# Multiuser Two-Way Relaying for Interference Limited Systems

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Abstract—We investigate multiuser two-way relaying strategies for interference limited systems where multiple pairs of users exchange information with their partners via an intermediate relay node in a two-phase communication scenario. To take advantage of the bidirectional communication structure, we propose that each pair of users share a common spreading signature instead of using distinct signatures as in a traditional CDMA setting. We design the jointly demodulate-and-XOR forward (JD-XOR-F) relaying scheme and derive the decision rule which enables the relay to generate the XORed symbol upon reception of the superposition of user symbols. When the relay has limited computational capability, amplify-and-forward relaying can be applied instead. We evaluate the BER performance for the proposed relaying schemes and show that they significantly outperform the traditional "one-way" CDMA systems.

#### I. INTRODUCTION

Two-way relay networks have attracted increasing research attention due to their potential of improving spectral efficiency upon one-way relaying systems. Recent work has proposed different protocols for two-way relay channels that outperform the traditional four-phase relaying communications, in terms of throughput [1], [2] and achievable rate [3]-[7]. It is proposed in [1], [2] that two users transmit sequentially and the relay broadcasts an XORed version of two users' data after decoding both of them. In the protocols presented in [5], [7]-[9], two users transmit simultaneously and the relay simply amplifies and forwards the received signal. The relay node can also generate an XORed or superposed symbol for two simultaneously transmitting users after jointly decoding them as in [6], [7], [10], [11]. Reference [7] also considers twoway relaying for multiple pairs of users assisted by multiple relays, where the number of relays has to satisfy the condition to orthogonalize the overall channel between each pair of users by zero-forcing. Wireless ad hoc networks of the near future are most likely to consist of many more than two nodes wishing to exchange information, potentially having to share intermediate relays. To that end, we propose a model where multiple user pairs exchanging information with the help of a *single* shared intermediate relaying node. Such a communication scenario, henceforth termed as multiuser twoway relaying, is what we propose and investigate in this paper.

In multiuser two-way communication systems, a choice to support multiple users is code division multiple access (CDMA) whose merits and limitations are well understood in the context of one-way cellular communications as well as some ad hoc settings [12], [13]. It is well known that CDMA systems are interference limited, and can provide reliable communication at a given quality of service, for a limited number of users, with a given processing gain. Multiuser two-way communication systems employing CDMA are naturally interference limited as well. However, clever transmit and relaying strategies, that take advantage of the two-way communication structure, can help reduce the multi access interference (MAI) significantly.

In this paper, we investigate multiuser two-way relaying strategies for interference limited systems. Motivated by the fact that in a two-way communication scenario, each user can recover its partner's symbol from the common signal broadcasted by the relay, we propose that each pair of users share a common spreading signature. This reduces the number of the spreading signatures in use by half, and the MAI experienced by each user, if used in conjunction with advanced reception techniques. The two-way communications are carried out in two phases. In phase one, all users transmit to the relay simultaneously. The relay node has two choices of relaying scheme in phase two: (1) If the relay node is able to perform multiuser detection, it can do joint demodulation and generate the XORed symbol to transmit for each user pair by deciding whether the two users have sent the same symbol, since the XOR operation of two binary symbols is equivalent to indicating whether they are same. We term this relaying scheme jointly demodulate-and-XOR forward (JD-XOR-F); (2) If the relay node has limited computational capability, it can simply amplify-and-forward (AF) the received signal. Utilizing the side information, i.e., its own symbol, each user can recover its partner's symbol from the common signal broadcasted from the relay.

We derive the decision rule for the relay to generate the XORed symbol for the JD-XOR-F scheme, and evaluate the end-to-end bit error rate (BER) performance for the relaying strategies analytically and numerically. We demonstrate that considerable performance gain can be achieved by the proposed relaying schemes as compared to the "traditional" CDMA where each user has a distinct signature. This shows that the proposed relaying strategies are a viable solution for heavy loaded multiuser two-way interference limited systems.



