Interference Management for Multiuser Two-Way Relaying

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***Abstract*—We consider a *multiuser two-way relay* network where multiple pairs of users communicate with their pre-assigned partners, using a common intermediate relay node, in a two-phase communication scenario. In this system, a pair of partners transmit to the relay sharing a common spreading signature in the first phase, and the relay broadcasts an estimate of the XORed symbol for each user pair in the second phase employing the relaying scheme termed jointly demodulate-and-XOR forward (JD-XOR-F) in [1]. We investigate the joint power control and receiver optimization problem for this multiuser two-way relay system with JD-XOR-F relaying. We show that the total power optimization problem decouples into two subproblems, one for each phase. We construct the distributed power control and receiver updates in each phase which converge to the corresponding unique optimum. Simulation results are presented to demonstrate the significant power savings of the multiuser two-way relay system with the proposed iterative power control and receiver optimization algorithms, as compared to the designs with a "one-way" communication perspective.**

***Index Terms*—Two-way relaying, power control, MMSE-MUD**

I. Introduction

Two-way relay networks where nodes exchange information via the help of intermediate relay node(s) have attracted recent attention thanks to their potential performance improvement upon one-way relaying systems by making use of the bi-directional nature of communication. A number of different protocols for two-way relay channels have been proposed that outperform the traditional four-phase relaying communications in terms of achievable rates [2]–[8]. The proposed schemes include two and three phase protocols that allow digital or analog network coding, i.e., protocols that have two users wishing to exchange information transmit sequentially or simultaneously to a relay which broadcasts an XORed version of two users' symbols after decoding them; or those where the relay amplifies and forwards the received signal from both parties [4]–[12]. Two-way relaying is applied to multi-relay systems in [6], where the number of relays has to satisfy the minimum relay configuration to orthogonalize the overall channel between each pair of users by zero-forcing.

Wireless ad hoc networks of the near future are most likely to consist of many more than two nodes wishing to exchange information, potentially having to share intermediate relays. To that end, we have defined a model to address the communication scenario where one intermediate relaying node assists multiple user pairs, and termed it *multiuser two-way relaying* in [1]. We have employed code division multiple access (CDMA) to accommodate simultaneous communications of multiple users and have shown that with the appropriate detection and relaying strategies, each pair of users can share a common signature waveform, potentially doubling the user capacity of the system. We have proposed the jointly demodulate-and-XOR forward (JD-XOR-F) relaying scheme where the relay broadcasts an estimate of the XORed symbol of each pair of users after all users transmit their symbols simultaneously to the relay. As the focus was in establishing the resource sharing in the form of resulting signature sequences, we have assumed equal powers [1].

For a multiuser two-way relaying system, interference management is a key design issue, and that is what we set out to accomplish in this present paper. Interference management herein refers to the reduction and control of the interference experienced by each end user via careful choice of transmit and receive strategies as well as the relaying scheme, so as to optimize a system-wide performance. Specifically, we aim to jointly design the transmit power control algorithm, the receiver structure and the relaying scheme, such that the QoS requirement for each pair of users exchanging information is satisfied with a minimum total transmit power of the system.

Iterative power control algorithms for (one-way) CDMA systems with receiver optimization have been studied extensively in the past [13]–[15]. At the outset, one might be tempted to think that a direct application is possible for the two-way system at hand executed in two phases. However, as we show in this paper, the very advantage that the bi-directional communication presents, i.e., the use of a common signature per pair in conjunction with JD-XOR-F, is the reason why a direct application does not work. In particular, in the first phase, the probability that the relay estimates an incorrect XORed symbol of a pair of users becomes a function of the received signal-to-interference ratio (SIR) of both partners, and we no longer have a one-to-one mapping between the error probability and the partners' received SIRs. Hence, care must be exercised to find the optimal *transmit power levels* as well as the *receivers* of all nodes exchanging information.

