relay over this node, in such a way to obtain the best performance. Optimum power allocation for relay networks is studied up-to-date in [5–8] for several relay transmission schemes *with a single source-destination pair.* In contrast, in this paper, we will consider the uplink of a relay assisted TDMA network with *multiple sources and relays*, and address the optimum power allocation problem at the relay nodes performing decode-and-forward and amplify-and-forward relay transmissions while considering the sum capacity as the performance metric.

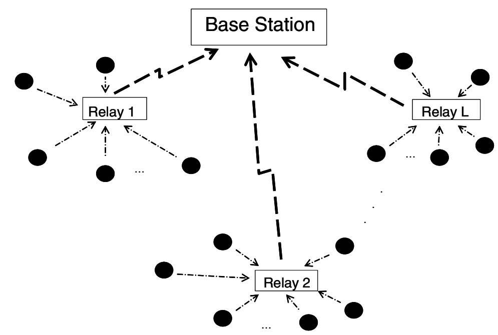

Figure 1: System Model

II. System Model

We consider the uplink of a relay assisted TDMA system with K users and L relay nodes and a destination node, e.g. a base station or an access point (Figure 1). We assume that each user has a pre-assigned relay node that will assist its transmission. The data transmission of each user occurs in *two* time slots where the user broadcasts its signal in the first time slot, and the preassigned relay node transmits this user's information in the second time slot. All time slots of all users and relay nodes are distinct and nonoverlapping. The signal received by the destination in the ith user's first time slot is

$$y_{di1} = \sqrt{P_{si}}\beta_i x_{si} + n_{di1} \quad (1)$$

where x_{si} is the symbol transmitted by user i, P_{si} is the transmit power of user i and β_i denotes the normalized channel gain from user i to the destination with $n_{d,i}$ as the zero mean AWGN with unit variance. Similarly, the received signal at the relay node k which user i is assigned, is

$$y_{ri} = \sqrt{P_{si}}\alpha_i x_{si} + n_{ri} \quad (2)$$

where α_i is the normalized channel gain from user i to the assigned relay node k, and n_{ri} is the zero mean AWGN with unit variance. In the second time slot, kth relay node transmits x_{ri} and the corresponding received signal at the destination is

$$y_{di2} = \sqrt{P_{ri}}\gamma_k x_{ri} + n_{di2} \tag{3}$$

where x_{ri}, P_{ri} and γ_k denote the signal transmitted for user i from the kth relay node, the transmit power of the kth relay node dedicated to user i and the normalized channel gain from the kth relay node to the destination with a zero mean and unit variance AWGN n_{di2}, respectively. We assume that each relay node has a total power constraint $\sum_{i\in A_k} P_{ri} \leq P_{Rk,total}$ where A_k denotes the set of users that relays their information through node k.

We consider three different relay transmission schemes at the relay nodes and address the optimum power allocation in each case individually.

- **Regenerative Decode-and-Forward (RDF)**: When the transmission from the user is received reliably at the relay node, the relay node decodes the signal, re-encodes it with the same codebook used in the original user's transmission and transmits the signal in the second time slot of the user [3, 5, 7].
- **Nonregenerative Decode-and-Forward (NDF)**: Similar to RDF, the relay decodes the signal, but re-encodes it with a codebook different than the original user and transmits it in the second time slot of the user [8].
- **Amplify-and-Forward (AF)**: The signal received at the relay node is amplified and forwarded in the second time slot of the user [3, 6, 7].

III. Optimal Power Allocation

In this work, we aim to optimally distribute the power of each relay node between the users' transmissions to be relayed by that node. Our goal is to maximize the sum capacity of the system. The individual capacities of the users are functions of the relay transmission scheme. In the case of RDF relay transmission, the individual capacity of user i is

$$C_{i,RDF} = \frac{1}{2K}log(1 + P_{si}\beta_i^2 + P_{ri}\gamma_k^2) \tag{4}$$

when assigned to the relay node k. Similarly, for the case of NDF relay transmission, the individual capacity expression is

$$C_{i,NDF} = \frac{1}{2K}log(1 + P_{si}\beta_i^2) + \frac{1}{2K}log(1 + P_{ri}\gamma_k^2) \tag{5}$$