### **II. SYSTEM DESCRIPTION**

We consider a multiuser two-way communication system shown in Figure 1, which consists of K pairs of users and an intermediate relay node. User  $k_1$  and  $k_2$  ( $k \in [1, K]$ ) are a pair of partners who wish to communicate with each other via the relay node indexed as 0. We remark that each node wishes to communicate with its pre-assigned partner only. We assume all the users and the relay node are half-duplex and equipped with single antenna, and there is no direct link between users. The information exchange between the partners via the relay is accomplished in two phases. Phase one is dedicated to the transmissions from the users to the relay, and phase two is dedicated to the transmission from the relay to the users.

To accommodate the communications of multiple pairs of users simultaneously, direct sequence (DS)-CDMA is employed. For clarity of exposition, we assume a synchronous DS-CDMA system employing non-orthogonal signatures with spreading gain N. If CDMA is employed in the traditional sense in this system, i.e., each user's symbol is spread by a distinct signature, 2K signature waveforms are needed. On the other hand, we know that, in two-way communication, each user can recover its partner's symbol from a common signal broadcasted from the relay, utilizing the side information which is its own symbol. Therefore, considering the bidirectional communication structure, we propose that in the multiuser two-way interference limited systems, each pair of users transmit with the same signature waveform. This way, only K signature waveforms are needed, and the MAI present in the system can be (potentially) reduced.

In the first phase, all the users transmit their symbols to the relay simultaneously, each pair of users  $k_1$  and  $k_2$  using their common signature waveform  $s_k(t)$ . We assume users can adjust their transmit powers such that the signals at the relay node from a pair of users  $k_1$  and  $k_2$  have the same received power level  $P_{k,0}^{-1}$ . Thus, the received signal at the relay is

$$r_0(t) = \sum_{k=1}^{K} \sqrt{P_{k,0}} b_k s_k(t) + n_0(t)$$
(1)

where  $b_k=b_{k_1}+b_{k_2}$  is the superposition of the partners' symbols, and  $n_0(t)$  denotes the additive white Gaussian noise (AWGN) at the relay, with zero mean and variance  $\sigma_{n_0}^2$ . We assume that we have  $b_{k_1}, b_{k_2} \in \{-1, +1\}$  with equal probability,

consequently,  $b_k \in \{-2,0,2\}$  with probability  $\{1/4,1/2,1/4\}$ . The discrete-time equivalent received signal at the output of the chip matched filter is

$$\mathbf{r}_0 = \sum_{k=1}^{K} \sqrt{P_{k,0}} b_k \mathbf{s}_k + \mathbf{n}_0 \tag{2}$$

where  $\mathbf{s}_k$  denotes the unit norm spreading sequence,  $\mathbf{n}_0$  is the zero-mean Gaussian random vector with  $E[\mathbf{n}_0\mathbf{n}_0^{\mathsf{T}}] = \sigma_{n_0}^2\mathbf{I}_N$  where  $(\cdot)^{\mathsf{T}}$  denotes transpose operation, and  $\mathbf{I}_N$  denotes the *N*-by-*N* identity matrix. In the sequel, we will use this discrete-time representation.

Upon receiving  $\mathbf{r}_0$ , the relay node can choose to employ different relaying schemes in the second phase, and the users employ the corresponding method to recover their partners' symbols from the common signal broadcasted from the relay. Next, we propose two relaying schemes that support this multiuser two-way relay network.

# III. JOINTLY DEMODULATE-AND-XOR FORWARD Relaying

It is shown in [1], [2], [6] that two-way relay networks benefit from the simple network coding technique using digital XOR operation at the relay, which allows the relay to combine the decoded users' data before broadcasting. For our proposed system with 2K users as well, in the second phase, if the relay transmits an XORed symbol to each pair of users, the MAI experienced by each user will be reduced since the number of interfering users is reduced from 2K-1 to K-1.