In this paper, we first show that the power control problem over the total user and relay transmit power can be decoupled into two subproblems, one for each phase. We then carefully construct the power control and the receiver updates in each

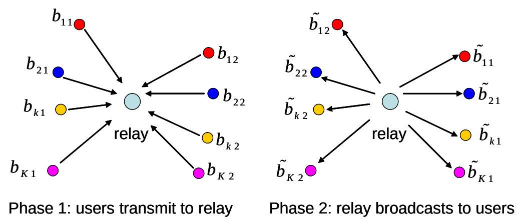

Fig. 1. System model.

phase which are shown to converge to the corresponding unique optimum. Together, the two algorithms produce the overall minimum power solution. We validate our theoretical findings by showing numerical examples where the proposed joint optimum interference management for the multiuser two-way system is observed to provide significant power savings over designs with a "one-way" communication perspective.

II. System Model

We consider a multiuser two-way relaying system shown in Fig. 1, which consists of K pairs of users and an intermediate relay node. User $i1$ and $i2$ ($i \in [1, K]$) are a pair of partners who wish to communicate with each other via the relay node indexed as 0. We remark that each node wishes to communicate with one (pre-assigned) partner only and is uninterested in the remaining transmissions (as in an interference channel). We assume the users and the relay node are half-duplex and equipped with single antenna, and there is no direct link between partners. The information exchange between partners via the relay is accomplished in two phases. The first phase is dedicated to the transmissions from all users to the relay, and the second phase is dedicated to the transmissions from the relay to all the users. For clarity of exposition, we assume a synchronous DS-CDMA system employing non-orthogonal signatures with spreading gain N.

In the first phase, all users spread and transmit their symbols to the relay simultaneously, the ith pair of users, $i1$ and $i2$, using their common signature waveform $s_i(t)$ [1], with transmit powers p_{i1} and p_{i2}, respectively. The channel gain from user $i1$ and $i2$ to the relay are denoted by h_{i1} and h_{i2}, and reciprocal channels are assumed. The channel gains stay constant for the duration of the communication. The received signal at the relay is given by

$$r_0(t) = \sum_{i=1}^{K} (\sqrt{p_{i1} h_{i1}} b_{i1} + \sqrt{p_{i2} h_{i2}} b_{i2}) s_i(t) + n_0(t) \quad (1)$$

where b_{i1} and b_{i2} are the symbols of the user $i1$ and $i2$, and $n_0(t)$ denotes the additive white Gaussian noise (AWGN) at the relay, with zero mean and power spectral density $\sigma_{n_0}^2$. We assume that we have $b_{i1}, b_{i2} \in \{-1, +1\}$ with equal probability. The discrete-time equivalent received signal at the output of the chip matched filter is

$$\mathbf{r}_0 = \sum_{i=1}^{K} (\sqrt{p_{i1} h_{i1}} b_{i1} + \sqrt{p_{i2} h_{i2}} b_{i2}) \mathbf{s}_i + \mathbf{n}_0 \quad (2)$$

where $\mathbf{s}_i$ denotes the unit norm spreading sequence, $\mathbf{n}_0$ is the zero-mean Gaussian random vector with $E[\mathbf{n}_0 \mathbf{n}_0^\intercal] = \sigma_{n_0}^2 \mathbf{I}_N$ where $(\cdot)^\intercal$ denotes transpose operation, and $\mathbf{I}_N$ denotes the N-by-N identity matrix. In the sequel, we will use this discrete-time representation.

We consider the JD-XOR-F scheme [1] at the relay. That is, upon receiving $\mathbf{r}_0$, the relay employs a linear filter $\mathbf{c}_i$ on $\mathbf{r}_0$ to obtain the decision variable $y_i = \mathbf{c}_i^\intercal \mathbf{r}_0$ and then makes a hard decision on y_i to obtain $\hat{b}_i$, the estimate of $b_i = b_{i1} \oplus b_{i2}$ for the ith user pair, where $\oplus$ represents XOR, the bitwise exclusive operation. In the second phase, the relay spreads $\hat{b}_i$ with $\mathbf{s}_i$, for $i = 1, ..., K$, and broadcasts $x_0 = \sum_{i=1}^{K} \sqrt{p_{0i}} \hat{b}_i \mathbf{s}_i$ to all users, where p_{0i} is the transmit power to broadcast $\hat{b}_i$ at the relay. The received signal at user im in the second phase is

