Adaptive CDMA Cell Sectorization with Linear Multiuser Detection

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Abstract— **We consider the problem of adaptive cell sectorization in a CDMA system when the receiver employs a linear multiuser detector. Specifically, given the number of sectors and terminal locations, we investigate how to appropriately sectorize the cell, such that the total transmit power is minimized, while each user has acceptable quality of service, under the assumption that the base station employs linear multiuser detectors. The resulting joint optimization problem has high complexity for arbitrary signature sequences, so we concentrate on the tractable case where the square of the crosscorrelation values between the signature sequences are equal. We formulate the sectorization problem for different linear detectors. The optimum sectorization problem with linear multiuser detector has the same complexity as the one with the matched filter. Numerical results suggest that significant power saving is achieved by incorporating better receiver structures, i.e. MMSE detector, or decorrelator.**

I. INTRODUCTION

The demand for high capacity flexible wireless services is ever growing. CDMA shows promise in meeting this demand [8]. It is well known that CDMA systems are interference limited, so interference management techniques are necessary to improve the capacity of CDMA systems. Many techniques that control or suppress interference in CDMA systems such as transmit power control, multiuser detection and cell sectorization have been proposed to date [1]–[5]. In this work, we investigate adaptive cell sectorization with multiuser detection. Specifically, given the number of sectors and terminal locations, the problem we investigate is to appropriately sectorize the cell, such that the total transmit power is minimized, while each user has acceptable quality of service, under the assumption that the base station employs linear multiuser detection.

Conventional cell sectorization, where the cell is sectorized to equal angular regions, may not perform well especially in systems where the user distribution is nonuniform. Adaptive cell sectorization, where the cell is sectorized in response to the user distribution, is a promising method to improve the capacity in CDMA systems [1]. Previous work showed that, if the aim is to minimize the total received power of all users, the best sectorization arrangement is to assign equal number of users to each sector, whereas the solution for the minimum total transmit power sectorization is achieved by solving a shortest path problem [1]. Reference [1] reported substantial power savings as compared to conventional cell sectorization. We note that sectorization related previous work is based on the assumption that the receiver structure is the conventional matched filter (MF), see [1] and references therein.

Multiuser detection is the process of demodulating the signals of the users in a multiple access environment. It was shown in [5] that the optimum multiuser detector has computational complexity which increases exponentially with the number of users. This result prompted the development of several suboptimum detectors. Two key linear multiuser detectors are the decorrelating detector (DD) and the Minimum Mean Square Error (MMSE) detector. The decorrelating detector chooses the linear filter to completely eliminate the multiple access interference (MAI); the MMSE detector chooses the filter to minimize the mean squared error, or equivalently maximize the signal to interference ratio (SIR) [5].

In this work, we consider adaptive cell sectorization when the receiver is equipped with linear multiuser detector (MUD) for all users. The premise of this work is that further capacity improvement is possible with adaptive cell sectorization when matched filters are replaced by linear multiuser detectors. Thus, we address the problem of finding the best sectorization arrangement with minimum transmit power and linear receiver filters. The resulting joint optimization problem has high complexity for arbitrary signature sequences, so we concentrate on the tractable case where the square of the crosscorrelation values between the users' signatures are equal. Two such cases are considered. First, we consider a large system employing random signatures as is considered in [6]. We formulate the sectorization problem for different linear multiuser detectors for the large system. Second, for finite size deterministic systems, we consider m-sequences, which have equal crosscorrelations. In both cases, the optimum sectorization problem with linear MUD has the same complexity as the same problem with MF.

We provide performance comparisons between different linear detectors in uniform distribution and nonuniform distribution environment. We observe that the incorporation of a better receiver structure, such as the MMSE detector, or the decorrelating detector, provides significant power savings, and user capacity can be improved by employing linear multiuser receivers in conjunction with adaptive sectorization.

II. Model

A single cell DS-CDMA system with processing gain G and K users is considered. The user locations in the cell are assumed to be known. This is a reasonable assumption in very slow mobility environment. For example, in a Wireless Local Loop (WLL), this information is readily available through the addresses of the subscribers. We assume the cell is to be sectorized into N sectors and there is no intersector interference. All users within the same sector interfere with each other and each user has a pre-assigned signature sequence.

Conventional cell sectorization, or equal angular width cell sectorization may not perform well especially in systems where the user distribution is nonuniform. Adaptive cell sectorization, where the cell is sectorized in response to the user distribution, is a promising method to improve the capacity of CDMA systems.

