Downlink Throughput Maximization: TDMA versus CDMA

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Abstract — **We consider the downlink throughput maximization problem for interference limited multiuser systems. Our goal is to characterize the optimum base station transmission policy (TDMA or CDMA), and find the corresponding power allocation. We model the interference by the aid of the orthogonality factor and determine the transmission strategy for a range of the values of the orthogonality factor, and users' channel gains, so as to maximize the system throughput, subject to a total power constraint. Although the resulting optimization problem may turn out to be non-convex depending on the value of the orthogonality factor, we show that valuable observations regarding the optimum solution can be obtained based on the properties of the problem. In particular, we propose an exact and a near-exact algorithm to determine whether TDMA is the optimum strategy, or simultaneous users (CDMA) should be transmitted to.**

I. Introduction

Wireless services are becoming more data centric and often require higher data rate for downlink communications [1, 2]. Data services typically require high reliability but are delay tolerant. The delay tolerance can be exploited by effectively scheduling transmissions to users in favorable channel conditions to increase the overall system throughput [3].

In this paper, we consider the power allocation problem for the downlink of delay tolerant CDMA systems, by taking the channels of the users into account. For the CDMA downlink, typically, orthogonal codes are used to communicate to each user [4]. However, due to multipath fading, orthogonality between codes is not preserved at the mobile side. In such cases, a parameter defined as the orthogonality factor is often used to designate the level of multipath fading, representing the degree of interference from other users [5]. We focus on this scenario, where the orthogonality between the codes is not maintained. Specifically, our aim is to find the optimum transmission policy, i.e., which user(s) will be transmitted to and the power allocation in each time slot so as to maximize the total system utility, subject to a total power constraint at the base station.

The total system utility is defined simply as the sum of the individual utilities. The individual utility we consider in this paper is the transmission rate to a user, which is an increasing function of the user's transmit power. Utility based power control for data services has previously been considered in [6, 7], for the uplink, where the quality of service (QoS) of each user, e.g. signal to interference ratio (SIR), is defined as the utility.

In references [8, 9], one-at-a-time transmission for downlink throughput maximization is considered for delay tolerant service, and the optimality of this solution is shown. All available power from the base station is transmitted for the sake of increased throughput, and no compensation for the intracell interference is needed [8, 9].

To overcome the possible unfairness that may result from the one-at-a-time transmission strategy, reference [10] imposes a minimum service rate requirement for each user, and considers the throughput maximization problem for downlink multirate CDMA system where the multirate is achieved by either multicode or orthogonal variable spreading factor (OVSF). The optimality of greedy power allocation is shown in this scenario [10]. This policy is intuitively pleasing when the user rate and power have a linear relationship such as in a CDMA system with variable processing gain or multicodes. Reference [10] also provides numerical results which show the effect of orthogonality factor to the system performance.

Agrawal et al. in [11] consider the optimization of the sum of the individual weighted throughput values in the downlink of a multirate CDMA system with orthogonal codes. It is assumed that the orthogonality is preserved at the receiver side. The resulting problem is a convex program, which for the special case of the equal weight scenario, i.e., throughput maximization, produces one-at-a-time transmission as the optimum policy [11].

In contrast with [11], we consider the downlink of an *interference limited* CDMA system and determine the optimum transmission strategy. The key observation is that the optimum policy is a function of the orthogonality factor. Specifically, given the channel realization of the users, we determine whether the TDMA-mode, i.e., the base station transmits to one user at a time, or the CDMA-mode, i.e., simultaneous transmissions, should be chosen. If the CDMA-mode is optimum, that is, if multiple interfering users should be served by the base station simultaneously, then the corresponding power allocation is also found.

The paper is organized as follows. Section II describes the system model as well as the problem formulation and the performance metric. Optimum power allocation is considered in section III. Section IV provides numerical examples that support our analysis, and Section V summarizes the results and concludes the paper.

