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Joint Power Scheduling and Estimator Design for Sensor Networks Across Parallel Channels

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***Abstract*—This paper addresses the joint estimator and power optimization problem for a sensor network whose mission is to estimate an unknown parameter. We assume a two-hop network where each sensor collects observations from the source that transmits the quantity to be estimated, then amplifies and forwards its observations to a fusion center. The fusion center combines the observations using a Linear Minimum Mean Squared Error (LMMSE) estimator. We study the scenario where multiple parallel channels are available between the source and each sensor as well as between the sensors and the fusion center. We find the global optimal power allocation and estimator design for this network model. We present two practical scenarios of interest that utilize spatial and temporal diversity for which this solution applies, namely, a clustered network model and a single cluster model with an ergodic fading channel.**

I. INTRODUCTION

Wireless sensor networks pose a number of new design challenges primarily due to their distributed nature and the fact that they are energy limited. In this work, we consider a scenario where a sensor network is deployed to estimate an unknown scalar parameter. In this problem, a transmission scheme local at each sensor and a scheme at the fusion center are sought to minimize the total power subject to a given distortion requirement. The problem of scalar estimation is the simplest conceivable estimation problem, yet, it still embodies the potential to draw interesting insights on the interaction between the estimation and the communication schemes. In fact, determining the optimal local scheme at each sensor and fusion center estimator schemes remain an open problem [1].

A large research effort has been devoted to this problem and as a result optimal power and estimator designs under several different estimator structures have been found. Power minimization using the Best Linear Unbiased Estimate (BLUE) is investigated in [2]–[4]. In [2], the optimum sensor power schedule is found under analog amplify and forward, while in [3], a digital approach is used to find the optimal number of bits to quantize each sensor's measurement. A comparison of digital and analog techniques is given in [4] and shows that the analog approach is more energy efficient than digital systems without coding and in some cases more efficient than those with coding. Power minimization under Linear Minimum Mean Square Error (LMMSE) estimation is considered in [5]. The optimal power schedule using analog amplify and forward is given. It is shown that this solution achieves significant savings in power consumption in a non-homogeneous network. We note that these previous works considered the availability of a single channel in each hop.

One might envision that an interesting scenario is the case when multiple parallel channels exist between the observed object, the sensors, and the fusion center. Multiple parallel channels in each hop allow diversity gain in observations hence improving the accuracy of the estimate. These parallel channels may be needed when the unknown parameter has to be reconstructed with a high accuracy (or low distortion) at the fusion center. The existence of multiple parallel channels in each hop models a class of meaningful physical scenarios. In particular, these non-interfering parallel channels may correspond either to a time slot in the temporal domain or a sensor cluster in the spatial domain. In either case, in order to satisfy stringent distortion requirements, data from the parallel channels must be fused and a power scheduling scheme must be devised. Thus, resource allocation among the parallel channels to minimize total power under a given distortion constraint arises as a fundamental problem. The main result we report in this paper is the solution of this problem, i.e. we find the globally optimal solution for the multiple parallel channel power scheduling problem.

The remainder of the paper is organized as follows. We introduce the system model in Section II and formulate the optimization problem which is promptly observed to be non-convex. We then show in Section III that we can find the global optimum for this non-convex problem by using the solution of the single channel case [5] as a building block. Specifically, we prove that the minimum sum power of each channel is a convex function of the received SNR requirement of that channel. Using this result, the problem can be reformulated as a convex programming problem and global optimality claimed. In Section IV we present two applications of the main result of our work: cluster based networks and the ergodic fading channel. The first model exploits spatial diversity resulting from clusters of sensors and the second model exploits time diversity resulting from the time varying channel in each hop. Section V provides the summary of our results.