To decode users' symbols at the relay prior to the XOR operation, references [1], [2] and [6] propose using orthogonal transmissions or channel coding techniques. In the uncoded two-way relay network, however, the relay node can still generate the XORed symbol for a pair of users even when they transmit simultaneously [10], [11]. Based on this observation, we propose a multiuser two-way relaying scheme termed *jointly demodulate-and-XOR forward* (JD-XOR-F), where the relay first decodes the superposition of two users' symbols using multiuser detection for each pair of users, then generates the XORed symbol based on the decision on whether the two users have sent the same symbol. Focusing on the linear multiuser detectors for manageable complexity, we consider the minimum mean-square error (MMSE) receiver at both the relay node and the users.

Next, we describe the relaying scheme and analyze its BER performance in detail.

#### A. Optimum Decision Rule at the Relay in Phase One

To minimize the BER of the estimated XORed symbol, the relay should employ the optimum decision rule on a twohypothesis model, and make a decision on whether a pair of users have sent the same symbol or not. We first derive the optimum decision rule for a system with a single pair, and see that it can be easily extended to multiuser two-way relay systems with MMSE receiver, since the MAI can be accurately approximated as a Gaussian [15].

<sup>&</sup>lt;sup>1</sup>We assume equal received powers and linear MMSE receivers in this paper as the focus is in establishing the resource sharing in terms of signature sequences. We further investigate the power allocation and the receiver optimization problem for the multiuser two-way relay systems in [14].

Consider two users transmitting their symbols  $b_1$  and  $b_2$ , which are both binary  $(\pm 1)$  with equal probability, to the relay simultaneously. The received signal at the relay is

$$y = \sqrt{P_1}b_1 + \sqrt{P_2}b_2 + n$$
 (3)

where  $P_1$  and  $P_2$  are the received power of  $b_1$  and  $b_2$  respectively, and n is the AWGN term with variance  $\sigma^2$ . Based on y, the relay wants to choose one of two hypotheses, i.e.,  $b_1$  and  $b_2$  are same or different, and output the decision  $\hat{b}$  using one bit,  $\hat{b}=-1$  when it decides  $b_1=b_2$  and  $\hat{b}=1$  otherwise. Equivalently,  $\hat{b}$  is the estimate of  $b_1\oplus b_2$ . Following the maximum aposteriori probability (MAP) criterion, the optimum decision rule can be expressed as

$$\frac{\Pr\{y|b_1 = b_2\}\Pr\{b_1 = b_2\}}{\Pr\{y|b_1 \neq b_2\}\Pr\{b_1 \neq b_2\}} \stackrel{\hat{b}=-1}{\underset{\hat{b}=1}{\gtrsim}} 1$$
(4)

Based on (4), we find the optimum decision region  $R_{-1} = \{y | y < -a \cup y > a\}$  for  $\hat{b} = -1$ , and  $R_1 = \{y | -a < y < a\}$  for  $\hat{b} = 1$ , where the decision threshold *a* is the positive root<sup>2</sup> of the following equation:

$$1 + e^{\frac{2a(\sqrt{P_1} + \sqrt{P_2})}{\sigma^2}} = \left(e^{\frac{2a\sqrt{P_1}}{\sigma^2}} + e^{\frac{2a\sqrt{P_2}}{\sigma^2}}\right)e^{\frac{2\sqrt{P_1}\sqrt{P_2}}{\sigma^2}}$$
(5)

The BER of the estimated XORed symbol  $\hat{b}$  using the optimum decision rule is calculated as

$$q_{\hat{b}} = \frac{1}{2} \int_{R_1} \Pr\{y|b_1 = b_2\} dy + \frac{1}{2} \int_{R_{-1}} \Pr\{y|b_1 \neq b_2\} dy$$
$$= \frac{1}{2} \left[ Q(\frac{-a + \sqrt{P_1} + \sqrt{P_2}}{\sigma}) + Q(\frac{-a - \sqrt{P_1} - \sqrt{P_2}}{\sigma}) + Q(\frac{a - \sqrt{P_1} + \sqrt{P_2}}{\sigma}) + Q(\frac{a + \sqrt{P_1} - \sqrt{P_2}}{\sigma}) - 1 \right]$$
(6)

where  $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ . When two users have the same received power, i.e.,  $P_1 = P_2 = P$ , the analytical expression for a can be found as [10]

$$a = \ln\left(\sqrt{\exp(\frac{4P}{\sigma^2}) - 1} + \exp(\frac{2P}{\sigma^2})\right)\frac{\sigma^2}{2\sqrt{P}}$$
(7)

Note that since  $b \in \{-2,0,2\}$  are not equally likely, the decision threshold is a function of the noise power.

