Outage Performance of Cognitive Wireless Relay Networks

Kyounghwan Lee Aylin Yener Wireless Communications and Networking Laboratory Electrical Engineering Department The Pennsylvania State University University Park, PA 16802 Telephone: (814) 865 - 4337 Fax: (814) 863 - 5341 kxl251@psu.edu yener@ee.psu.edu

Abstract—In this paper, we investigate the outage performance of cognitive wireless relay networks where source nodes communicate to their destinations via multiple hops facilitated by intermediate cognitive nodes able to acquire spectrum holes. Specifically, we consider a model that consists of a source node, a destination node, and a group of network clusters each consisting of a number of cognitive (unlicensed) relay nodes and a primary (licensed) node. Cognitive nodes relay information from the source depending on their geographical proximity and their ability to acquire the spectrum hole successfully. We investigate the high SNR approximation of the outage probability of the resulting two-hop system to obtain the diversity order. We show that full diversity is achieved only if each relay node successfully identifies the spectrum hole unoccupied by the corresponding primary node in the cluster, and that the diversity order can be significantly less for imperfect spectrum acquisition. Thus, we set out to improve the outage performance by incorporating a specific intra-cluster cooperation scheme where neighboring cognitive relay nodes in a cluster collaborate with a desired cognitive relay node. We show that the combination of this intra-cluster cooperation along with the system level cooperation via relaying through cognitive nodes improves the outage performance significantly, and the full diversity can be achieved if the proper number of neighboring relay nodes participate in the intra-cluster cooperation.

I. INTRODUCTION

Due to the increasingly wide deployment of high speed wireless networks for services such as mobile internet and multi-media applications, the demand for spectrum is expected to grow rapidly in the near future. The allocation of the spectrum currently is regulated. The spectrum bands are licensed and sharing of bands is not allowed. Because this inflexible spectrum allocation policy may result in the underutilization of overall spectrum [1], more flexible alternatives for better utilization of the spectrum should be sought.

Cognitive radios are amenable to employ a more open spectrum policy [2], [3]. A cognitive radio allows a unlicensed user to access a spectrum hole unoccupied by a primary (licensed) user. By doing so, spectrum utilization can be improved significantly while reducing the white spaces in the spectrum [3]. A requirement for this system is seamless operation; thus, the cognitive users must detect the presence of the spectrum hole, i.e., equivalently, detect the presence of the primary users transmission [3], [4]. The common approach for detecting these unknown signals is to use an energy detector: whose performance has been investigated for various channel conditions in [5], [6]. Recent research effort investigates cooperative spectrum sensing where a group of neighboring nodes cooperate with a desired cognitive user. It has been shown that the cooperative spectrum sensing provides more reliable and faster detection of primary users [7]–[9].

Future wireless networks will continue to evolve towards allowing mobile nodes to communicate without the need of infrastructure while providing more reliability and capacity increase. To that end, relay networks, where a source node is assisted by intermediate nodes offer a significant performance gain advantage [10]–[14].

Inspired by these two futuristic aspects of wireless networks, i.e., cognitive radios and relay networks, in this paper, we investigate a *cognitive wireless relay network* which is defined by a source node, a destination node, and a group of network clusters each of which consists of a number of cognitive relay nodes and a primary node. We assume that the cognitive relay nodes are grouped as clusters based on their geographical proximity. Specifically, we aim to understand the impact of spectrum acquisition performance of the cognitive relay nodes on the outage performance of this system. We analyze the high SNR approximation of the outage performance in order to examine the diversity order of these networks.

We consider three different scenarios depending on the spectrum acquisition capability of a cognitive relay node in each cluster. First, we consider the scenario where the cognitive relay node always acquires the spectrum hole successfully whenever it is available, and observe that with this idealistic setup, full diversity is achieved. However, for the realistic scenario where spectrum acquisition is not always guaranteed¹, we observe that the outage performance is dependent on the spectrum acquisition capability of the relay nodes. Motivated by this performance deficiency of the imperfect spectrum acquisition scenario, we propose an intra-cluster cooperation scheme which exploits the idea of cooperative

¹Henceforth we refer to this scenario as the imperfect spectrum acquisition scenario.

spectrum sensing [7] to improve the spectrum acquisition capability. It is shown that if we allow neighboring nodes in each cluster to collaborate with the potential relay node, the outage performance is improved and full diversity can be achieved when a proper number of neighboring nodes participate in the intra-cluster cooperation.