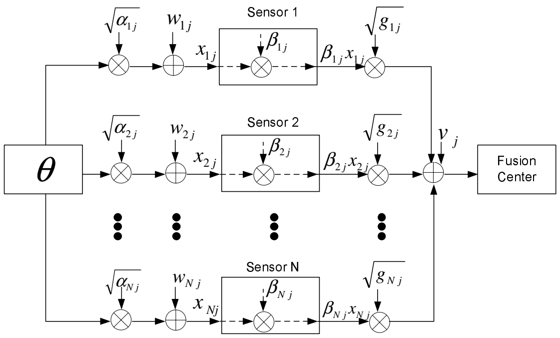

Fig. 1. System Model

II. System Model

The network model given in Figure 1 shows a snapshot of the j^{th} sub-channel for a network comprised of N sensors. We assume that X such parallel channels are available. The subscript j is used to denote variables related to the j^{th} sub-channel, $j = 1, \ldots, X$. Communication in each sub-channel consists of two hops. In the first hop, the parameter, which is a realization of the random variable θ with variance σ_θ^2, is broadcasted to N sensors. In the second hop all sensors communicate to the fusion center (FC), which forms an MMSE estimate of θ. The source-to-sensors and sensors-to-fusion center channel gains are fixed and given by α_{ij} and g_{ij} for $i = 1, \ldots, N; \quad j = 1, \ldots, X$, respectively. The randomness in the sensors' observation is modeled as additive noise, w_{ij}. The received signal noise is v_j. Both w_{ij} and v_j are modeled as i.i.d. Gaussian random variables with zero mean and variance σ_w^2 and σ_v^2, respectively. The i.i.d. assumption implicitly assumes the observations are independent, conditioned on the value of the parameter θ. Each sensor has an amplifier with an adjustable gain, β_{ij}. Sensors only have access to their own (local) measurements.

From Figure 1, the received signal at sensor i from sub-channel j is

$$x_{ij} = \sqrt{\alpha_{ij} P_s}\theta + w_{ij} \tag{1}$$

and the signal received at the fusion center from sub-channel j is given by

$$\begin{aligned} y_j &= v_j + \sum_{i=1}^{N} \sqrt{g_{ij}}\left(\beta_{ij} x_{ij}\right) & (2)\\ &= v_j + \sum_{i=1}^{N} \left(\sqrt{\frac{g_{ij}\alpha_{ij}\ P_s P_{ij}}{\alpha_{ij}P_s + \sigma_w^2}}\ \theta + \sqrt{\frac{g_{ij}P_{ij}}{\alpha_{ij}P_s + \sigma_w^2}} w_{ij} \right) & (3) \end{aligned}$$

where the amplification factor, $\beta_{ij} = \sqrt{\frac{P_{ij}}{\alpha_{ij}P_s + \sigma_w^2}}$ for each sensor is adjusted to yield the transmit power, P_{ij} of sensor i for sub-channel j. We further simplify notation by defining $q_{ij}^2 = \frac{\gamma_{ij} g_{ij}}{1+\gamma_{ij}} = \frac{\alpha_{ij} P_s g_{ij}}{\alpha_{ij}P_s + \sigma_w^2}$ and $\gamma_{ij} = \alpha_{ij}P_s/\sigma_w^2$, where γ_{ij} is the local received SNR at every sensor. The received signal at the fusion center becomes,

$$y_j = v_j + \sum_{i=1}^{N} \left(q_{ij}\sqrt{P_{ij}}\ \theta + \frac{q_{ij}\sqrt{P_{ij}} w_{ij}}{\sqrt{\alpha_{ij} P_s}} \right) \tag{4}$$

based on which the fusion center forms the linear estimate.

$$\hat{\theta} = \sum_{j=1}^{X} a_j y_j \tag{5}$$

The end-to-end $\mathrm{MSE} = \mathrm{E}\left[\left(\theta - \hat{\theta}\right)^2\right]$ can be expressed as

$$\begin{aligned} \mathrm{MSE} &= \mathbf{e}^T \left(\sigma_\theta^2 \mathbf{Q}^T \mathbf{a}\mathbf{a}^T \mathbf{Q} + E\left[\tilde{\mathbf{W}}^T \mathbf{a}\mathbf{a}^T \tilde{\mathbf{W}}\right] \right) \mathbf{e} \\ &+ \mathbf{a}^T \sigma_v^2 \mathbf{I}\mathbf{a} - 2\sigma_\theta^2 \mathbf{a}^T \mathbf{Q}\mathbf{e} + \sigma_\theta^2 \end{aligned} \tag{6}$$

