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Power Allocation for Multi-Access Two-Way Relaying

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***Abstract*—We consider a multi-access two-way relay network where multiple pairs of users exchange information with their pre-assigned partners with the assistance of an intermediate relay node. Each pair is assumed to have a shared channel which is orthogonal to the channels used by the remaining pairs. We investigate the relay power allocation problem for two-way relaying protocols that allow a variety of forwarding mechanisms, such as decode-and-superposition-forward (DSF), decode-and-XOR-forward (DXF), amplify-and-forward (AF) and compress-and-forward (CF). Different from one-way communications, in two-way relaying, the rates of the two communication directions between a pair of partners constrain each other, and the relay power allocated to one user pair simultaneously affects the rates of both directions. For each relaying scheme, we solve the problem of optimally allocating relay's power among the user pairs such that an arbitrary weighted sum rate of all users is maximized. Simulation results are presented to demonstrate performance of optimum relay power allocation, as well as the comparison among different two-way relaying schemes.**

I. Introduction

Two-way relay networks, where nodes exchange information via the help of the relay node(s), have recently brought new interests to the research on relay-assisted communications, due to their ability of improving the spectral efficiency upon one-way relaying. A variety of two-way relaying protocols have been proposed to date that allow two nodes exchange information over a relay in an efficient manner, e.g., [1]–[5].

While most work on two-way relaying concentrates on single pair of partners, two-way relaying is naturally expected to improve the system performance in multiuser scenarios. Reference [2] considers multiple pairs of users communicating via a sufficiently large number of relays such that the transmissions between different pairs are orthogonalized by zero forcing. Earlier, we have studied an uncoded interference limited two-way relay network with multiple user pairs assisted by a shared relay node [6]–[8]. In that scenario, we considered nodes employing code division multiple access (CDMA) with non-orthogonal spreading sequences to support simultaneous communications, and designed the jointly optimal relaying scheme, transmit power and receiver structure of all communicating nodes. While our earlier work in [6]–[8] advocated the transmission and resource management strategies for this practical interference limited multiple access system, we must recognize that under various scenarios and availability of bandwidth, when the nodes have limited computational capability or multiuser detection is undesired, using orthogonal channels to avoid interference is a valid design choice. Therefore, in this paper, we focus on the communication scenario where multiple user pairs communicate with the relay's help, using orthogonal channels per pair, and study this *multi-access two-way relay network*.

In ad hoc networks where nodes are powered by battery and frequent battery replacement is undesired, power efficiency becomes a critical concern, especially for a shared relay node that serves multiple user pairs that wish to communicate. Optimum relay power allocation strategies for such scenarios have been studied in references [9], [10] for *unidirectional* relay-assisted communications.

In this paper, we investigate the optimum relay power allocation problem for the multi-access two-way relay networks. We consider decode-and-forward type of relaying protocols, where the relay broadcasts a superposition or an XORed version of the messages of each user pair after decoding them, termed decode-and-superposition-forward (DSF), and decode-and-XOR-forward (DXF) relaying, respectively. We also consider relay cooperation without decoding, namely, amplify-and-forward (AF) and compress-and-forward (CF) two-way relaying. For each scheme, we address the problem of optimally allocating relay's power among the user pairs such that an arbitrary weighted sum rate of all users is maximized. We show that the power allocation algorithm for one-way communications derived in [10] has a direct application only for three-phase DSF two-way relaying, but does not apply any of the other schemes. This is due to the fact that the rates of the two communication directions between two partners constrain each other in the two-phase DSF and DXF relaying schemes, and the relay power allocated to one user pair simultaneously affects the rates of both directions in the DXF, AF and CF schemes. We formulate and solve the power allocation problem as one or a set of convex problems for each relaying scheme. In the numerical results section, we demonstrate performance of optimum relay power allocation and the comparison among different two-way relaying schemes.

II. System Description

We consider a multi-access two-way relay network shown in Figure 1, which consists of K pairs of users and an intermediate relay node. User a_i and b_i ($i \in [1, K]$) are a

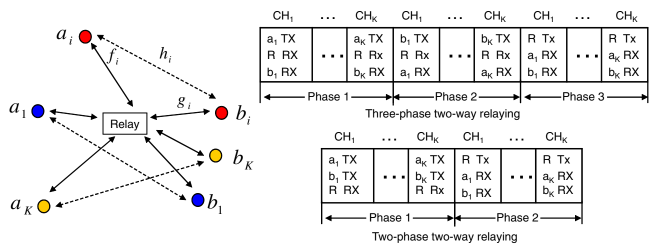

Figure 1. System model.