$$\mathbf{r}_{im} = \sqrt{h_{im}} \cdot (\sum_{j=1}^{K} \sqrt{p_{0j}} \hat{b}_j \mathbf{s}_j) + \mathbf{n}_{im}, \; i = 1, ..., K, \; m = 1, 2 \quad (3)$$

where $\mathbf{n}_{im}$ is the AWGN vector with covariance matrix $\sigma_{n_{im}}^2 \mathbf{I}_N$. User im applies its linear filter $\mathbf{c}_{im}$ on $\mathbf{r}_{im}$, and obtain $\hat{b}_{im}$, the hard decision estimate of $\hat{b}_i$, from $y_{im} = \mathbf{c}_{im}^\intercal \mathbf{r}_{im}$. Next, it performs another XOR operation on $\hat{b}_{im}$ with its own symbol b_{im} to recover its partner's symbol. When $\hat{b}_i$ at the relay and $\hat{b}_{im}$ at user im are both correct or both wrong, user im can correctly recover its partner's symbol.

III. Problem Formulation

In this paper, we consider the joint optimization of the transmit power levels and the receivers for the multiuser two-way relay network employing JD-XOR-F at the relay as described in the previous section. We seek to expend the minimum total transmit power in the system while satisfying the bit error rate (BER) requirement on both phases of the communication:

$$\min_{\{p_{i1}, p_{i2}, p_{0i}, \mathbf{c}_i, \mathbf{c}_{i1}, \mathbf{c}_{i2}\}} \quad \sum_{i=1}^{K} (p_{i1} + p_{i2} + p_{0i}) \quad (4)$$

$$s.t. \quad Pe1_i \le \rho1_i, Pe2_{i1} \le \rho2_{i1}, Pe2_{i2} \le \rho2_{i2} \quad (5)$$

$$p_{i1} \ge 0, \quad p_{i2} \ge 0, \quad p_{0i} \ge 0 \quad (6)$$

$$\mathbf{c}_i \in \mathbb{R}^N, \quad \mathbf{c}_{i1} \in \mathbb{R}^N, \quad \mathbf{c}_{i2} \in \mathbb{R}^N, \quad \forall i \quad (7)$$

where $Pe1_i$ is the probability in phase one that the relay makes an incorrect decision on $b_i = b_{i1} \oplus b_{i2}$, $Pe2_{i1}$ and $Pe2_{i2}$ are the error probabilities in phase two that user $i1$ and $i2$ make incorrect decisions on the symbol $\hat{b}_i$ broadcasted by the relay respectively, and $\rho1_i$, $\rho2_{i1}$ and $\rho2_{i2}$ are the corresponding system QoS requirements which are given. Note that the quantities $\{Pe1_i\}_{i=1}^K$ depend only on the users transmit power $\{p_{i1}, p_{i2}\}$ and the filters $\{\mathbf{c}_i\}$ in phase one, and $\{Pe2_{i1}, Pe2_{i2}\}_{i=1}^K$ depend only on the relay transmit power $\{p_{0i}\}$ and filters $\{\mathbf{c}_{i1}, \mathbf{c}_{i2}\}$ in phase two. Therefore, the above problem (4)-(7) can be decoupled into two subproblems, one for each phase[1]. The first phase subproblem is:

$$\min_{\{p_{i1}, p_{i2}, \mathbf{c}_i\}} \quad \sum_{i=1}^{K} (p_{i1} + p_{i2}) \quad (8)$$

$$s.t. \quad Pe1_i \le \rho1_i \quad (9)$$

$$p_{i1} \ge 0, \quad p_{i2} \ge 0, \quad \mathbf{c}_i \in \mathbb{R}^N, \quad \forall i \quad (10)$$

[1] We remark that this is a consequence of the detection rule at the relay and that we need to impose the QoS constraints in both phases.

and the second phase subproblem is:

$$\min_{\{p_{0i},\mathbf{c}_{i1},\mathbf{c}_{i2}\}} \quad \sum_{i=1}^{K} p_{0i} \tag{11}$$

$$s.t. \quad Pe2_{i1} \le \rho2_{i1}, \quad Pe2_{i2} \le \rho2_{i2} \tag{12}$$

$$p_{0i} \ge 0, \quad \mathbf{c}_{i1} \in \mathbb{R}^N, \quad \mathbf{c}_{i2} \in \mathbb{R}^N, \quad \forall i \tag{13}$$

We next solve these two subproblems.