For both RDF and NDF cases, the designated relay node must reliably decode the signal. Thus, the individual capacity of a relay assisted user can not exceed the capacity of the user-to-relay link. This constraint leads two important results in terms of optimum power allocation. When the direct link, β_i^2, is better than the relay link, α_i^2 for user i, the minimum of the capacity upper bounds of the direct link and the user-to-relay link is the latter. In this case, the capacity of the direct transmission is higher than that of the relay assisted transmission. Since the direct transmission of user i will both maximize the individual capacity of user i, and have the potential to improve the sum capacity of the remaining users, it will be chosen. In this case, the relay node will not be able to decode the signal of user i. Thus, the power allocated to user i is

$$P_{ri} = 0 \quad if \quad \alpha_i^2 < \beta_i^2, \quad \forall i = 1, ..., K \tag{6}$$

For clarity of exposition, we denote the set of users that are served by the kth relay node and have $\beta_i^2 \leq \alpha_i^2$ as A'_k in the sequel. Additionally, the maximum individual capacity that can be achieved by user i with the help of relay node is upper bounded by

$$C_{i,RDF} \leq C_{i,NDF} \leq C_{upperDF} = \frac{1}{2K}log(1 + P_{si}\alpha_i^2), \forall i \tag{7}$$

due to the decodability constraint of decode-and-forward (DF) relay transmission schemes. When AF relay transmission is used, the individual capacity of user i is

$$C_{i,AF} = \frac{1}{2K}log(1 + P_{si}\beta_i^2 + \frac{P_{si}\alpha_i^2 P_{ri}\gamma_k^2}{P_{si}\alpha_i^2 + P_{ri}\gamma_k^2 + 1}) \tag{8}$$

The optimum power allocation problem at the relay nodes is posed as

$$\max_{\{P_{ri}\}_{i=1,\cdots,K}} \quad C_{sum} = \sum_{i=1}^{K} C_{i,*} \tag{9}$$

$$\text{s.t.} \quad \sum_{i\in A_k} P_{ri} \leq P_{Rk,total}, \quad \forall i,k \tag{10}$$

where $*$ can be replaced with RDF, NDF or AF according to the relay transmission scheme chosen. We note that when RDF or NDF is used, constraints (6) and (7) are added to the optimization problem.

Since the power allocation at each relay node does not affect the individual capacities of the users that are served by other relay nodes, we focus on the sum capacity optimization problem at each relay node. For RDF relay networks, we have the following theorem:

Theorem 1. The optimal power allocation for RDF relay networks results in four user sets, namely *equal users, low potential users, oversized users* and *nonrelayed users* for each relay node.

1. Equal users set is the set of users that have the same individual capacities after the optimal power allocation, and the corresponding relay powers dedicated to each user and achieved individual capacities are

$$P_{ri} = (\frac{1}{\mu_{k,RDF}} - \frac{1 + P_{si}\beta_i^2}{\gamma_k^2})^+ \tag{11}$$

$$C_{i,RDF} = \frac{1}{2K}log(\gamma_k^2/\mu_{k,RDF}) \tag{12}$$

   where $(.)^+ = max(., 0)$ and $\mu_{k,RDF}$ is the water level for the kth RDF relay node that satisfies $\sum_{i\in A_k} P_{ri} = P_{Rk,total}$.

2. Low potential users set is the set of users that achieve the maximum individual capacities indicated in (7). The power allocation and corresponding individual capacities are

$$P_{rj} = \frac{P_{sj}(\alpha_j^2 - \beta_j^2)}{\gamma_k^2} < (\frac{1}{\mu_{k,RDF}} - \frac{1 + P_{sj}\beta_j^2}{\gamma_k^2})^+ \tag{13}$$

$$C_{j,RDF} = \frac{1}{2K}log(1 + P_{sj}\alpha_j^2) < \frac{1}{2K}log(\gamma_k^2/\mu_{k,RDF}) \tag{14}$$

3. Oversized users set is the set of the users that are not relayed due to high individual capacities achieved without the help of the relay nodes. $P_{rl} = 0$ for the users in this set and corresponding individual capacities are $C_{l,RDF} \geq \frac{1}{2K}log(\gamma_k^2/\mu_{k,RDF})$.

4. The nonrelayed users set involves the users that has better direct links than the relay link, $\alpha_n^2 < \beta_n^2$, and the $P_{rn} = 0$ for this set.