pair of pre-assigned partners who wish to communicate with each other. The relay node is willing to devote its resources to assist the communications of all the user pairs. We assume that the users and the relay node are half-duplex and each equipped with one antenna. We assume reciprocal channels and denote f_i, g_i and h_i the channel coefficients of the links between a_i and the relay, b_i and the relay, and a_i and b_i on the ith channel, and assume all channels stay constant for the duration of the communication. Without loss of generality, at each receiver in different phase, we assume i.i.d. additive white Gaussian noise $n_{rx,p} \sim \mathcal{CN}(0,1)$, where $rx \in \{a_i, b_i, r_i\}$ denotes the receiver on the ith channel and $p \in \{1,2,3\}$ denotes the phase.

III. Two-way relaying schemes and their achievable rate regions

We study three- and two-phase two-way relaying strategies. Each user pair is assigned an orthogonal channel in each phase, by means of non-overlapping time/frequency slots with equal time duration/bandwidth, as shown in Figure 1.

In three-phase protocols, user a_i transmits, and its partner b_i and the relay listen in the ith channel in phase one. The signals received at the relay and at the user b_i are:

$$y_{r_i,1} = f_i\sqrt{P_{a_i}}x_{a_i} + n_{r_i,1}, \quad y_{b_i,1} = h_i\sqrt{P_{a_i}}x_{a_i} + n_{b_i,1} \tag{1}$$

Similarly, user b_i transmits in the second phase, and the signals received at the relay and at the user a_i on the ith channel are:

$$y_{r_i,2} = g_i\sqrt{P_{b_i}}x_{b_i} + n_{r_i,2}, \quad y_{a_i,2} = h_i\sqrt{P_{b_i}}x_{b_i} + n_{a_i,2} \tag{2}$$

where $\sqrt{P_{a_i}}x_{a_i}$ and $\sqrt{P_{b_i}}x_{b_i}$ denote the transmitted signals of users a_i and b_i with transmit power P_{a_i} and P_{b_i}, and x_{a_i} and x_{b_i} are drawn from Gaussian codebooks.

Upon receiving the signals, the relay decodes the messages m_{a_i} and m_{b_i}. To help forward the messages in phase three, it can employ DSF, i.e., re-encode the messages individually and transmit $t_{r_i} = \sqrt{P_{r_{a_i}}}x_{r_{a_i}} + \sqrt{P_{r_{b_i}}}x_{r_{b_i}}$; or employ DXF [3], i.e., encode the message $m_i = m_{a_i} \oplus m_{b_i}$ and transmit $t_{r_i} = \sqrt{P_{r_i}}x_{r_i}$, where $P_{r_{a_i}}$, $P_{r_{b_i}}$ and P_{r_i} denote the corresponding relay transmit power, and $x_{r_{a_i}}$, $x_{r_{b_i}}$ and x_{r_i} are drawn from Gaussian codebooks. The received signals at the users in phase three are

$$y_{a_i,3} = f_i t_{r_i} + n_{a_i,3}, \; y_{b_i,3} = g_i t_{r_i} + n_{b_i,3} \tag{3}$$

Knowing its own message, each user can subtract its self-interference from the received signal and decode its partner's message. We denote R_{ab_i} and R_{ba_i} the rate from user a_i to b_i and from b_i to a_i, respectively. Note that when the direct link has higher channel gain than the user-to-relay link, i.e., $|f_i|^2 \leq |h_i|^2$ or $|g_i|^2 \leq |h_i|^2$, the direct transmission achieves higher rate than the relay assisted transmission. We define sets S_a and S_b where $S_a = \{i||f_i|^2 > |h_i|^2\}$ and $S_b = \{i||g_i|^2 > |h_i|^2\}$, i.e., relay-assisted transmission can increase R_{ab_i} and R_{ba_i} for user pairs in S_a and S_b, respectively. Thus, we can describe the achievable rate region for 3pDSF when different codebooks are used at the relay and the users as:

$$R_{ab_i} \leq \begin{cases} \frac{1}{3K}\min(C(P_{a_i}|f_i|^2), C(P_{a_i}|h_i|^2) + C(P_{r_{a_i}}|g_i|^2)), \\ \qquad\qquad \forall i \in S_a \\ \frac{1}{3K}C(P_{a_i}|h_i|^2), \text{otherwise} \end{cases} \tag{4}$$

$$R_{ba_i} \leq \begin{cases} \frac{1}{3K}\min(C(P_{b_i}|g_i|^2), C(P_{b_i}|h_i|^2) + C(P_{r_{b_i}}|f_i|^2)), \\ \qquad\qquad \forall i \in S_b \\ \frac{1}{3K}C(P_{b_i}|h_i|^2), \text{otherwise} \end{cases} \tag{5}$$

where $C(x) = \log(1+x)$. The achievable rate region for 3pDXF relaying can be obtained by replacing both $P_{r_{a_i}}$ and $P_{r_{b_i}}$ in (4) and (5) by P_{r_i} [3].

