On Resource Allocation for the Multi-Band Relay Channel

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Abstract — **In this paper, we investigate the information-theoretic performance of the multi-band relay (MBR) channel which is defined by a source node, a relay node, and a destination node communicating over multiple orthogonal bands. The model is motivated by the vision of hybrid wireless networks where links operating with different communication standards cooperate to relay information. Depending on how these different standards utilize bandwidth, we consider two system models. First model considers the case where the available bandwidth is equally allocated between channels. Second model considers sharing bandwidth between the different systems involved. We concentrate on the tractable case of two orthogonal bands from the source and one from the relay node. Next, we find the optimum power and bandwidth allocation strategies that maximize the lower bounds on the capacity. It is shown that sharing the bandwidth between different systems leads to higher achievable rates. It is also shown that the upper and lower bounds for each model become tighter as the link quality between source to relay improves comparing to that of source to destination.**

I. Introduction

Future wireless networks are expected to enable nodes to communicate over multiple technologies and multiple hops. Recent advances in the development of software defined radios [1] support the vision where agile radios are employed at each node that utilize multiple standards and communicate seamlessly. Indeed, an intense research effort is being directed towards having multiple communication standards coexist within one system, e.g. the cellular network and IEEE 802.11 WLAN as in [2, 3]. We refer to a group of nodes capable of employing a number of communication technologies in an effort to find the best multi-hop route between the source-destination pairs, as a *hybrid wireless network*.

In this paper, we consider a simple hybrid wireless network with a source destination pair, and aim at understanding the bounds of its information theoretic capacity with optimum resource allocation. In particular, we consider a scenario where the source node can communicate over multiple frequency bands to its destination, and a node that overhears the source transmission acts as a relay. We assume that the frequency bands that the source utilizes as well the ones used by the relay node are mutually orthogonal. The different bands are assumed to represent links that operate with different wireless communication standards.

There has been considerable research effort towards characterizing the information theoretic capacity of relay channels in the past [4], [5], which has been rejuvenated with the recent advances in multi-hop ad hoc network architectures [6–9]. Cover and El Gamal provide upper and lower bounds on the Gaussian relay channel and find the capacity of the degraded Gaussian relay channel [5]. The work of Schein and Gallager investigates upper and lower bounds on the Gaussian parallel relay channel, where two relay channels exist [7]. The study of obtaining an achievable rate region in a relay network of arbitrary size and topology has been investigated in [9].

Most of the earlier work on relay channel capacity assume that the simultaneous transmission and reception over time or frequency at the relay is possible. As this is difficult to implement, recent work considers employing orthogonality at the relay via time-division or frequency-division [10–12]. To compensate for the loss of spectral efficiency caused by this architecture and increase the capacity, optimum resource allocation has been considered in [10, 11, 13]. In [10], optimum bandwidth allocation for the Gaussian frequency-division relay channel was studied to maximize the capacity lower bound. The optimum power and time slot duration allocation for the time-division relay channel has been considered in [11].

In this paper, we investigate the capacity bounds for a simple hybrid network. For the case where the source has two bands and the relay has a single band available, we find the optimum bandwidth allocation between these three bands in an effort to determine the potential capacity benefit of nodes borrowing frequency resources from each other. We find that sharing the total available bandwidth, i.e., employing optimum bandwidth allocation between the source bands and the relay band, in addition to optimum power allocation at the source, improves the capacity significantly.

The rest of the paper is organized as follows. Section II presents system model for MBR. Section III derives upper and lower bounds for two different models and investigates the optimum resource allocation for each models. In Section IV, we present the numerical results. We conclude the paper in Section V.

II. System Model

We consider a three node relay network where multiple frequency bands available from the source and the relay are mutually orthogonal. We term this the *multi-band* relay (MBR) channel. In particular, when there are m channels available for the source node and $k-m$ for the relay node, we call this a (k, m)-MBR. The source node transmits desired information over m orthogonal channels to the relay node and destination node. The relay node decodes and re-encodes the received data to relay to the destination node. The (k, m)-MBR input-output signal model is given by

$$\begin{aligned} \tilde{\mathbf{Y}}_{\mathbf{SR}} &= \mathbf{X}_{\mathbf{S}} + \tilde{\mathbf{Z}}_{\mathbf{SR}} \\ \mathbf{Y}_{\mathbf{RD}} &= \tilde{\mathbf{X}}_{\mathbf{R}} + \mathbf{Z}_{\mathbf{RD}} \\ \mathbf{Y}_{\mathbf{SD}} &= \mathbf{X}_{\mathbf{S}} + \mathbf{Z}_{\mathbf{SD}} \end{aligned} \qquad (1)$$

where $\mathbf{X_S} = [X_1, X_2, \cdots, X_m]^{\mathrm{T}}$ and $\tilde{\mathbf{X}}_{\mathbf{R}} = [\tilde{X}_{m+1}, \tilde{X}_{m+2}, \cdots, \tilde{X}_k]^{\mathrm{T}}$ are the transmitted signal vectors from the source node and the relay node, respectively. $\mathbf{Y_{SD}} = [Y_1, Y_2, \cdots, Y_m]^{\mathrm{T}}$ and $\tilde{\mathbf{Y}}_{\mathbf{SR}} = [\tilde{Y}_1, \tilde{Y}_2, \cdots, \tilde{Y}_m]^{\mathrm{T}}$ are the received signal vectors at the destination node and the relay node when the signal is transmitted from the source node. $\mathbf{Y_{RD}} = [Y_{m+1}, Y_{m+2}, \cdots, Y_k]^{\mathrm{T}}$ is the received signal vector at the destination from the relay node. $\tilde{\mathbf{Z}}_{\mathbf{SR}} = [\tilde{Z}_1, \tilde{Z}_2, \cdots, \tilde{Z}_m]^{\mathrm{T}}$ is the zero-mean independent additive white Gaussian noise (AWGN) vector with $E[\tilde{\mathbf{Z}}_{\mathbf{SR}}\tilde{\mathbf{Z}}_{\mathbf{SR}}^{\mathrm{T}}] = \mathrm{diag}\{\tilde{N}_1/2, \tilde{N}_2/2, \cdots, \tilde{N}_m/2\}$ at the relay node. $\mathbf{Z_{SD}} = [Z_1, Z_2, \cdots, Z_m]^{\mathrm{T}}$ is the zero-mean independent AWGN vector with $E[\mathbf{Z_{SD}}\mathbf{Z_{SD}^{\mathrm{T}}}] = \mathrm{diag}\{N_1/2, N_2/2, \cdots, N_m/2\}$ at the destination node. $\mathbf{Z_{RD}} = [Z_{m+1}, Z_{m+1}, \cdots, Z_k]^{\mathrm{T}}$ is the zero-mean independent AWGN vector with $E[\mathbf{Z_{RD}}\mathbf{Z_{RD}^{\mathrm{T}}}] = \mathrm{diag}\{N_{m+1}/2, N_{m+2}/2, \cdots, N_k/2\}$ at the destination. $[\cdot]^{\mathrm{T}}$ is the transpose operation of a vector.