**BER Upper Bound for the Equal Received Power Case:** It can be easily seen that a in (7) is a monotonically increasing function of  $\sigma^2$ . We also observe that a is lower bounded as

$$a > \ln\left(\exp(\frac{2P}{\sigma^2})\right)\frac{\sigma^2}{2\sqrt{P}} = \sqrt{P}$$
 (8)

and a converges to  $\sqrt{P}$  as  $\sigma^2 \rightarrow 0$ , i.e.,  $\lim_{\sigma^2 \rightarrow 0} a = \sqrt{P}$ . That is, in the high SNR scenario, the decision threshold approaches to  $\sqrt{P}$  which does not depend on the noise power. When applying  $a = \sqrt{P}$  to all SNR levels, we achieve an upper bound on the error probability as

$$q_{\hat{b}} < q_{\hat{b}}^{upper} = \frac{1}{2} \left[ 3Q(\frac{\sqrt{P}}{\sigma}) + Q(\frac{-3\sqrt{P}}{\sigma}) - 1 \right]$$
(9)

 $^{2}$ It can be shown that the equation has two roots, both are real and their absolute values are same.

Next, we apply the decision rule to the relay in the multiuser two-way system. We rewrite the received signal at the relay in phase one as  $\mathbf{r}_0 = \sum_{k=1}^{K} b_k \mathbf{c}_{k,0} + \mathbf{n}_0$  where  $\mathbf{c}_{k,0} = \sqrt{P_{k,0}} \mathbf{s}_k$ . The relay first employs the MMSE filter to output the soft decision estimate of  $b_l$  for l=1, ..., K. Note that  $E[b_l]=0$  and  $E[b_l^2]=2$  since  $b_l \in \{-2,0,2\}$  with probability  $\{1/4, 1/2, 1/4\}$ . The MMSE filter for  $b_l$  at the relay can be derived as

$$\mathbf{g}_{l,0} = \left(\sum_{k=1}^{K} 2\mathbf{c}_{k,0}\mathbf{c}_{k,0}^{\mathsf{T}} + \sigma_{n_0}^2 \mathbf{I}_N\right)^{-1} \cdot 2\mathbf{c}_{l,0} \qquad (10)$$

and the output of the *l*th MMSE filter is

$$y_{l,0} = \mathbf{g}_{l,0}^{\mathsf{T}} \mathbf{r}_0 = b_l(\mathbf{g}_{l,0}^{\mathsf{T}} \mathbf{c}_{l,0}) + \sum_{k \neq l} b_k \mathbf{g}_{l,0}^{\mathsf{T}} \mathbf{c}_{k,0} + \mathbf{g}_{l,0}^{\mathsf{T}} \mathbf{n}_0 \quad (11)$$

We note that, to generate the estimate of the XORed symbol  $\hat{b}_l$  from  $y_{l,0}$ , the relay still needs to search all  $3^K$  possible  $\{b_l\}_{l=1}^K$  which is computationally prohibitive when K is large. This is because  $b_l \in \{-2, 0, 2\}$  are not uniformly distributed, resulting in a decision threshold that depends on both the interference and the noise power. Since the MMSE receiver is employed, the interference plus noise term can be approximated as a Gaussian random variable [15]  $N_{l,0} \approx \sum_{k \neq l} b_k \mathbf{g}_{l,0}^\mathsf{T} \mathbf{c}_{k,0} + \mathbf{g}_{l,0}^\mathsf{T} \mathbf{n}_0$ , whose variance is

$$\sigma_{N_{l,0}}^2 = \sum_{k \neq l} 2(\mathbf{g}_{l,0}^{\mathsf{T}} \mathbf{c}_{k,0})^2 + \sigma_{n_0}^2 \mathbf{g}_{l,0}^{\mathsf{T}} \mathbf{g}_{l,0}$$
(12)