Different from the three-phase schemes, in the two-phase protocols, two nodes a_i and b_i transmit simultaneously on the ith channel in phase one. The received signal at the relay is:

$$y_{r_i,1} = f_i\sqrt{P_{a_i}}x_{a_i} + g_i\sqrt{P_{b_i}}x_{b_i} + n_{r_i,1} \tag{6}$$

Since users are half-duplex nodes, they do not hear from each other on the direct link in phase one. In the second phase, the relay can employ DSF or DXF relaying after decoding m_{a_i} and m_{b_i}, same as in the three-phase schemes. The achievable rate region for 2pDSF is described as [2], [4]:

$$R_{ab_i} \leq \frac{1}{2K}\min(C(P_{a_i}|f_i|^2), C(P_{r_{a_i}}|g_i|^2)), \forall i \tag{7}$$

$$R_{ba_i} \leq \frac{1}{2K}\min(C(P_{b_i}|g_i|^2), C(P_{r_{b_i}}|f_i|^2)), \forall i \tag{8}$$

$$R_{ab_i} + R_{ba_i} \leq \frac{1}{2K}C(P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2), \forall i \tag{9}$$

The achievable rate region for 2pDXF relaying can be obtained by replacing both $P_{r_{a_i}}$ and $P_{r_{b_i}}$ in (7) and (8) by P_{r_i} [3].

Instead of decoding, in phase two the relay can amplify and forward the received signal from phase one, and the signal transmitted by the relay is $t_{r_i} = \alpha_i y_{r_i,1}$ where α_i is a scalar such that the relay's transmit power for the ith user pair is P_{r_i}, i.e., $\alpha_i = \sqrt{\frac{P_{r_i}}{P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2 + 1}}$. The achievable rate region for AF relaying is given as [2]:

$$R_{ab_i} \leq \frac{1}{2K}C(\frac{P_{r_i}|f_i|^2|g_i|^2 P_{a_i}}{P_{r_i}|g_i|^2 + P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2 + 1}), \forall i \tag{10}$$

$$R_{ba_i} \leq \frac{1}{2K}C(\frac{P_{r_i}|f_i|^2|g_i|^2 P_{b_i}}{P_{r_i}|f_i|^2 + P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2 + 1}), \forall i \tag{11}$$

Similarly, the relay can compress the received signal and broadcast the quantized version $\hat{x}_{r_i}$ with power P_{r_i}, i.e., $t_{r_i} = \sqrt{P_{r_i}}\hat{x}_{r_i}$. The achievable rate region for CF is given as [5]:

$$R_{ab_i} \leq \frac{1}{2K}C(\frac{P_{r_i}|f_i|^2|g_i|^2 P_{a_i}}{P_{r_i}|g_i|^2 + P_{a_i}|f_i|^2 + 1}), \forall i \tag{12}$$

$$R_{ba_i} \leq \frac{1}{2K}C(\frac{P_{r_i}|f_i|^2|g_i|^2 P_{b_i}}{P_{r_i}|f_i|^2 + P_{b_i}|g_i|^2 + 1}), \forall i \tag{13}$$

IV. The Relay Power Allocation Problem

In this section, we formulate and solve the relay power allocation problem for each two-way relaying protocol. The optimum power allocation problem is posed as allocating the relay power to different user pairs such that an arbitrary weighted sum rate of all user pairs is maximized. Specifically,

$$\max_{\{P_{r_i}\}_{i=1}^K} \quad \sum_{i=1}^K (w_{ab_i} R_{ab_i} + w_{ba_i} R_{ba_i}) \tag{14}$$

$$s.t. \quad \sum_{i=1}^K P_{r_i} \leq P_{r,total} \tag{15}$$

$$\{R_{ab_i}, R_{ba_i}\} \in R_* \tag{16}$$

where $P_{r,total}$ is the given total relay power, R_* is the achievable rate region of one of the relaying schemes described in Section III. For DSF schemes the optimization variables are $\{P_{r_{a_i}}, P_{r_{b_i}}\}$ instead of $\{P_{r_i}\}$. The non-negative weights $\{w_{ab_i}, w_{ba_i}\}$ are to indicate the priority of the traffic amongst different directions and pairs, a larger wight indicating priority. The resulting weighted sums for all $\{w_{ab_i}, w_{ba_i}\}$ clearly allow us to trace the boundary of the achievable rate region.