Proof: Using the fact that $P_{ri} = 0$ for the users that have $\beta_i^2 > \alpha_i^2$ the optimization problem at the kth relay node can be expressed as

$$\max_{\{P_{ri}\}_{i\in A'_k}} \sum_{i\in A'_k} \frac{1}{2K} log(1 + P_{si}\beta_i^2 + P_{ri}\gamma_k^2) \quad (15)$$

$$\text{s.t.} \sum_{i\in A'_k} P_{ri} \le P_{Rk,total} \quad (16)$$

$$\frac{1}{2K} log(1 + P_{si}\beta_i^2 + P_{ri}\gamma_k^2) \le \frac{1}{2K} log(1 + P_{si}\alpha_i^2), \forall i \in A'_k \quad (17)$$

The constraint in (17) is a simple upper bound for the $\{P_{ri}\}$

$$0 \le P_{ri} \le \frac{P_{si}(\alpha_i^2 - \beta_i^2)}{\gamma_k^2} \quad (18)$$

Thus, the Lagrangian is

$$L(\{P_{ri}\}, \mu_{k,RDF}, \{\rho_{i,RDF}\}) =$$
$$\sum_{i\in A'_k} \frac{1}{2K} log(1 + P_{si}\beta_i^2 + P_{ri}\gamma_k^2) + \mu_{k,RDF}(\sum_{i\in A'_k} P_{ri} - P_{Rk,total})$$
$$+ \sum_{i\in A'_k} \rho_{i,RDF}(P_{ri} - \frac{P_{si}(\alpha_i^2 - \beta_i^2)}{\gamma_k^2})$$

where $\mu_{k,RDF}$ and $\rho_{i,RDF}$ are the Lagrange multipliers associated with the total transmit power constraint of the relay node k and the upper bound for the relay power used for user i. The cost function is a concave function and the $\{P_{ri}\}$ set is a convex set. Thus, simply using the KKT conditions, we arrive at the optimum relay power for user i as

$$P_{ri} = min((\frac{1}{\mu_{k,RDF}} - \frac{1 + P_{si}\beta_i^2}{\gamma_k^2})^+, \frac{P_{si}(\alpha_i^2 - \beta_i^2)}{\gamma_k^2}) \quad (19)$$

The users for which the upper bounds in (17) are inactive and $P_{ri} = (\frac{1}{\mu_{k,RDF}} - \frac{1+P_{si}\beta_i^2}{\gamma_k^2}) > 0$ achieve the same individual capacities and form the equal users set. When the upper bound is active, $P_{ri} = \frac{P_{si}(\alpha_i^2 - \beta_i^2)}{\gamma_k^2}$, forming the low potential users. Finally, the users with $(\frac{1}{\mu_{k,RDF}} - \frac{1+P_{si}\beta_i^2}{\gamma_k^2}) < 0$ form the oversized users. □

The optimum power allocation for RDF networks is a modified water-filling solution where each user has a base level and an upper bound. The base level for each user is $\frac{1+P_{si}\beta_i^2}{\gamma_k^2}$ whereas the upper bound is $\frac{1+P_{si}\alpha_i^2}{\gamma_k^2}$. Such a power allocation scheme is demonstrated in Figure 2 with five users. In this example, users 1 and 2 are the low potential users which relay node allocates maximum power for each user to achieve the maximum individual capacities. However, the resulting individual capacities are lower than the individual capacities of the users 3 and 4 that are in the equal users set. Note that user 5 is an oversized user and is not allocated any power due to high quality direct link, and its individual capacity is higher than the equal users' individual capacities. Observe that the optimum power allocation tries to equalize the individual capacities achieved by each user, also increasing the symmetric capacity of the system. The users that are not equalized in terms of individual capacities are the users that have low quality user to relay link (low potential users set, nonrelayed users set) or very high direct link (oversized users). Even if the transmit power of the relay nodes are increased, the relay benefits obtained by the low potential users and nonrelayed users do not increase. Thus, an appropriate relay selection strategy for RDF relay networks will be selecting the relay nodes that will provide high quality user to relay links.

When the relays operate in the NDF mode, we have the following theorem for the optimal power allocation.

Theorem 2. The optimal power allocation for NDF relay networks results in three user sets, namely *equal benefit users*, *low potential users* and *nonrelayed* users for each relay node.

1. Equal benefit users set are the set of users that benefit from the relay transmission equally. For this set of users, the optimum power allocation does not depend on the transmission of the users in the first time slots. The optimum power allocation dedicated to each user and corresponding individual capacities are

$$P_{ri} = \frac{1}{\mu_{k,NDF}} \quad (20)$$

$$C_{i,NDF} = \frac{1}{2K} log(1 + P_{si}\beta_i^2) + \frac{1}{2K} log(1 + \frac{\gamma_k^2}{\mu_{k,NDF}}) \quad (21)$$

where $\mu_{k,NDF}$ is the water level for the kth NDF relay node that satisfies the power constraint of the relay.