In the following, we present the upper and lower bounds for the capacity of (k,m)-MBR. Since the multiple channels are independent, the channel transition probability mass function is given by

$$P(y_1, y_2, \cdots, y_k, \tilde{y}_1, \tilde{y}_2, \cdots, \tilde{y}_m | x_1, x_2, \cdots, x_m, \tilde{x}_{m+1}, \cdots, \tilde{x}_k) = \prod_{i=1}^{m} P(y_i, \tilde{y}_i | x_i) \prod_{j=m+1}^{k} P(\tilde{y}_j | \tilde{x}_j) \quad (2)$$

For the channel transition probability mass function given in (2), we obtain the capacity bounds for (k,m)-MBR as follows.

Theorem 1. *The upper and lower bounds for the capacity of (k,m)-MBR are given by*

$$C_{low} = \sup_{P(\cdot)} \min\{\sum_{i=1}^{m} I(X_i; Y_i) + \sum_{i=m+1}^{k} I(\tilde{X}_i; Y_i), \sum_{i=1}^{m} I(X_i; \tilde{Y}_i)\} \quad (3)$$

$$C_{up} = \sup_{P(\cdot)} \min\{\sum_{i=1}^{m} I(X_i; Y_i) + \sum_{i=m+1}^{k} I(\tilde{X}_i; Y_i), \sum_{i=1}^{m} I(X_i; \tilde{Y}_i, Y_i)\} \quad (4)$$

where $I(X;Y)$ denotes the mutual information between X and Y and the input joint distribution $P(\cdot)$ is given by

$$P(x_1, \cdots, x_m, \tilde{x}_{m+1}, \cdots, \tilde{x}_k) = P(x_1)P(x_2)\cdots P(\tilde{x}_k) \quad (5)$$

III. Capacity Bounds and Optimum Resource Allocation

In this paper, we investigate the performance of a tractable scenario when three different communication standards coexist: the source node has access to two distinct bands and a second node that overhears the source information relays to the destination using a separate band. This case which corresponds to a $(3,2)$-MBR shown in Fig.1. We have the input-output signal model is given by (1) with $k = 3$ and $m = 2$ under power constraints

$$E[X_1]^2 \le \alpha P_s, \quad E[X_2]^2 \le (1-\alpha)P_s \quad (6)$$

where P_s is the total available transmitted power at the source node and $\alpha \in [0,1]$ is the power allocation factor. We assume that the relay node uses its available full power P_r. We do not have a total power constraint between source and relay and assume each has its own power supply.

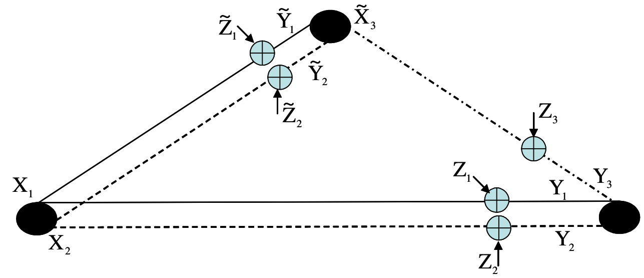

Figure 1: (3,2) Multi-Band Relay Channel

Under these assumptions, we employ resource allocation to optimize the capacity bounds for two cases: (i) the case termed No-Bandwidth-Allocation (NBA): each band has a fixed and equal width, which leaves the total source power as the only resource parameter to be optimized, and (ii) the case termed With-Bandwidth-Allocation (WBA): the bands can "borrow" bandwidth from each other, which leads to optimally allocating both the source power among the two source bands, and the total bandwidth among the three bands. The upper and lower bounds for the capacity of $(3,2)$-MBR WBA are now given as follows.

Theorem 2. *The upper and lower bounds for the capacity of $(3,2)$-MBR WBA are given by*

$$C_{low}^{WBA} = \max_{0 \le \alpha, \phi \le 1} \min\{\phi_1 \log\left(1 + \alpha_1 \frac{\gamma_{sd}^1}{\phi_1}\right) + \phi_2 \log\left(1 + \alpha_2 \frac{\gamma_{sd}^2}{\phi_2}\right) + \phi_3 \log\left(1 + \frac{\gamma_{rd}^3}{\phi_3}\right), \phi_1 \log\left(1 + \alpha_1 \frac{\gamma_{sr}^1}{\phi_1}\right) + \phi_2 \log\left(1 + \alpha_2 \frac{\gamma_{sr}^2}{\phi_2}\right)\} \quad (7)$$

$$C_{up}^{WBA} = \max_{0 \le \alpha, \phi \le 1} \min\{\phi_1 \log\left(1 + \alpha_1 \frac{\gamma_{sd}^1}{\phi_1}\right) + \phi_2 \log\left(1 + \alpha_2 \frac{\gamma_{sd}^2}{\phi_2}\right) + \phi_3 \log\left(1 + \frac{\gamma_{rd}^3}{\phi_3}\right), \phi_1 \log\left(1 + \alpha_1 \frac{\gamma_{sd}^1 + \gamma_{sr}^1}{\phi_1}\right) + \phi_2 \log\left(1 + \alpha_2 \frac{\gamma_{sd}^2 + \gamma_{sr}^2}{\phi_2}\right)\} \quad (8)$$