A. Three-Phase Decode-and-Superposition-Forward (3pDSF)

Note that the relay power should only be allocated to user pairs that belong to sets S_a or S_b defined in Section III. The power allocation problem can be expressed as:

$$\max_{\{P_{r_{a_i}}, P_{r_{b_i}}\}_{i=1}^K} \sum_{i\in S_a} \frac{w_{ab_i}}{3K}\left(C(P_{a_i}|h_i|^2) + C(P_{r_{a_i}}|g_i|^2)\right) + \sum_{i\in S_b} \frac{w_{ba_i}}{3K}\left(C(P_{b_i}|h_i|^2) + C(P_{r_{b_i}}|f_i|^2)\right) \tag{17}$$

$$s.t. \quad \sum_{i=1}^K P_{r_{a_i}} + P_{r_{b_i}} \leq P_{r,total} \tag{18}$$

$$C(P_{a_i}|h_i|^2) + C(P_{r_{a_i}}|g_i|^2) \leq C(P_{a_i}|f_i|^2), \ \forall i \in S_a \tag{19}$$

$$C(P_{b_i}|h_i|^2) + C(P_{r_{b_i}}|f_i|^2) \leq C(P_{b_i}|g_i|^2), \ \forall i \in S_b \tag{20}$$

$$P_{r_{i_a}}, P_{r_{i_b}} \geq 0, \ \forall i; \ P_{r_{i_a}} = 0, \forall i \notin S_a; \ P_{r_{i_b}} = 0, \forall i \notin S_b \tag{21}$$

Note that 3pDSF is an extension of the one-way relaying philosophy, which superposes the codewords to both directions in phase three. Therefore, the traffic in the two directions can be considered independently, and hence, this problem has the same form as that of the one-way nonregenerative decode-and-forward relaying in [10]. Therefore, we have the similar form of the modified water-filling solution as in [10] and the optimum relay power allocation is obtained as:

$$P_{r_{a_i}} = \min\left((\tfrac{w_{ab_i}}{\mu_0} - \tfrac{1}{|g_i|^2})^+, P^{max}_{r_{a_i},3p}\right), \forall i \in S_a \tag{22}$$

$$P_{r_{b_i}} = \min\left((\tfrac{w_{ba_i}}{\mu_0} - \tfrac{1}{|f_i|^2})^+, P^{max}_{r_{b_i},3p}\right), \forall i \in S_b \tag{23}$$

$$P_{r_{i_a}} = 0, \forall i \notin S_a; \quad P_{r_{i_b}} = 0, \forall i \notin S_b \tag{24}$$

where $P^{max}_{r_{a_i},3p} = \frac{P_{a_i}(|f_i|^2 - |h_i|^2)}{|g_i|^2(1+P_{a_i}|h_i|^2)}$, $P^{max}_{r_{b_i},3p} = \frac{P_{b_i}(|g_i|^2 - |h_i|^2)}{|f_i|^2(1+P_{b_i}|h_i|^2)}$ result from the constraints (19) and (20), and μ_0 is the Lagrangian multiplier satisfying the constraint (18) with equality.

B. Two-Phase Decode-and-Superposition-Forward (2pDSF)

Recalling the achievable rate region for 2pDSF relaying in (7)-(9), we can express the power allocation problem as:

$$\max_{\{P_{r_{a_i}}, P_{r_{b_i}}\}_{i=1}^K} \quad \sum_{i=1}^K \frac{w_{ab_i}}{2K} C(P_{r_{a_i}}|g_i|^2) + \frac{w_{ba_i}}{2K} C(P_{r_{b_i}}|f_i|^2) \tag{25}$$

$$s.t. \quad \sum_{i=1}^K P_{r_{a_i}} + P_{r_{b_i}} \leq P_{r,total}, \tag{26}$$

$$C(P_{r_{a_i}}|g_i|^2) \leq C(P_{a_i}|f_i|^2), P_{r_{a_i}} \geq 0, \forall i \tag{27}$$

$$C(P_{r_{b_i}}|f_i|^2) \leq C(P_{b_i}|g_i|^2), P_{r_{b_i}} \geq 0, \forall i \tag{28}$$

$$C(P_{r_{a_i}}|g_i|^2) + C(P_{r_{b_i}}|f_i|^2) \leq C(P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2), \forall i \tag{29}$$