where we have

$$\tilde{\mathbf{q}}_{\mathbf{j}} = \left[q_{1j} \quad \cdots \quad q_{Nj} \right]^T;$$

$$\mathbf{Q} = \begin{bmatrix} \tilde{\mathbf{q}}_1^T & 0 & \cdots & 0 & 0 & 0 \\ 0 & \tilde{\mathbf{q}}_2^T & 0 & \cdots & 0 & 0 \\ \vdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & 0 & 0 & \tilde{\mathbf{q}}_X^T \end{bmatrix} \tag{7}$$

$$\tilde{\mathbf{w}}_{\mathbf{j}} = \left[\frac{q_{1j} w_{1j}}{\sqrt{\alpha_{1j} P_s}} \quad \cdots \quad \frac{q_{Nj} w_{Nj}}{\sqrt{\alpha_{Nj} P_s}} \right]^T;$$

$$\tilde{\mathbf{W}} = \begin{bmatrix} \tilde{\mathbf{w}}_1^T & 0 & \cdots & 0 & 0 & 0 \\ 0 & \tilde{\mathbf{w}}_2^T & 0 & \cdots & 0 & 0 \\ \vdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & 0 & 0 & \tilde{\mathbf{w}}_X^T \end{bmatrix} \tag{8}$$

$$\mathbf{e} = \left[\sqrt{P_{11}} \quad \cdots \quad \cdots \quad \sqrt{P_{1X}} \quad \cdots \quad \sqrt{P_{NX}} \right]^T \tag{9}$$

and

$$\mathbf{a} = \left[a_0 \quad a_1 \quad \ldots \quad a_{N-1} \right]^T; \tag{10}$$

$$E\left[\tilde{\mathbf{W}}^T \mathbf{a}\ \mathbf{a}^T \tilde{\mathbf{W}}\right] = E\left[\mathbf{u}\mathbf{u}^T\right] \quad \text{with} \tag{11}$$

$$\mathbf{u} = \left[\frac{q_{11} w_{11} a_1}{\sqrt{\alpha_{11} P_s}} \quad \cdots \quad \frac{q_{N1} w_{N1} a_1}{\sqrt{\alpha_{N1} P_s}} \quad \cdots \quad \frac{q_{NX} w_{NX} a_X}{\sqrt{\alpha_{NX} P_s}} \right]^T. \tag{12}$$

We would like to minimize the total energy used under a given MSE constraint. Thus, the optimization problem is:

$$\min_{\mathbf{e},\mathbf{a}} \quad \|\mathbf{e}\|^2 = \textstyle\sum_{i=1}^{N}\sum_{j=1}^{X} P_{i,j} \tag{13}$$

$$\text{s.t.} \qquad \mathrm{MSE} \le D_o \tag{14}$$

$$\mathbf{e} \ge \overline{\mathbf{0}} \tag{15}$$

where $\mathbf{e}$ is the vector of transmission powers given by equation (9), $\mathbf{a}$ is the set of linear estimator coefficients given by (10), MSE is given by (6) and D_o is the maximum allowable system MSE. This optimization is performed by the fusion center. We point out that this joint optimization problem over $\mathbf{e}$ and $\mathbf{a}$ is not a convex program. This can easily be seen from the fact that each KKT point for the case with $X-1$ sub-channels is also a KKT point for the case with X sub-channels. These KKT points can be highly suboptimal for the X sub-channel case, establishing that the problem has to be non-convex in $\mathbf{e}$ and $\mathbf{a}$.

