On the Energy-Delay Trade-off of a Two-Way Relay Network

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***Abstract*—We consider a three node network in which a pair of nodes with stochastic arrivals communicate with each other with the help of an intermediate relay. The bi-directional nature of the traffic, in this setting, poses a new energy delay trade-off. Namely, the relay node may choose to cache packets from one direction and send it only after packets from the other direction arrive, using an XOR network coding scheme. Doing so would save energy, but would also incur some delay for the packet. In this work, we analyze this trade-off when the relay node queues packets from each direction and uses a first-come-first-serve policy. We show that under an even traffic load where one would hope for the most energy savings, to achieve the minimum energy expenditure promised by the XOR network coding scheme, the average delay has to go to ∞.**

Keywords: Two-way relay, queuing, energy-delay trade-off

I. Introduction

The two-way relay network considered in this work consists of two nodes who wish to communicate with each other via a relay node. Significant recent effort has been dedicated to this model, in particular, to understanding its information-theoretic performance. The capacity region under two-phase/multiple-phase protocols and decode-and-forward relaying is presented in [1], [2]. Reference [2] also computes these for Gaussian fading channels. Achievable rates for the two-way relay network are given in [3]. Practical relay coding schemes are proposed in [4], [5] and power/rate optimization problems are presented in [6]. The model is extended to multiple relay nodes, and the capacity scaling law when the relay sum power is fixed and number of relays goes to infinity is considered in [7].

There are two implicit assumptions behind all these works: (i) Traffic arriving at the two source nodes are modeled as deterministic flows with constant rates. (ii) To obtain the information theoretic achievable rates, the code word length is assumed to be infinity. While these assumptions are widely used and are essential to simplify the analysis, there are certain scenarios under which they may not hold. A likely scenario is that the packets can arrive at the transmitters in a stochastic fashion. Hence, one can no longer guarantee that at a certain time instant, the relay would receive data from both directions. In this case, a relevant issue is that of the characterization of the stability region [8]–[10] under assumption (ii). In addition, commonly envisioned applications such as voice and video are delay sensitive. Therefore, the delay that the received data incurs at the relay node must be limited, rendering it impossible for the relay to code over long blocks.

To address the first problem, a natural solution is to allow the relay to cache the traffic in one direction and transmit only after traffic from the other direction arrives, and employ network coding [4]. Doing this is advantageous, since combining two (or more) transmissions into one will allow the relay to save energy. However, we immediately see that doing this will increase the delay of the traffic since we must wait for the packets arriving from the “more idle” direction. Therefore, there is a trade-off between energy consumption at the relay and the end-to-end communication delay.

Energy-delay trade-off has been considered previously for a number of different wireless communication scenarios. From the physical layer perspective, reference [11] considers the average energy consumption when channel coding may be done over a limited number of independent channel fading states for a multiple access channel. Reference [12] explores this for a relay network. From a medium access layer perspective as well, it is widely accepted to “wait” for good channel states and save energy at the cost of delay. This trade-off is considered in [13] and the lower bound of energy consumption when delay goes ∞ is given and shown to be tight. This result is extended to a multi-user scenario in [14]. We note that both results are restricted to single hop communication. The energy delay trade-off therein is a result of the concave relationship between the transmission power and the service rate. By comparison, the energy delay trade-off in two-way relay network considered here results from the lack of coordination of traffic demands at the two source nodes.

In this work, we investigate the energy delay trade-off in a bi-directional relay network when the relay node uses a first-come-first-serve policy. The details of the system model and the relevant assumptions are presented in Section II. In Section III, we analyze the stationary behavior of the queues at the relay node. The average packet delay and average energy consumption are related via the queue capacity at relay node. Our analysis shows that, in the case where we have a symmetric traffic load coming in from both directions, where one would expect to harvest the energy savings of the XOR combining the most, to achieve the minimum energy, the end-to-end delay goes to infinity. This is equivalent to saying the

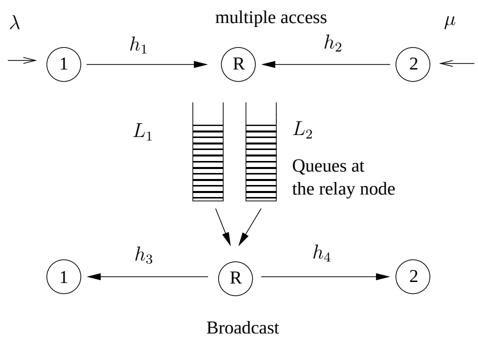

Fig. 1. System Model

queues become unstable and leads to the conclusion that the network coding protocol must be used in conjunction with other protocols in order to ensure the stability of the queues.