2. Similar to the RDF case, low potential users are the set of users that achieve the maximum individual capacities indicated in (7). The power allocation and corresponding individual capacities are

$$P_{rj} = \frac{P_{sj}(\alpha_j^2 - \beta_j^2)}{\gamma_k^2(1 + P_{sj}\beta_j^2)} < \frac{1}{\mu_{k,NDF}} \quad (22)$$

$$C_{j,NDF} = \frac{1}{2K} log(1 + P_{sj}\alpha_j^2) \quad (23)$$

3. The nonrelayed user set is similar to the RDF case for which every user in the set has a better direct link than the relay link, $\alpha_n^2 < \beta_n^2$, and $P_{rn} = 0$.

Proof: The power allocation problem at the kth NDF relay node can be expressed as

$$\max_{\{P_{ri}\}_{i\in A'_k}} \sum_{i\in A'_k} \frac{1}{2K} log(1 + P_{si}\beta_i^2) + \frac{1}{2K} log(1 + P_{ri}\gamma_k^2) \quad (24)$$

$$\text{s.t.} \sum_{i\in A'_k} P_{ri} \le P_{Rk,total} \quad (25)$$

$$\frac{1}{2K}[log(1 + P_{si}\beta_i^2) + log(1 + P_{ri}\gamma_k^2)] \le \frac{1}{2K} log(1 + P_{si}\alpha_i^2) \quad (26)$$

The constraint in (26) is a simple upper bound for P_{ri}

$$P_{ri} \le \frac{P_{si}(\alpha_i^2 - \beta_i^2)}{\gamma_k^2(1 + P_{si}\beta_i^2)} \quad (27)$$

The Lagrangian, $L(\{P_{ri}\}, \mu_{k,NDF}, \{\rho_{i,NDF}\})$, is

$$\frac{1}{2K} \sum_{i\in A'_k} [log(1 + P_{si}\beta_i^2) + log(1 + P_{ri}\gamma_k^2)] - \mu_{k,NDF} P_{Rk,total}$$
$$+ \sum_{i\in A'_k} \mu_{k,NDF} P_{ri} + \rho_{i,NDF}(P_{ri} - \frac{P_{si}(\alpha_i^2 - \beta_i^2)}{\gamma_k^2(1 + P_{si}\beta_i^2)})$$

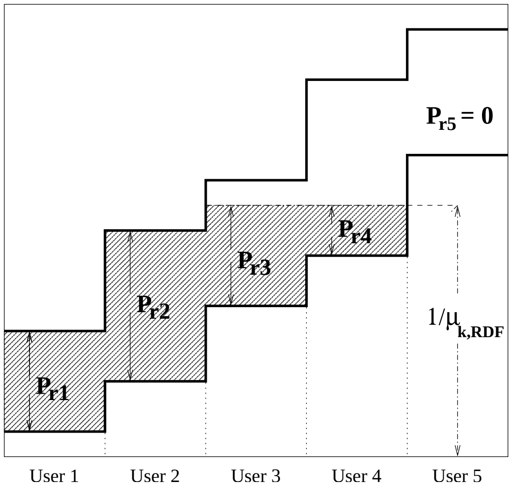

Figure 2: Optimum power allocation for RDF relaying

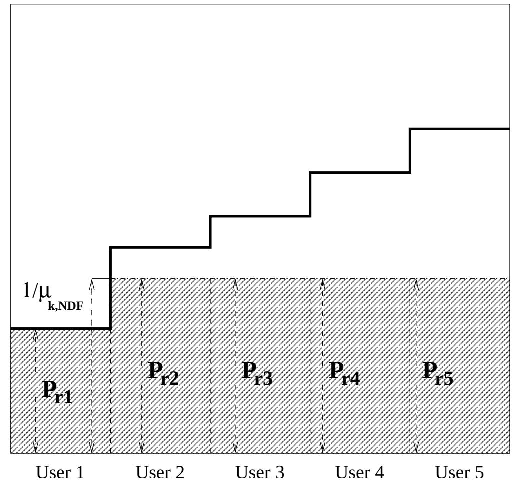

Figure 3: Optimum power allocation for NDF relaying

where $\mu_{k,NDF}$ and $\rho_{i,NDF}$ are the Lagrange multipliers associated with the power constraint of the relay node k and the upper bound for the relay power used for user i. The optimization problem is a concave function and the $\{P_{ri}\}$ set is convex. Thus, simply using KKT conditions, we arrive at the optimum relay power for user i as

$$P_{ri} = min(\frac{1}{\mu_{k,RDF}}, \frac{P_{si}(\alpha_i^2 - \beta_i^2)}{\gamma_k^2(1 + P_{si}\beta_i^2)}) \tag{28}$$