II. System Model and Problem Formulation

We consider a single cell CDMA system with K users. The base station has a maximum power budget, P_T. This power budget is assigned to each user such that $\sum_{i=1}^{K} p_i \leq P_T$, where p_i is the allocated power for user i. Figure 1 depicts the model of the CDMA downlink. An orthogonal code is assigned to each user and is used to modulate the signal transmitted to that user. However, due to multipath fading, orthogonality between the codes (users) is lost at the receiver side. The

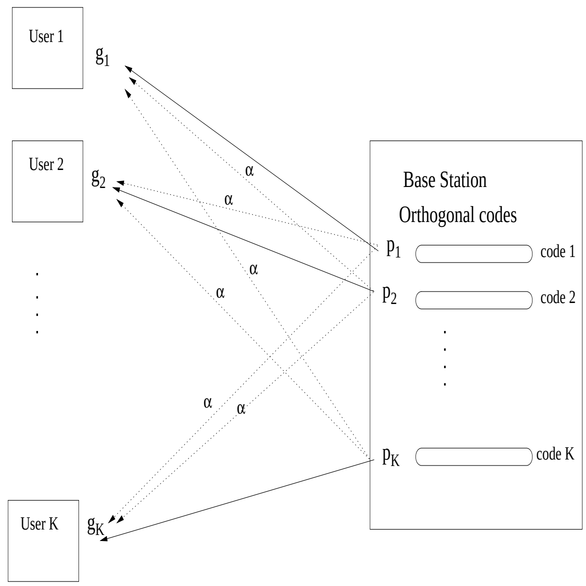

Fig. 1: Downlink System model

degree of nonorthogonality is described by the *orthogonality factor*, α $(0 \leq \alpha \leq 1)$ [5]. Under the assumption of independent identical channels for all users, it is reasonable to work with the same average orthogonality factor for all users. We assume that each user employs a matched filter receiver. The signal to interference ratio at the receiver of user i is expressed as

$$SIR_i = \frac{p_i g_i}{\alpha \sum_{j \neq i} p_j g_i + I} \tag{1}$$

where g_i denotes channel gain of user i. I denotes the sum of the thermal noise and the intercell interference.

We define the utility of user i as follows:

$$U_i(\mathbf{p}) = \log(1 + k\frac{p_i g_i}{\alpha \sum_{j \neq i} p_j g_i + I}) \tag{2}$$

where log denotes the natural logarithm. We note that the individual utility is a function of the power vector $\mathbf{p} = [p_1, p_2, ..., p_K]$. The utility is an achievable rate for user i, specifically by treating the interference as noise [9]. It is assumed that adaptive modulation is employed to enable the rate determined for each user [12]. The factor k captures the product of the adaptive modulation index and the fixed processing gain. This resembles the structure of HSDPA where multirate is achieved by multicode and adaptive modulation. We should also note that, in a practical system employing M-ary modulation, the transmission rate (utility) is a discrete quantity. For simplicity of analysis, and similar to previous work [13, 14], we will assume continuous rate values. We will examine the effect of this assumption in the numerical results.

The problem we consider is to determine the optimum allocation of the total transmit power from the base station to the users so as to maximize the overall system utility, i.e., sum of individual utilities. Formally, the optimization problem is formulated as:

$$\max_{\mathbf{p}} \quad \sum_{i=1}^{K} \log(1 + k\frac{p_i g_i}{\alpha \sum_{j \neq i} p_j g_i + I}) \tag{3}$$

$$\text{s.t.} \quad \sum_{i=1}^{K} p_i \leq P_T, \quad p_i \geq 0, \quad i = 1, \cdots, K \tag{4}$$

where we assume that $g_1 > g_2 > .... > g_K$, such that user with the lower index has a higher channel gain.

We note that the actual outcome of the above optimization problem is power vector. The optimum transmission strategy is implicitly included in the optimum power vector in that the users that belong to the subset of users that the base station transmits to, end up with non-zero power, and the rest with zero power. If the TDMA-mode turns out to be optimum, the power allocation is such that transmission power to only one user is positive. If the CDMA-mode turns out to be optimum, the power allocation is such that transmitted power values to multiple users are positive.

We first note that any power vector $\overline{\mathbf{p}}$ with $\sum_{i=1}^{K} \overline{p}_i < P_T$ can not be the optimum power vector. Define a power vector $\mathbf{p}$ with $p_i = \beta\overline{p}_i \quad (\beta > 1), i = 1, \cdots, K$ such that $\sum_{i=1}^{K} \beta p_i = P_T$. It is easy to see that $\mathbf{p}$ increases all individual utilities and hence the system utility as compared to $\overline{\mathbf{p}}$. Thus, (6) can be rewritten as:

$$\max_{\mathbf{p}} \quad \sum_{i=1}^{K} \log(1 + k\frac{p_i}{\alpha(p_T - p_i) + \frac{I}{g_i}}) \tag{5}$$

$$\text{s.t.} \quad \sum_{i=1}^{K} p_i = P_T, \quad p_i \geq 0, \quad i = 1, \cdots, K \tag{6}$$

Observe that the utility of user i is a function of the power for user i only, i.e., $U_i(\mathbf{p}) = U_i(p_i)$. Also note that, although the utility is concave in SIR, it need not be concave in power. Therefore, the optimization problem is in general not a convex program. Figure 3 emphasizes this point by plotting the individual utilities for users with different channel gains.