Our purpose is to investigate the best sectorization arrangement such that the total transmit power is minimized, while each user has acceptable quality of service, under the assumption that the base station employs linear multiuser detection. The quality of service is represented by signal to interference ratio (SIR). A user has an acceptable quality of service if its SIR is greater than a target SIR, γ^*.

III. Transmit Power Optimization

A. Problem Statement

For the synchronous system, using chip matched filtering and sampling the signal at the chip rate, the received signal vector at the front end of the receiver filter at the base station is given by

$$\mathbf{r} = \sum_{j=1}^{K} \sqrt{p_j}\sqrt{h_j} b_j \mathbf{s}_j + \mathbf{n} \quad (1)$$

where p_j, h_j, b_j and $\mathbf{s}_j$ denote the transmit power, the uplink gain, the information bit and the signature sequence of the jth user, $\mathbf{n}$ denotes a Gaussian random vector with $E[\mathbf{n}\mathbf{n}^\top] = \sigma^2\mathbf{I}$.

SIR for user j at the receiver filter output can be expressed as

$$\text{SIR}_j = \frac{(\mathbf{c}_j^\top \mathbf{s}_j)^2 p_j h_j}{\sum_{l\neq j}(\mathbf{c}_j^\top \mathbf{s}_l)^2 p_l h_l + \sigma^2(\mathbf{c}_j^\top \mathbf{c}_j)} \quad (2)$$

where $\mathbf{c}_j$ denotes the receiver filter for user j.

Equivalently, we may use bank of matched filters each of which is matched to the signature waveform of a user. In one symbol interval this yields

$$\mathbf{y} = \mathbf{S}^\top \mathbf{r} = \mathbf{R}\mathbf{A}\mathbf{b} + \sigma\mathbf{n} \quad (3)$$

where $\mathbf{R}$ denotes the normalized crosscorrelation matrix of users with $\mathbf{R}_{k,l} = \rho_{k,l} =< \mathbf{s}_k, \mathbf{s}_l >$, $\mathbf{S} = [\mathbf{s}_1, ..., \mathbf{s}_K]$, $\mathbf{A} = diag(\sqrt{p_1 h_1}, ..., \sqrt{p_K h_K})$, $\mathbf{b}$ is K-vector whose jth component is b_j, and $<,>$ denotes inner product.

The decision for user j will be made as

$$\hat{b}_k = sgn((\mathbf{L}\mathbf{y})_k)$$

with

$$\mathbf{L} = \mathbf{I} \quad \text{for matched filter}$$

$$\mathbf{L} = \mathbf{R}^{-1} \quad \text{for decorrelator}$$

$$\mathbf{L} = (\mathbf{R} + \sigma^2\mathbf{A}^{-2})^{-1} \quad \text{for MMSE detector}$$

where $\mathbf{I}$ is identity matrix.

SIR for user j at the output of a linear MUD $\mathbf{L}$ can be expressed as

$$\text{SIR}_j = \frac{(\mathbf{L}_{jj} + \sum_{k\neq j}\rho_{kj}\mathbf{L}_{jk})^2 p_j h_j}{\sum_{l\neq j}(\mathbf{L}_{jl} + \sum_{t\neq l}\rho_{tl}\mathbf{L}_{jt})^2 p_l h_l + \sigma^2(\mathbf{LRL})_{jj}}. \quad (4)$$

The sectorization problem we consider can be formulated as the transmit power optimization problem.

$$\min_{\theta,\ \mathbf{p}} \sum_{i=1}^{N} \sum_{j\in g_i(\theta)} p_j \quad (5)$$

$$s.t. \qquad \gamma_j = \frac{(\mathbf{c}_j^\top \mathbf{s}_j)^2 p_j h_j}{\sum_{l\neq j, l\in g_i(\theta)}(\mathbf{c}_j^\top \mathbf{s}_l)^2 p_l h_l + \sigma^2(\mathbf{c}_j^\top \mathbf{c}_j)} \geq \gamma^*$$

$$i = 1, ..., N \quad \mathbf{p} \geq \mathbf{0} \quad \mathbf{1}^T\theta = 2\pi$$

where γ_j is the SIR of the jth user, $g_i(\theta)$ is the set of users that reside in the area spanned by sector i, θ is the N-tuple vector that denotes the sector angles, $\mathbf{0}$ and $\mathbf{1}$ denote the all zero and all one vectors, respectively.