The users that have inactive upper bounds as in (26), will have $P_{ri} = \frac{1}{\mu_{k,NDF}}$ forming the equal benefit users set. When the upper bound is active, $P_{ri} = \frac{P_{si}(\alpha_i^2-\beta_i^2)}{\gamma_k^2(1+P_{si}\beta_i^2)}$, forming the low potential users. □

Observe that optimum power allocation for NDF relay networks tries to equalize the benefits obtained by relay transmission for each user. The optimum solution is a modified water-filling solution with upper bounds, $\frac{P_{sj}(\alpha_j^2-\beta_j^2)}{\gamma_k^2(1+P_{sj}\beta_j^2)}$ and identical base levels for each user. Such a power allocation scheme is demonstrated in Figure 3. In this example, user 1 is a low potential user and the other users are equal benefit users. The users in the set of low potential users and the nonrelayed users, can not benefit from relay transmission as the equal benefit users set due to low quality user to relay links. Similar to RDF case, even if the total transmit power of relay nodes are increased, the users in the low potential users set and nonrelayed users set will not be able to achieve higher individual capacities. Thus, the performance of the optimum resource allocation in NDF relay networks is very much dependent on the relay selection, and appropriate relay selection schemes which provide high quality user to relay links increase the performance of the NDF relay networks.

For AF relay networks, the following theorem identifies the optimum power allocation.

Theorem 3. The optimal power allocation for AF relay networks results in nonzero power allocation for some of the users assigned to the relay node, and the optimum power allocation for user i is

$$P_{ri} = (\frac{-(\frac{a_i}{b_i}+2) + \sqrt{(\frac{a_i}{b_i})^2 + \frac{4a_i}{\mu_{k,AF}}(1+\frac{a_i}{b_i})}}{2(a_i+b_i)})^+ \tag{29}$$

$$\text{where } a_i = \frac{P_{si}\alpha_i^2/(P_{si}\alpha_i^2+1)}{(1+\beta_i^2 P_{si})/\gamma_k^2} \text{ and } b_i = \frac{\gamma_k^2}{P_{si}\alpha_i^2+1} \tag{30}$$

where $\mu_{k,AF}$ is the water level for the kth AF relay node that satisfies $\sum_{i\in A_k} P_{ri} = P_{Rk,total}$.

Proof: The power allocation problem at the kth AF relay node can be expressed as

$$\max_{\{P_{ri}\}_{i\in A_k}} \sum_{i\in A_k} \frac{1}{2K} log(1 + P_{si}\beta_i^2 + \frac{P_{si}\alpha_i^2 P_{ri}\gamma_k^2}{P_{si}\alpha_i^2 + P_{ri}\gamma_k^2 + 1}) \tag{31}$$

$$\text{s.t.} \sum_{i\in A_k} P_{ri} \le P_{Rk,total} \tag{32}$$

The Lagrangian, $L(\{P_{ri}\}, \mu_{k,AF})$, is

$$\frac{1}{2K} log(1 + P_{si}\beta_i^2 + \frac{P_{si}\alpha_i^2 P_{ri}\gamma_k^2}{P_{si}\alpha_i^2 + P_{ri}\gamma_k^2 + 1}) \tag{33}$$

$$+\mu_{k,AF}(\sum_{i\in A_k} P_{ri} - P_{Rk,total}) \tag{34}$$

where $\mu_{k,AF}$ is the Lagrange multipliers associated with the total transmit power constraint of the relay node k. The optimization problem is a concave function and the $\{P_{ri}\}$ set is convex. Simply taking the derivative with respect to P_{ri} equating it to zero, we arrive at the optimum relay power for user i in (29). □

Observe that the optimal power allocation for the AF relay nodes results in nonzero power allocation to the users that satisfy $\mu_{k,AF} < a_i$. When the relay node that is very close to the users, then $a_i \approx \frac{\gamma_k^2}{P_{si}\beta_i^2+1}$ and $b_i \to 0$. This case corresponds to the case when the users' received SNR at the relay node are very high and the received signal is close to perfect. The optimal power allocation in this case reduces to the optimal power allocation in RDF as expected. It is important to note that in AF, the individual capacities of the users are not constrained by the capacity of the user-to-relay channel. The upper bound for the individual capacities of the users are

$$C_{i,AF} \le \frac{1}{2K} log(1 + P_{si}\beta_i^2 + P_{si}\alpha_i^2) \quad \forall i \tag{35}$$

Thus, AF relaying may perform better than the DF relaying.