IV. Iterative Interference Management for the Multiuser Two-way Relay Network

In this section, we investigate the power control and receiver optimization subproblems for phase one and two respectively.

A. Decision Rule and the BER at the Relay in Phase One

To solve the subproblem for phase one, we first consider the ith pair as the desired user pair and fix the filter $\mathbf{c}_i$ and the transmit power of the remainder of the users. The output of the filter $\mathbf{c}_i$ can be written as

$$\begin{aligned} y_i &= \mathbf{c}_i^{\mathsf{T}}\mathbf{r}_0 = (\sqrt{p_{i1}h_{i1}}b_{i1} + \sqrt{P_{i2}h_{i2}}b_{i2})\mathbf{c}_i^{\mathsf{T}}\mathbf{s}_i + \\ &\quad \sum_{j\neq i}^{K}(\sqrt{p_{j1}h_{j1}}b_{j1} + \sqrt{p_{j2}h_{j2}}b_{j2})\mathbf{c}_i^{\mathsf{T}}\mathbf{s}_j + \mathbf{c}_i^{\mathsf{T}}\mathbf{n}_0 \quad (14) \\ &= \sqrt{q_{i1}}b_{i1} + \sqrt{q_{i2}}b_{i2} + N_i \quad (15) \end{aligned}$$

where $q_{im} = p_{im}h_{im}(\mathbf{c}_i^{\mathsf{T}}\mathbf{s}_i)^2$, m=1,2 denotes the received power of user im at the output of the filter, and $N_i = \sum_{j\neq i}^{K}(\sqrt{p_{j1}h_{j1}}b_{j1}+\sqrt{p_{j2}h_{j2}}b_{j2})\mathbf{c}_i^{\mathsf{T}}\mathbf{s}_j+\mathbf{c}_i^{\mathsf{T}}\mathbf{n}_0$ denotes the interference plus noise term. Let $\sigma_i^2 = \sum_{j\neq i}(p_{j1}h_{j1}+p_{j2}h_{j2})(\mathbf{c}_i^{\mathsf{T}}\mathbf{s}_j)^2+\sigma_{n_0}^2\mathbf{c}_i^{\mathsf{T}}\mathbf{c}_i$ be the variance of N_i, which we approximate with a Gaussian. This is because the optimum linear filter, we will observe, as in the case of one-way communications, is the minimum mean squared error (MMSE) filter, whose BER can be well approximated by a Q-function [16].

Estimating b_i=$b_{i1}\oplus b_{i2}$ is equivalent to making a decision in favor of one of two hypotheses, i.e., whether b_{i1} and b_{i2} have the same or opposite sign. It is shown in [1] that the optimum decision rule does not have a close form solution in general, which may bring implementation difficulties in practice and the evaluation of the BER may become intractable. In this paper, we propose that the relay first jointly detects $(\hat{b}_{i1},\hat{b}_{i2})\in\{(-1,-1),(-1,1),(1,-1),(1,1)\}$ using the maximum aposteriori probability (MAP) rule and then generates $\hat{b}_i$ as $\hat{b}_i$=$\hat{b}_{i1}\oplus\hat{b}_{i2}$. This way, we obtain a simple decision rule for $\hat{b}_i$ as

$$\hat{b}_i = \begin{cases} 1, & \text{when} \quad y_i \in R = \{y_i| - y_{th} < y_i < y_{th}\} \\ -1, & \text{when} \quad y_i \in R_c \end{cases} \tag{16}$$

where y_{th}=$\sqrt{q_{i1}}$ when q_{i1}≥q_{i2}, and y_{th}=$\sqrt{q_{i2}}$ when q_{i1}<q_{i2}, and R_c denotes the complement set of R in $\mathbb{R}$. Under this decision rule, $Pe1_i$, the error probability of estimating b_i at the relay, can be obtained as