In (5), the minimum transmit power is achieved when the SIR constraints are satisfied with equality [4]. In general, the above optimization problem in (5) has high complexity for arbitrary signature sequences. However, when all the users have equal squared crosscorrelation, transmit power optimization can be formulated as a graph partitioning problem, and can be solved by a shortest path algorithm, see [1] and references therein. In this particular case, minimum power is achieved by assigning equal received powers to all users belonging to the same sector. Under this assumption, (5) is simplified as

$$\min_{\theta,\ \mathbf{p}} \sum_{i=1}^{N} q_i^* \sum_{j\in g_i(\theta)} \frac{1}{h_j} \quad (6)$$

where q_i^* is equal received power for sector i. This optimization problem can be solved by shortest path algorithm with computational complexity $O(K^3N)$ as we will see shortly.

In this work, we consider two cases where the optimum received power in each sector turns out to be identical for all users in that sector. First, we consider a large system employing random signature sequences as is considered in [6]. Second, for finite sized deterministic systems, we consider m-sequences, which have equal crosscorrelation between users.

B. Large System with Random Sequences

Reference [6] shows that if the signature sequences are random, as $N \to \infty$, and the ratio $\alpha = \frac{K}{N}$ is fixed, the empirical distribution of users' powers converges to a fixed distribution $F(P)$, and β_1^N will converge in probability to a deterministic value β_1^* which is the unique solution to the equation

$$\beta_1^* = \frac{P_1}{\sigma^2 + \alpha \int I(P, P_1, \beta_1^*)\, dF(P)} \tag{7}$$

$$I(P, P_1, \beta_1^*) = \frac{P P_1}{P_1 + P\beta_1^*} \quad \text{for MMSE receiver}$$

$$I(P, P_1, \beta_1^*) = P \quad \text{for Matched Filter}$$

$$\beta_1^* = \frac{P_1(1-\alpha)}{\sigma^2}; \quad \alpha < 1 \quad \text{for Decorrelator} \tag{8}$$

where P_1 and β_1^N are the received power and SIR of user 1, respectively. β_1 is deterministic in a large system and approximately satisfies

$$\beta_{1,MMSE} \simeq \frac{P_1}{\sigma^2 + \frac{1}{N}\sum_{i=2}^{K} \frac{P_i P_1}{P_1 + P_i \beta_1^*}} \tag{9}$$

$$\beta_{1,MF} \simeq \frac{P_1}{\sigma^2 + \frac{1}{N}\sum_{i=2}^{K} P_i} \tag{10}$$

$$\beta_{1,Decorrelator} \simeq \frac{P_1(1-\alpha)}{\sigma^2} \qquad \alpha < 1. \tag{11}$$

Also it is shown that minimum power solution is achieved by assigning the same received power $P(\beta^*)$ to all users for a given SIR β^*. In this case, the minimum received powers for the three receivers are

$$P_{MF}(\beta^*) = \frac{\beta^* \sigma^2}{1 - \alpha\beta^*} \qquad \alpha < \frac{1}{\beta^*} \tag{12}$$

$$P_{MMSE}(\beta^*) = \frac{\beta^* \sigma^2}{1 - \alpha\frac{1+\beta^*}{\beta^*}} \qquad \alpha < \frac{1+\beta^*}{\beta^*} \tag{13}$$

$$P_{DD}(\beta^*) = \frac{\beta^* \sigma^2}{1 - \alpha} \qquad \alpha < 1. \tag{14}$$

Under the large system assumption, the cost function of our transmit power optimization problem (6) for three linear receivers can be expressed as

$$\min_{\theta,\, \mathbf{p}} \sum_{i=1}^{N} P_{MF,i}(\beta^*) \sum_{j \in g_i(\theta)} \frac{1}{h_j} \tag{15}$$

$$\min_{\theta,\, \mathbf{p}} \sum_{i=1}^{N} P_{DD,i}(\beta^*) \sum_{j \in g_i(\theta)} \frac{1}{h_j} \tag{16}$$

$$\min_{\theta,\, \mathbf{p}} \sum_{i=1}^{N} P_{MMSE,i}(\beta^*) \sum_{j \in g_i(\theta)} \frac{1}{h_j} \tag{17}$$

where $P_{MF,i}(\beta^*)$, $P_{DD,i}(\beta^*)$ and $P_{MMSE,i}(\beta^*)$ are minimum received powers for the sector i.