II. SYSTEM MODEL

We consider a cognitive wireless relay network where a source node is assisted by a number of relay nodes. The relay nodes are cognitive (unlicensed) users and are grouped in clusters, based on their geographical proximity. Each cluster consists of a number of cognitive relay nodes (a potential relay node and neighboring relay nodes) and one primary (licensed) node, all capable of communicating over different frequency bands. The system model is depicted in Figure 1. Among the nodes in a cluster, one node is chosen as a potential relay node and can assist the source node, if the potential node can both decode the message from the source, and acquire the spectrum hole unoccupied by its primary user successfully. We define the relaying set R(s) to be the set of successful potential relays that meet both of these requirements. We assume that the spectrum used by each primary user is non-overlapping. The primary user broadcasts a beacon signal over a side channel to inform relay nodes of the availability of a frequency band. We note that even though the beacon requires additional spectrum and power, it is resource efficient for the relay nodes in that it prevents the potential nodes from browsing the entire frequency range.

The communication takes place in two phases. In the first phase, the source node broadcasts its information. In the second phase, the successful potential relay nodes decode and transmit it to the destination node over the acquired spectrum, using the same codebook used in the source node, i.e., the relays employ regenerative decode-and-forward (RDF).

The transmission of the potential relay nodes from different clusters to the destination node is orthogonal because the acquired spectra from different clusters are non-overlapping. The received signal at each relay during the first phase is

$$Y_{r_i} = h_{sr_i} X_s + N_{sr_i} \tag{1}$$

where X_s is the transmitted signal from the source node. The received signal at the destination during the first phase is

$$Y_d = h_{sd}X_s + N_{sd} \tag{2}$$

During the second phase, the destination receives the signal from the successful potential relay node, in channel *i*, i.e., $r_i \in R(s)$:

$$Y_{d_i} = h_{r_i d} X_{r_i} + N_{r_i d} \tag{3}$$

where X_{r_i} is the transmitted signal from the successful potential relay node *i*. We assume that X_{r_i} contains the information of the frequency range acquired from primary node, so that the destination node knows which licensed channels are used by the potential relay nodes. N_{sr_i} , N_{sd} , and N_{r_id} are receiver noise and modeled as zero-mean mutually



Fig. 1. System Model

independent, circularly symmetric, complex Gaussian random variables with variance N_0 , and h_{sr_i} , h_{sd} and h_{r_id} are the fading coefficients, which as zero-mean, independent, circularly symmetric complex Gaussian random variables with variances $1/\lambda_{sr_i}$, $1/\lambda_{sd}$, and $1/\lambda_{r_id}$, respectively. The source and relays all transmit with power P. Then, we define the received SNR at the relay and the destination as,

$$\gamma_{sr_i} = SNR|h_{sr_i}|^2, \ \gamma_{sd} = SNR|h_{sd}|^2, \ \gamma_{r_id} = SNR|h_{r_id}|^2 \tag{4}$$

where $SNR = P/N_0W$.

III. SPECTRUM ACQUISITION IN RAYLEIGH FADING

Since the potential relay nodes are cognitive users, it is important that each potential relay node detects the unused spectrum hole. Since the primary user broadcasts a beacon whenever the spectrum is available, the spectrum detection problem is a hypothesis testing problem with the following two hypotheses:

$$Y_{pr} = \begin{cases} N_{pr}, & H_0 \\ h_{pr} X_p + N_{pr}, & H_1 \end{cases}$$
(5)

where H_0 corresponds to the event where a spectrum hole is not available, i.e., the beacon is not present and H_1 corresponds to the event where a spectrum hole is available, i.e., the beacon is present. Y_{pr} is the signal received by potential relay node from its primary node, X_p is the transmitted beacon signal, N_{pr} is the received noise at the potential relay, and h_{pr} is a fading coefficient which is a zero-mean circularly symmetric complex Gaussian random variable with $1/\lambda_{pr}$.