$$Pe1_i = \begin{cases} Q(\sqrt{qr_{i2}}) + 0.5Q(2\sqrt{qr_{i1}} - \sqrt{qr_{i2}}) \\ \quad -0.5Q(2\sqrt{qr_{i1}} + \sqrt{qr_{i2}}), \ \text{when } qr_{i1} \ge qr_{i2} \\ Q(\sqrt{qr_{i1}}) + 0.5Q(2\sqrt{qr_{i2}} - \sqrt{qr_{i1}}) \\ \quad -0.5Q(2\sqrt{qr_{i2}} + \sqrt{qr_{i1}}), \ \text{when } qr_{i1} < qr_{i2} \end{cases} \tag{17}$$

where qr_{i1}=q_{i1}/σ_i^2 and qr_{i2}=q_{i2}/σ_i^2 are the received SIR of user $i1$ and $i2$ at the relay, and the Q-function is $Q(x)$=$\int_x^{\infty}\frac{1}{\sqrt{2\pi}}e^{-t^2/2}dt$. Note that the decision rule in (16) is near-optimum which has negligible performance loss compared with the optimum one, as we will show in Section V numerically.

***Lemma** 1:* The error probability function $Pe1_i(qr_{i1}, qr_{i2})$ is a quasiconvex function of (qr_{i1}, qr_{i2}) on both R_1 and R_2, where R_1=$\{(qr_{i1}, qr_{i2})|qr_{i1} > 0,\ qr_{i2} > 0,\ qr_{i1} \ge qr_{i2}\}$, and R_2=$\{(qr_{i1}, qr_{i2})|qr_{i1} > 0,\ qr_{i2} > 0,\ qr_{i1} < qr_{i2}\}$[2].

Proof: First, we prove that $Pe1_i(qr_{i1}, qr_{i2})$ is a quasiconvex on R_1. Let $f = Pe1_i$, $x_1 = qr_{i1}$, $x_2 = qr_{i2}$ and $\mathbf{x} = [x_1, x_2]^{\mathsf{T}}$ for simplicity. A sufficient condition for f to be quasiconvex on R_1 is that for each $\mathbf{x} \in R_1$, $\det(B_n(\mathbf{x})) < 0$ for $n = 1, 2$ [17], where $\det(\cdot)$ is the determinant of the matrix, and B_n denotes the nth submatrix of the bordered Hessian of f, i.e.,

$$B_1 = \begin{bmatrix} f_{11} & f_1 \\ f_1 & 0 \end{bmatrix} \quad \text{and} \quad B_2 = \begin{bmatrix} f_{11} & f_{12} & f_1 \\ f_{21} & f_{22} & f_2 \\ f_1 & f_2 & 0 \end{bmatrix} \tag{18}$$

where $f_m = \frac{\partial f}{\partial x_m}$ and $f_{mn} = \frac{\partial^2 f}{\partial x_m \partial x_n}$ with $n, m \in \{1, 2\}$. Using the fact that $x_1 \ge x_2$ and $x_1, x_2 > 0$, it can be shown that $f_1 < 0$, $f_2 < 0$, $f_{11} > 0$, $f_{22} > 0$ and $f_{12} = f_{21} < 0$. Therefore, the determinants of the submatrices of the bordered Hessian are both less than zero, i.e.,

$$\det(B_1) = -(f_{11})^2 < 0 \tag{19}$$

$$\det(B_2) = -(f_1)^2 f_{22} + 2f_1 f_2 f_{12} - (f_2)^2 f_{11} < 0 \tag{20}$$

Thus, $f(\mathbf{x})$, equivalently, $Pe1_i(qr_{i1}, qr_{i2})$, is quasiconvex on R_1 according to the sufficient condition. By switching qr_{i1} and qr_{i2} in the above proof for R_1, we can show that $Pe1_i(qr_{i1}, qr_{i2})$ is quasiconvex on R_2 as well. ■

B. Power Control and Receiver Optimization in Phase One

In this section, we first convert the error probability constraint in (9) to its equivalent total transmit power constraint, by solving the optimal power allocation problem for the ith user pair. The iterative power control and receiver optimization algorithm in [14] then can be applied for the first phase of the multiuser two-way relay system.