Thus,  $y_{l,0}$  can be approximated as  $y_{l,0} \approx \sqrt{P'_{l,0}b_l + N_{l,0}}$  where  $P'_{l,0} = (\mathbf{g}_{l,0}^{\mathsf{T}} \mathbf{c}_{l,0})^2$  is the received power of  $b_l$  at the output of the MMSE receiver. This way, the relay can obtain  $\hat{b}_l$  from  $y_{l,0}$  using the decision threshold derived in (7), by replacing P and  $\sigma$  by  $P'_{l,0}$  and  $\sigma_{N_{l,0}}$  respectively. The BER of the estimate  $\hat{b}_l$ ,  $q_{l,0}$ , can be derived from (6) accordingly.

To further simplify the decision threshold, we can use the threshold limit in (8) derived in high SNR scenario. By replacing P and  $\sigma$  in (8) and (9) by  $P'_{l,0}$  and  $\sigma_{N_{l,0}}$ , respectively, we obtain an BER upper bound of the estimated XORed symbol  $\hat{b}_l$  at the relay, which we will evaluate numerically in Section V.

## B. Communication in Phase Two

At the beginning of the second phase, the relay spreads  $\hat{b}_k$ , also a binary symbol, by  $\mathbf{s}_k$  and broadcasts it with power  $P_k$ , for k=1,...,K. Let  $\mathbf{x}_0$  denote the transmitted signal from the relay, which is the sum of K signals, i.e.,

$$\mathbf{x}_0 = \sum_{k=1}^K \sqrt{P_k} \hat{b}_k \mathbf{s}_k \tag{13}$$

Without loss of generality, we consider the signal received at user  $l_1$ , K

$$\mathbf{r}_{l_1} = \sqrt{h_{l_1}} \mathbf{x}_0 + \mathbf{n}_{l_1} = \sum_{k=1}^{N} \hat{b}_k \mathbf{c}_{k,l_1} + \mathbf{n}_{l_1}$$
(14)

where  $\mathbf{c}_{k,l_1} = \sqrt{h_{l_1} P_k} \mathbf{s}_k$ ,  $h_{l_1}$  is the channel gain from the relay to user  $l_1$ , and  $\mathbf{n}_{l_1}$  is the cero-mean Gaussian random vector with  $E[\mathbf{n}_{l_1}\mathbf{n}_{l_1}^{\mathsf{T}}] = \sigma_{n_{l_1}}^2 \mathbf{I}_N$ . The MMSE receiver at user  $l_1$  to detect the binary symbol  $\hat{b}_l$  is

$$\mathbf{g}_{l,l_1} = \left(\sum_{k=1}^{K} \mathbf{c}_{k,l_1} \mathbf{c}_{k,l_1}^{\mathsf{T}} + \sigma_{n_{l_1}}^2 \mathbf{I}_N\right)^{-1} \cdot \mathbf{c}_{l,l_1} \qquad (15)$$

The estimate of  $\hat{b}_l$  at user  $l_1$  is  $\tilde{b}_{l,l_1} = \operatorname{sign}(\mathbf{g}_{l,l_1}^{\mathsf{T}} \mathbf{r}_{l_1})$ . Next, user  $l_1$  performs an XOR operation on  $b_{l,l_1}$  and its own symbol  $b_{l_1}$  to obtain an estimate of its partner's symbol  $b_{l_2}$ , i.e.,  $b_{l_2} = b_{l,l_1} \oplus b_{l_1}$ . If  $b_{l,l_1}$  is correct, user  $l_1$  can correctly recover  $b_{l_2}$ , otherwise,  $b_{l_2}$  is in error. The BER of receiving  $\hat{b}_l$  at user  $l_1$  can be approximated as  $q_{l,l_1} = Q\left(\sqrt{SIR_{l,l_1}}\right)$  with

$$SIR_{l,l_1} = \frac{\left(\mathbf{g}_{l,l_1}^{\mathsf{T}} \mathbf{c}_{l,l_1}\right)^2}{\sum_{k \neq l} \left(\mathbf{g}_{l,l_1}^{\mathsf{T}} \mathbf{c}_{k,l_1}\right)^2 + \sigma_{n_{l_1}}^2 \mathbf{g}_{l,l_1}^{\mathsf{T}} \mathbf{g}_{l,l_1}}$$
(16)