where all logarithms are base 2. Let $\alpha_1 = \alpha$, $\alpha_2 = 1 - \alpha$, and $\sum_{i=1}^{3} \phi_i = 1$ denote the power and bandwidth allocation parameters, respectively. Note that NBA is a special case of WBA with $\phi_1 = \phi_2 = \phi_3 = 1/3$. We assume that the system has total bandwidth W and the input and output signals are sampled every $1/2W$ seconds. We define the received SNR at the relay and destination over channel 1 and 2 as,

$$\gamma_{sr}^i \triangleq \frac{P_s}{\tilde{N}_i W}, \quad \gamma_{sd}^i \triangleq \frac{P_s}{N_i W}, \quad i = 1, 2 \quad (9)$$

and the received SNR at the destination over channel 3 as,

$$\gamma_{rd}^3 \triangleq \frac{P_r}{N_3 W} \quad (10)$$

Note that the actual received SNR values are the scaled versions of (9) and (10) depending on the power and bandwidth allocation parameters. For example, the actual received SNR at the relay from channel 1, which is allocated α_1 fraction of the source power and ϕ_1 fraction of the bandwidth, is simply $\frac{\alpha_1 \gamma_{sr}^1}{\phi_1}$.

Given the received SNRs, which are assumed to be available at the source and relay, we can maximize the capacity lower bound by optimally choosing the power and the bandwidth. The optimum power and bandwidth choices are a function of the received SNRs at the relay and destination. We identify the optimum values for different ranges of SNRs. The following notation is used throughout the rest of the paper.

- We use $C_{\text{low1}}^{\text{NBA}}$ and $C_{\text{low2}}^{\text{NBA}}$ to indicate the multi-access cut and the broadcast cut of the lower bound for NBA.
- We use $C_{\text{low1}}^{\text{WBA}}$ and $C_{\text{low2}}^{\text{WBA}}$ to indicate the multi-access cut and the broadcast cut of the lower bound for WBA.
- We denote α^*_{Clow1} and α^*_{Clow2} as the α that maximizes $C_{\text{low1}}^{\text{WBA}}$ and $C_{\text{low2}}^{\text{WBA}}$, respectively.
- We denote $\overline{\phi^*}_{\text{Clow1}}$ and $\overline{\phi^*}_{\text{Clow2}}$ as the $\Phi = (\phi_1, \phi_2, \phi_3)$ that maximizes $C_{\text{low1}}^{\text{WBA}}$ and $C_{\text{low2}}^{\text{WBA}}$, respectively.
- Let α_{int} denote the α value where $C_{\text{low1}}^{\text{NBA}}$ (or $C_{\text{low1}}^{\text{WBA}}$) and $C_{\text{low2}}^{\text{NBA}}$ (or $C_{\text{low2}}^{\text{WBA}}$) intersect.
- Let $\overline{\phi}_{\text{int}}$ denote the Φ where $C_{\text{low1}}^{\text{WBA}}$ and $C_{\text{low2}}^{\text{WBA}}$ intersect.
- $A \cap B$ represents intersection of A and B.

We now discuss how to maximize the capacity lower bound and find the associated optimum resource allocation parameters given the received SNRs. The observations follow from the investigation of the graphical behavior of $C_{\text{low1}}^{\text{WBA}}$ (or $C_{\text{low1}}^{\text{NBA}}$) and $C_{\text{low2}}^{\text{WBA}}$ (or $C_{\text{low2}}^{\text{NBA}}$) as a function of α.

Lemma 1.

$$\text{For} \quad \gamma_{sd}^i \geq \gamma_{sr}^i, \quad i = 1, 2 \tag{11}$$

we have

$$C_{low}^{NBA} = \max_{0 \leq \alpha \leq 1} C_{low2}^{NBA}, \qquad \alpha^* = \frac{1}{6}\left(\frac{1 + 3\gamma_{sr}^2}{\gamma_{sr}^2} - \frac{1}{\gamma_{sr}^1}\right) \tag{12}$$

$$C_{low}^{WBA} = \max_{0 \leq \alpha, \phi \leq 1} C_{low2}^{WBA}, \quad (\alpha^*, \overline{\phi^*}) = (\alpha^*_{Clow2}, \overline{\phi^*}_{Clow2}) \tag{13}$$

Proof. We note that for $\gamma_{sd}^i \geq \gamma_{sr}^i$ $i = 1, 2$, the multi-access cut $C_{\text{low1}}^{\text{WBA}}$ (or $C_{\text{low1}}^{\text{NBA}}$) is always larger than the broadcast cut $C_{\text{low2}}^{\text{WBA}}$ (or $C_{\text{low2}}^{\text{NBA}}$) regardless of γ_{rd}^3, α, and $\phi_i, i = 1, 2, 3$. Therefore, the max-min of (7) is $C_{\text{low2}}^{\text{WBA}}$ (or $C_{\text{low2}}^{\text{NBA}}$). The corresponding optimum power allocation factor α^* is readily found. The optimum resource allocation parameters $(\alpha^*_{\text{Clow2}}, \overline{\phi^*}_{\text{Clow2}})$ are (see Appendix for the derivation)

$$(\alpha^*_{\text{Clow2}}, \overline{\phi^*}_{\text{Clow2}}) = \begin{cases} (1, 1, 0, 0) & \text{if } \gamma_{sr}^1 > \gamma_{sr}^2 \\ (0, 0, 1, 0) & \text{if } \gamma_{sr}^1 < \gamma_{sr}^2 \end{cases} \tag{14}$$

□

Lemma 1 is easily found because $C_{\text{low1}}^{\text{WBA}}$ (or $C_{\text{low1}}^{\text{NBA}}$) is always larger than $C_{\text{low2}}^{\text{WBA}}$ (or $C_{\text{low2}}^{\text{NBA}}$) regardless of γ_{rd}^3, α, and $\phi_i, i = 1, 2, 3$. We note that Lemma 1 is the case where the relay is not useful. We allocate all resources to the channel with better relay channel because the max-min of (7) is always $C_{\text{low2}}^{\text{WBA}}$. When (11) is not satisfied, the max-min of (7) is more involved and requires us to investigate the relationship between all dependent variables.