II. System Model and Assumptions

We consider a packet-based communication system, depicted in Figure 1. The channel is assumed to be static, and the channel state information is assumed to be globally known. We assume the packets arrive at node 1 and node 2 from two independent Poisson processes with parameters λ and μ respectively. The added redundancy and the energy of a packet are chosen in such a way that each packet can be decoded reliably at the relay node, even if packets arrive at the relay node at the same time. Under this assumption, the traffic flow at the relay node from node 1 and node 2 are also Poisson processes with the same parameters.

At the relay, the decoded traffic flows from node 1 and node 2 are stored in two queues, with buffer capacity L_1 and L_2 respectively. The policy employed by the relay to process the packets from the queues is described below.

1) When a packet arrives, if the queue holding the traffic from the opposite direction is not empty, the relay sends out the coded version of the binary sum of this packet and the packet in the queue from the opposite direction immediately. For the XOR network coding to work, we assume each packet contains same number of information bits. Again, we assume the redundancy and the energy of each packet are chosen such that they can be successfully decoded at both node 1 and node 2.
2) When a packet arrives, if the queue holding the traffic from the opposite direction is empty, then the relay stores the packet in the queue that contains packets for the arrival direction. If this queue is full, the relay sends out the oldest packet in the queue immediately and makes room to store this latest packet. Again, we assume the redundancy and energy of each packet are chosen such that it can be successfully decoded at its intended destination.

In both cases, we assume the packet size is small enough such that the transmission delay is negligible. In other words, the transmissions can be modeled as points on the time axis. Therefore, the average delay experienced by the packets equals the average amount of time they spend in the queue at the relay. In the next section, we will explore the relationship of the average delay with the average transmission power of the relay.

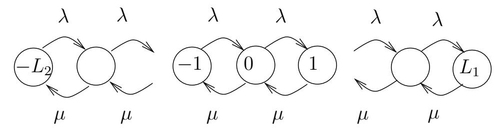

Fig. 2. State Transition Diagram

Remark 1: We assume that the traffic generated from the two ends are independent. This assumption may not hold in a scenario where the traffic of node 2 is triggered by the response from node 1. The assumption on the other hand is appropriate when the two end nodes are simply routers in an ad hoc network forwarding data to each other.

Remark 2: We assume the queues at the relay node have finite capacity. This is a seemingly different starting point as compared to previous work that assumes infinite buffer size [8]–[10]. The connection between these works will become apparent when we let the capacity of the queue, i.e., the buffer size, go to ∞ in order to minimize the energy consumption.

III. Energy Delay Trade-off

A. Queuing Model and Stationary Distribution

Based on the transmission policy at the relay described in the previous section, we remark that at most one queue at the relay node can be non-empty. Therefore, the state of queues at the relay node can be characterized with a single number $S \in [-L_2, L_1]$. Since the future state is independent from its past given the current state, S is a continuous time finite state Markov chain.

Recall that the traffic flows arriving at the relay node are two independent Poisson processes with parameters λ and μ respectively. Therefore, for a given time interval Δt, we have the transition probabilities:

$$P\left(S\left(t+\Delta t\right)=a+1|S\left(t\right)=a\right)=\lambda\Delta t+o\left(\Delta t\right), \quad a\in\left[-L_2,L_1-1\right] \tag{1}$$

$$P\left(S\left(t+\Delta t\right)=a-1|S\left(t\right)=a\right)=\mu\Delta t+o\left(\Delta t\right), \quad a\in\left[-L_2+1,L_1\right] \tag{2}$$

The resulting continuous time Markov chain is shown in Figure 2. We note that Figure 2 is tantamount to a $M/M/1/(L_1+L_2+1)$ system [15]. Therefore, they have the same stationary distribution, given by (3) below, where $u=\lambda/\mu$.