We assume that each potential relay node performs energy detection [5]–[9]. The expression for the probability of detection (P_d) under a Rayleigh fading channel is given by [6]:

$$P_{d} = e^{\frac{-\lambda}{2}} \sum_{k=0}^{u-2} \frac{1}{k!} \left(\frac{\lambda}{2}\right)^{k} + \left(\frac{1+\lambda_{pr}}{\lambda_{pr}}\right)^{u-1} \\ \times \left(e^{\frac{-\lambda}{2(1+\lambda_{pr})}} - e^{\frac{-\lambda}{2}} \sum_{k=0}^{u-2} \frac{1}{k!} \left(\frac{\lambda\lambda_{pr}}{2(1+\lambda_{pr})}\right)^{k}\right)$$
(6)

where u = TW is time-bandwidth product and an equal bandwidth is assumed for all the licensed channels. λ is the decision threshold found by the false-alarm probability, P_f as

follows.

$$P_f = \frac{\Gamma(n, \lambda/2)}{\Gamma(n)} \tag{7}$$

where $\Gamma(.)$ and $\Gamma(.,.)$ are the complete and incomplete gamma functions, respectively [15].

IV. OUTAGE PROBABILITY ANALYSIS

For a slow fading channel, an appropriate metric is the outage capacity: one can talk about a tradeoff between the outage probability and the supportable rate [16]. The outage occurs when the mutual information falls below a certain rate. For a rate R, the outage probability is defined as [16]

$$P_{out} = Pr[I < R] \tag{8}$$

The high SNR analysis of the outage probability of a relay network was developed in [13], [14]. In the sequel, we focus on this limiting analysis of the outage probability for our system model. Then, the mutual information is given by [14]:

$$I = \frac{1}{M+1} \log \left(1 + SNR |h_{sd}|^2 + SNR \sum_{r_i \in R(s)} |h_{r_id}|^2 \right)$$
(9)

where all logarithms are base 2. M is the total number of clusters. We note that $|h_{sd}|^2$ and $|h_{r_id}|^2$ are exponentially distributed with parameter λ_{sd} and λ_{r_id} , respectively. The outage probability is then given by

$$P_{out}[I < R] = \sum_{R(s)} Pr[I < R|R(s)]Pr[R(s)]$$
(10)

We consider three scenarios depending on the spectrum acquisition capability of cognitive relay nodes in each cluster.

1) **Perfect Spectrum Acquisition**: This is the idealistic scenario where we assume that the potential relay nodes always acquire the spectrum holes successfully whenever they are available. Thus, this system is equivalent to a "regular" cooperative diversity scenario [13], [14] where relays have dedicated resources. Its performance is considered for the purpose of obtaining a benchmark, i.e., an upper bound on the performance of a spectrum-sensing cognitive relay network. In this case, the potential relay nodes are only required to meet the decoding constraint given as

$$Pr[r_i \in R(s)] = exp\left(-\lambda_{sr_i}\frac{2^{(M+1)R} - 1}{SNR}\right)$$
(11)

Using the approximation technique developed in [14] as $SNR \rightarrow \infty$ we obtain

$$Pr[R(s)] = \prod_{r_i \in R(s)} exp\left(-\lambda_{sr_i} \frac{2^{(M+1)R} - 1}{SNR}\right)$$
$$\times \prod_{r_i \notin R(s)} \left(1 - exp\left(-\lambda_{sr_i} \frac{2^{(M+1)R} - 1}{SNR}\right)\right)$$
$$\sim \left(\frac{2^{(M+1)R} - 1}{SNR}\right)^{M - |R(s)|} \prod_{r_i \notin R(s)} \lambda_{sr_i}$$
(12)

where |R(s)| is the cardinality of the set, R(s). Conditioned on R(s), the outage probability is given by [14]

$$Pr[I < R|R(s)] \sim \left[\frac{2^{(M+1)R} - 1}{SNR}\right]^{|R(s)|+1} \times \lambda_{sd} \prod_{r_i \in R(s)} \lambda_{r_i d} \frac{1}{(|R(s)|+1)!}$$
(13)