Lemma 2. *For given γ_{sd}, γ_{sr}, γ_{rd}, and $\phi_i, i = 1, 2, 3$ satisfying the following conditions:*

$$\phi_1 \log\left(1 + \frac{\gamma_{sd}^1}{\phi_1}\right) + \phi_3 \log\left(1 + \frac{\gamma_{rd}^3}{\phi_3}\right) < \phi_1 \log\left(1 + \frac{\gamma_{sr}^1}{\phi_1}\right) \tag{15}$$

$$\phi_2 \log\left(1 + \frac{\gamma_{sd}^2}{\phi_1}\right) + \phi_3 \log\left(1 + \frac{\gamma_{rd}^3}{\phi_3}\right) > \phi_2 \log\left(1 + \frac{\gamma_{sr}^2}{\phi_2}\right) \tag{16}$$

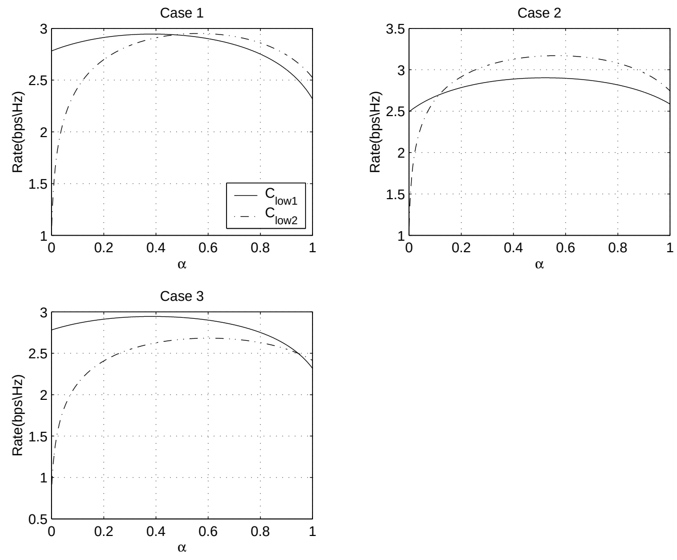

Figure 2: Cases of Lemma 2

The maximum lower bound and its corresponding optimum resource allocation parameters are given by

Case 1: $\alpha^*_{Clow1} < \alpha_{int} < \alpha^*_{Clow2}$

$$C_{low}^{NBA} = C_{low1}^{NBA} \cap C_{low2}^{NBA}, \qquad \alpha^* = \alpha_{int} \tag{17}$$

$$C_{low}^{WBA} = \max\left(C_{low1}^{WBA} \cap C_{low2}^{WBA}\right), \quad (\alpha^*, \overline{\phi^*}) = (\alpha_{int}, \overline{\phi}_{int}) \tag{18}$$

Case 2: $\alpha_{int} < \alpha^*_{Clow1}, \ \alpha^*_{Clow2}$

$$C_{low}^{NBA} = \max_{0 \leq \alpha \leq 1} C_{low1}^{NBA}, \quad \alpha^* = \frac{1}{6}\left(\frac{1 + 3\gamma_{sd}^2}{\gamma_{sd}^2} - \frac{1}{\gamma_{sd}^1}\right) \tag{19}$$

$$C_{low}^{WBA} = \max_{0 \leq \alpha, \phi \leq 1} C_{low1}^{WBA}, \quad (\alpha^*, \overline{\phi^*}) = (\alpha^*_{Clow1}, \overline{\phi^*}_{Clow1}) \tag{20}$$

Case 3: $\alpha_{int} > \alpha^*_{Clow1}, \ \alpha^*_{Clow2}$

$$C_{low}^{NBA} = \max_{0 \leq \alpha \leq 1} C_{low2}^{NBA}, \quad \alpha^* = \frac{1}{6}\left(\frac{1 + 3\gamma_{sr}^2}{\gamma_{sr}^2} - \frac{1}{\gamma_{sr}^1}\right) \tag{21}$$

$$C_{low}^{WBA} = \max_{0 \leq \alpha, \phi \leq 1} C_{low2}^{WBA}, \quad (\alpha^*, \overline{\phi^*}) = (\alpha^*_{Clow2}, \overline{\phi^*}_{Clow2}) \tag{22}$$

Proof. We first provide the proof for the NBA case. We note that (15) indicates that $C_{\text{low2}}^{\text{NBA}}$ is larger than $C_{\text{low1}}^{\text{NBA}}$ for $\alpha = 1$. Also note that (16) means that $C_{\text{low2}}^{\text{NBA}}$ is smaller than $C_{\text{low1}}^{\text{NBA}}$ for $\alpha = 0$. We note that (15) and (16) satisfy the following condition of the received SNRs for NBA.

$$\gamma_{sr}^1 \geq \gamma_{rd}^3 + \gamma_{sd}^1 + 3\gamma_{sd}^1\gamma_{rd}^3 \tag{23}$$

$$\gamma_{sr}^2 \leq \gamma_{rd}^3 + \gamma_{sd}^2 + 3\gamma_{sd}^2\gamma_{rd}^3, \tag{24}$$

Since it is readily shown that $C_{\text{low1}}^{\text{NBA}}$ and $C_{\text{low2}}^{\text{NBA}}$ are strictly concave functions in $\alpha \in [0, 1]$, we know that $C_{\text{low1}}^{\text{NBA}}$ and $C_{\text{low2}}^{\text{NBA}}$ must intersect at α_{int}. There are three different possible cases for how they intersect as shown in Fig.2. For case 1, the max-min of (7) can not be one of the maximum of $C_{\text{low1}}^{\text{NBA}}$ or $C_{\text{low2}}^{\text{NBA}}$ and must be the intersection value at $\alpha^* = \alpha_{\text{int}}$. For case 2, the max-min of (7) is $C_{\text{low1}}^{\text{NBA}}$. Case 3 is the opposite of case 2 and the max-min of (7) is $C_{\text{low2}}^{\text{NBA}}$.