$$P\left(S=i\right)=\begin{cases}\frac{(1-u)u^{i+L_2}}{1-u^{(L_1+L_2+1)}}, & u\neq 1\\ \frac{1}{L_1+L_2+1}, & u=1\end{cases} \quad i=-L_2...L_1 \tag{3}$$

B. Average Power

The average power expended by the relay node is defined as:

$$\begin{aligned} E[P] &= \lim_{\Delta t\to 0} E\left[\frac{\Delta\varepsilon}{\Delta t}\right] \\ &= \lim_{\Delta t\to 0}\sum_{i,j}\frac{P(S(t)=i)\,P(S(t+\Delta t)=j|S(t)=i)}{\Delta t}\varepsilon_{ij} \end{aligned} \tag{4}$$

Here ε_{ij} is the energy cost associated with state transition $i \to j$. From the model described in the previous section, we know that ε_{ij} is decided by the coding scheme used within each packet and the channel conditions.

Let ε_a be the energy cost to successfully multi-cast the XOR-ed packet to both node 1 and node 2. ε_b is the energy cost to successfully send the packet from the relay to node 2. Similarly, ε_c is the energy cost to send the packet from relay to node 1. How $\varepsilon_a, \varepsilon_b, \varepsilon_c$ are computed is not relevant to our discussion. In practice, they may be approximated by the following equations:

$$\varepsilon_b \sim N_1\left(2^{R_1}-1\right)/h_4 \tag{5}$$

$$\varepsilon_c \sim N_2\left(2^{R_2}-1\right)/h_3 \tag{6}$$

$$\varepsilon_a = \max\{\varepsilon_b, \varepsilon_c\} \tag{7}$$

where N_1, N_2 are the variance of the additive Gaussian noise seen by node 1 and node 2. h_4, h_3 are the channel gains from the relay node to node 1 and to node 2 respectively (see Figure 1). R_i is the rate used from node $i, i = 1, 2$.

For the state diagram in Figure 2, since there are only finite number of states, we may manipulate (4) by interchanging the summation and the limit, as shown below:

$$\lim_{\Delta t\to 0}\sum_{i,j}\frac{P(S(t)=i)\,P(S(t+\Delta t)=j|S=i)}{\Delta t}\varepsilon_{ij} \tag{8}$$

$$=\sum_{i,j}\lim_{\Delta t\to 0}\frac{P(S(t)=i)\,P(S(t+\Delta t)=j|S=i)}{\Delta t}\varepsilon_{ij} \tag{9}$$

$$=\sum_{|i-j|\le 1} P(S=i)\,P(S(t+\Delta t)=j|S=i)\,\varepsilon_{ij} \tag{10}$$

$$\begin{aligned} &=\sum_{i=1}^{L_1} P(S=i)\,\mu\varepsilon_a + P(S=L_1)\,\lambda\varepsilon_b \\ &\quad+\sum_{i=1}^{L_2} P(S=-i)\,\lambda\varepsilon_a + P(S=-L_2)\,\mu\varepsilon_c \end{aligned} \tag{11}$$

Substituting (3) into (11), for $u \neq 1$, we obtain:

$$\begin{aligned} E[P] &= \frac{u^{L_1+L_2+1}}{u^{L_1+L_2+1}-1}\mu\varepsilon_a - \frac{1}{u^{L_1+L_2+1}-1}\lambda\varepsilon_a \\ &\quad+\frac{(u-1)\,u^{L_1+L_2}}{u^{L_1+L_2+1}-1}\lambda\varepsilon_b + \frac{u-1}{u^{L_1+L_2+1}-1}\mu\varepsilon_c \end{aligned} \tag{12}$$

Remark 3: From (12), we readily see that the average power consumption by the relay node depends only on the traffic load from the source nodes and the total storage capacity at the relay, $L_1 + L_2$. It does not depend on the service order.