III. Optimum Transmit Strategy and Power Allocation

We examine the throughput maximization problem in terms of the individual utility and the system utility, which we term the user centric approach, and the system-wide approach, respectively. We first consider the user centric approach, and we observe the behavior of the individual utility in power by utilizing the derivative of the individual utility function. Our observations motivate us to consider the system-wide approach, in which we investigate the system utility in terms of the best user's power. The system-wide approach is useful to characterize the system utility when the utilities of all users are not concave in power. Figure 3 depicts such a case.

III.A The User Centric Approach

Consider the utility function of user i and its first and second derivatives in terms of p_i, the power of user i. The first derivative of the utility function is positive, i.e., $U_i(p_i)$ is an increasing function of p_i. We are interested in whether the utility $U_i(p_i)$ is an increasing concave or convex because this plays a crucial role in characterizing the behavior of the utility in power. Whether $U_i(p_i)$ is concave ($\frac{\partial^2 U_i(p_i)}{\partial p_i^2} < 0$) or convex ($\frac{\partial^2 U_i(p_i)}{\partial p_i^2} > 0$) depends on the $sign(A_i)$ of the numerator of $\frac{\partial^2 U_i(p_i)}{\partial p_i^2}$. A_i is given by

$$A_i = (\alpha - k)(\alpha(p_T - p_i) + \frac{I}{g_i}) + \alpha(\alpha(p_T - p_i) + \frac{I}{g_i} + kp_i).$$

By letting $I_{ti} = \alpha(p_T - p_i) + \frac{I}{g_i}$, we have $A_i = (\alpha - k)I_{ti} + \alpha(I_{ti} + kp_i)$. Thus, we need to have

$$\frac{p_i}{I_{ti}} > \frac{1}{\alpha} - \frac{2}{k} \tag{7}$$

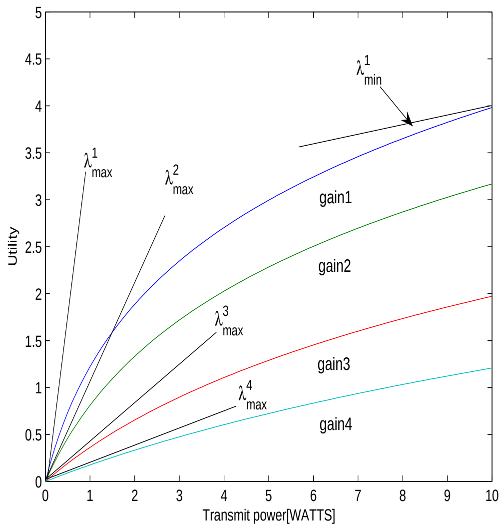

Fig. 2: $gain_1 > gain_2 > gain_3 > gain_4$. An example where individual utility functions for all users are concave.

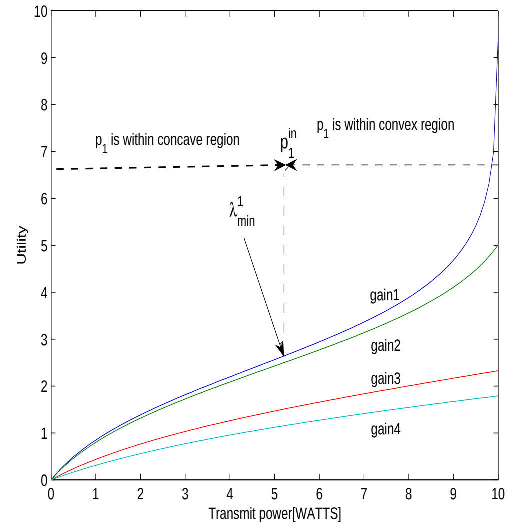

Fig. 3: $gain_1 > gain_2 > gain_3 > gain_4$. An example where individual utility functions for some users are not concave.