C. Finite Size System with m-Sequences

We now consider m-sequences with processing gain G with equal crosscorrelation between users, $\rho = -\frac{1}{G}$. In this case, the SIR for user j in a sector i at the output of linear MUD $\mathbf{L}$ can be expressed as

$$\text{SIR}_j = \frac{(\mathbf{L}_{jj} + \rho \sum_{k \neq j} \mathbf{L}_{jk})^2 p_j h_j}{\sum_{l \neq j} (\mathbf{L}_{jl} + \rho \sum_{t \neq l} \mathbf{L}_{jt})^2 p_l h_l + \sigma^2 (\mathbf{LRL})_{jj}} \tag{18}$$

The transmit power optimization problem (5) is

$$\min_{\theta,\, \mathbf{p}} \sum_{i=1}^{N} \sum_{j \in g_i(\theta)} p_j \tag{19}$$

$$s.t.\ \gamma_j = \frac{(\mathbf{L}_{jj} + \rho \sum_{k \neq j} \mathbf{L}_{jk})^2 p_j h_j}{\sum_{l \neq j} (\mathbf{L}_{jl} + \rho \sum_{t \neq l} \mathbf{L}_{jt})^2 p_l h_l + \sigma^2 (\mathbf{LRL})_{jj}} \geq \gamma^*$$

$$l, k \in g_i(\theta) \;\; i = 1, ..., N \;\; \mathbf{p} \geq \mathbf{0} \;\; \mathbf{1}^T \theta = 2\pi.$$

Minimum power is achieved when all users within the sector have the same received power. In this case, (19) is given by

$$\min_{\theta,\, \mathbf{p}} \sum_{i=1}^{N} q_i^* \sum_{j \in g_i(\theta)}^{K_i} \frac{1}{h_j}. \tag{20}$$

For Matched Filter and Decorrelator, q_i^*, the received power for the users in sector i, is given by

$$q_i^* = \frac{\gamma^* \sigma^2}{1 - \gamma^* \frac{(K_i - 1)}{\rho^2}}, \quad q_i \geq 0 \tag{21}$$

$$q_i^* = \gamma^* \sigma^2 R_{11}^{-1}, \quad \rho > \frac{-1}{K_i - 1} \tag{22}$$

where K_i denotes the number of users in sector i. For MMSE receiver, q_i^* satisfies the following three equations.

$$q_i^* = \frac{(\mathbf{L}_{jj} + \rho \sum_{k \neq j, k \in g_i(\theta)} \mathbf{L}_{1k})^2 p_j h_j}{\sum_{l \neq j, l \in g_i(\theta)} (\mathbf{L}_{1l} + \rho \sum_{t \neq l, t \in g_i(\theta)} \mathbf{L}_{1t})^2 p_l h_l + \sigma^2 (\mathbf{LRL})_{jj}} \tag{23}$$

$$L_{jj} = [1 + \frac{\sigma^2}{q_i^*} + (K_i - 1)\rho]^{-1} [1 + \frac{\rho(K_i - 1)}{1 - \rho + \frac{\sigma^2}{q_i^*}}] \tag{24}$$

$$L_{jk} = [1 + \frac{\sigma^2}{q_i^*} + (K_i - 1)\rho]^{-1} [\frac{-\rho}{1 - \rho + \frac{\sigma^2}{q_i^*}}]. \tag{25}$$

D. Graph Theoretic Formulation of the Problem

Transmit power optimization problem (TP) does not have a closed form solution. However, when the minimum received powers in a sector are equal, through an appropriate transformation, TP can be formulated as a graph partitioning problem which is guaranteed to find the optimal solution in polynomial time. First, the angular and radial coordinate is assigned to each terminal in a cell. The angular position is the angular distance of the terminal to a reference terminal. The radial position is the distance of the terminal to the base station. Each terminal in a cell is represented by a vertex along the ring. The position of vertex is determined by the angular position of the terminal regardless of its radial position. This constitutes the elements of an undirected connected graph with vertex set V and edge set E, $G = (V, E)$. The vertex set and the edge set are $V = \{v_1, ..., v_K\}$ and $E = \{(v_i, v_{i+1})|i = 1, ..., K-1\} \bigcup \{(v_K, v_1)\}$, respectively. Vertex i has weight $w_i = \frac{1}{h_i}$, where h_i is the uplink gain of the ith terminal. Sectorization problem for minimum transmit power is to find optimum connected partition on this ring. The set of terminals are partitioned into disjoint subsets $\{V_1, ..., V_N\}$ along the ring. Subset i has weight $q_i^* \sum_{j \in g_i(\theta)} \frac{1}{h_i}$ and the cost of all subsets is $\sum_{i=1}^{N} q_i^* \sum_{j \in g_i(\theta)} \frac{1}{h_i}$. The graph partitioning problem is to find a feasible partition such that some given cost is minimized.

