Further Results on Adaptive CDMA Cell Sectorization with Linear Multiuser Detection

(Invited Paper)

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Abstract-We consider the adaptive sectorization problem for a CDMA system under the imperfect directional antenna. Specifically, given the number of sectors and terminal locations, and assuming base station employs linear multiuser detection, we investigate how to appropriately sectorize the cell, such that the total transmit power is minimized, while each user has acceptable quality of service. We observe uplink/downlink duality under the assumption of matched filter and perfect antenna response. We also investigate adaptive sectorization problem in the more realistic scenario of imperfect directional antenna response. We employ MMSE power control to suppress both the intrasector interference and the intersector interference. As the optimum solution for arbitrary signature sets may have high complexity, we propose simpler suboptimum methods. The results suggest that by intelligently combining adaptive cell sectorization, power control and temporal linear multiuser detection, we are able to increase the uplink user capacity of the cell. We provide numerical results showing the robustness of optimum sectorization against Gaussian channel estimation error.

I. INTRODUCTION

CDMA shows promise in meeting the demand for future wireless services [1]. It is well known that CDMA systems are interference limited and the capacity of CDMA systems can be improved by various interference management techniques. These techniques include transmit power control, multiuser detection and cell sectorization [2]–[6]. In this work, we consider the adaptive cell sectorization problem for the uplink of a CDMA system under the imperfect directional antenna. Given the number of sectors and terminal locations and the fact that the base station employs linear multiuser detection, the problem we consider is to appropriately sectorize the cell, such that the total transmit power is minimized, while each terminal has acceptable quality of service which is defined by the received signal to interference ratio (SIR) at the base station.

Conventional cell sectorization, where the cell is sectorized to equal angular regions, may not perform sufficiently well especially in systems where user distribution is nonuniform [2]. Previous work, where the perfect directional antenna model is assumed, has shown that, adaptive cell sectorization where sector boundaries are adjusted in response to terminal locations improves the uplink user capacity [2]. Uplink capacity is further improved when adaptive cell sectorization is employed in conjunction with linear multiuser detection [7]. Adaptive sectorization, where users are grouped in spatial orthogonal channels by means of directional antennas, is in general a combinatorial optimization problem. In the special case when the system employs random signatures or an equicorrelated signature set, the minimum received power in each sector is achieved when all users' received powers are equal. In this case, the transmit power optimization problem can be transformed into a graph partitioning problem that can be solved by a shortest path algorithm in polynomial time. References [2] and [7] considered such cases when matched filters and linear multiuser detectors are employed at the base station. Both references assumed perfect directional antenna response, i.e., complete orthogonality between sectors. In practical scenarios, directional antenna response is imperfect which leads to intersector interference.

In this paper, we further investigate adaptive sectorization in practical scenarios such as imperfect directional antenna patterns and the presence of channel estimation errors and report our results. We observe a duality in terms of total transmit power in uplink and downlink under the assumption of perfect directional antenna response and matched filters. This is, however, no longer the case, for the optimization problem where we have the flexibility of designing the linear receiver filters. Furthermore, when imperfect directional antennas are present, the intersector interference (ISecI) patterns (uplink and downlink) resulting from the imperfect directional antenna are quite different [8].

Jointly optimal power control and MMSE multiuser detection has been proposed in [4]. We utilize MMSE power control in conjunction with adaptive sectorization to suppress both the intrasector interference and the ISecl. The optimum solution of our problem turns out to have considerable computational complexity. Hence, we propose simpler methods that are nearoptimum. Numerical results show that the uplink capacity significantly benefits from intelligently combining receiver filtering and adaptive sectorization.

Finally, we consider the effect of channel estimation errors on adaptive sectorization and observe in our numerical results that optimum sectorization is robust against users' channel estimation errors.

II. ANTENNA PATTERN AND SYSTEM MODEL

A single cell uplink DS-CDMA system with processing gain G, and K users is considered. The locations and the channel gains of the users in the cell are assumed to be known. This is a reasonable assumption in a slow mobility environment such as fixed wireless. In Section V-B, we investigate the performance of adaptive cell sectorization in the presence of channel estimation errors. We assume the cell is to be sectorized to N sectors.