for $U_i(p_i)$ to be an increasing convex function of p_i and

$$\frac{p_i}{I_{ti}} < \frac{1}{\alpha} - \frac{2}{k} \tag{8}$$

for $U_i(p_i)$ to be an increasing concave function of p_i. Clearly, by observing $\frac{\partial^2 U_i(p_i)}{\partial p_i^2}|_{p_i=P_T}$, we can identify the behavior of function $U_i(p_i)$. When $\frac{\partial^2 U_i(p_i)}{\partial p_i^2}|_{p_i=P_T} < 0$, $U_i(p_i)$ is an increasing concave function in $0 \leq p_i \leq P_T$ as in Figure 2. When $\frac{\partial^2 U_i(p_i)}{\partial p_i^2}|_{p_i=P_T} > 0$, $U_i(p_i)$ is an increasing concave function in $0 \leq p_i \leq p_i^{in}$, while it is increasing convex function in $p_i^{in} \leq p_i \leq P_T$, where p_i^{in} denotes the inflection point in $U_i(p_i)$ as in $U_1(p_1)$ shown in Figure 3. Figure 2 shows the individual utilities of different channel gains. In this example, the individual utilities of all users are concave in power. It is clear that when the individual utility is concave in power, among multiple users, rather than allocating the entire power P_T to one user, sharing the power P_T will yield a higher total utility. In this case, the optimization problem in (6) is a convex program. However, this is no longer the case when the individual utilities of some users are not concave, as depicted in Figure 3. In this case, we need to have a closer look at the components that contribute to the system utility. The following terminology is used extensively in the sequel.

- $U_i(p_i)$ is convex if $\frac{\partial^2 U_i(p_i)}{\partial p_i^2} > 0$ in $0 \leq p_i \leq P_T$
- $U_i(p_i)$ is concave/convex if $\frac{\partial^2 U_i(p_i)}{\partial p_i^2}|_{p_i=0} < 0$ and $\frac{\partial^2 U_i(p_i)}{\partial p_i^2}|_{p_i=P_T} > 0$
- $U_i(p_i)$ is concave if $\frac{\partial^2 U_i(p_i)}{\partial p_i^2}|_{p_i=P_T} < 0$
- p_i^* is said to be within the concave region of $U_i(p_i)$, if $\frac{\partial^2 U_i(p_i)}{\partial p_i^2}|_{p_i=p_i^*} < 0$
- p_i^* is said to be within the convex region of $U_i(p_i)$, if $\frac{\partial^2 U_i(p_i)}{\partial p_i^2}|_{p_i=p_i^*} > 0$

Figure 3 depicts two possible power regions of user 1 (concave/convex user). Following from the above, we define

$$\lambda_{min}^1 = \frac{\partial U_1(p_1)}{\partial p_1}|_{p_1=p^*} \text{ when } U_1(p_1) \text{ is concave/convex,} \tag{9}$$
$$\text{and } \frac{\partial^2 U_1(p_1)}{\partial p_1^2}|_{p_1=p^*} = 0 \quad ;$$
$$\lambda_{min}^1 = \frac{\partial U_1(p_1)}{\partial p_1}|_{p_1=p^T} \text{ when } U_1(p_1) \text{is concave} \tag{10}$$
$$\lambda_{max}^i = \frac{\partial U_i(p_i)}{\partial p_i}|_{p_i=0} \tag{11}$$

and,

$$p_i(\lambda) = \arg_{p_i}\left(\frac{\partial U_i(p_i)}{\partial p_i}|_{p_i=p_i(\lambda)} = \lambda\right) \quad \text{when } \lambda < \lambda_{max}^i \tag{12}$$
$$= 0 \quad \text{when } \lambda > \lambda_{max}^i \tag{13}$$

where λ_{min}^1 implies the minimum derivative value of utility of user 1, while λ_{max}^i denotes the maximum derivative value of utility of user i. Figures 2 and 3 depict λ_{min}^1 and λ_{max}^i. Given λ, power of user i $p_i(\lambda)$ is obtained by optimizing the individual utility $U_i(p_i)$ over p_i. The optimum policy is a function of the orthogonality factor as well as users' channel gains in general. Below we state the propositions that lead to characterizing the optimum policies. The proofs of all the propositions in this paper can be found in [15].

Our first observation states that when the orthogonality factor is larger than a certain threshold, TDMA-mode is always optimum, regardless of users' channel gains:

Proposition III.1: *If $\alpha \geq \frac{k}{2}$, TDMA-mode with $p_1 = P_T$ yields the optimum system utility.*

While, it is clear that the TDMA-mode is optimum when $\alpha \geq \frac{k}{2}$, and no further power allocation is necessary, when $\alpha \leq \frac{k}{2}$, we need to characterize the optimum policy which consists of the transmission strategy, i.e., TDMA-mode or CDMA-mode, and the corresponding power allocation. First, we observe the following.