III. Optimal Power Allocation

We begin by describing the relationship between the received SNR at the fusion center and the end-to-end MMSE. Following (4), we express the received SNR at the fusion center for sub-channel j as

$$\Gamma_j = \left(E\left[\left(\frac{v_j + \sum_{i=1}^{N} \frac{q_{ij}\sqrt{P_{ij}}w_{ij}}{\sqrt{\alpha_{ij}P_s}}}{\sum_{i=1}^{N} q_{ij}\sqrt{P_{ij}}} \right)^2 \right] \right)^{-1} \quad (16)$$

$$= \frac{\left(\sum_{i=1}^{N} q_{ij}\sqrt{P_{ij}}\right)^2}{\sigma_{v_j}^2 + \sum_{i=1}^{N} \frac{q_{ij}^2 P_{ij}\sigma_{w_{ij}}^2}{\alpha_{ij}P_s}} = \frac{\left(\sum_{i=1}^{N} \hat{P}_{ij}\right)^2}{\sigma_{v_j}^2 + \sum_{i=1}^{N} \frac{1}{\gamma_{ij}}\hat{P}_{ij}^2} \quad (17)$$

When MMSE estimation is performed at the fusion center, the resulting maximum SNR and the minimum MSE has the following relationship [6]:

$$\mathrm{MSE} = \frac{1}{\sum_{j=1}^{X} \Gamma_j + 1} \quad (18)$$

Using (18), the MSE constraint $\mathrm{MSE} \leq D_0$ can be rewritten as a sum received SNR constraint: $\sum_{j=1}^{X} \Gamma_j \geq \Gamma_o$, where $\Gamma_o = \frac{1}{D_o} - 1$. Therefore, the MSE constraint (14) in the optimization problem (13)-(15) can be reformulated with the sum SNR constraint and the following constraint for each sub-channel obtained from (17).

$$(S_j)^2 \geq \Gamma_j \left(\sum_{i=1}^{N} \frac{1}{\gamma_{ij}} \hat{P}_{ij}^2 + \sigma_{v_j}^2 \right) j = 1, \ldots, X \quad (19)$$

$$S_j = \sum_{i=1}^{N} \hat{P}_{ij};\ \hat{P}_{ij} \geq 0 \quad (20)$$

We recognize that for a fixed Γ_j, constraints (19) and (20) characterize a convex set of S_j, $\hat{P}_{ij}$, where $i = 1, \ldots, N$ and $j = 1, \ldots, X$. Therefore, for each sub-channel with a fixed SNR, the problem is a convex programming problem [5]. On the other hand, when Γ_j is no longer a constant but instead is a part of the optimization problem, this is no longer the case.

We show next that the multiple parallel channel power scheduling problem can be solved by examining the relationship between the sum power in the sub-channel j and the required SNR in that sub-channel Γ_j. This relationship which follows from [5] is shown next:

$$\sum_{i=1}^{N} \frac{\left(\frac{g_{ij}\gamma_{ij}}{(1+\gamma_{ij})\sigma_{v_j}^2}\right) P_{tot,j}(\Gamma_j)}{1 + \left(\frac{g_{ij}}{(1+\gamma_{ij})\sigma_{v_j}^2}\right) P_{tot,j}(\Gamma_j)} = \Gamma_j \quad (21)$$

The total power in sub-channel j is $P_{tot,j} = \sum_{i=1}^{N} P_{ij}$ and is written as $P_{tot,j}(\Gamma_j)$ to emphasize the functional relationship between Γ_j and $P_{tot,j}$.

Taking the derivative of (21) we have:

$$\frac{\partial P_{tot,j}(\Gamma_j)}{\partial \Gamma_j} = \frac{1}{\sum_{i=1}^{N} \frac{\left(\frac{g_{ij}\gamma_{ij}}{(1+\gamma_{ij})\sigma_{v_j}^2}\right)}{\left(\left(\frac{g_{ij}}{(1+\gamma_{ij})\sigma_{v_j}^2}\right) P_{tot,j}(\Gamma_j) + 1\right)^2}} \geq 0 \quad (22)$$

since $P'_{tot,j} \geq 0$, $P_{tot,j}$ increases with Γ_j. Using (22), we find $P'_{tot,j}$ increases with Γ_j which means $P''_{tot,j} \geq 0$. Hence we have the following observation.