Remark 4: Minimal energy consumption is achieved when $L_1 + L_2$ goes to ∞. If $L_1 + L_2 \to \infty$, we observe

$$E[P] \to (\lambda-\mu)\varepsilon_b + \mu\varepsilon_a \tag{13}$$

C. Average Delay

Let us focus on the delay experienced by a packet sent from node 1 to node 2. The case of the other direction can be addressed in a similar fashion. Suppose that the packet arrives at τ. Then the distribution of the delay T is given by

$$P(T\le t) = \sum_i P(T\le t|S(\tau)=i)\,P(S(\tau)=i) \tag{14}$$

Note that there are only two possible ways for the packet to leave the queue.

1) Enough packets from the opposite direction arrive so that it gets served with XOR-ed with a packet from the other queue.
2) Too many packets from node 1 arrive after this packet so that it needs to be transmitted by itself.

Therefore each term $P(T \le t|S(\tau) = i)$ in the summation can be computed as:

$$\begin{aligned} P(T\le t|S(\tau)=i) &= P(n_2(t)\ge i+1|\,S(\tau)=i) \\ &\quad+P(n_2(t)<i+1, n_1(t)\ge L|S(\tau)=i) \end{aligned} \tag{15}$$

where $n_i(t)$ is the number of packets received from node i during time $(\tau, t + \tau]$.

The first term in (15) is given by:

$$P(n_2(t)\ge i+1|S(\tau)=i) = 1-\sum_{k=0}^{i}\frac{(\mu t)^k}{k!}e^{-\mu t} \tag{16}$$

The second term in (15) can be computed as:

$$\begin{aligned} &P(n_2(t)<i+1, n_1(t)\ge L_1|S(\tau)=i) \\ =&P(n_2(t)<i+1)\,P(n_1(t)\ge L_1) \\ =&\left(\sum_{k=0}^{i}\frac{(\mu t)^k}{k!}e^{-\mu t}\right)\left(1-\sum_{n=0}^{L_1-1}\frac{(\lambda t)^n}{n!}e^{-\lambda t}\right) \end{aligned} \tag{17}$$

Note that, with the Poisson flows, the residual time [15] the packet has to wait to "see" a packet from the opposite direction is an exponential random variable with parameter $1/\mu$.

With these preparations, we are now ready to compute $E[T]$.

$$E[T] = \int_0^\infty t\frac{d}{dt}P(T<t)\,dt \tag{18}$$

$$=\sum_i\int_0^\infty t\frac{d}{dt}P(T<t|S=i)\,dt\,P(S=i) \tag{19}$$

Each term of the summation in (19) can be expressed as:

$$\int_0^\infty t\frac{d}{dt}P(T<t|S=i)\,dt \tag{20}$$

$$=-\int_0^\infty t\frac{d}{dt}\left(1-P(T<t|S=i)\right)dt \tag{21}$$

$$=\int_0^\infty 1-P(T<t|S=i)\,dt \tag{22}$$

We notice that $1-P(T<t|S=i)$ is the sum of terms of the form $ct^a e^{-bt}$. Therefore the integral can be computed using the following equation, where a is a non-negative integer.

$$\int_0^\infty x^a e^{-\lambda x} dx = \lambda^{-(a+1)} a! \tag{23}$$

Using this result, along with (17), (16), (22) becomes:

$$\int_0^\infty \left(\sum_{w=0}^{i} \sum_{k=0}^{L_1-1} \frac{(\mu t)^w}{w!} \frac{(\lambda t)^k}{k!} \right) e^{-(\lambda+\mu)t} dt \tag{24}$$

$$= \sum_{w=0}^{i} \sum_{k=0}^{L_1-1} \left(\frac{\lambda}{\lambda+\mu}\right)^k \frac{\mu^w}{(\lambda+\mu)^{w+1}} \binom{k+w}{w} \tag{25}$$

where $\binom{k+w}{w} = \frac{(k+w)!}{k!w!}$. Substituting (25) and (3) back to (19), we obtain the average delay from node 1 to node 2 as (26) for $u \neq 1$. The delay seen by traffic from node 2 to node 1 is similar, as shown in (27).