Proposition III.2: *The optimum power allocation $\mathbf{p}^* = (p_1^*, p_2^*, ..., p_K^*)$ is such that $p_1^* > p_2^* > ... > p_K^*$ when $p_i^* > 0$.*

When the individual utilities of multiple users are concave/convex as in Figure 3, we have the following observation.

Proposition III.3: *Suppose $\mathbf{p}^*$ is the optimum power allocation. Then, at most one user's power (the user with the best channel gain) is within the convex region of the utility function.*

From the preceding observations, we conclude that the optimum power allocation $\mathbf{p}^* = [p_1^*, p_2^*, ..., p_K^*]$ is one of two cases in terms of the best user, i.e., p_1^* can be either within the concave region or within the convex region of $U_1(p_1)$. This observation motivates our approach to focus on the the system utility in terms of the best user.

III.B System-Wide Approach

We define the system utility in terms of the best user, i.e., p_1:

$$J(p_1) = U_1(p_1) + Z(p_1) \tag{14}$$

$$Z(p_1) = \max[\textstyle\sum_{j=2}^{K} U_j(p_j)]_{\sum_{j=2}^{K} p_j = P_T - p_1}. \tag{15}$$

We note that $J(p_1)$ expends all available power, P_T, p_1 for user 1 and $P_T - p_1$ for the rest of the users. Our aim is to find the maximizer of $J(p_1)$ over $0 \leq p_1 \leq P_T$. Proposition 3 guarantees that $U_j(p_j)$ $(j \geq 2)$ is always concave at optimum power allocation. Hence, $Z(p_1)$ can be maximized with the power constraint $\sum_{j=2}^{K} p_j(\lambda) = P_T - p_1$ where λ is the optimum Lagrange multiplier. We note that no power is assigned to the user i when $\lambda_{max}^i < \lambda$. Depending on user 1's channel condition, $U_1(p_1)$ can be either concave or concave/convex. When $U_1(p_1)$ is concave, the resulting optimization problem is convex and the optimum power allocation is such that $\sum_{j=1}^{K} p_j(\lambda) = P_T$. If $U_1(p_1)$ is concave/convex as in Figure 3, the optimum power allocation is one of three possibilities.

Observation III.4: *If $U_1(p_1)$ is concave/convex as in Figure 3, the optimum power allocation is one of three local optimum solutions: (i) p_1 is within the concave region with $0 \leq p_1 < p_1(\lambda_{min}^1)$, (ii) p_1 is within the convex region with $p_1(\lambda_{min}^1) < p_1 < P_T$, or (iii) $p_1 = P_T$.*

Transmission strategy for the first two cases is CDMA, while the third case ($p_1 = P_T$) is TDMA. Given the fact that when $U_1(p_1)$ is concave/convex, the resulting optimization problem is non-convex, the three possible cases described above need to be considered in detail to maximize $J(p_1)$.

Consider $\sum_{i=1}^{K} p_i(\lambda)$, i.e., the sum of the powers transmitted to all users, given λ. Observing whether $\sum_{i=1}^{K} p_i(\lambda) < P_T$, or $\sum_{i=1}^{K} p_i(\lambda) > P_T$ provides us with the information whether $J(p_1)$ is increasing or decreasing at $p_1 = p_1(\lambda)$:

Proposition III.5: *If $\sum_{i=1}^{K} p_i(\lambda) < P_T$ for a given λ, $J(p_1)$ is an increasing function at $p_1 = p_1(\lambda)$.*

Proposition III.6: *If $\sum_{i=1}^{K} p_i(\lambda) > P_T$ for a given λ, $J(p_1)$ is a decreasing function at $p_1 = p_1(\lambda)$.*

From the previous two propositions, by observing $\sum_{i=1}^{K} p_i(\lambda)$, we can always determine whether $J(p_1)$ is an increasing function or decreasing function at $p_1 = p_1(\lambda)$. For instance, if $\sum_{i=1}^{K} p_i(\lambda_{min}^1) > P_T$, $J(p_1)$ is a decreasing function at $p_1 = p_1(\lambda_{min}^1)$ and the optimum power allocation is one of three possibilities as described by observation III.4.