We model the antenna pattern following reference [10]. Figure 1 shows the uplink antenna pattern model. Due to the imperfect antenna pattern, interference (ISecI) results from adjacent sectors. Main lobe between $+\theta_1$ and $-\theta_1$ (within the sector) has constant antenna gain, and side lobe between θ_1 and θ_2 , $-\theta_2$ and $-\theta_1$ (out of sector) has linear attenuated antenna gain in dB, which induces ISecI. Increase in $\theta_2 - \theta_1$ causes a large area to be spanned by the sector antenna which increases ISecI. Increased users (out of sector users) in between θ_1 and θ_2 , $-\theta_1$ and $-\theta_2$ increases ISecI. In case of perfect directional antenna, there is no side lobe ($\theta_2 - \theta_1 = 0^\circ$).

III. UPLINK/DOWNLINK DUALITY

In this section, we assume the perfect directional antenna, i.e., no ISecI. In adaptive uplink/downlink cell sectorization problem, our aim is to minimize the total transmit power, given SIR constraints for uplink (UL) and downlink (DL).

$$\min_{\theta, \mathbf{p}} \sum_{k=1}^{N} \sum_{i \in g_k(\theta)} p_i \qquad (UL) \quad (1)$$

s.t.
$$\gamma_i = \frac{(\mathbf{c}_i^{\top} \mathbf{s}_i)^2 p_i h_i}{\sum_j (\mathbf{c}_i^{\top} \mathbf{s}_j)^2 p_j h_j + \sigma^2 (\mathbf{c}_i^{\top} \mathbf{c}_i)} \ge \gamma^*$$

$$\min_{\theta, \mathbf{q}} \sum_{k=1}^N \sum_{i \in g_k(\theta)} q_i \qquad (DL) \quad (2)$$

s.t.
$$\gamma_i = \frac{(\mathbf{c}_i^{\top} \mathbf{s}_i)^2 q_i h_i}{\sum_j (\mathbf{c}_i^{\top} \mathbf{s}_j)^2 q_j h_i + \sigma^2 (\mathbf{c}_i^{\top} \mathbf{c}_i)} \ge \gamma^*$$

 $\neq i, j \in g_k(\theta) \quad k = 1, ..., N \quad \mathbf{p}, \mathbf{q} \ge 0 \quad \mathbf{1}^{\top} \theta = 2\pi$

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where p_i , q_i , h_i , b_i , γ_i , c_i and s_i denote the uplink transmit power, downlink transmit power, the uplink (or downlink) gain, the information bit, SIR, receiver filter and the signature sequence of the *i*th user. γ^* denotes the target SIR. θ is the N-tuple vector that denotes the sector angles. p and q denote uplink and downlink power vectors, respectively. $g_k(\theta)$ is the set of users that reside in the area spanned by sector k. 0 and 1 denote the all zero and all one vector, respectively. In (1), (2), the minimum transmit power is achieved when the SIR constraints are satisfied with equality [5], [9]. We observe uplink downlink duality in terms of total transmit power.

Lemma 3.1. Under no ISecI, and the assumption that matched filter receivers are employed, same signature is used for uplink and downlink for each user, and the same noise power is present at each receiver, the sector transmit powers for uplink/downlink are equal. Consequently, cell powers for uplink/downlink are equal [8].

Proposition 3.2: Under the same assumption as lemma (3.1), optimum sectorization arrangements in terms of minimum transmit power are equal for both uplink/downlink [8].

Using the above uplink/downlink duality, downlink optimum sectorization arrangement can be directly determined by the uplink result or vice versa.