IV. Cooperative Communications

In this section, we investigate the optimum power allocation for cooperative communications of two users where each user is both a source and a relay node. In such a case, each user should devote some of its power to its own data transmission and the remaining part to the transmission of the relay traffic. Considering the individual power constraints and the relay transmission scheme used, we investigate the optimum cooperation level of each user that maximizes the sum capacity of the users.

In a two user cooperative communication model, there exists only two sources and two relay nodes. We assume that the total power of each user is P_{total} and the sum of the powers dedicated to the individual data and relay traffic should be less than P_{total}, i.e., $P_{s1} + P_{r2} \leq P_{total}$ for user 1 and vice versa. We denote the channel gain between two users as α. Defining the portion of the total power dedicated to the individual data transmission $\pi_i = P_{si}/P_{total}$ the sum capacity optimization problem for a two user cooperative communication model can be expressed as

$$\max_{0 \leq \pi_1, \pi_2 \leq 1} \qquad C_{sum} = C_{1,*} + C_{2,*} \tag{36}$$

where $*$ can be replaced with RDF, NDF or AF according to the relay transmission scheme chosen. For RDF or NDF relay transmission, additional constraints in (6) and (7) for the decodability of the signal at the relay nodes should be considered.

For RDF and NDF cases, when the channels to the destination are better than inter-user channel, i.e., $\beta_i^2 > \alpha^2$ for both of the users, the users can not cooperate due to the decodability constraints of DF relay transmission schemes as expected. When $\beta_i^2 > \alpha^2$ for only one of the users, one user can help the other for the transmission in both RDF and NDF cases. Thus, in these cases, the optimum power allocations for the two user cooperative communication model are simple extensions of Theorem 1 and 2 and can be summarized as in the following lemma for RDF and NDF.

Lemma 1. When $\beta_2^2 > \alpha^2 > \beta_1^2$, the optimum power allocation for RDF and NDF cooperative communications results in $\pi_1 = 1$ for the first user and

$$\pi_2(RDF) = max(\frac{1}{2}(\frac{\beta_1^2}{\beta_2^2} + 1), 1 - \frac{\alpha^2 - \beta_1^2}{\beta_2^2}) \tag{37}$$

$$\pi_2(NDF) = max(\frac{1}{2}, 1 - \frac{\alpha^2 - \beta_1^2}{\beta_2^2(1 + P_{total}\beta_1^2)}) \tag{38}$$

for the second user.

For the RDF cooperative communications, when the constraints (6) and (7) are inactive, the maximum sum capacity is

$$C_{sum,RDF} = \frac{1}{K} log(1 + P_{total}(\beta_1^2 + \beta_2^2)/2) \tag{39}$$

where each user achieves equal single-user capacities. It is identified in Lemma 1 that such a power allocation exists for $\beta_2^2 > \alpha^2 > \beta_1^2$ as

$$\pi_1(RDF) = 1 \qquad \pi_2(RDF) = \frac{1}{2}(\frac{\beta_1^2}{\beta_2^2} + 1) \tag{40}$$

if $\alpha^2 \geq \frac{\beta_1^2 + \beta_2^2}{2}$. Observe that (40) is an optimal power allocation also for the case $\beta_1^2 \leq \beta_2^2 \leq \alpha^2$ that provides equal single-user capacities and a sum capacity as in (39). It is important to note that the optimum power allocation is not unique. There exist other (π_1, π_2) pairs that achieve the same sum capacity with equal single-user capacities. However, the proposed power allocation is easy to implement and saves one time slot of the nonrelaying user. Observe that in RDF cooperation, the user with higher channel gain to the destination should assist the other user to maximize the sum capacity.