If $\sum_{i=1}^{K} p_i(\lambda_{min}^1) < P_T$, this implies $\sum_{i=1}^{K} p_i(\lambda) < P_T$ for $\lambda > \lambda_{min}^1$ where $p_1(\lambda)$ is within the concave region, since $p_i(\lambda_{min}^1) > p_i(\lambda)$ when $U_i(p_i)$ is increasing concave. In this case, $J(p_1)$ is an increasing function when p_1 is within the concave region, i.e., $0 \leq p_1 \leq p_1(\lambda_{min}^1)$. Hence, no p_1 value within the concave region can be optimum. The optimum power allocation dictates that p_1 be either $p_1(\lambda_{min}^1) < p_1 < P_T$, or P_T. Finding the local optimum solution within the convex region of $U_1(p_1)$ requires intensive computational complexity. If we find a way to identify $p_1^* = P_T$, i.e., the optimality of TDMA, we can reduce the complexity of identifying the optimum policy.

Consider the case of $\sum_{i=1}^{K} p_i(\lambda) < P_T$ for $\forall\lambda$ where $p_1(\lambda)$ is within the convex region of $U_1(p_1)$. In this case, $J(p_1)$ is an increasing function for $0 \leq p_1 < P_T$.

Observation III.7: *If $\sum_{i=1}^{K} p_i(\lambda) < P_T$ for $\forall\lambda$ where $p_1(\lambda)$ is within the convex region of $U_1(p_1)$, then $p_1 = P_T$ is optimum power allocation.*

Observation III.7 guarantees the optimality of TDMA, but still requires considerable computational complexity. Notice that, as described above, when $\sum_{i=1}^{K} p_i(\lambda_{min}^1) < P_T$, the optimum power allocation does not fall within the concave region of $U_1(p_1)$. By adopting TDMA as the optimum policy whenever $\sum_{i=1}^{K} p_i(\lambda_{min}^1) < P_T$, we can significantly reduce computational complexity in finding local optimum power allocation in the convex region of $U_1(p_1)$. In the next section, we will term this short-cut, the *near-exact algorithm*. The numerical results in Section IV validate the accuracy of near-exact algorithm.

III.C Algorithms for Transmit Strategy and Power Allocation

Following the preceding discussion in Section III, we conclude that the optimality of TDMA is determined by checking $\sum_{i=1}^{K} p_i(\lambda) < P_T$ for all λ where $p_1(\lambda)$ is within the convex region of $U_1(p_1)$. Instead of this potentially computationally intensive search, we propose to simply check $\sum_{i=1}^{K} p_i(\lambda_{min}^1) < P_T$.

Clearly, the performance gap between the optimum solution and the near-optimum solution utilizing the near-exact algorithm of determining TDMA optimality results from the case when the optimum power allocation is such that $p_1(\lambda_{min}^1) < p_1 < P_T$. This is because the near-exact algorithm decides that TDMA-mode, i.e., $p_1 = P_T$ is optimum only if $\sum_{i=1}^{K} p_i(\lambda_{min}^1) < P_T$, regardless of $\max_{p_1} J(p_1)$ for $p(\lambda_{min}^1) < p_1 \leq P_T$. In our numerical results, we have observed that such cases, i.e., the optimum power allocation is such that $p_1(\lambda_{min}^1) < p_1 < P_T$, are rare and that in most cases, the optimum power allocation is either $0 < p_1 < p_1(\lambda_{min}^1)$ or $p_1 = P_T$. The power allocation step is done according to the mode that is designated to be the optimum strategy. When TDMA-mode is optimum, no further power allocation is necessary. When CDMA-mode is optimum, the optimum power allocation needs to be found. We should note that observation III.7 is a sufficient condition for TDMA optimality. Hence, even though $\sum_{j=1}^{K} p_j(\lambda_{min}^1) > P_T$, TDMA can be optimum. However, as the orthogonality factor increases, this condition accounts for almost all of the cases where TDMA is optimum, as demonstrated by numerical results in Section IV. The following summarizes the proposed algorithm to maximize the system utility, which we term the system utility maximizer, A-SUM.

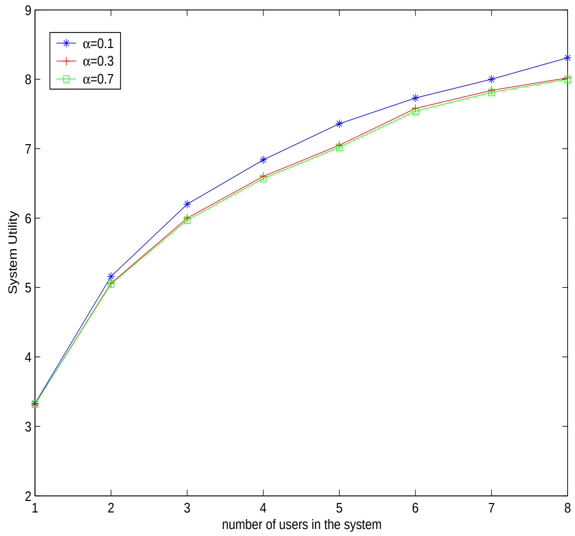

Fig. 4: System Utility versus Number of users in the system.