Substituting (12) and (13) into (10), we get the outage probability for the perfect spectrum acquisition as

$$P_{out} \sim \left[\frac{2^{(M+1)R} - 1}{SNR}\right]^{M+1} \Gamma_P \tag{14}$$

where Γ_P is given by [14]

$$\Gamma_P = \lambda_{sd} \sum_{R(s)} \prod_{r_i \in R(s)} \lambda_{r_i d} \prod_{r_i \notin R(s)} \lambda_{sr_i} \frac{1}{(|R(s)| + 1)!}$$
(15)

where $|R(s)| \in [0, M]$. We observe that the full diversity of order M+1 is achieved. For $\lambda_{r_id} = \lambda_{rd}$ and $\lambda_{sr_i} = \lambda_{sr}$, $\forall i$, Γ_P is given by

$$\lambda_{sd} \sum_{|R(s)|=0}^{M} \begin{pmatrix} M \\ |R(s)| \end{pmatrix} (\lambda_{rd})^{|R(s)|} (\lambda_{sr})^{M-|R(s)|} \times \frac{1}{(|R(s)|+1)!}$$
(16)

2) *Imperfect Spectrum Acquisition:* A more realistic scenario is that the potential relay nodes may not always be able to acquire the spectrum hole successfully. In this case, each potential relay node is required to meet the decoding constraint and acquire the spectrum hole successfully. Then, the probability of each relay being in the relaying set is

$$Pr[r_i \in R(s)] = exp\left(-\lambda_{sr_i} \frac{2^{(M+1)R} - 1}{SNR}\right) P_d \qquad (17)$$

where we assume that each relay node acquires the spectrum hole with detection probability of P_d given in (6). The outage probability becomes

$$P_{out} = \sum_{k=0}^{M} \sum_{R(s)} Pr[I < R|R(s), K = k] \times Pr[R(s)|K = k] Pr[K = k]$$
(18)

where K is the number of successful potential relay nodes and Pr[K = k] and Pr[R(s)|K = k] are given by

$$Pr[K=k] = \begin{pmatrix} M \\ k \end{pmatrix} P_d^k (1-P_d)^{M-k}$$
(19)

$$Pr[R(s)|K=k] \sim \left(\frac{2^{(M+1)R}-1}{SNR}\right)^{k-|R(s)|} \prod_{r_i \in R(s)} P_d \prod_{r_i \notin R(s)} \lambda_{sr_i}$$
(20)

$$Pr[I < R|R(s), K = k]$$
 as $SNR \to \infty$ is given by

$$\left[\frac{2^{(M+1)R}-1}{SNR}\right]^{|R(s)|+1} \times \lambda_{sd} \prod_{r_i \in R(s)} \lambda_{r_i d} \frac{1}{(|R(s)|+1)!}$$
(21)

Substituting (19), (20), and (21) into (18), the outage probability of the imperfect spectrum acquisition is given by

$$P_{out} \sim \sum_{k=0}^{M} \left[\frac{2^{(M+1)R} - 1}{SNR} \right]^{k+1} {\binom{M}{k}} P_d^k (1 - P_d)^{M-k} \Gamma_I$$
(22)

In (22), Γ_I is given by

$$\lambda_{sd} \sum_{R(s)} \prod_{r_i \notin R(s)} \lambda_{sr_i} \prod_{r_i \in R(s)} P_d \lambda_{r_i d} \frac{1}{(|R(s)|+1)!}$$
(23)

with $|R(s)| \in [0, k]$. For $\lambda_{r_i d} = \lambda_{r d}$ and $\lambda_{sr_i} = \lambda_{sr}$, $\forall i, \Gamma_I$ is given by

$$\lambda_{sd} \sum_{|R(s)|=0}^{k} \binom{k}{|R(s)|} (\lambda_{rd})^{|R(s)|} (\lambda_{sr})^{k-|R(s)|} \times P_{d}^{|R(s)|} \frac{1}{(|R(s)|+1)!}$$
(24)

In (18), instead of averaging over all possible K, by simply taking $E[K] = MP_d$ as the number of relaying nodes, a simple approximation of the outage probability as $SNR \to \infty$ can be obtained:

$$P_{out} \sim \left[\frac{2^{(M+1)R} - 1}{SNR}\right]^{\lfloor MP_d + 1 \rfloor} \Gamma_A \tag{25}$$

where Γ_A is given by

$$\lambda_{sd} \sum_{R(s)} \prod_{r_i \notin R(s)} \lambda_{sr_i} \times \prod_{r_i \in R(s)} P_d \lambda_{r_i d} \frac{1}{(|R(s)| + 1)!}$$
(26)

with $|R(s)| \in [0, \lfloor MP_d \rfloor]$. For $\lambda_{r_i d} = \lambda_{rd}$ and $\lambda_{sr_i} = \lambda_{sr}, \forall i, \Gamma_A$ is given by

$$\lambda_{sd} \sum_{|R(s)|=0}^{\lfloor MP_d \rfloor} \begin{pmatrix} \lfloor MP_d \rfloor \\ |R(s)| \end{pmatrix} (\lambda_{rd})^{|R(s)|} (\lambda_{sr})^{\lfloor MP_d \rfloor - |R(s)|} \times P_d^{|R(s)|} \frac{1}{(|R(s)|+1)!}$$
(27)