The proof for the WBA is similar to that of NBA. It is also readily shown that for a fixed Φ, $C_{\text{low1}}^{\text{WBA}}$ and $C_{\text{low2}}^{\text{WBA}}$ are strictly concave functions in $\alpha \in [0, 1]$. Therefore, we have the same three possible cases as NBA, depending on the intersections. The difference is that there is a set of feasible Φ values satisfying (15) and (16). Therefore, for case 1, the maximum lower bound is the maximum value of possible intersections which leads to the corresponding optimum resource allocation parameters. The optimum resource allocation parameters $(\alpha^*_{\text{Clow1}}, \overline{\phi^*}_{\text{Clow1}})$ are

$$(\alpha^*_{\text{Clow1}}, \overline{\phi^*}_{\text{Clow1}}) =$$

$$\begin{cases} \left(1, 1-\frac{\gamma_{rd}^3}{\gamma_{rd}^3+\gamma_{sd}^1}, 0, \frac{\gamma_{rd}^3}{\gamma_{rd}^3+\gamma_{sd}^1}\right) & \text{if } \gamma_{sd}^1 > \gamma_{sd}^2 \\ \left(0, 0, 1-\frac{\gamma_{rd}^3}{\gamma_{rd}^3+\gamma_{sd}^2}, \frac{\gamma_{rd}^3}{\gamma_{rd}^3+\gamma_{sd}^2}\right) & \text{if } \gamma_{sd}^1 < \gamma_{sd}^2 \end{cases} \quad (26)$$

□

The steps for solving for (25) are similar to those in the Appendix.

Lemma 3. *For given γ_{sd}, γ_{sr}, γ_{rd}, and $\phi_i, i = 1,2,3$ satisfying the following conditions:*

$$\phi_1 \log\left(1+\frac{\gamma_{sd}^1}{\phi_1}\right) + \phi_3 \log\left(1+\frac{\gamma_{rd}^3}{\phi_3}\right) > \phi_1 \log\left(1+\frac{\gamma_{sr}^1}{\phi_1}\right) \quad (27)$$

$$\phi_2 \log\left(1+\frac{\gamma_{sd}^2}{\phi_1}\right) + \phi_3 \log\left(1+\frac{\gamma_{rd}^3}{\phi_3}\right) < \phi_2 \log\left(1+\frac{\gamma_{sr}^2}{\phi_2}\right) \quad (28)$$

The maximum lower bound and its corresponding optimum resource allocation parameters are given by

Case 1: $\alpha^*_{Clow1} < \alpha_{int} < \alpha^*_{Clow2}$

$$C^{NBA}_{low} = C^{NBA}_{low1} \cap C^{NBA}_{low2}, \qquad \alpha^* = \alpha_{int} \quad (29)$$

$$C^{WBA}_{low} = max\left(C^{WBA}_{low1} \cap C^{WBA}_{low2}\right), \ (\alpha^*, \overline{\phi^*}) = (\alpha_{int}, \overline{\phi}_{int}) \quad (30)$$

Case 2: $\alpha_{int} < \alpha^*_{Clow1}, \ \alpha^*_{Clow2}$

$$C^{NBA}_{low} = \max_{0\leq\alpha\leq1} C^{NBA}_{low2}, \qquad \alpha^* = \frac{1}{6}\left(\frac{1+3\gamma_{sr}^2}{\gamma_{sr}^2} - \frac{1}{\gamma_{sr}^1}\right) \quad (31)$$

$$C^{WBA}_{low} = \max_{0\leq\alpha,\phi\leq1} C^{WBA}_{low2}, \quad (\alpha^*, \overline{\phi^*}) = (\alpha^*_{Clow2}, \overline{\phi^*}_{Clow2}) \quad (32)$$

Case 3: $\alpha_{int} > \alpha^*_{Clow1}, \ \alpha^*_{Clow2}$

$$C^{NBA}_{low} = \max_{0\leq\alpha\leq1} C^{NBA}_{low1}, \qquad \alpha^* = \frac{1}{6}\left(\frac{1+3\gamma_{sd}^2}{\gamma_{sd}^2} - \frac{1}{\gamma_{sd}^1}\right) \quad (33)$$

$$C^{WBA}_{low} = \max_{0\leq\alpha,\phi\leq1} C^{WBA}_{low1}, \quad (\alpha^*, \overline{\phi^*}) = (\alpha^*_{Clow1}, \overline{\phi^*}_{Clow1}) \quad (34)$$

Proof. We note that (27) indicates that $C^{\text{NBA}}_{\text{low2}}$ is smaller than $C^{\text{NBA}}_{\text{low1}}$ for $\alpha = 1$. Also note that (28) means that $C^{\text{NBA}}_{\text{low2}}$ is larger than $C^{\text{NBA}}_{\text{low1}}$ for $\alpha = 0$. This indicates that Lemma 3 is the opposite of Lemma 2 by switching the role of $C^{\text{NBA}}_{\text{low1}}$ and $C^{\text{NBA}}_{\text{low2}}$ for every cases of Fig.2. Therefore, the proof of Lemma 3 is analogous with the proof of Lemma 2. In addition, (27) and (28) satisfy the following condition of the received SNRs for NBA.

$$\gamma_{sr}^1 \leq \gamma_{rd}^3 + \gamma_{sd}^1 + 3\gamma_{sd}^1\gamma_{rd}^3 \quad (35)$$

$$\gamma_{sr}^2 \geq \gamma_{rd}^3 + \gamma_{sd}^2 + 3\gamma_{sd}^2\gamma_{rd}^3, \quad (36)$$