$$\arg \min_{\{V_1, \ldots, V_N\}} \sum_{i=1}^{N} q_i^* \sum_{j \in g_i(\theta)} \frac{1}{h_i}. \qquad (26)$$

TP in (6) is a graph partitioning problem. Graph theoretic formulation returns the optimal sectors in terms of a list of terminals that belong to each sector. In the previous subsection we already formulated the TP, which is suitable to be transformed into the problem of graph partitioning. When the cost function is separable as is in (26), the problem of graph partitioning a string can be reduced to a shortest path problem with computational complexity $O(K^2N)$ [1]. The cost function in (26) is separable, but the corresponding graph is a ring instead of string. When disconnected at some arbitrary edge, the ring is easily transformed into a string. A shortest path algorithm can be applied to the resulting network constructed out of the string. By repeating the same procedure for each K edge in E, graph partitioning problem for the ring can be solved by a shortest path algorithm with complexity $O(K^3N)$.

IV. Numerical Results

We provide sectorization results by three linear detectors for transmit power optimization. Figure 1 and 2 show sector boundaries for a large system with random signature sequences in uniform and nonuniform user distribution, respectively. In addition to the benefit from exploiting other users' power and crosscorrelation, linear multiuser detectors have more flexible choice for selecting sector boundaries than MF, which has a strict constraint for maximum number of users supportable. This leads to significant power saving of linear multiuser detectors over the MF. Table I shows the optimum transmit power of the large system with random signature sequences. The MMSE detector has 65%, 43% power savings over MF in the case of uniform and nonuniform distribution, respectively. Figure 3 and 4 show sector boundaries for a finite size system with m-sequences in uniform and nonuniform user distribution, respectively. M-sequences has low squared crosscorrelation, which is beneficial to MF. Table II shows the optimum transmit power of a finite size system with m-sequences. The MMSE detector has 40%, 20% power savings over MF in case of uniform and nonuniform distribution, respectively.

V. Conclusion

In this paper, we studied the joint transmit power optimization and adaptive cell sectorization under the assumption that the base station employs linear multiuser detectors. In general, this joint optimization problem has high complexity, so tractable cases where the squares of crosscorrelation between users are equal was investigated. First, a large system with random signature sequences was considered. In second case, we considered the finite size system with m-sequences. We note that besides these two cases, any sequence set with a crosscorrelation matrix where non-diagonal entries are $\pm\rho$ fits in the same framework.

We provided numerical results for uniform and nonuniform distribution cases. In both cases, the MMSE detector has significant power saving over the MF, while the optimum sectorization problem with linear multiuser detection has the same complexity as the same problem with MF. We conclude that the incorporation of a better receiver structure, such as the MMSE detector, or the decorrelator, provides significant power savings, and user capacity can be improved by employing linear multiuser receivers in conjunction with adaptive sectorization.[1]

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TABLE I

OPTIMUM TOTAL TRANSMIT POWERS[WATTS] OF A LARGE SYSTEM WITH RANDOM SIGNATURE SEQUENCES FOR THREE LINEAR DETECTORS.

User-distribution	Uniform	Nonuniform
MF	11.46	17.02
DD	4.04	9.70
MMSE	3.92	9.62

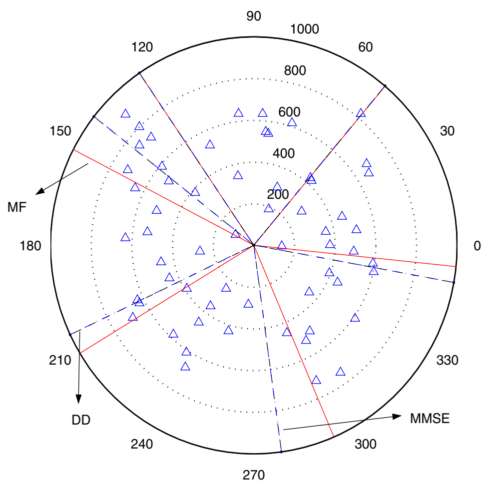

Fig. 1. Sector boundaries in a cell for a large system with random signature sequences with Matched Filter(MF), Decorrelator(DD) and MMSE detector(MMSE) where the number of users, K=60, number of sectors, N=6, processing gain, G=64, target SIR, γ^{*}=5 and noise power, σ^2= 10^{-13}. Distribution of terminals is uniform.