Constraint (29) is nonconvex set over $(P_{r_{a_i}}, P_{r_{b_i}})$. Fortunately, a simple change of variables overcomes this hardship. Let $q_{a_i} = C(P_{r_{a_i}}|g_i|^2), q_{b_i} = C(P_{r_{b_i}}|f_i|^2)$, and the problem becomes

$$\max_{\{q_{a_i}, q_{b_i}\}_{i=1}^K} \quad \frac{1}{2K}\sum_{i=1}^K w_{ab_i} q_{a_i} + w_{ba_i} q_{b_i} \tag{30}$$

$$s.t. \quad \sum_{i=1}^K (e^{q_{a_i}} - 1)/|g_i|^2 + (e^{q_{b_i}} - 1)/|f_i|^2 \leq P_{r,total} \tag{31}$$

$$0 \leq q_{a_i} \leq C(P_{a_i}|f_i|^2), \forall i \tag{32}$$

$$0 \leq q_{b_i} \leq C(P_{b_i}|g_i|^2), \forall i \tag{33}$$

$$q_{a_i} + q_{b_i} \leq C(P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2), \forall i \tag{34}$$

It can be easily verified that the above problem is convex, and therefore the optimum solution can be found via convex optimization techniques [11]. In [12], we provide a subgradient optimization based iterative algorithm that converges to the optimum solution of (30)-(34) which can also be applied for the convex problems in sections IV-C, IV-D and IV-E.

C. Three-Phase Decode-and-XOR-Forward (3pDXF)

Unlike the DSF scheme where the portions of the relay power for forwarding the messages to two partners individually control the rates on two directions, in DXF relaying, the relay forwards a single XORed message with power P_{r_i} for a pair of users, and consequently, P_{r_i} simultaneously controls the rate on both directions.

Similar as in 3pDSF two-way relaying, the relay assisted transmission only helps users in S_a or S_b and hence no power is allocated to users in $\overline{S_a \cup S_b}$. We let $S_{ab} = S_a \cap S_b$ and further partition S_{ab} as S_{ab1} and S_{ab2}, where $S_{ab1} = \{i | i \in S_{ab} \text{ and } P^{max}_{r_{a_i},3p} \geq P^{max}_{r_{b_i},3p}\}$, and $S_{ab2} = S_{ab} \backslash S_{ab1}$ with $P^{max}_{r_{a_i},3p}$ and $P^{max}_{r_{b_i},3p}$ given in Section IV-A. In set S_{ab1}, increasing P_{r_i} beyond $P^{max}_{r_{b_i},3p}$ but below $P^{max}_{r_{a_i},3p}$ will increase the data rate R_{ab_i} but not R_{ba_i} since it has reached the upper bound. Similarly, in set S_{ab2}, increasing P_{r_i} beyond $P^{max}_{r_{a_i},3p}$ but below $P^{max}_{r_{b_i},3p}$ will increase R_{ba_i} but not R_{ab_i}. We also define sets $S'_a = S_a \backslash S_{ab}$ and $S'_b = S_b \backslash S_{ab}$, and observe that the relay can only increase R_{ab_i} for user pairs in S'_a and only increase R_{ba_i} for user pairs in S'_b. Having the above observations, we can express the relay power allocation problem for 3pDXF two-way relaying as:

$$\max_{\{P_{r_i}, \hat{P}_{r_i}\}_{i=1}^K} \sum_{i\in S_{ab1}} \left(\frac{w_{ab_i}}{3K} C(P_{r_i}|g_i|^2) + \frac{w_{ba_i}}{3K} C(\hat{P}_{r_i}|f_i|^2)\right) + \sum_{i\in S_{ab2}} \left(\frac{w_{ab_i}}{3K} C(\hat{P}_{r_i}|g_i|^2) + \frac{w_{ba_i}}{3K} C(P_{r_i}|f_i|^2)\right) + \sum_{i\in S'_a} \frac{w_{ab_i}}{3K} C(P_{r_i}|g_i|^2) + \sum_{i\in S'_b} \frac{w_{ba_i}}{3K} C(P_{r_i}|f_i|^2) \tag{35}$$

$$s.t. \sum_{i=1}^{K} P_{r_i} \leq P_{r,total} \quad (36)$$

$$\hat{P}_{r_i} \leq P_{r_i}, \hat{P}_{r_i} \leq P^{max}_{r_{b_i},3p}, P_{r_i} \leq P^{max}_{r_{a_i},3p}, \forall i \in S_{ab1} \quad (37)$$

$$\hat{P}_{r_i} \leq P_{r_i}, \hat{P}_{r_i} \leq P^{max}_{r_{a_i},3p}, P_{r_i} \leq P^{max}_{r_{b_i},3p}, \forall i \in S_{ab2} \quad (38)$$

$$P_{r_i} \leq P^{max}_{r_{a_i},3p}, \forall i \in S'_a; \quad P_{r_i} \leq P^{max}_{r_{b_i},3p}, \forall i \in S'_b \quad (39)$$

$$\hat{P}_{r_i} \geq 0, P_{r_i} \geq 0, \forall i; \quad P_{r_i} = 0, \forall i \in \overline{S_a \cup S_b} \quad (40)$$