□

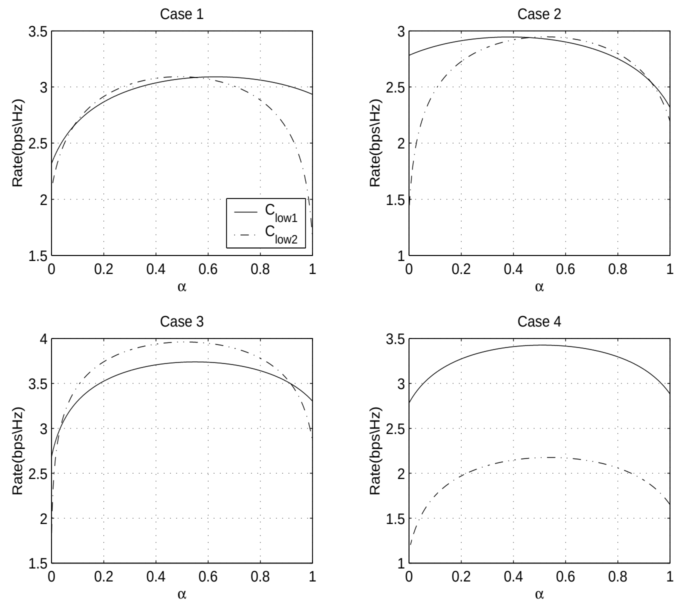

Figure 3: Cases of Lemma 4

Lemma 4. *For given γ_{sd}, γ_{sr}, γ_{rd}, and $\phi_i, i = 1,2,3$ satisfying the following conditions:*

$$\phi_1 \log\left(1+\frac{\gamma_{sd}^1}{\phi_1}\right) + \phi_3 \log\left(1+\frac{\gamma_{rd}^3}{\phi_3}\right) > \phi_1 \log\left(1+\frac{\gamma_{sr}^1}{\phi_1}\right) \quad (37)$$

$$\phi_2 \log\left(1+\frac{\gamma_{sd}^2}{\phi_1}\right) + \phi_3 \log\left(1+\frac{\gamma_{rd}^3}{\phi_3}\right) > \phi_2 \log\left(1+\frac{\gamma_{sr}^2}{\phi_2}\right) \quad (38)$$

The maximum lower bound and its corresponding optimum resource allocation parameters are given by

Case 1: $\alpha^*_{Clow2} < \alpha_{int1} < \alpha^*_{Clow1}$

$$C^{NBA}_{low} = C^{NBA}_{low1} \cap C^{NBA}_{low2}, \qquad \alpha^* = \alpha_{int1} \quad (39)$$

$$C^{WBA}_{low} = max\left(C^{WBA}_{low1} \cap C^{WBA}_{low2}\right), \ (\alpha^*, \overline{\phi^*}) = (\alpha_{int1}, \overline{\phi}_{int}) \quad (40)$$

Case 2: $\alpha^*_{Clow1} < \alpha_{int2} < \alpha^*_{Clow2}$

$$C^{NBA}_{low} = C^{NBA}_{low2} \cap C^{NBA}_{low2}, \qquad \alpha^* = \alpha_{int} \quad (41)$$

$$C^{WBA}_{low} = max\left(C^{WBA}_{low1} \cap C^{WBA}_{low2}\right), \ (\alpha^*, \overline{\phi^*}) = (\alpha_{int2}, \overline{\phi}_{int}) \quad (42)$$

Case 3: $\alpha_{int1} > \alpha^*_{Clow1}, \ \alpha^*_{Clow2}$ *and* $\alpha_{int2} < \alpha^*_{Clow1}, \ \alpha^*_{Clow2}$

$$C^{NBA}_{low} = \max_{0\leq\alpha\leq1} C^{NBA}_{low1}, \qquad \alpha^* = \frac{1}{6}\left(\frac{1+3\gamma_{sd}^2}{\gamma_{sd}^2} - \frac{1}{\gamma_{sd}^1}\right) \quad (43)$$

$$C^{WBA}_{low} = \max_{0\leq\alpha,\phi\leq1} C^{WBA}_{low1}, \quad (\alpha^*, \overline{\phi^*}) = (\alpha^*_{Clow1}, \overline{\phi^*}_{Clow1}) \quad (44)$$

Case 4: $\alpha_{int} = NULL$

$$C^{NBA}_{low} = \max_{0\leq\alpha\leq1} C^{NBA}_{low2}, \qquad \alpha^* = \frac{1}{6}\left(\frac{1+3\gamma_{sr}^2}{\gamma_{sr}^2} - \frac{1}{\gamma_{sr}^1}\right) \quad (45)$$

$$C^{WBA}_{low} = \max_{0\leq\alpha,\phi\leq1} C^{WBA}_{low2}, \quad (\alpha^*, \overline{\phi^*}) = (\alpha^*_{Clow2}, \overline{\phi^*}_{Clow2}) \quad (46)$$

Proof. We note that (37) and (38) indicate that $C^{\text{NBA}}_{\text{low2}}$ is smaller than $C^{\text{NBA}}_{\text{low1}}$ for $\alpha = 0$ and 1. Thus, there must exist two intersections at α_{int1} and α_{int2} ($\alpha_{\text{int1}} > \alpha_{\text{int2}}$). Otherwise, there is no intersection at all. When there is no intersection that corresponds to case 4 of Fig.3, it is shown that $C^{\text{NBA}}_{\text{low1}}$ is always larger than $C^{\text{NBA}}_{\text{low2}}$ over $\alpha \in [0,1]$. Therefore, the max-min of (7) is $C_{\text{low2}}^{\text{NBA}}$. When they intersect, there are three different possible cases as shown in Fig.3.

For case 1, we observe that the intersection value at α_{int1} is smaller than the maximum of $C_{\text{low1}}^{\text{NBA}}$ or $C_{\text{low2}}^{\text{NBA}}$ (see case 1 of Fig.3). Thus, the max-min of (7) must be the intersection value at $\alpha^* = \alpha_{\text{int1}}$. For case 2, we observe that the intersection value at α_{int2} is smaller than the maximum of $C_{\text{low1}}^{\text{NBA}}$ and $C_{\text{low2}}^{\text{NBA}}$. Thus, max-min of (7) must be the intersection value at $\alpha^* = \alpha_{\text{int2}}$. For case 3, the maximum value of $C_{\text{low1}}^{\text{NBA}}$ and $C_{\text{low2}}^{\text{NBA}}$ are larger than two intersection values. Therefore, the max-min of (7) is $C_{\text{low1}}^{\text{NBA}}$. We note that (37) and (38) satisfy the following condition of the received SNRs for NBA.

$$\gamma_{sr}^1 \le \gamma_{rd}^3 + \gamma_{sd}^1 + 3\gamma_{sd}^1\gamma_{rd}^3 \tag{47}$$
$$\gamma_{sr}^2 \le \gamma_{rd}^3 + \gamma_{sd}^2 + 3\gamma_{sd}^2\gamma_{rd}^3, \tag{48}$$