Following a similar approach for the NDF cooperative communications, when (6) and (7) are not active, the optimum power allocation is simply $\pi_1 = \pi_2 = 1/2$ and the maximum sum capacity is

$$C_{i,NDF} = \frac{1}{K} log(1 + \frac{P_{total}}{2}\beta_1^2) + \frac{1}{K} log(1 + \frac{P_{total}}{2}\beta_2^2) \tag{41}$$

and the condition that the (6) and (7) are not active is

$$\beta_1^2 + \beta_2^2 + \frac{P_{total}}{2}\beta_1^2\beta_2^2 \leq \alpha^2 \tag{42}$$

In such a case, the optimum solution is unique and provides equal single-user capacities for each user.

It is important to note that the maximum sum capacity achievable by the NDF cooperative transmission is simply the sum capacity of a TDMA system where users do not cooperate and transmit independent symbols at each time slot. Due to the fact that the NDF cooperative communication performs better than RDF cooperative communication, both RDF and NDF cooperation do not provide any improvement of the sum capacity over conventional non-cooperative systems. However, they improve the single-user capacities of the worst users providing fairness to the system.

When AF cooperative communications is investigated in the asymptotic case of $\alpha >> \beta_i$, as expected, we observe that the power allocation should be done similar to RDF case without considering the decodability constraints. In such a scenario, the power allocation can be done as $\pi_1 = \pi_2 = 1/2$. When the decodability constraints are inactive, the RDF cooperative transmission will perform better than the AF cooperative transmission due to the noise propagation nature of the AF relaying.

The performance of cooperative communications is a function of the quality of the inter-user channel [4,4]. Thus, to facilitate the potential gain of cooperative communications the users that will cooperate should be selected carefully. Especially the DF type cooperative communications require high quality inter-user channels. Thus, the users that have good inter-user channels should cooperate to fully facilitate the gain provided by the cooperative communications.

V. Numerical Results and Conclusion

In this section, we present numerical results related to the performance of the TDMA multiuser relay network with optimum power allocation. For numerical results, we consider a TDMA multiuser relay network with 5 users and one relay node that serves all. The link SNRs of the users used throughout the simulations are $\{(P_{si}\beta_i^2, P_{si}\alpha_i^2)\} = \{(1,5), (6,10), (8,11), (12,15), (14,17)\}$ dB. We investigate the individual capacities achieved by each relay transmission scheme with different values of power constraints for the relay node.

Figures 4, 5 and 6 show the performance of the relay transmission for RDF, NDF and AF relay nodes with optimum power allocation. We observe that the individual capacities are improved as the relay power is increased up to a threshold for each user as expected. In the RDF case, when the relay