Observation 1 *The sum power $P_{tot,j}$ is a convex function of received SNR Γ_j.*

From this observation, we can recalibrate the optimization problem in terms of the total power value expended in each sub-channel as follows:

$$\min_{\{P_{tot,j}\}_{j=1}^{X}} \sum_{j=1}^{X} P_{tot,j} \quad (23)$$

$$\text{s.t.} \quad \sum_{j=1}^{X} \Gamma_j \geq \frac{1}{D_o} - 1 \quad (24)$$

$$\sum_{i=1}^{N} \frac{\frac{g_{ij}\gamma_{ij}}{(1+\gamma_{ij})\sigma_{v_j}^2} P_{tot,j}}{1 + \frac{g_{ij}}{(1+\gamma_{ij})\sigma_{v_j}^2} P_{tot,j}} \geq \Gamma_j, \quad P_{tot,j} \geq 0 \ \forall\ j \quad (25)$$

Observe that (23)-(25) is a convex programming problem. The solution has a waterfilling-like structure as shown by (27) and (28). Recall that the right hand side of (25) is a monotonically increasing function of $P_{tot,j}$. For a given λ_o, there is a unique $P_{tot,j}$ such that (28) holds. Therefore, the optimal solution $\{P_{tot,j}\}_{j=1}^{X}$ is governed by a single Lagrange parameter λ_0.

$$P'_{tot,j}\big|_{\Gamma_j=0, P_{tot,j}=0} = \frac{1}{\sum_{i=1}^{N} \frac{g_{ij}\gamma_{ij}}{(1+\gamma_{ij})\sigma_{v_j}^2}} \quad (26)$$

$$\text{If} \quad \lambda_o < P'_{tot,j}\big|_{\Gamma_j=0} \text{ then } P_{tot,j} = 0 \quad (27)$$

$$\text{If} \quad \lambda_o > P'_{tot,j}\big|_{\Gamma_j=0}$$

$$\text{then} \quad \frac{1}{\lambda_o} = \frac{\partial P_{tot,j}(\Gamma_j)}{\partial \Gamma_j} = \sum_{i=1}^{N} \frac{\frac{g_{ij}\gamma_{ij}}{(1+\gamma_{ij})\sigma_{v_j}^2}}{\left(\left(\frac{g_{ij}\gamma_{ij}}{(1+\gamma_{ij})\sigma_{v_j}^2}\right) P_{tot,j} + 1\right)^2} \quad (28)$$

From (27) and (28), we can find the received SNR Γ_j of each sub-channnel. In essence, by this method, the multiple

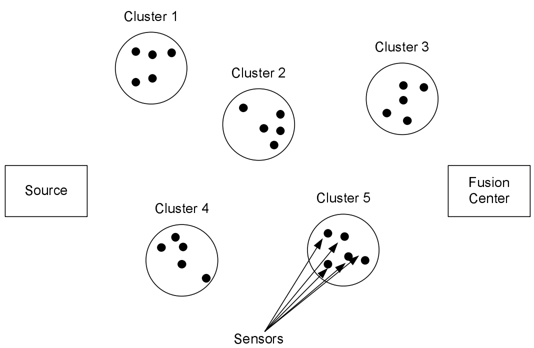

Fig. 2. Cluster Sensor Network

parallel channel power scheduling problem decomposes into a sequence of single sub-channel optimization problems which can be solved as shown in [5].

IV. Applications

So far we presented a general solution for optimum power allocation for two-hop sensor networks with parallel channels. A natural follow-up is to consider practical communication scenarios for which this solution is applicable. In this section, we present two such scenarios.

A. Cluster based network

Cooperative beamforming among sensors over the entire network can be difficult due to synchronization challenges. A more practical approach is to organize sensors into clusters and perform cooperative beamforming within each cluster. The fusion center communicates with each cluster in a TDMA fashion. This system is depicted in Figure 2. Because each time the fusion center only talks with one cluster, the clusters equate to non-interfering sub-channels as described in Figure 1. In general, the quality of observations from each cluster will be different. Thus, by considering power allocation among clusters, the overall performance can be improved [7]. Aiming specifically for the jointly optimum power allocation and estimators, we can immediately observe that this problem is mathematically identical to our parallel channel model. We can apply the globally optimal solution directly when each cluster corresponds to a parallel channel.