The approximate outage probability for the imperfect scenario can serve as a upper bound of the exact outage probability and a simple method for evaluating the diversity performance of the system. We note that the imperfect spectrum acquisition scenario *does not achieve full diversity*.

3) Imperfect Spectrum Acquisition with Intra-Cluster Cooperation: We have observed that the previous scenario does not achieve full diversity due to less than perfect spectrum acquisition performance. The natural question to ask then is whether we can improve the outage performance by having neighboring nodes cooperate with the potential cognitive node by sharing spectrum sensing information. This is termed *intracluster cooperation*. We assume that all cooperating nodes employ the same energy detector and the same decision rule as described in Section III. In this paper, we will adopt the decision rule that for N cooperating nodes independently sensing the beacon, the potential relay node decides in favor of the presence of a spectrum hole if at least one of the



Fig. 2. Outage Probabilities at high SNR region: R = 1, $P_f = 10^{-2}$, $\lambda_{pr} = 10dB$, M = 4, u = 5, $\lambda_{sd} = \lambda_{sr_i} = \lambda_{r_id} = 1$

cooperating nodes detects it. The probabilities of detection and false-alarm for the intra-cluster cooperation are then given by

$$C_d = 1 - (1 - P_d)^{N+1}, \quad C_f = 1 - (1 - P_f)^{N+1}$$
 (28)

where P_d and P_f are given in (6) and (7). The outage performance as $SNR \rightarrow \infty$ is obtained by (22) with the new probabilities of detection and false-alarm. With the intracluster cooperation, as more nodes cooperate, the outage performance improves and the system gets close to obtaining full diversity.

It can be shown that given M and C_d , full diversity can be achieved if there are N cooperating nodes, in each cluster satisfying the following:

$$\Omega(M, SNR, C_d, N) \approx 1 \tag{29}$$

where $\Omega(M, SNR, C_d, N)$ is given by

$$\frac{1}{\Gamma_P} \sum_{k=0}^{M} \left[\frac{2^{(M+1)R} - 1}{SNR} \right]^{k-M} \begin{pmatrix} M \\ k \end{pmatrix} C_d^k (1 - C_d)^{M-k} \Gamma_I$$
(30)

Equation (29) serves as a design rule: we can determine the number of cooperating nodes necessary to have (30) sufficiently close to 1 and conclude that having at least that many cooperating nodes will achieve full diversity. It can easily be seen that when C_d becomes close to one, (29) is satisfied and we can find the corresponding necessary number of cooperating nodes numerically.

V. NUMERICAL RESULTS AND DISCUSSION

In this section, we present numerical results to support our analysis. Figure 2 shows the outage probability of a cognitive wireless relay network. As expected, unlike the perfect spectrum acquisition scenario, the imperfect scenario does not achieve full diversity.

Figure 3 shows the outage probabilities of all three scenarios and the impact of intra-cluster cooperation. We observe that as the total number of cooperating nodes, N, increases, the



Fig. 3. Outage Probabilities at high SNR region: R = 1, $C_f = 10^{-2}$, $\lambda_{pr} = 10dB$, M = 4, u = 5, $\lambda_{sd} = \lambda_{sr,i} = \lambda_{r,i} d = 1$



Fig. 4. The number of required cooperating nodes for full diversity: R = 1, $C_f = 10^{-2}$, $\lambda_{pr} = 10dB$, M = 4, u = 5, $\lambda_{sd} = \lambda_{sr_i} = \lambda_{r_id} = 1$, SNR = 40dB

outage performance improves. Specifically, for N = 10, the imperfect scenario achieves full diversity.