$$E_{12}[T] = \left(\sum_{i=0}^{L_1} \sum_{w=0}^{i} \sum_{k=0}^{L_1-1} \left(\frac{\lambda}{\lambda+\mu}\right)^k \frac{\mu^w}{(\lambda+\mu)^{w+1}} \binom{k+w}{w} u^i \right) \frac{u^{L_2+1}-u^{L_2}}{u^{L_1+L_2+1}-1} \tag{26}$$

$$E_{21}[T] = \left(\sum_{i=0}^{L_2} \sum_{w=0}^{i} \sum_{k=0}^{L_2-1} \left(\frac{\mu}{\lambda+\mu}\right)^k \frac{\lambda^w}{(\lambda+\mu)^{w+1}} \binom{k+w}{w} u^{-i} \right) \frac{u^{L_2+1}-u^{L_2}}{u^{L_1+L_2+1}-1} \tag{27}$$

D. Energy Delay Trade-off

An immediate thought following the analysis above is the selection of L_1 and L_2 to minimize traffic delay under a relay power constraint. Before doing that, we prove a monotonic property of the average delay, which will be useful later.

We know from (12) that there is a bijection between the average power and total storage capacity $L_1 + L_2$. Also, if L_1+L_2 increases, the average power consumption decreases. Therefore, we only need to examine the behavior of the average delay (26) (27) under fixed L_1+L_2. If L_1+L_2 is fixed and L_1 increases, from (27), we readily see that $E_{21}[T]$ will decrease. What is less obvious is that $E_{12}[T]$ will increase, as shown below. Suppose L_1 increases by 1. Then , as shown in (31), the first term in (26) will increase by at least u fold. Since L_2 decreases by 1, the remaining part in (26) will decrease at least by u fold. Therefore, their product $E_{12}[T]$ will increase. By a similar reasoning, it can be shown that if L_2 is fixed, then $E_{12}[T]$ will increase with L_1. If L_1 is fixed, then $E_{21}[T]$ will increase with L_2.

$$\sum_{i=0}^{(L_1+1)} \sum_{w=0}^{i} \sum_{k=0}^{(L_1+1)-1} \left(\frac{\lambda}{\lambda+\mu}\right)^k \frac{\mu^w}{(\lambda+\mu)^{w+1}} \binom{k+w}{w} u^i \tag{28}$$

$$> \sum_{i=1}^{(L_1+1)} \sum_{w=0}^{i} \sum_{k=0}^{L_1} \left(\frac{\lambda}{\lambda+\mu}\right)^k \frac{\mu^w}{(\lambda+\mu)^{w+1}} \binom{k+w}{w} u^i \tag{29}$$

$$> \sum_{i=1}^{(L_1+1)} \sum_{w=0}^{i-1} \sum_{k=0}^{L_1-1} \left(\frac{\lambda}{\lambda+\mu}\right)^k \frac{\mu^w}{(\lambda+\mu)^{w+1}} \binom{k+w}{w} u^i \tag{30}$$

$$= u \sum_{i=0}^{L_1} \sum_{w=0}^{i} \sum_{k=0}^{L_1-1} \left(\frac{\lambda}{\lambda+\mu}\right)^k \frac{\mu^w}{(\lambda+\mu)^{w+1}} \binom{k+w}{w} u^i \tag{31}$$

The minimum delay $\bar{T}$ under the average power constraint is defined as (32) below. We are interested in $\bar{T}$ as a function of $\bar{P}$, which will yield the optimal energy delay trade-off curve achievable under this relay policy.

$$\begin{aligned} \bar{T} = \min_{L_1, L_2} \max \{E_{12}[T], E_{21}[T]\} \\ s.t.\ E[P] \geq \bar{P} \end{aligned} \tag{32}$$

It follows from the previous discussion that the power constraint can be translated to a lower bound on $L_1 + L_2$. From the monotonic properties of $E_{12}[T]$ and $E_{21}[T]$ we have just argued, we know that the equivalent constraint on $L_1 + L_2$ must be binding. Therefore $L_1 + L_2$ is fixed. Under this condition, we have shown above that $E_{12}[T]$ is a strictly increasing function of L_1, and $E_{21}[T]$ is a strictly decreasing function of L_1. Also, it is obvious to see that when $L_1 = 0$, $E_{12}[T] = 0$ and when $L_2 = 0$, $E_{21}[T] = 0$. Therefore, the optimal solution can only be of the following form and can be easily found via a bisection method:

L_1 is either $L_{1,a}$ or $L_{1,a} + 1$. $L_{1,a}$ is chosen such that $E_{12}[T](L_{1,a}) \geq E_{21}[T](L_{1,a}), E_{12}[T](L_{1,a}+1) \leq E_{21}[T](L_{1,a}+1)$.