We examine the performance of the global optimal solution and compare it to equal power allocation among clusters as shown in Figure 3. In the case of equal power allocation, each cluster is allocated power separately using the solution given by [5]. Each channel is assumed to be Rayleigh flat fading. The variance σ^2 of the Rayleigh distribution is set to be d^{-2}, where d is the distance between the transmitter and the receiver. The location of the clusters are generated randomly over a 10×10 field. Sensors are distributed uniformly within a circle centered on the location of the cluster. The radius of the cluster is 0.5. There are 5 clusters, each with 5 sensors. We observe a 10% decrease in minimum achievable MSE using the optimal power allocation among clusters.

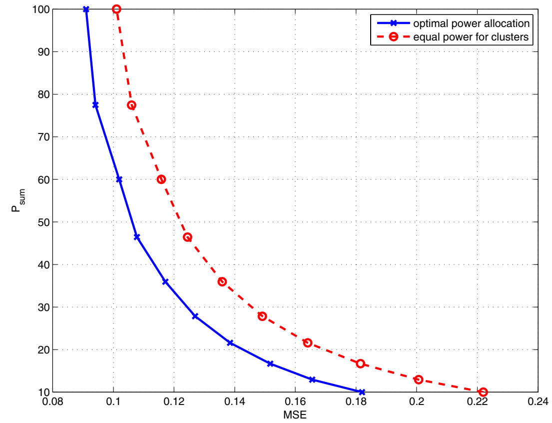

Fig. 3. Power Allocation

B. Ergodic Block Fading Channel

We consider ergodic block fading in a system similar to Figure 1, where each link is a block fading channel. Suppose that the observation time is sufficiently long such that the channel gains appear to be ergodic within the observation period. In addition, the noise coherence time introduced by the sensor amplifier is much shorter as compared with the coherence time of the channel gains. Thus, each fading state can be viewed as a sub-channel corrupted by independent noise. We wish to determine the power allocation for each fading state.

Before the problem can be addressed, the meaning of optimality must be refined. Since an infinite number of observations are involved, MSE alone is not a valid constraint because equal (non-zero) power allocation among fading states would yield a MSE $= 0$. For this application, we are more interested in how MSE will approach 0. Therefore, we impose constraints on the asymptotic behavior of MSE. The optimization problem is stated below:

$$\begin{aligned} &\min \lim_{X\to\infty} \frac{1}{X}\sum_{j=1}^{X} P_{tot,j} \\ &\text{s.t.} \lim_{X\to\infty} X \ \text{MSE} \le \alpha, \end{aligned} \tag{29}$$

where $\alpha > 0$.

Observe that from (18), we have:

$$\lim_{X\to\infty} X \ \text{MSE} = \lim_{X\to\infty} \frac{X}{\sum_{j=1}^{X}\Gamma_j + 1} = \lim_{X\to\infty} \frac{X}{\sum_{j=1}^{X}\Gamma_j} \tag{30}$$

Substituting (30) into (29) and defining $1/\alpha = \alpha'$ we have:

$$\begin{aligned} &\min \lim_{X\to\infty} \frac{1}{X}\sum_{j=1}^{X} P_{tot,j} \\ &\text{s.t.} \lim_{X\to\infty} \frac{\sum_{j=1}^{X}\Gamma_j}{X} \ge \alpha' \end{aligned} \tag{31}$$

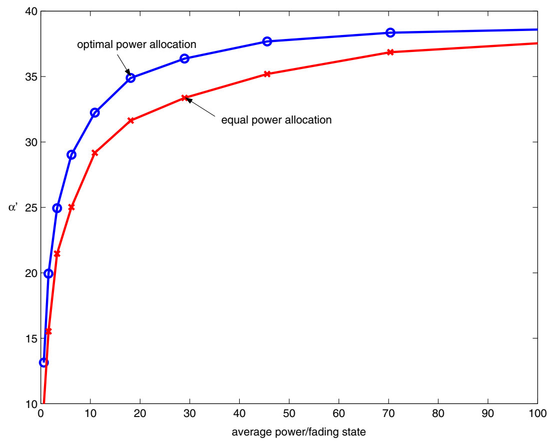

Fig. 4. Performance under strongly correlated channel gains (4 sensors)