Note from (16) that, the number of interfering users using JD-XOR-F relaying is K-1, while it would have been 2K-1if the traditional CDMA were employed, i.e., if each user used a distinct spreading signature without considering the bidirectional communication structure of the multiuser twoway relay system. Finally, the end-to-end BER of receiving  $b_{l_2}$  at user  $l_1$  using JD-XOR-F relaying scheme is

$$q_{l_2,l_1}^{JD-XOR-F} = (1 - q_{l,0}) \cdot q_{l,l_1} + q_{l,0} \cdot (1 - q_{l,l_1})$$
(17)

## IV. AMPLIFY-AND-FORWARD RELAYING

In the previous section, we assumed that the relay node is able to employ linear multiuser detection. When the relay node has limited computational capability, a valid alternative is amplify-and-forward (AF) relaying. Upon receiving  $r_0$ , the relay node broadcasts a signal  $\mathbf{x}_0 = \alpha \cdot \mathbf{r}_0$  in the second phase, where the scalar  $\alpha$  is chosen such that the total transmit power of the relay node is  $P_{R,total}$ , i.e.,

$$\alpha = \sqrt{\frac{P_{R,total}}{\sum_{k=1}^{K} 2P_{k,0} + \sigma_{n_0}^2 N}}$$
(18)

The signal received at user  $l_1$  is

$$\mathbf{r}_{l_1} = \sqrt{h_{l_1}} \mathbf{x}_0 + \mathbf{n}_{l_1} = \sum_{k=1}^{K} b_k \mathbf{c}_{k,l_1} + \mathbf{n}'_{l_1}$$
(19)

where  $\mathbf{c}_{k,l_1} = \alpha \sqrt{h_{l_1} P_{k,0}} \mathbf{s}_k$ ,  $\mathbf{n}'_{l_1} = \alpha \sqrt{h_{l_1}} \mathbf{n}_0 + \mathbf{n}_{l_1}$  and  $E[\mathbf{n}'_{l_1}\mathbf{n}'_{l_1}^{\mathsf{T}}] = \sigma^2_{n'_{l_1}} \mathbf{I}_N$  with  $\sigma^2_{n'_{l_1}} = \alpha^2 h_{l_1} \sigma^2_{n_0} + \sigma^2_{n_{l_1}}$ . Knowing its own symbol  $\dot{b}_{l_1}$ , user  $l_1$  can subtract its self-interference  $b_{l_1}\mathbf{c}_{l,l_1}$  from  $\mathbf{r}_{l_1}$  before recovering its partner's symbol  $b_{l_2}$ . The received signal after this subtraction becomes

$$\mathbf{r}_{l_1}' = b_{l_2} \mathbf{c}_{l,l_1} + \sum_{k \neq l} b_k \mathbf{c}_{k,l_1} + \mathbf{n}_{l_1}'$$
(20)

The MMSE receiver to recover  $b_{l_2}$  at user  $l_1$  is thus

$$\mathbf{g}_{l_2,l_1} = \left(\sum_{k=1}^{K} 2\mathbf{c}_{k,l_1} \mathbf{c}_{k,l_1}^{\mathsf{T}} - \mathbf{c}_{l,l_1} \mathbf{c}_{l,l_1}^{\mathsf{T}} + \sigma_{n_{l_1}'}^2 \mathbf{I}_N\right)^{-1} \cdot \mathbf{c}_{l,l_1}$$
(21)