When the power levels of all the other users and the linear filter $\mathbf{c}_i$ are fixed, the optimization problem in (8)-(10) reduces to the following optimization problem over the transmit power of the ith user pair only with fixed $\sigma_i^2/(\mathbf{c}_i^{\mathsf{T}}\mathbf{s}_i)^2$,

$$\min_{\{qr_{i1},qr_{i2}\}} \quad g(qr_{i1}, qr_{i2}) = \left(\frac{qr_{i1}}{h_{i1}} + \frac{qr_{i2}}{h_{i2}}\right)\frac{\sigma_i^2}{(\mathbf{c}_i^{\mathsf{T}}\mathbf{s}_i)^2} \tag{21}$$

$$s.t. \quad Pe1_i \le \rho1_i \tag{22}$$

$$qr_{i1} > 0, \quad qr_{i2} > 0 \tag{23}$$

Note that since $Pe1_i$ is expressed as a function of (qr_{i1}, qr_{i2}) in (17), we replace the variables p_{i1} and p_{i2} in (8) by qr_{i1} and qr_{i2}, and let $g(qr_{i1}, qr_{i2})$ denote the objective function.

[2]Having qr_{i1}=0 or qr_{i2}=0 leads to $Pe1_i$=0.5, which is not a desired situation in communication systems. Hence, in the sequel, we will exclude the case qr_{i1}=qr_{i2}= 0, or equivalently, p_{i1}=p_{i2}= 0.

Let the feasible set of the problem in (21)-(23) be S, which can be partitioned into two sets, $S_1 = \{(qr_{i1}, qr_{i2}) | (qr_{i1}, qr_{i2}) \in S, \text{ and } qr_{i1} \geq qr_{i2}\}$ and $S_2 = \{(qr_{i1}, qr_{i2}) | (qr_{i1}, qr_{i2}) \in S, \text{ and } qr_{i1} < qr_{i2}\}$. Since it is proved in Lemma 1 that $Pe1_i$ is quasiconvex on R_1 and R_2, the corresponding lower level sets S_1 and S_2 are both convex sets. Therefore, replacing S by S_1 and S_2 respectively in problem (21)-(23) leads to two convex problems, each having a unique optimum solution, $(qr_{i1}^{\dagger}, qr_{i2}^{\dagger})$ and $(qr_{i1}^{\dagger\dagger}, qr_{i2}^{\dagger\dagger})$, respectively, due to the fact that each problem minimizes a linear objective function g over a convex feasible set. Thus, the optimum solution of problem (21)-(23), (qr_{i1}^*, qr_{i2}^*), can be obtained as

$$(qr_{i1}^*, qr_{i2}^*) = \underset{\{qr_{i1}, qr_{i2}\}}{\arg\min} \left(g(qr_{i1}^{\dagger}, qr_{i2}^{\dagger}), g(qr_{i1}^{\dagger\dagger}, qr_{i2}^{\dagger\dagger}) \right) \quad (24)$$

Next, we define two quantities γ_i and α_i as

$$\gamma_i = \frac{qr_{i1}^*}{h_{i1}} + \frac{qr_{i2}^*}{h_{i2}} \quad \text{and} \quad \alpha_i = \frac{p_{i1}^*}{p_{i2}^*} = \frac{qr_{i1}^*}{h_{i1}} \frac{h_{i2}}{qr_{i2}^*} \quad (25)$$

Note that (qr_{i1}^*, qr_{i2}^*) depends only on $\rho 1_i$ and h_{i1}/h_{i2}, therefore, γ_i and α_i are functions of $(h_{i1}, h_{i2}, \rho 1_i)$ and independent of $(\mathbf{c}_i, \sigma_i^2)$. We have the following observation.