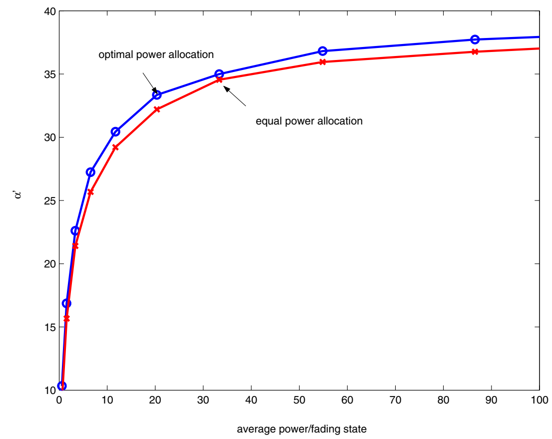

Fig. 5. Performance under independent channel gains (4 sensors)

Since the channel is ergodic, (31) is equivalent to

$$\begin{aligned} &\min E\left[P_{tot,j}\right] \\ &\text{s.t. } E\left[\Gamma_j\right] \geq \alpha' \end{aligned} \tag{32}$$

where the expectation is over the channel gains.

If the channel gains are discrete random variables with a finite support, then (32) can be solved with the same method we used to solve (23). When that is not the case, care must be excercised to find the solution.

First, we observe that (32) has a mathematical form similar to (23). As shown in (27) and (28), the solution of (23) is governed by a single Lagrangian multiplier λ_0. The solution of (32) has the same property. Once λ_0 is fixed, the optimal $P_{tot,j}$ and Γ_j of the sub-channel j can be computed via (27) and (28). $E\left[P_{tot,j}\right]$ and $E\left[\Gamma_j\right]$ are then approximated by averaging $P_{tot,j}$ and Γ_j over the channel gains. Hence, once we determine λ_0, we can get $E\left[P^*_{tot,j}\right]$ and $E\left[\Gamma^*_j\right]$, where the superscript $*$ denotes the optimal solution. In practice, a lookup table of $E\left[P^*_{tot,j}\right]$ and λ_0 under different values of $E\left[\Gamma^*_j\right]$ can be generated. Because the constraint $E\left[\Gamma_j\right] \geq \alpha'$ in (32) must be binding, $E\left[\Gamma^*_j\right]$ is equal to α'. Therefore, the lookup table can be used to find λ_0 under a given α', which yields the power power allocation for any given fading state under (27) and (28).

The approach described above is used to compute the minimum average power per fading state under a given α' in Figures 4 and 5. Figure 4 presents a case in which the channel gains seen by different sensors are set to be the same realization of Rayleigh random variable. Figure 5 presents the case in which the channel gains seen by different sensors are independent Rayleigh random variables. Real-world scenarios lie between these two extreme cases. The Rayleigh random variables have unit variance. The variance of all additive white noise is 0.1. The variance of the unknown parameter is 1. As seen in the perfectly correlated case (Figure 4), the optimal solution achieves a greater α' as compared with equal power allocation when the average power per fading state is 30. In the case of independent channel gains (Figure 5), the performances are similar due to spatial diversity. From this experiment, we observe that, depending on the physical reality of the system, optimal power allocation may signifcantly outperform equal power allocation which validates it as a design choice. On the other hand, there may be scenarios where the optimal solution provides modest gains over equal power allocation, in which case equal power allocation may be preferred for its ease of implementation.

V. Conclusion

In this work, we have presented the globally optimum power allocation and estimator design for the sensor network based scalar estimation problem with multiple parallel channels between the observed object, the sensors, and the fusion center. The sensors use an amplify and forward scheme. This problem cannot be solved by simply combining the constraints of all sub-channels as the resulting problem is non-convex. Instead, we examine the optimal solution of each sub-channel and make use of the fact that the minimum sum power spent on each sub-channel is a convex function of the received SNR of that channel.

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