User  $l_1$  can obtain the estimate of  $b_{l_2}$  directly from  $\mathbf{r}'_{l_1}$  as  $\tilde{b}_{l_2} = \operatorname{sign}(\mathbf{g}_{l_2,l_1}^{\mathsf{T}}\mathbf{r}'_{l_1})$ . The BER of receiving  $b_{l_2}$  at user  $l_1$  using AF relaying can be approximated as

$$q_{l_2,l_1}^{AF} = Q(\sqrt{SIR_{l_2,l_1}}) \tag{22}$$

where

here  

$$SIR_{l_2,l_1} = \frac{\left(\mathbf{g}_{l_2,l_1}^{\mathsf{T}} \mathbf{c}_{l,l_1}\right)^2}{\sum_{k \neq l} 2\left(\mathbf{g}_{l_2,l_1}^{\mathsf{T}} \mathbf{c}_{k,l_1}\right)^2 + \sigma_{n_{l_1}}^2 \mathbf{g}_{l_2,l_1}^{\mathsf{T}} \mathbf{g}_{l_2,l_1}} \quad (23)$$
V. NUMERICAL RESULTS

In this section, we present numerical results demonstrating the BER performance of the proposed multiuser two way relaying strategies. First, we consider a system having K pairs of users where  $K \in [3, 11]$  with spreading gain N=20. We assume that the channel gains follow the path-loss model, i.e.,  $h_{k_m} = d_{k_m}^{-\alpha}$  for k=1,...,K and m=1,2, where  $d_{k_m}$ , the distance between user  $k_m$  and the relay, is set as 500m for all users, and  $\alpha$  is set as 4. The variance of all AWGN terms is set to  $10^{-13}$ . We simulate the system to obtain the endto-end BER which is averaged over 100 sets of randomly generated sequences, and the BER of each set of sequences is averaged over 1000 realizations of the AWGN channels. We also calculate the approximate BER using the corresponding Q-function expressions, denoted by the subscript "approx." in Figure 2 and 3. We observe that the BER approximations match the simulation results.

In Figure 2, we plot the end-to-end BER of a user receiving its partner's symbol in the multiuser two-way relay system using different relaying schemes. The transmit power of each user in phase one is set as 0.0625 Watts for all schemes, so that the received SNR of each user symbol at the relay is 10dB. The relay transmit power in phase two is 0.0625 Watts for each XORed symbol in JD-XOR-F scheme, 0.0625 Watts for each user symbol in the traditional CDMA, and  $2K \times 0.0625$  Watts as the total relay power in the AF scheme. We observe that the proposed JD-XOR-F scheme provides a remarkable BER performance gain upon traditional CDMA as a pair of users share a common signature so that the relay can jointly demodulate them and transmit one binary symbol for them. The AF scheme outperforms traditional CDMA as K increases beyond 6, since the AF scheme acts as one-hop communication with an enlarged noise term which no longer dominates the BER when MAI becomes larger. The JD-XOR-F scheme outperforms the AF scheme due to the fact that the former reduces the MAI by transmitting an XORed symbol from the relay for each user pair in the second phase.

In Figure 3, we compare the BER of the JD-XOR-F scheme with its upper bound obtained by using the high SNR decision threshold limit at the relay, for different user transmit power levels such that the received SNR (RX-SNR) of each user at the relay is  $\{7, 8.5, 10\}$ dB in phase one, while the relay transmit power remains the same as that in Figure 2. We observe that as the user transmit power gets larger, the upper bound becomes tighter. In this scenario, applying the high SNR threshold limit results in negligible performance loss while it largely reduces the complexity of the decision threshold.

Next, in Figure 4 we evaluate the maximum number of pairs of users,  $K_{max}$ , that the system can support with an end-to-end BER that is no greater than the system requirement of 0.01, for different processing gain  $N = \{8, 16, 24, 32, 40\}$ , using JD-



XOR-F two-way relaying and traditional CDMA, respectively. The transmit power levels are the same as those in Figure 2. We observe that the multiuser two-way relay system with JD-XOR-F scheme can support almost twice of the number of user pairs that can be supported by the traditional CDMA. This is expected as the multiuser two-way relay system utilizes the bi-directional communication structure and needs only half number of the signatures compared with the traditional CDMA.