Figure 4 plots $\Omega(M, SNR, C_d, N)$ for M = 4 and SNR = 40dB. We observe that $N \ge 10$ is sufficient to achieve fulldiversity, consistent with observation in Figure 3. We also observe that increasing N well beyond 10 does not help. We note that (30) requires calculation of Γ_I and Γ_P . One can get an approximation by ignoring these terms. We observe that this approximation, also shown in Figure 4, is quite close to the exact value.

VI. CONCLUSION

In this paper, we have analyzed the outage performance of cognitive wireless relay networks where a single sourcedestination pair is assisted by a group of cognitive relay nodes employing regenerative decode and forward. We have considered three scenarios depending on how the cognitive relay nodes acquire the spectrum holes. We have shown that when the potential relay nodes always acquire the available spectrum hole successfully, full diversity is achieved. On the other hand, when the spectrum acquisition is not always guaranteed, full diversity is not achieved. Inspired by this performance deficiency, we have investigated intra-cluster cooperation, which allows the neighboring relay nodes in a cluster to collaborate with a potential relay node in acquiring the spectrum hole. The intra-cluster cooperation is shown to improve the outage performance. Full diversity is achieved if the proper number of neighboring relay nodes in each cluster participate in intra-cluster cooperation.

REFERENCES

- [1] R. W. Broderson, A. Wolisz. D. Cabric. S. M. Mishra. and D. Willkomm. CORVUS : A cognitive radio of unlisenced approach for usage virtual spectrum. http://bwrc.eecs.berkeley.edu/Research/MCMA/CRWhitepaperfinal1.pdf.
- [2] J. Mitola and G. Q. Maguire Jr. Cognitive radio: making software radios more personal. *IEEE Personal Communications*, 6(4):13 – 18, August 1999.
- [3] S. Haykin. Cognitive radio: Brain-empowered wireless communications. *IEEE Journal on Selected Areas in Communications*, 23(2):201 – 220, February 2005.
- [4] A. Sahai, N. Hoven, and R. Tandra. Some fundamental limits on cognitive radio. In *Allerton conference on Coomunications, Control,* and Computing,, 2004.
- [5] H. Urkowitz. Energy detection of unknown deterministic signals. Proceedings of the IEEE, 55(4):523 – 531, April 1967.
- [6] F. F. Digham, M. S. Alouini, and M. K. Simon. On the energy detection of unknown signals over fading channels. In *IEEE International Conference on Communications, ICC'03*, pages 3575 – 3579, May 2003.
- [7] A. Ghasemi and E. S. Sousa. Collaborative spectrum sensing for opportunistic access in fading environments. In *First IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks*, *DySPAN'05*, pages 131 – 136, November 2005.
- [8] G. Ganesan and Y. Li. Cooperative spectrum sensing in cognitive radio networks. In *First IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks, DySPAN'05*, pages 137 – 143, November 2005.
- [9] G. Ganesan and Y. Li. Agility improvement through cooperative diversity in cognitive radio. In *IEEE Global Telecommunications Conference*, *GLOBECOM*' 05, pages 2505 – 2509, 28 Nov.-2 Dec 2005.
- [10] A. Sendonaris, E. Erkip, and B. Aazhang. User cooperation diversity part I: System description; part II: Implementation, aspects and performance analysis. *IEEE Transactions on Communications*, 51(11):1927 – 1948, November 2003.
- [11] A. Høst-Madsen and J. Zhang. Capacity bounds and power allocation in wireless relay channel. *IEEE Transactions on Information Theory*, 51(6):2020 – 2040, June 2005.
- [12] Y. Jing and B. Hassibi. Cooperative diversity in wireless relay networks with multiple-antenna nodes. In *IEEE International Symposium on Information Theory, ISIT 05*', pages 815 – 819, September 2005.
- [13] J. N. Laneman, D. N. C. Tse, and G. W. Wornell. Cooperative diversity in wireless networks: Efficient protocols and outage behavior. *IEEE Transactions on Information Theory*, 50(12):3062 – 3080, December 2004.
- [14] J. N. Laneman and G. W. Wornell. Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks. *IEEE Transactions on Information Theory*, 49(10):2415 – 2425, October 2003.
- [15] D. Zwillinger. Standard Mathematical Tables and Formulae. CRC Press, 1996.
- [16] L. H. Ozarow, S. Shamai, and A. D. Wyner. Information theoretic considerations for cellular mobile radio. *IEEE Transactions on Vehicular Technology*, 33(2):359 – 378, May 1994.