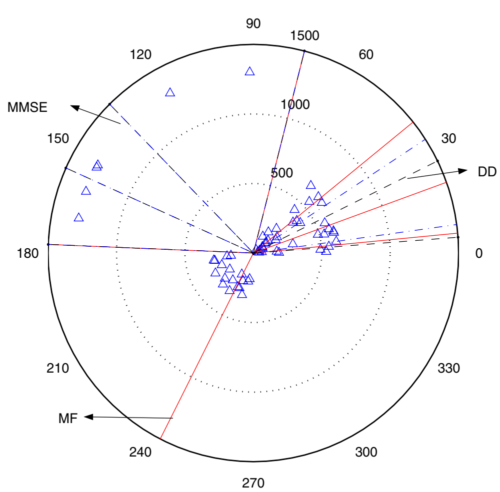

Fig. 2. Sector boundaries in a cell for a large system with random signature sequences with Matched Filter(MF), Decorrelator(DD) and MMSE detector(MMSE) where the number of users, K=60, number of sectors, N=6, processing gain, G=64, target SIR, γ^{*}=5 and noise power, σ^2= 10^{-13}. Distribution of terminals is nonuniform.

TABLE II

OPTIMUM TOTAL TRANSMIT POWERS[WATTS] OF A FINITE SIZE SYSTEM WITH M-SEQUENCES FOR THREE LINEAR DETECTORS.

User-distribution	Uniform	Nonuniform
MF	3.20	6.03
DD	2.18	5.06
MMSE	1.89	4.80

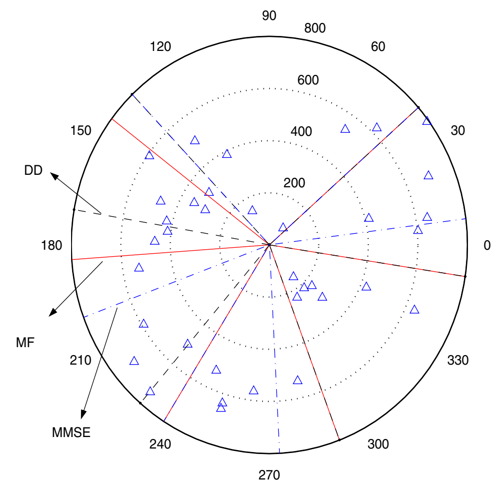

Fig. 3. Sector boundaries in a cell for finite size system with m-sequences with Matched Filter(MF), Decorrelator(DD) and MMSE detector(MMSE) where the number of users, K=36, number of sectors, N=6, processing gain, G=7, target SIR, γ^{*}=5 and noise power, σ^2= 10^{-13}. Distribution of terminals is uniform.

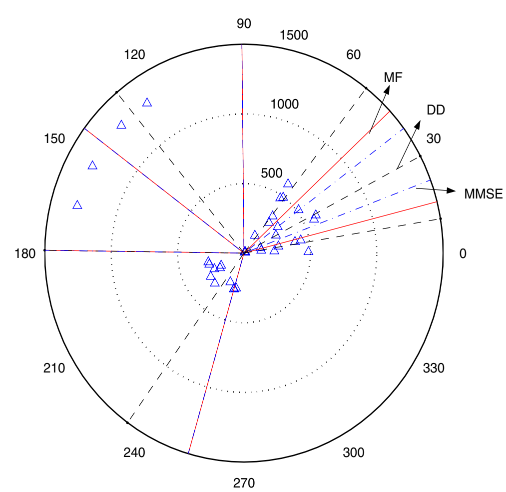

Fig. 4. Sector boundaries in a cell for finite size system with m-sequences with Matched Filter(MF), Decorrelator(DD) and MMSE detector(MMSE) where the number of users, K=36, number of sectors, N=6, processing gain, G=7, target SIR, γ^{*}=5 and noise power, σ^2= 10^{-13}. Distribution of terminals is nonuniform.