## VI. CONCLUSIONS

In this paper, we have proposed multiuser two-way relaying strategies for interference limited systems. The proposed system model enables bidirectional communication between multiple pairs of users via a shared intermediate relay node. We have shown that using a spread spectrum multiple access technique, in conjunction with judicious choices of transmission, reception, and relaying, can achieve remarkable improvement upon traditional settings in terms of BER performance.

Bidirectional communication scenarios, appropriate especially given the promising future of wireless ad hoc networks, raise interesting questions in system design, particularly pertaining to the design of lower network layers. We took one representative scenario with a shared relay and addressed physical layer design issues by adopting CDMA as the multiple access technique. We believe that interesting observations and revealing insights remain to be discovered for general multiuser two-way communication systems on various design aspects, such as resource allocation and partner pairing.

#### REFERENCES

- P. Larsson, N. Johansson, and K.-E. Sunell, "Coded bi-directional relaying," in *Proceedings of the IEEE 63rd Vehicular Technology Conference*, VTC'06-Spring, May 2006.
- [2] Y. Wu, P. A. Chou, and S. Kung, "Information exchange in wireless networks with network coding and physical-layer broadcast," in *Proceedings of the Conference on Information Sciences and Systems*, CISS'05, March 2005.
- [3] B. Rankov and A. Wittneben, "Achievable rate regions for the two-way relay channel," in *Proceedings of the IEEE International Symposium on Information Theory, ISIT'06*, July 2006.
- [4] T. J. Oechtering, C. Schnurr, I. Bjelakovic, and H. Boche, "Achievable rate region of a two phase bidirectional relay channel," in *Proceedings* of the 41st Annual Conference on Information Sciences and Systems, CISS'07, March 2007.



Figure 4. Maximum number of user pairs that can be supported with end-to-end BER  $\leq 0.01$ .

- [5] S. Katti, I. Maric, A. Goldsmith, D. Katabi, and M. Medard, "Joint relaying and network coding in wireless networks," in *Proceedings of the IEEE International Symposium on Information Theory, ISIT'07*, June 2007.
- [6] S. J. Kim, P. Mitran, and V. Tarokh, "Performance bounds for bi-directional coded cooperation protocols." [Online]. Available: http://arxiv.org/abs/cs/0703017
- [7] B. Rankov and A. Wittneben, "Spectral efficient protocols for halfduplex fading relay channels," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 2, pp. 379–389, February 2007.
- [8] S. Katti, S. Gollakota, and D. Katabi, "Embracing wireless interference: Analog network coding," in *Proceedings of the ACM SIGCOMM*'07, 2007.
- [9] P. Popovski and H. Yomo, "Wireless network coding by amplify-andforward for bi-directional traffic flows," *IEEE Communications Letters*, vol. 11, no. 1, pp. 16–18, January 2007.
- [10] S. Zhang, S. Liew, and P. P. Lam, "Physical-layer network coding," in Proceedings of the ACM MOBICOM'06, 2006.
- [11] P. Popovski and H. Yomo, "The anti-packets can increase the achievable throughput of a wireless multi-hop network," in *Proceedings of the IEEE International Conference on Communications, ICC'06*, June 2006.
- [12] A. Goldsmith, Wireless Communications. Cambridge University Press, 2005.
- [13] L. Sanjay and E. S. Sousa, "Distributed resource allocation for DS-CDMA-based multimedia ad hoc wireless LANs," *IEEE Journal on Selected Areas in Communications*, vol. 17, no. 5, pp. 947–967, May 1999.
- [14] M. Chen and A. Yener, "Interference management for multiuser two-way relaying," in *Proceedings of the Conference on Information Sciences and Systems, CISS'08*, March 2008.
- [15] S. Verdú, Multiuser Detection. Cambridge University Press, 1998.