Note that, in the above problem, we omit the constant terms $C(P_{a_i}|h_i|^2)$ and $C(P_{b_i}|h_i|^2)$ in the objective function for simplicity. We use the variables $\{\hat{P}_{r_i}\}$ to ensure that the upper bound of R_{ba_i} of user pairs in S_{ab1} and that of R_{ab_i} of user pairs in S_{ab2} are not violated in the problem formulation. This way, the optimum power allocation problem is easily seen to be convex and can be solved. Note that in the optimum solution, $\hat{P}_{r_i} = \min(P_{r_i}, P^{max}_{r_{b_i},3p})$ for $i \in S_{ab1}$ and $\hat{P}_{r_i} = \min(P_{r_i}, P^{max}_{r_{a_i},3p})$ for $i \in S_{ab2}$.

D. Two-Phase Decode-and-XOR-Forward (2pDXF)

We first define the thresholds $P^{max}_{r_{a_i},2p} = P_{a_i}|f_i|^2/|g_i|^2$ and $P^{max}_{r_{b_i},2p} = P_{b_i}|g_i|^2/|f_i|^2$. We note that the relay assisted transmission can potentially increase R_{ab_i} when $P_{r_i} \leq P^{max}_{r_{a_i},2p}$, and R_{ba_i} when $P_{r_i} \leq P^{max}_{r_{b_i},2p}$, under the sum rate constraint $R_{ab_i} + R_{ba_i} \leq C(P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2)/2K$. This sum rate constraint is equivalent to $P_{r_i} \leq P^{max}_{r_i,2p}$ with

$$P^{max}_{r_i,2p} = \begin{cases} P^{maxa}_{r_i,2p}, \text{ if } P_{r_i} \geq \min(P^{max}_{r_{a_i},2p}, P^{max}_{r_{b_i},2p}) = P^{max}_{r_{b_i},2p} \\ P^{maxb}_{r_i,2p}, \text{ if } P_{r_i} \geq \min(P^{max}_{r_{a_i},2p}, P^{max}_{r_{b_i},2p}) = P^{max}_{r_{a_i},2p} \\ P^{maxc}_{r_i,2p}, \text{ if } P_{r_i} < \min(P^{max}_{r_{a_i},2p}, P^{max}_{r_{b_i},2p}) \end{cases} \quad (41)$$

where $P^{maxa}_{r_i,2p} = P_{a_i}|f_i|^2/|g_i|^2(1 + P_{b_i}|g_i|^2)$, $P^{maxb}_{r_i,2p} = P_{b_i}|g_i|^2/|f_i|^2(1 + P_{a_i}|f_i|^2)$ and $P^{maxc}_{r_i,2p} = \frac{-(|g_i|^2+|f_i|^2)+\sqrt{(|g_i|^2+|f_i|^2)^2+4|g_i|^2|f_i|^2(P_{a_i}|f_i|^2+P_{b_i}|g_i|^2)}}{2|g_i|^2|f_i|^2}$.

Next, we partition all user pairs as sets $\tilde{S}_{ab}$, $\tilde{S}_a$ and $\tilde{S}_b$, where $\tilde{S}_{ab} = \{i|P^{maxc}_{r_i,2p} \leq \min(P^{max}_{r_i,2p}, P^{max}_{r_{b_i},2p})\}$, $\tilde{S}_a = \{i|P^{maxc}_{r_i,2p} > \min(P^{max}_{r_{a_i},2p}, P^{max}_{r_{b_i},2p}) = P^{max}_{r_{b_i},2p}\}$ and $\tilde{S}_b = \{i|P^{maxc}_{r_i,2p} > \min(P^{max}_{r_{a_i},2p}, P^{max}_{r_{b_i},2p}) = P^{max}_{r_{a_i},2p}\}$. We also define sets $\tilde{S}_{a1} \subseteq \tilde{S}_a$, $\tilde{S}_{b1} \subseteq \tilde{S}_b$, $\tilde{S}_{a2} = \tilde{S}_a \backslash \tilde{S}_{a1}$ and $\tilde{S}_{b2} = \tilde{S}_b \backslash \tilde{S}_{b1}$. The relay power allocation problem can thus be expressed as:

$$\max_{\{P_{r_i}\},\tilde{S}_{a1},\tilde{S}_{b1}} \sum_{i \in \tilde{S}_{ab} \cup \tilde{S}_{a1} \cup \tilde{S}_{b1}} \frac{w_{ab_i}}{2K} C(P_{r_i}|g_i|^2) + \frac{w_{ba_i}}{2K} C(P_{r_i}|f_i|^2)$$

$$+ \sum_{i \in \tilde{S}_{a2}} \frac{w_{ab_i}}{2K} C(P_{r_i}|g_i|^2) + \sum_{i \in \tilde{S}_{b2}} \frac{w_{ba_i}}{2K} C(P_{r_i}|f_i|^2) \quad (42)$$

$$s.t. \quad \sum_{i=1}^{K} P_{r_i} \leq P_{r,total} \quad (43)$$

$$0 \leq P_{r_i} \leq P^{maxc}_{r_i,2p}, \forall i \in \tilde{S}_{ab} \quad (44)$$

$$0 \leq P_{r_i} \leq P^{max}_{r_{b_i},2p}, \forall i \in \tilde{S}_{a1}; \quad 0 \leq P_{r_i} \leq P^{max}_{r_{a_i},2p}, , \forall i \in \tilde{S}_{b1} \quad (45)$$

$$P^{max}_{r_{b_i},2p} < P_{r_i} \leq P^{maxa}_{r_i,2p}, \forall i \in \ \tilde{S}_{a2} \quad (46)$$

$$P^{max}_{r_{a_i},2p} < P_{r_i} \leq P^{maxb}_{r_i,2p}, \forall i \in \tilde{S}_{b2} \quad (47)$$

$$\tilde{S}_{a1} \subseteq \tilde{S}_a, \quad \tilde{S}_{b1} \subseteq \tilde{S}_b \quad (48)$$

Note that for fixed $\tilde{S}_{a1}$ and $\tilde{S}_{b1}$, the above problem is convex. The optimum solution can be found by comparing the solutions corresponding to different $(\tilde{S}_{a1}, \tilde{S}_{b1})$.

E. Amplify-and-Forward (AF)/Compress-and-Forward (CF)

The relay power allocation problem for AF relaying can be expressed as:

$$\max_{\{P_{r_i}\}_{i=1}^K} \sum_{i=1}^{K} \frac{w_{ab_i}}{2K} C(\frac{P_{r_i}|f_i|^2|g_i|^2 P_{a_i}}{P_{r_i}|g_i|^2 + P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2 + 1})$$

$$+ \frac{w_{ba_i}}{2K} C(\frac{P_{r_i}|f_i|^2|g_i|^2 P_{b_i}}{P_{r_i}|f_i|^2 + P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2 + 1}) \quad (49)$$

$$s.t. \quad \sum_{i=1}^{K} P_{r_i} \leq P_{r,total}; \quad P_{r_i} \geq 0, \forall i \quad (50)$$

Similarly, the problem for CF can be expressed by replacing the two corresponding rate expressions with the RHS of (12) and (13) respectively. It can be shown that for both AF and CF, the objective function is a concave function with respect to $\{P_{r_i}\}$, therefore, the problem is convex and the optimum solution can be found. Note that, unlike the decode-and-forward relaying schemes, the AF and CF schemes do not have decodability constrains. Also, note that for both AF and CF, the objective function is an increasing function of P_{r_i} and approaches to the upper bound $\sum_{i=1}^{K} \frac{w_{ab_i}}{2K} C(P_{a_i}|f_i|^2) + \frac{w_{ab_i}}{2K} C(P_{b_i}|g_i|^2)$ as P_{r_i} goes to infinity.

V. Numerical Results

In this section, we present numerical results to demonstrate the performance of the optimum power allocation for multi-access two-way relaying. We first consider a network with 3 user pairs and one relay node as shown in Figure 2. We assume the channel gains $\{|f_i|^2, |g_i|^2, |h_i|^2\}$ are proportional to the fourth power of the corresponding distances between the nodes. We set the users' transmit powers to $P_{a_i} = P_{b_i} = 0.1\ Watts$, and assume all AWGN terms have variance 10^{-10}. Setting $w_{ab_i} = w_{ba_i} = 1$ for all i, we investigate the sum rate of all users achieved by the optimum relay power allocation with different multi-access two-way relaying schemes, for a range of relay total power constraints.