IV. TRANSMIT POWER OPTIMIZATION

A. Problem Statement

The received signal at the front end of receiver filter for user i in sector k at the base station is

$$\begin{aligned} \varphi_i(t) &= \sqrt{p_i h_i} b_i s_i(t) + \sum_{j \neq i, j \in g_k(\theta)} \sqrt{p_j h_j} b_j s_j(t) \\ &+ \sum_{l \notin g_k(\theta)} \sqrt{p_l h_l v_{li}} b_l s_l(t) + n(t) \end{aligned}$$
(3)

where p_j , h_j , b_j , $s_j(t)$ are transmit power, uplink gain, information bit and signal waveform for user j. The signature waveforms can be represented by G orthonormal basis waveforms $\{\psi_j(t)\}_{j=1}^G$, such that $s_i(t) = \sum_{j=1}^G s_{ij}\psi_j(t)$, with $s_{ij} = \langle s_i(t), \psi_j(t) \rangle$. Therefore, the signal in (3) can be expressed in an equivalent vector form as [6]:

$$\mathbf{r}_{i} = \sqrt{p_{i}h_{i}}b_{i}\mathbf{s}_{i} + \sum_{j\neq i,j\in g_{k}(\theta)}\sqrt{p_{j}h_{j}}b_{j}\mathbf{s}_{j} + \sum_{l\notin g_{k}(\theta)}\sqrt{p_{l}h_{l}v_{li}}b_{l}\mathbf{s}_{l} + \mathbf{n}$$
(4)

where $\mathbf{s}_i = [s_{i1}, ..., s_{iG}]$ is the signature sequence of user *i*. **n** denotes the Gaussian noise random vector with $E(\mathbf{nn}^{\top}) = \sigma^2 \mathbf{I}_G$. Note that the second term represents the intrasector interference while the third term represents the ISecl. v_{li} is antenna gain between interferer *l* and user *i*. If user *i* experiences no ISecI, $v_{li} = 0$. SIR for user *i* in sector *k* at the receiver filter output can be expressed as

$$SIR_{i} = \frac{P_{i,S}}{P_{i,INTRA} + P_{i,INTER} + P_{i,NOISE}}$$
(5)
th
$$P_{i,S} = p_{i}h_{i}(\mathbf{c}_{i}^{T}\mathbf{s}_{i})^{2}$$

with

$$P_{i,INTRA} = \sum_{j \neq i, j \in g_k(\theta)} p_j h_j (\mathbf{c}_i^{\mathsf{T}} \mathbf{s}_j)^2$$
$$P_{i,INTER} = \sum_{l \notin g_k(\theta)} p_l h_l v_{li} (\mathbf{c}_i^{\mathsf{T}} \mathbf{s}_l)^2$$
$$P_{i,NOISE} = \sigma^2 (\mathbf{c}_i^{\mathsf{T}} \mathbf{c}_i).$$

Our aim is to investigate the best sectorization arrangement such that the total transmit power is minimized, while each user has acceptable quality of service. A user is said to have an acceptable quality of service if its SIR is greater than a target SIR, γ^* . The sectorization problem we consider can be formulated as the transmit power optimization problem



Fig. 1. Uplink Antenna Pattern Model

$$\min_{\theta, \mathbf{p}} \sum_{k=1}^{N} \sum_{i \in g_k(\theta)} p_i \tag{6}$$

s.t.
$$p_i \ge \min_{\mathbf{c}_i} \frac{\gamma^* (P_{i,INTRA} + P_{i,INTER} + P_{i,NOISE})}{h_i (\mathbf{c}_i^\top \mathbf{s}_i)^2}$$

 $k = 1, ..., N \qquad \mathbf{p} \ge \mathbf{0} \qquad \mathbf{1}^\top \theta = 2\pi.$ (7)

The solution to the above optimization problem does not have a closed form and hence has to be found iteratively. We note that for each sectorization arrangement, an iterative algorithm that finds the minimum power solution along with the best linear filters is easily obtained as proposed in [4] as outlined below.