Observation 1: The error probability constraint in (9), $Pe1_i(p_{i1}, p_{i2}) \leq \rho 1_i$, is satisfied when the transmit power pair (p_{i1}, p_{i2}) satisfies the following condition:

$$p_{i1} + p_{i2} \geq \gamma_i \frac{\sigma_i^2}{(\mathbf{c}_i^{\intercal} \mathbf{s}_i)^2} \quad \text{and} \quad p_{i1}/p_{i2} = \alpha_i \quad (26)$$

Equivalently, (26) is a sufficient condition for $Pe1_i \leq \rho 1_i$, with the minimum total transmit power requirement on the ith user pair.

Therefore, we replace $Pe1_i \leq \rho 1_i$ by (26) in the optimization problem (8)-(10) and rewrite it as

$$\min_{\{p_{i1}, p_{i2}, \mathbf{c}_i\}} \quad \textstyle\sum_{i=1}^{K} (p_{i1} + p_{i2}) \quad (27)$$

$$s.t. \quad p_{i1} + p_{i2} \geq \gamma_i \frac{\sigma_i^2}{(\mathbf{c}_i^{\intercal} \mathbf{s}_i)^2} \quad \text{and} \quad p_{i1}/p_{i2} = \alpha_i \quad (28)$$

$$p_{i1} > 0, \quad p_{i2} > 0, \quad \mathbf{c}_i \in \mathbb{R}^N, \quad \forall i \quad (29)$$

Letting $p_{i1} = \alpha_i p_{i2}$, $h_i' = \alpha_i h_{i1} + h_{i2}$, $\gamma_i' = \frac{\gamma_i}{1+\alpha_i}$, and recalling that $\sigma_i^2 = \sum_{j \neq i} (p_{j1} h_{j1} + p_{j2} h_{j2}) (\mathbf{c}_i^{\intercal} \mathbf{s}_j)^2 + \sigma_{n_0}^2 \mathbf{c}_i^{\intercal} \mathbf{c}_i$, the above optimization problem becomes

$$\min_{\{p_{i2}, \mathbf{c}_i\}} \quad \textstyle\sum_{i=1}^{K} (1 + \alpha_i) p_{i2} \quad (30)$$

$$s.t. \quad p_{i2} \geq \gamma_i' \frac{\sum_{j \neq i} h_j' p_{j2} (\mathbf{c}_i^{\intercal} \mathbf{s}_j)^2 + \sigma_{n_0}^2 \mathbf{c}_i^{\intercal} \mathbf{c}_i}{(\mathbf{c}_i^{\intercal} \mathbf{s}_i)^2} \quad (31)$$

$$p_{i2} > 0, \quad \mathbf{c}_i \in \mathbb{R}^N, \quad \forall i \quad (32)$$

We note that this optimization problem is in the similar form as that in [14]. Therefore, we can now define

$$I_i(\mathbf{p}_2, \mathbf{c}_i) = \gamma_i' \frac{\sum_{j \neq i} h_j' p_{j2} (\mathbf{c}_i^{\intercal} \mathbf{s}_j)^2 + \sigma_{n_0}^2 \mathbf{c}_i^{\intercal} \mathbf{c}_i}{(\mathbf{c}_i^{\intercal} \mathbf{s}_i)^2} \quad (33)$$

$$T1_i(\mathbf{p}_2) = \min_{\mathbf{c}_i \in \mathbb{R}^N} I_i(\mathbf{p}_2, \mathbf{c}_i) \quad (34)$$

and the iterative power control algorithm

$$\mathbf{p}_2(n+1) = \mathbf{T1}(\mathbf{p}_2(n)) \quad (35)$$

where $\mathbf{p}_2 = [p_{12}, ..., p_{K2}]^{\intercal}$, $\mathbf{T1} = [T1_1(\mathbf{p}), ..., T1_K(\mathbf{p})]^{\intercal}$, and n is the iteration index. It is worth emphasizing that h_i' and γ_i' are calculated prior to the iterative updates, using γ_i and α_i in (25), which are obtained by solving the problem in (21)-(23) for the ith user pair. For each iteration, the optimum filter $\mathbf{c}_i$ that minimizes $I_i(\mathbf{p}_2, \mathbf{c}_i)$ is found as