The proof for the WBA is similar to that of NBA because of the strict concavity of $C_{\text{low1}}^{\text{WBA}}$ and $C_{\text{low2}}^{\text{WBA}}$ in $\alpha \in [0,1]$ for a fixed Φ. Therefore, we also have the same three possible cases as NBA depending the intersections. The difference is that there is a set of feasible $\phi_i, i = 1,2,3$ satisfying (37) and (38). Therefore, for case 1 and case 2, the maximum lower bound is the maximum value of possible intersections which leads to the corresponding optimum resource allocation parameters. □

Lemma 5. *For given γ_{sd}, γ_{sr}, γ_{rd}, and $\phi_i, i = 1,2,3$ satisfying the following conditions:*

$$\phi_1 \log\left(1 + \frac{\gamma_{sd}^1}{\phi_1}\right) + \phi_3 \log\left(1 + \frac{\gamma_{rd}^3}{\phi_3}\right) < \phi_1 \log\left(1 + \frac{\gamma_{sr}^1}{\phi_1}\right) \tag{49}$$
$$\phi_2 \log\left(1 + \frac{\gamma_{sd}^2}{\phi_1}\right) + \phi_3 \log\left(1 + \frac{\gamma_{rd}^3}{\phi_3}\right) < \phi_2 \log\left(1 + \frac{\gamma_{sr}^2}{\phi_2}\right) \tag{50}$$

The maximum lower bound and its corresponding optimum resource allocation parameters are given by

Case 1: $\alpha_{Clow1}^* < \alpha_{int1} < \alpha_{Clow2}^*$

$$C_{low}^{NBA} = C_{low1}^{NBA} \cap C_{low2}^{NBA}, \qquad \alpha^* = \alpha_{int1} \tag{51}$$
$$C_{low}^{WBA} = max\left(C_{low1}^{WBA} \cap C_{low2}^{WBA}\right), \ (\alpha^*, \overline{\phi^*}) = (\alpha_{int1}, \overline{\phi}_{int}) \tag{52}$$

Case 2: $\alpha_{Clow2}^* < \alpha_{int2} < \alpha_{Clow1}^*$

$$C_{low}^{NBA} = C_{low1}^{NBA} \cap C_{low2}^{NBA}, \qquad \alpha^* = \alpha_{int2} \tag{53}$$
$$C_{low}^{WBA} = max\left(C_{low1}^{WBA} \cap C_{low2}^{WBA}\right), \ (\alpha^*, \overline{\phi^*}) = (\alpha_{int2}, \overline{\phi}_{int}) \tag{54}$$

Case 3: $\alpha_{int1} > \alpha_{Clow1}^*, \ \alpha_{Clow2}^*$ *and* $\alpha_{int2} < \alpha_{Clow1}^*, \ \alpha_{Clow2}^*$

$$C_{low}^{NBA} = \max_{0\le\alpha\le1} C_{low2}^{NBA}, \qquad \alpha^* = \frac{1}{6}\left(\frac{1+3\gamma_{sr}^2}{\gamma_{sr}^2} - \frac{1}{\gamma_{sr}^1}\right) \tag{55}$$
$$C_{low}^{WBA} = \max_{0\le\alpha,\phi\le1} C_{low2}^{WBA}, \quad (\alpha^*, \overline{\phi^*}) = (\alpha_{Clow2}^*, \overline{\phi^*}_{Clow2}) \tag{56}$$

Case 4: $\alpha_{int} = NULL$

$$C_{low}^{NBA} = \max_{0\le\alpha\le1} C_{low1}^{NBA}, \qquad \alpha^* = \frac{1}{6}\left(\frac{1+3\gamma_{sd}^2}{\gamma_{sd}^2} - \frac{1}{\gamma_{sd}^1}\right) \tag{57}$$
$$C_{low}^{WBA} = \max_{0\le\alpha,\phi\le1} C_{low1}^{WBA}, \quad (\alpha^*, \overline{\phi^*}) = (\alpha_{Clow1}^*, \overline{\phi^*}_{Clow1}) \tag{58}$$

Proof. We note that the inequalities (49) and (50) indicate that $C_{\text{low2}}^{\text{NBA}}$ is larger than $C_{\text{low1}}^{\text{NBA}}$ for $\alpha = 0$ and 1. This indicates that Lemma 5 is the opposite of Lemma 4 by switching the role of $C_{\text{low1}}^{\text{NBA}}$ and $C_{\text{low2}}^{\text{NBA}}$ for every cases in Fig.3. Therefore, the proof of Lemma 5 is identical to the proof of Lemma 4. In addition, we note that (49) and (50) satisfy the following condition of the received SNRs for NBA. □

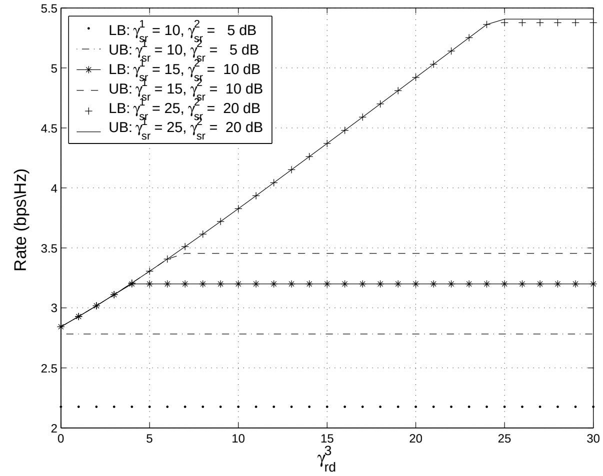

Figure 4: Upper and Lower bounds of NBA with $\gamma_{sd}^1 =$ 10dB and $\gamma_{sd}^2 = 5$dB