Consider the minimum total power solution, given a feasible sectorization arrangement. Define the power vector for all users in the cell $\mathbf{P} = [p_1, ..., p_{N_1}, p_1, ..., p_{N_2}, p_1, ..., p_{N_N}]$ where N_i is number of user in the sector i, and

$$I_{ki}(\mathbf{P}, \mathbf{c}_i) = \frac{\gamma^*(P_{i,INTRA} + P_{i,INTER} + P_{i,NOISE})}{h_i(\mathbf{c}_i^{\mathsf{T}}\mathbf{s}_i)^2}$$
(8)

$$I_{ki}(\mathbf{P}) = \min_{\mathbf{c}_i} I_{ki}(\mathbf{P}, \mathbf{c}_i).$$
(9)

The interference function $I(\mathbf{P})$ is

$$I(\mathbf{P}) = [I_{11}(\mathbf{P}), ..., I_{1N_1}(\mathbf{P}), ..., I_{N1}(\mathbf{P}), ..., I_{NN_N}(\mathbf{P})].$$
(10)

Reference [5] showed that the power control algorithm in the form of $\mathbf{P}(\mathbf{n} + 1) = \mathbf{I}(\mathbf{P}(\mathbf{n}))$ converges to the minimum power solution if $\mathbf{I}(\mathbf{P})$ is a standard interference function. It is straightforward to show that $\mathbf{I}(\mathbf{P})$ in (10) is a standard interference function. The resulting power control algorithm first finds the receiver filter for user *i* to be the MMSE filter for fixed power vector. The power for user *i* is then adjusted to meet the SIR constraint:

$$\mathbf{A}_{ki}(\mathbf{P}(n)) = \sum_{j \neq i, j \in g_k(\theta)} p_j h_j \mathbf{s}_j \mathbf{s}_j^\top + \sum_{l \notin g_k(\theta)} p_l h_l v_{li} \mathbf{s}_l \mathbf{s}_l^\top + \sigma^2 \mathbf{I}$$
(11)

$$\mathbf{c}_i = \frac{\sqrt{p_i(n)}}{1 + p_i(n)\mathbf{s}_i^\top \mathbf{A}_{ki}^{-1}(\mathbf{P}(n))\mathbf{s}_i} \mathbf{A}_{ki}^{-1}(\mathbf{P}(n))\mathbf{s}_i \qquad (12)$$

$$\mathbf{P}(n+1) = \mathbf{I}(\mathbf{P}(n)). \tag{13}$$

We should note that due to the presence of ISecI, the iterative power control algorithms that are run in each sector for a given arrangement interact with each other. However, cell-wide convergence is guaranteed no matter which order the sector power updates are executed thanks to the asynchronous convergence theorem in [5]. The resulting MMSE filters suppresses both the intrasector interference and the ISecI each user experiences.

B. Optimum Sectorization

First, for each sectorization arrangement that satisfies the maximum angle constraints, the minimum total transmit power solution is obtained via MMSE power control described in the previous section. Second, the best sectorization arrangement with minimum total transmit power solution is selected. The resulting optimum solution has high complexity which motivates us to look for solutions with reduced complexity that result in near optimum performance. Such an algorithm is presented next.

C. Near-optimum Sectorization

The intuition behind the reduced complexity solution we present in this section is to try to equalize the "load" per sector as much as possible. Equal load per sector solution is simply equal number per sector solution which can be obtained simply by determining the angular boundaries of sectors such that an equal number of users reside in each sector with respect to a reference user and finding the minimum power solution via MMSE power control. This process can be repeated K/N times by shifting the reference point with 0° angle to the next user from the previous reference point. The sectorization arrangement in terms of minimum total transmit power is selected as the best "equal number of users per sector solution".

When the terminal distribution is uniform, equal load solution works well. However, as the terminal distribution becomes nonuniform, equal load solution needs to be improved to achieve near optimum performance. We have observed that the following algorithm improves the equal loading scenario and works near-optimum in a range of scenarios. Once the equal loading scenario (equal number of users per sector) that yields the minimum (cell) total power is found, we move the boundaries of the sectors with the minimum total power to include users from neighboring cells in an effort to try to shift a user that may cause substantial increase in transmit power within a sector to the neighboring sector that has the least power expenditure. Hence we try to maximize the minimum P_k where P_k is the sector received power in kth sector antenna. Although it is difficult to draw general conclusions under the assumption of a general system with no particular channel or signature matrix structure, we find that running a couple of the above iteration improved the performance in all our simulation scenarios considerably as compared to equal number of users per sector and performed near optimum.