Notice that $\bar{T}$ can be reduced by employing time sharing between $L_1 = L_{1,a}$ and $L_1 = L_{1,a}+1$ while keeping L_1+L_2 fixed. The resulting energy delay trade-off curve is given in Figure 3 for $\varepsilon_a = \varepsilon_b = \varepsilon_c = 1$. The horizontal dotted lines depict the average relay power consumption when $L_1+L_2 \to \infty$ for different λ, μ. We observe that when λ and μ are close to each other, a wider trade-off between energy and delay can be achieved. When the traffic load becomes less even, the achievable trade-off range becomes smaller.

E. The Symmetric Case $\lambda = \mu$

In Figure 3, we observe that when $\lambda = \mu$, to achieve the lower bound on average power, the average delay will go to ∞. We next prove this observation formally via the following lemma.

Lemma 1: $\bar{P} - \mu\varepsilon_a \sim O(L^{-1}), \bar{T} \sim O(L)$ with $L_1 = L_2 = L$

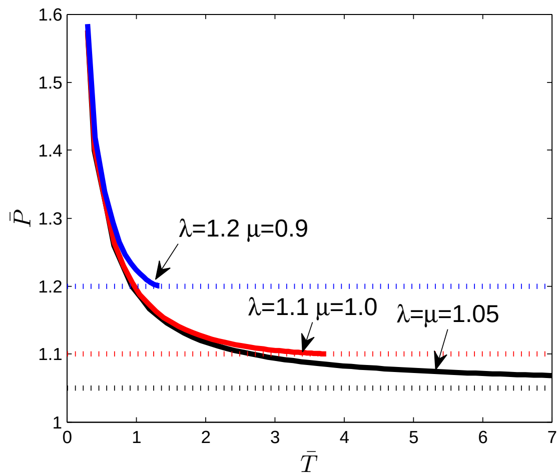

Fig. 3. Energy Delay Trade-off Curve when $L_1 + L_2 \to \infty$

Proof: First we examine the average delay $\bar{T}$. We notice that the solution of (32) yields $L_1 = L_2 \triangleq L$. Therefore, it suffices to derive an upper bound and lower bound of (26) when $L \to \infty$.

$$\bar{T} = \frac{\sum_{i=0}^{L_1}\sum_{w=0}^{i}\sum_{k=0}^{L_1-1}\left(\frac{1}{2}\right)^{k+w}\binom{k+w}{w}}{(\lambda+\mu)(L_1+L_2+1)} \tag{33}$$

$$\bar{T} \le \frac{\sum_{i=0}^{L_1}\sum_{\substack{w\ge0,k\ge0\\ w+k\le i+L_1-1}}\left(\frac{1}{2}\right)^{k+w}\binom{k+w}{w}}{(\lambda+\mu)(L_1+L_2+1)} \tag{34}$$

$$= \sum_{i=0}^{L_1}\frac{(i+L_1)}{(\lambda+\mu)(L_1+L_2+1)} \tag{35}$$

$$= \frac{3(L+1)}{2(\lambda+\mu)(2L+1)}L \tag{36}$$

$$\sim O(L) \tag{37}$$

$$\bar{T} \ge \frac{\sum_{i=0}^{L_1}\sum_{\substack{w\ge0,k\ge0\\ w+k\le i}}\left(\frac{1}{2}\right)^{k+w}\binom{k+w}{w}}{(\lambda+\mu)(L_1+L_2+1)} \tag{38}$$

$$= \frac{\sum_{i=0}^{L_1}(i+1)}{(\lambda+\mu)(L_1+L_2+1)} \tag{39}$$

$$= \frac{(1+L_1+1)(L_1+1)}{2(\lambda+\mu)(L_1+L_2+1)} \tag{40}$$

$$= \frac{(L+2)(L+1)}{2(\lambda+\mu)(2L+1)} \tag{41}$$

$$\sim O(L) \tag{42}$$

Where (35) and (39) follow from the binomial expansion formula.