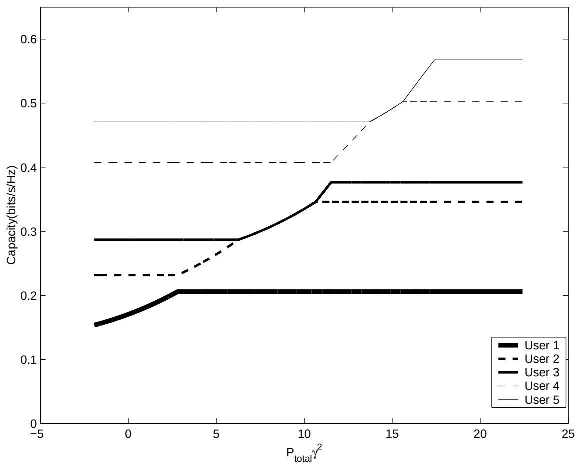

Figure 4: RDF relay networks

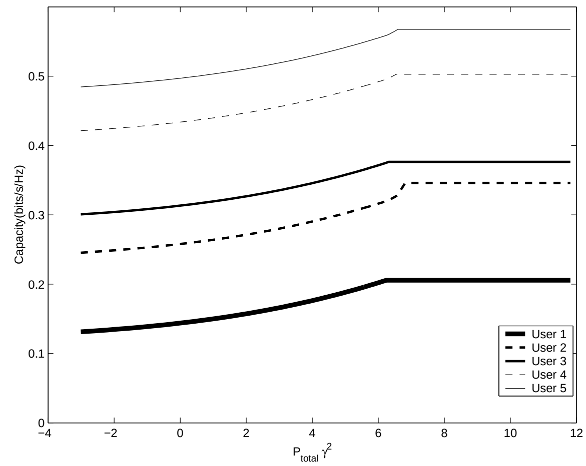

Figure 5: NDF relay networks

node has relatively low SNR, the relay node helps only the first user that has the worst direct link, since the rest of the users are oversized in such a scenario. As the available power at the relay increases, the first user's potential is reached, and the relay node starts to help the rest of the users. We observe that user 2 and 3; user 4 and 5 become *equal* users for larger relay power values. We also observe that after a threshold, increasing the relay power does not help since all users already achieve the maximum single-user capacities. In the NDF case, we again observe that the sum capacity is improved as the relay power is increased up to a threshold. Since NDF performs better than RDF, this threshold is much lower than the threshold in the RDF case, i.e., for the maximum sum capacity, NDF requires less power at the relay node as compared to RDF. In the NDF relay scheme, we observe that the relay tries to help all the users equally and simultaneously without considering the performance of the direct links. However, the benefit that can be provided by the relay node is limited by the quality of the user-to-relay link. In Figure 6, we observe that the benefit obtained by the AF relay nodes converges to its maximum point gradually for each user. We also observe that in the AF mode, both individual capacities and the resulting sum capacities may be higher than that of the DF relay transmission. This is due to the fact that DF relaying has the decodability constraints in the user-to-relay links whereas the AF relaying does not.

In this work, we have obtained optimal power allocation schemes for the relay nodes when each relay node serves several users. We have observed that the optimum power allocation in RDF relay nodes helps the users with low quality direct links first, and tries to equalize the individual capacities of the users bringing fairness among the users. We identified that the optimum power allocation in NDF relay networks provides equal improvements to the individual capacities of the users when the qualities of the user-to-relay links are high. We observe that the AF relay nodes provide higher sum capacities than the DF relay nodes in high SNR regime due to the decodability constraints in DF relaying. For cooperative communications, it is observed that RDF, NDF and AF type cooperation do not provide any improvement in the sum capacity over conventional non-cooperative TDMA systems. However, NDF type cooperation with appropriate cooperative user selection have the potential to achieve the same sum capacity as the non-cooperative TDMA systems while providing fairness among users.

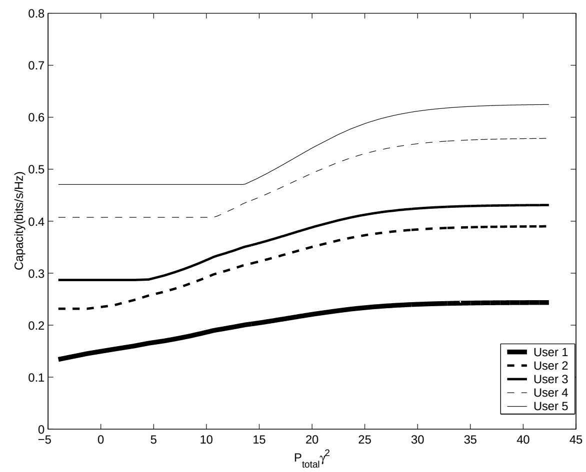

Figure 6: AF relay networks

References

[1] E. Van der Meulen. A survey of multi-way channels in information theory: 1961-1976. *IEEE Tran. on Info. Theory*, 23(1):1 – 37, Jan. 1977.

[2] T. M. Cover and A. A. El Gamal. Capacity theorems for the relay channel. *IEEE Tran. on Info. Theory*, 25(5):572 – 584, Sept. 1979.

[3] J. N. Laneman, D. N. C. Tse, and G. W. Wornell. Cooperative diversity in wireless networks: Efficient protocols and outage behavior. *IEEE Tran. on Info. Theory*, 50(12):3062 – 3080, Dec. 2004.

[4] A. Sendonaris, E. Erkip, and B. Aazhang. User cooperation diversity parts I & II. *IEEE Tran. on Comm.*, 51(11):1927 – 1948, Nov. 2003.

[5] D. R. Brown III. Resource allocation for cooperative transmission in wireless networks. In *38th Asilomar Conference on Signals, Systems and Computers*, Nov. 2004.

[6] A. Ribeiro, X. Cai, and G. B. Giannakis. Symbol error probabilities for general cooperative links. In *IEEE ICC'04*, pages 3369 – 3373, June 2004.

[7] M. O. Hasna and M. S. Alouini. Optimal power allocation for relayed transmissions over rayleigh-fading channels. *IEEE Tran. on Wireless Comm.*, 3(6):1999 – 2004, Nov. 2004.

[8] I. Maric and R. Yates. Forwarding strategies for parallel-relay networks. In *CISS'04*, March 2004.