V. NUMERICAL RESULTS

A. Perfect channel estimation

We consider a synchronous uplink CDMA system with processing gain G = 16 and number of users K = 25. The cell is to be partitioned to N = 6 sectors. For the antenna pattern model, we set $\theta_2 - \theta_1 = 15^\circ$, P = -10dB, and the maximum angle constraint $(max(2\theta_1)) = 120^\circ$. We assume no channel estimation error in this section.

In the equal loading scenario, sectors 1 through 5 have 4 users each and the remaining sector has 5 users. We then try to maximize the minimum sector power to improve the performance as described in Section IV. Figures 2 and 3 show sector boundaries for uniform and nonuniform user distributions, respectively. Tables I and II show the total transmit powers and sectorization arrangements of the optimum sectorization (OS) and the near-optimum sectorization (NS) in uniform and nonuniform distributions, respectively. We can observe from Tables I and II, that as users' distributions become nonuniform, the performance gap between OS and NS slowly increases.

To assess the benefit of adaptive uplink cell sectorization with multiuser detection, we compared our results with (i) conventional sectorization (equal angular partition) when the base station employs MMSE multiuser detection (EAP), and (ii) adaptive optimum sectorization when the base station uses matched filters (AMF) in Figures 2 and 3 and in Tables I and II. In nonuniform terminal distribution as in figure 3, EAP can not efficiently sectorize the hot spot region, consequently, requires about 3dB more transmit power than OS and NS. It is observed that the uplink performance is improved by combining adaptive cell sectorization and multiuser detection. That is, we can accommodate more users and or/higher SIR targets (higher bit rates) as compared if only one of these methods were employed.

B. Channel Estimation Error

The adaptive cell sectorization concept relies on the fact that users' channels/physical locations are known. Hence it is appropriate to investigate the robustness of the methods against channel estimation errors. In this section, we provide numerical results to show the robustness of optimum sectorization against Gaussian channel estimation error. Estimated pathloss gain \hat{g} is modeled as

$$\hat{h} = h + e; \quad \frac{E(\hat{h} - h)^2}{h^2} = \sigma_h^2.$$
 (14)

Figure 4 shows Probability(SIR > Target SIR) vs. Fading SIR. Fading SIR is the actual target SIR value assigned to γ^* in MMSE power control. Under the estimation errors, a fading SIR target value that is higher than the original target SIR is required to compensate the channel estimation errors. More users' SIR satisfy the target SIR by increasing fading SIR target value. We set the Fading SIR to the value that satisfies Probability(SIR > Target SIR) = 0.9 in figure 4, and term it effective target SIR, $\overline{\gamma}$. At an effective target SIR, about 90% of users' SIR satisfy the target SIR. Tables III and IV show the total transmit power for different σ_h^2 values. As expected, increased estimation error variance increases total transmit power. Tables V and VI show the robustness of optimum sectorization against Gaussian channel estimation error. The percentages shown represent the percentages of channel estimation error realizations that yield the same optimum adaptive cell sectorization arrangement as the ones that use the perfect channels estimates. For example, at $\sigma_h^2 = 0.01$ in nonuniform distribution, almost all cases, optimum sectorization arrangement does not change by slightly increasing the total transmit power, which shows the robustness of optimum sectorization against estimation errors. It is observed that the scenario with the uniform distribution of users is more vulnerable to estimation errors as compared to the nonuniform distribution which appears to be fairly robust to estimation errors. This may be attributed to the fact that, when the users are uniformly distributed in the cell, the number of feasible sectorization arrangements is a lot higher than the case of nonuniform distribution. Note that adaptive cell sectorization is in general less beneficial in the uniform user distribution scenario as compared to the nonuniform user distribution scenario.