Remark 5: The lower bound (42) implies that $\bar{T} \to \infty$ as $L \to \infty$.

Next, let us examine $\bar{P} - \mu\varepsilon_a$. Substituting (3) into (11), we obtain:

$$\begin{aligned}\bar{P} &= \left(\frac{L_1+L_2}{L_1+L_2+1}\right)\mu\varepsilon_a + \frac{\mu(\varepsilon_b+\varepsilon_c)}{L_1+L_2+1}\\ \bar{P}-\mu\varepsilon_a &= \frac{\mu(\varepsilon_b+\varepsilon_c-\varepsilon_a)}{L_1+L_2+1} = \frac{\mu(\varepsilon_b+\varepsilon_c-\varepsilon_a)}{2L+1}\end{aligned} \tag{43}$$

From (43) and the upper bound for $\bar{T}$, we observe that if $\bar{T}$ increases at a rate of $O(L)$, the average power $\bar{P}$ will *not* decrease faster than $O(L^{-1})$.

■

Remark 6: As shown by Lemma 1, for the symmetric traffic load $\lambda = \mu$, the average delay to achieve the minimum energy $\to \infty$.

Remark 7: If each packet must be paired along with a packet from the opposite direction for transmission, thus achieving minimal energy, then first-come-first-serve policy does achieve the minimal sum delay $E_{12}[T]+E_{21}[T]$. This can be shown as follows: Consider two sets of packets (A_1, B_1) and (A_2, B_2) paired as such to be transmitted. Suppose A_1 arrived before A_2, but B_1 arrived after B_2, in other words, the packets are not paired according to first-come-first-serve. It is then easy to verify that pairing these packets as (A_1, B_2) and (A_2, B_1) instead will not incur a greater sum delay for these four packets.

On the other hand, due to the symmetry of the system, we have $E_{12}[T] = E_{21}[T]$. This means achieving minimal sum delay is equivalent to min max delay $\bar{T}$. Therefore we find that first-come-first-serve policy is indeed the optimal policy. This means to achieve minimal energy consumption, $\bar{T}$ will go to ∞ regardless of what service policy is in use.

Remark 8: Our assumptions in section II dictates that the system operates at a rate that belongs to the achievable rate region $\mathcal{C}$ with the coding scheme used. Therefore, it is rather surprising to see the queues become unstable, since there are many known rate allocation policies [8]–[10] which stabilize queues for all rate points inside $\mathcal{C}$. However, a closer look shows none of these policies use network coding as the only coding scheme. Reference [8] uses the superposition coding scheme for the broadcast phase. In [9], the stability proof of the opportunistic network coding scheduling algorithm relies on the fact that a multi-hopping scheme is used along with network coding scheme. The policy in [10] is also a hybrid scheme; network coding is used along with direct transmission so that the resulting stability region has a nonempty interior. The stability of the queues is henceforth guaranteed via the CMDB policy [10].

An insight obtained is that a deterministic network coding scheme must be used along with other coding schemes. This hybrid approach was considered previously to obtain a larger achievable rate region of the underlying coding scheme [9], [10]. However, it is easy to see for a symmetric channel, i.e., when the relay is in the middle of the two source nodes, this approach will not increase the maximal rate or the sum rate. Nevertheless, we show here that a hybrid approach is still necessary. A multi-hopping scheme must be considered in a network coding protocol in order to stabilize the queues.

IV. Conclusion

In this paper, we have investigated the energy-delay trade-off in a two-way relay network when the relay node uses a first-come-first-serve policy and aims to harvest the energy savings by employing XOR combining of the data arriving from the sources. The trade-off is a result of the stochastic nature of the traffic from the source nodes. We have proved for the case with an even traffic load from both directions, to fully achieve the energy saving promised by XOR network coding scheme, the average delay will go to ∞.

The energy-delay trade-off curve here is derived under a specific policy used at the relay node. In that sense it is an achievability result. When a different policy is used, a different curve may result. It is of interest to derive an lower bound for this energy-delay trade-off curve as future work, along with the optimal strategy that will achieve this lower bound.

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