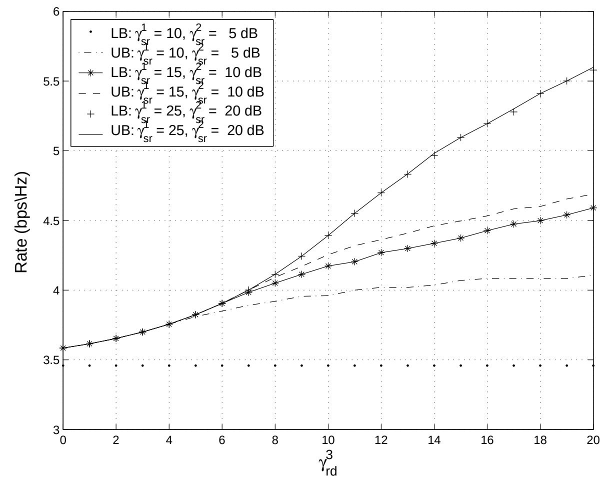

Figure 5: Upper and Lower bounds of WBA with $\gamma_{sd}^1 =$ 10dB and $\gamma_{sd}^2 = 5$dB

$$\gamma_{sr}^1 \ge \gamma_{rd}^3 + \gamma_{sd}^1 + 3\gamma_{sd}^1\gamma_{rd}^3 \tag{59}$$
$$\gamma_{sr}^2 \ge \gamma_{rd}^3 + \gamma_{sd}^2 + 3\gamma_{sd}^2\gamma_{rd}^3, \tag{60}$$

IV. Numerical Results

In this section, we present numerical results to support our analysis. Specifically, we plot the capacity lower bounds obtained by optimum resource allocation parameters found for each case, as well as the capacity upper bounds with the same resource allocation parameters.

Fig.4 plots upper bound (UB) and lower bound (LB) of NBA with optimum power allocation for different values of γ_{sr}^1 and γ_{sr}^2. When γ_{sr}^1 and γ_{sr}^2 are smaller than or equal to γ_{sd}^1 and γ_{sd}^2 respectively, the lower bound does not increase and saturates immediately even if γ_{rd}^3 increases. This is expected, since using the relay is not beneficial when the source to relay channel is worse than the source to destination channel. In this case, using the direct link between the source and the destination yields a higher rate.

In contrast, when γ_{sr}^1 and γ_{sr}^2 are larger than γ_{sd}^1 and γ_{sd}^2 respectively, the lower bound increases as γ_{rd}^3 increases and saturates after a certain threshold of γ_{rd}^3. This threshold γ_{rd}^3 becomes larger as the quality of the source to relay links improve as compared to source to destination links, i.e., as γ_{sr}^1 and γ_{sr}^2 gets larger compared to γ_{sd}^1 and γ_{sd}^2. Indeed, the fact that we can achieve higher rates when the source to relay channel is better than the source to destination channel is intuitively pleasing as the power allocation becomes more effective when we have a better source to relay channel. It is noticeable that the upper and lower bounds approach each other as the source to relay link quality improves as compared to that of source to destination.

Fig.5 shows the upper and lower bounds for WBA with jointly optimum power and bandwidth allocation. We observe that the lower bound does not saturate when the source to relay links are better than the source to destination links. This additional improvement is achieved by the bandwidth allocation. By comparing Fig.4 and Fig.5, we observe that the achievable rate of WBA is always larger than NBA, sometimes by a significant margin. This indicates that the combined power and bandwidth optimization yields to better performance than power optimization only, hence promoting the idea of different wireless technologies lending each other frequency resources to improve the capacity.

V. Conclusions

In this paper, we investigated the information-theoretic performance of a simple hybrid wireless network where the source, with the help of a relay node, communicates to the destination via multiple orthogonal channels (MBR). We considered two scenarios: one in which the source power is shared between multiple channels available from the source (NBA), and one in which the total bandwidth is dynamically allocated between the multiple channels from the source and the relay in addition to the source power (WBA). In particular, we derived the optimum power and bandwidth allocation parameters in order to maximize the capacity lower bound. For both scenarios, we observed that the upper and lower bounds approach each other as the source to relay channel condition improves as compared to the source to destination channel condition. We observed that joint power and bandwidth optimization always yields better performance than power optimization only.

The simple MBR investigated in this paper can be considered as a building block for more complex hybrid wireless networks. From the system design point of view, we conclude that, for this simple network, higher achievable rates can be obtained by optimally allocating resources between multiple standards. It is of interest to determine a more general set of conditions under which using multiple communication links and optimum sharing of resources would be beneficial for the capacity of the hybrid wireless network.

VI. Appendix

We provide the derivation of (14). We start with the following objective function

$$J(\alpha,\phi) = \phi \log\left(1+\alpha\frac{\gamma_{sr}^1}{\phi}\right) + (1-\phi)\log\left(1+(1-\alpha)\frac{\gamma_{sr}^2}{1-\phi}\right) \tag{61}$$

Differentiating (61) with respect to ϕ and equating it to 0 yields the following equation for ϕ:

$$\phi = \frac{\alpha\gamma_{sr}^1}{\alpha\gamma_{sr}^1 + (1-\alpha)\gamma_{sr}^2} \tag{62}$$

Using (62), we obtain the new cost function from (61) as follows

$$J(\alpha,\phi) = \log\left(1+\alpha\frac{\gamma_{sr}^1}{\phi}\right) \tag{63}$$

Substituting (62) into (63), we obtain

$$J(\alpha) = \log\left(1+\alpha\gamma_{sr}^1 + (1-\alpha)\gamma_{sr}^2\right) \tag{64}$$

We note that (64) is maximized by the following optimum power allocation

$$\alpha^*_{\text{Clow2}} = \begin{cases} 1 & \text{if } \gamma_{sr}^1 > \gamma_{sr}^2, \\ 0 & \text{if } \gamma_{sr}^1 < \gamma_{sr}^2. \end{cases} \tag{65}$$

The optimum bandwidth allocation is obtained from (62) and (65).

$$\phi^* = \begin{cases} 1 & \text{if } \gamma_{sr}^1 > \gamma_{sr}^2, \\ 0 & \text{if } \gamma_{sr}^1 < \gamma_{sr}^2. \end{cases} \tag{66}$$

With the variable change ($\phi_1^* = \phi^*$ and $\phi_2^* = 1-\phi^*$), we obtain (14).

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