In Figure 3, we compare the sum rate of all users achieved by different relaying schemes. We observe that different relaying schemes outperform one another for different range of relay power. When the relay has a low power budget, three-phase schemes outperform two-phase ones due to the dominating contribution from the direct links. As the relay power increases, two-phase schemes become better since the relay assisted transmissions dominate the rates and the pre-log factor is $1/2$ for two-phase schemes while it is $1/3$ for three-phase schemes. While the relay power keeps increasing, all schemes eventually reach (DSF/DXF) or approach (AF/CF) their upper bounds. We note that DXF schemes outperform the corresponding DSF schemes until they reach the upper bounds, because forwarding an XORed message to both parties is more power efficient than forwarding individual messages. We also observe that CF always outperforms AF as expected. From the above observations, we conclude that, given a relay power budget, we can always choose the relaying scheme and the

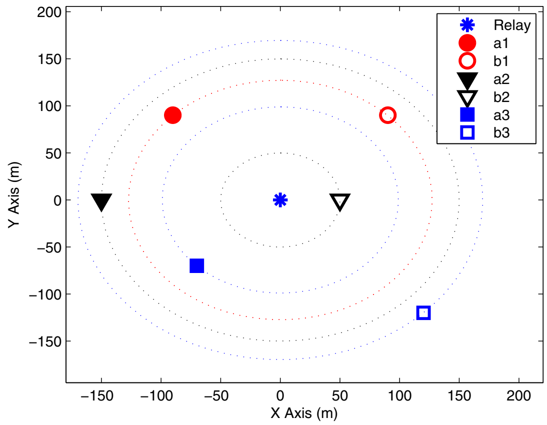

Figure 2. A multi-access two-way relay network.

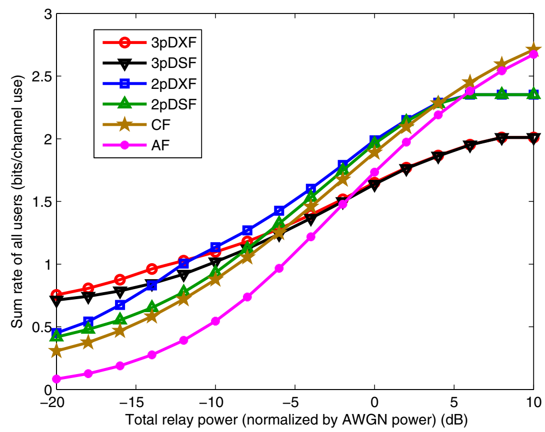

Figure 3. Comparison of different two-way relaying schemes with optimum power allocation.

corresponding power allocation algorithm to obtain the highest weighted sum rate.

Next, we present the performance gain of the optimum power allocation upon equal power allocation which equally distributes the relay power to assist all user pairs. To see the average performance gain, we generate 100 network topologies with one relay at the origin and 3 user pairs randomly distributed in the area of $[-250m, 250m]^2$. For each network, we find the sum rate achieved respectively by the optimum power allocation and the equal power allocation. For different relay power values, we choose the relaying scheme that achieves the highest sum rate with the optimum and the equal power allocation algorithm, respectively. In Figure 4, we observe that the optimum power allocation achieves a significant sum rate performance gain upon the equal power allocation, especially when the relay power is low.

VI. Conclusion

In this paper, we have investigated the optimum relay power allocation problem for a multi-access two-way relay network where a single relay assists bi-directional traffic of multiple pairs of nodes over orthogonal channels. We have considered a variety of two-way relaying protocols including DSF, DXF, AF and CF relaying. We have formulated and solved for the

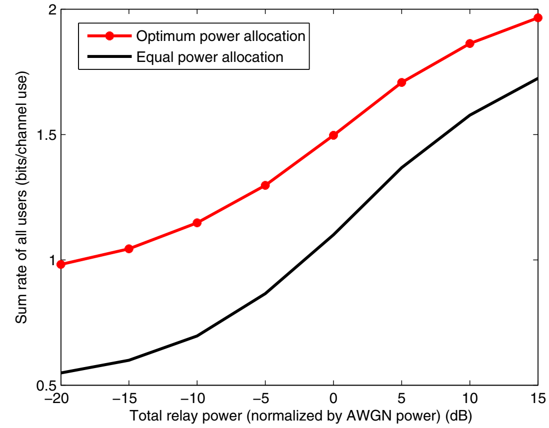

Figure 4. Comparison of optimum power allocation and equal power allocation.

relay power allocation that maximizes an arbitrary weighted sum of rates in the network, thereby tracing the boundary of the achievable rate region for each relaying scheme. We have shown and compared the performance of different two-way relaying schemes with optimum power allocation, and demonstrated their significant performance gain over equal power allocation. We have thus provided design guidelines for multi-access two-way relay networks to fully utilize the available relay resources in terms of power and improve the system-wide performance.

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