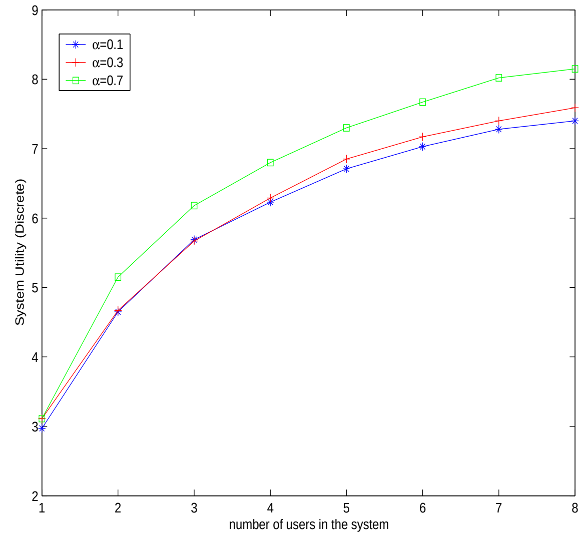

Fig. 5: System Utility versus Number of users in the system.

A-SUM:

STEP 1. If $\alpha \geq \frac{k}{2}$, then TDMA mode is optimum, **STOP**.
STEP 2. Find λ^i_{max} for $i = 1, \cdots, K$, and λ^1_{min}.
STEP 3. (convex region) If $\sum_{j=1}^{K} p_j(\lambda^1_{min}) < P_T$, then, TDMA is optimum, **STOP**.
STEP 4. (concave region) Find the power allocation in the concave region
$J^*(p_1(\lambda)) = \max(J(p(\lambda))), \quad 0 \leq p_1(\lambda) \leq p_1(\lambda^1_{min})$
via A-PACR.
STEP 5. Choose $\max(J^*(p_1(\lambda)), J(P_T))$.

The *near*-exactness of the algorithm arises from the fact that in STEP 3, we simply check the sufficient condition for optimality of TDMA, instead of solving for $\max_{p_1} J(p_1)$, $p_1(\lambda^1_{min}) < p_1 < P_T$.

The optimum power allocation (STEP 4) entails selecting the users to which the base station transmits with positive power. In other words, if zero power is assigned to one user, that implies that user is not *scheduled* for transmission. The algorithm to determine the power allocation in the concave region (A-PACR) is summarized below. It is assumed in A-PACR that $\sum_{j=1}^{K} p_j(\lambda^1_{min}) > P_T$. If this is not the case, there is no solution in the concave region, and A-PACR simply determines TDMA, i.e., $p_1 = P_T$.

A-PACR:

STEP 1. Find $\hat{k} = \arg_k \max \sum_{j=1}^{k} p_j(\lambda^k_{max})$, subject to $\sum_{j=1}^{k} p_j(\lambda^k_{max}) \leq P_T$, $k = 1, \cdots, K$.

STEP 2. Find $p_j(\lambda)$ $(1 \leq j \leq \hat{k})$ such that $\sum_{j=1}^{\hat{k}} p_j(\lambda) = P_T$ where $\lambda^1_{min} < \lambda < \lambda^{\hat{k}}_{max}$.

The optimum number of users to which the base station transmits is selected in STEP 1. Then, optimum power values are allocated in STEP 2. Note that a numerical method, e.g., bisection [16] can be used to determine the value of λ.

IV. Numerical Results

We provide simulation results for a range of the orthogonality factor values. Maximum total power at the base station is $P_T = 10$ watts. Total noise and intercell interference is $I = 10^{-11}$ watts. Factor k in the utility function is $k = 1.2$. Thus, when $\alpha \geq 0.6$, the individual utilities of all users are convex in power, and the TDMA-mode is optimum, regardless of users' channel gains. Channel gain from a base station to mobile i is modeled as $g_i = \frac{r_i}{d_i^4}$ where d_i denotes the distance between mobile i and the base station, which is uniformly distributed between 100m and 1000m. r_i is the realization of the lognormal fading coefficient with variance $8dB$.