VI. CONCLUSION

In this paper, we mainly focused on the adaptive cell sectorization for uplink CDMA system under the imperfect directional antenna when the base station is employing linear multiuser detectors. We posed the optimum sectorization problem where the angular sector boundaries are optimized along with transmit power values and the receiver filters under the imperfect directional antenna model. The system is optimized to suppress not only the intrasector interference but also the intersector interference that results from imperfect directional antenna patterns. We evaluated the results of the optimum solution as well as a heuristic reduced complexity solution that performs near optimum. We have observed that the uplink user capacity is improved through the cooperation between the three interference management methods. Finally, we have observed that we can compensate for channel estimation errors by a slight elevation in the total transmit power.

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TABLE I

UNIFORM TERMINAL DISTRIBUTION. TOTAL TRANSMIT POWER[WATTS], SECTOR ARRANGEMENT (3 8 10 17 20 23) MEANS SECTOR 1:(USER 3,4,5,6,7),SECTOR 2:(USER 8,9),SECTOR 3:(USER

10,11,12,13,14,15,16), SECTOR 4: (USER 17,18,19), SECTOR 5: (USER 20,21,22), SECTOR 6: (USER 23,24,25,1,2).

Method	Total Trans. Power	Sector Arrangement
OS	1.7812	3 8 10 17 20 23
NS	1.8622	3 8 10 15 19 23
EAP	2.2532	1 6 8 15 20 23
AMF		Infeasible

TABLE II

NONUNIFORM TERMINAL DISTRIBUTION, TOTAL TRANSMIT POWER[WATTS]

[Method	Total Trans. Power	Sector Arrangement
	OS	9.1039	3 6 13 15 21 25
.[NS	10.5815	2 6 10 13 18 21
ĺ	EAP	20.6124	1 3 5 7 20 25
	AMF 1		Infeasible

TOTAL POWER (TP) OF UNIFORM TERMINAL DISTRIBUTION, TARGET SIR=5

;	σ_h^2	0.001	0.01	0.05	0.1	0.15	Í	
	$\overline{\gamma}$	5.4	5.8	6.6	7.2	7.8	i	
	TP	1.9327	2.0965	2.5086	3.0674	4.5773	1	

TABLE IV Total Power (TP) of Nonuniform terminal distribution, Target SIR=5

σ_h^2	0.001	0.01	0.05	0.1	0.15
$\overline{\gamma}$	5.2	5.8	6.6	7.2	8.0
TP	9.5024	10.7366	13.0274	16.3130	23.4325

TABLE V
ROBUSTNESS OF OPTIMUM SECTORIZATION AGAINST GAUSSIAN
ESTIMATION ERRORS, UNIFORM TERMINAL DISTRIBUTION

σ_h^2	0.001	0.01	0.05	0.1	0.15
Robustness	85%	65 %	54%	49%	46%

TABLE VI

ROBUSTNESS OF OPTIMUM SECTORIZATION AGAINST GAUSSIAN ESTIMATION ERRORS, NONUNIFORM TERMINAL DISTRIBUTION

σ_h^2	0.001	0.01	0.05	0.1	0.15
Robustness	99.9%	99.9%	94%	87%	82%



Fig. 2. Sector boundaries in an uplink CDMA system for uniform terminal distribution. Number of users, K=25, Processing gain, G=16, Number of sectors N=6, noise power, $\sigma^2 = 10^{-13}$.



Fig. 3. Sector boundaries in an uplink CDMA system for nonuniform terminal distribution. Number of users, K=25, Processing gain, G=16, Number of sectors, N=6, noise power, $\sigma^2 = 10^{-13}$.



Fig. 4. Probability(SIR > Target SIR) vs. Fading SIR for Gaussian channel estimation errors $\sigma_h^2 = 0.001, 0.01, 0.05, 0.1, 0.15$, where minimum constraint Target SIR is 5.