We have first examined the accuracy of the near-exact algorithm determining TDMA optimality for 10,000 channel gain realizations. The ratio of the near exact algorithm which correctly determines the TDMA optimality in percentage are 97.5%, 97.5%, 99.9% for $\alpha = 0.1, 0.2, 0.3$, respectively. As α increases, as expected, the accuracy increases, i.e., the case when the optimum power allocation is such that $p_1(\lambda^1_{min}) \leq p_1 < P_T$ disappears. For example, when α=0.3, the near-exact algorithm almost always correctly determines the optimality of TDMA.

Figure 4 shows the system utility, given α and the number of users K in the system. The system utility is averaged over 10,000 channel gain realizations. The optimum policy selects the number of users and the power levels for the scheduled users. In general, not all users in the system are simultaneously scheduled. When $\alpha = 0.1$, i.e., a low orthogonality factor, simultaneous transmissions do not result in much interference between users, and the optimum policy chooses CDMA-mode. However, if the best user has a much higher channel gain as compared to the rest, the optimum policy may end up to allocating all power to that best user. As α increases, simultaneous transmissions result in higher interference, and TDMA-mode becomes the preferred mode so as to avoid the interference. For example, for $\alpha > 0.5$, TDMA-mode is optimum in most cases. For value of $\alpha > 0.6$, for example, when $\alpha = 0.7$, i.e., all individual utilities are convex, and TDMA-mode is always optimum. Figure 4 shows that the system utility, when $\alpha = 0.7$, is lower than the system utility value for smaller α values that facilitate simultaneous transmissions. The gap between the system utility for α ($\alpha < 0.6$) and the system utility for $\alpha = 0.7$ can be interpreted as the gain of optimum policy that results in hybrid CDMA/TDMA over the one that results in TDMA, as a result of the difference in the orthogonality factor values.

Figure 5 shows the system utility where the resulting individual utility is discretized to the integer value. With this, we attempt to investigate the performance of the proposed algorithm in a practical setting where only discrete rates are available. We observe that discretization does not lead to a significant loss in system utility especially for large α values. This is because for large α, TDMA results most often, and the quantization loss in utility is due to one user only.

Figure 6 shows the system utility gain of CDMA-mode over TDMA-mode. When the orthogonality factor is low, simultaneous transmissions has a gain over the one-at-a-time transmission. The gain increases as the number of users in the system increases up to a certain limit. As the orthogonality factor increases, however, we observe that the gain disappears, as eventually TDMA-mode becomes optimum.

V. Conclusion

In this paper, we considered the optimum policy which consists of the user(s) the base station selects to transmit to, and the power allocation for total utility maximization in the downlink. The individual utility we considered is the rate to the user. Depending on the channel conditions, i.e., the orthogonality factor and the users' channel gains, the optimum policy chooses either TDMA mode or CDMA mode and the corresponding power allocation. The higher the orthogonality factor, the larger the interference caused by simultaneous transmissions. This causes the TDMA-mode being optimal when the orthogonality factor is high. In contrast, simultaneous transmissions yield a higher overall utility when the orthogonality factor is low. We observed that depending on the channel conditions, the overall system utility may or may not be a concave function and care must be given to characterizing the solution.

We took the approach of examining the system utility in terms of the best user, and observed properties that helped us identify the optimal policy. We presented numerical results to support our analytical findings, in particular to the benefit of allocating transmission to simultaneous users in certain scenarios, and evaluate the frequency of TDMA mode optimality for different system parameters.

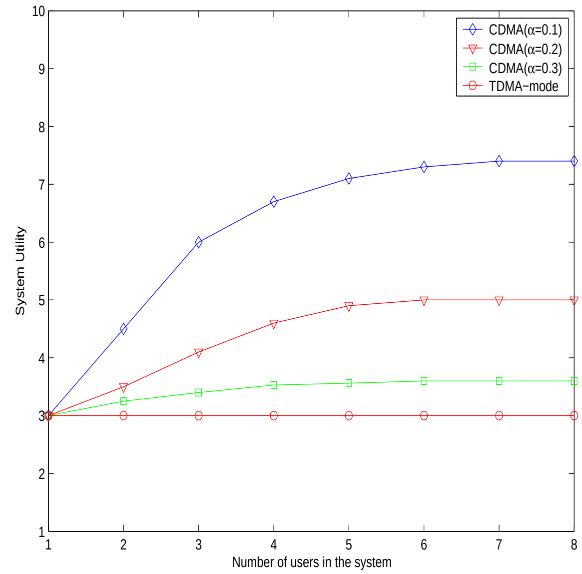

Fig. 6: CDMA-mode gain over TDMA-mode for different α value.

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