$$\mathbf{c}_i^* = (\sum_{j=1}^{K} h_j' p_{j2} \mathbf{s}_j \mathbf{s}_j^{\intercal} + \sigma_{n_0}^2 \mathbf{I})^{-1} h_i' p_{i2} \mathbf{s}_i \quad (36)$$

which is the MMSE filter that is:

$$\mathbf{c}_i^* = \underset{\mathbf{c}_i \in \mathbb{R}^N}{\arg\min} \; E[((\sqrt{p_{i1} h_{i1}} b_{i1} + \sqrt{p_{i2} h_{i2}} b_{i2}) - \mathbf{c}_i^{\intercal} \mathbf{r}_0)^2] \quad (37)$$

Finally, p_{i1} can be obtained from p_{i2} by the relationship $p_{i1} = \alpha_i p_{i2}$. Note that the implementation of this algorithm in a distributed fashion requires only a pair of partners to know each other's channel gain, but none of the interferers.

C. Power Control and Receiver Optimization in Phase Two

In this section, we investigate the power control and receiver optimization subproblem in (11)-(13) for the second phase. The output of the linear filter $\mathbf{c}_{im}$ at user im is

$$y_{im} = \sqrt{p_{0i} h_{im}} (\mathbf{c}_{im}^{\intercal} \mathbf{s}_i) \hat{b}_i + \sum_{j \neq i} \sqrt{p_{0j} h_{im}} (\mathbf{c}_{im}^{\intercal} \mathbf{s}_j) \hat{b}_j + \mathbf{c}_{im}^{\intercal} \mathbf{n}_{im},$$

$$\text{for} \quad i = 1, ..., K, \; m = 1, 2 \quad (38)$$

and the received SIR is

$$SIR_{im} = \frac{p_{0i} h_{im} (\mathbf{c}_{im}^{\intercal} \mathbf{s}_i)^2}{\sum_{j \neq i} p_{0j} h_{im} (\mathbf{c}_{im}^{\intercal} \mathbf{s}_j)^2 + \sigma_{n_{im}}^2 \mathbf{c}_{im}^{\intercal} \mathbf{c}_{im}} \quad (39)$$

The error probability of recovering $\hat{b}_i$ at user im can be well approximated by $Pe2_{im} = Q(\sqrt{SIR_{im}})$ [16]. Hence, the error probability requirement $Pe2_{im} \leq \rho 2_{im}$ is equivalent to the SIR requirement $SIR_{im} \geq \gamma_{im}$ with γ_{im} satisfying $\rho 2_{im} = Q(\sqrt{\gamma_{im}})$. Therefore, we have

$$p_{0i} \geq T2_{im}(\mathbf{p}_0) = \min_{\mathbf{c}_{im} \in \mathbb{R}^N} I_{mi} \quad (40)$$

where $\mathbf{p}_0 = [p_{01}, ..., p_{0K}]^{\intercal}$ and

$$I_{im} = \frac{\gamma_{im}}{h_{im}} \cdot \frac{\sum_{j \neq i} p_{0j} h_{im} (\mathbf{c}_{im}^{\intercal} \mathbf{s}_j)^2 + \sigma_{n_{im}}^2 \mathbf{c}_{im}^{\intercal} \mathbf{c}_{im}}{(\mathbf{c}_{im}^{\intercal} \mathbf{s}_i)^2} \quad (41)$$

The solution of the optimization problem on the RHS in (40) for fixed power levels can be found as

$$\mathbf{c}_{im}^* = \left(\sum_{j=1}^{K} h_{jm} p_{0j} \mathbf{s}_j \mathbf{s}_j^{\intercal} + \sigma_{n_{im}}^2 \mathbf{I} \right)^{-1} \sqrt{h_{im} p_{0i}} \mathbf{s}_i \quad (42)$$

which is the MMSE filter at user im that is:

$$\mathbf{c}_{im}^* = \underset{\mathbf{c}_{im} \in \mathbb{R}^N}{\arg\min} \; E[(\sqrt{p_{0i} h_{im}} \hat{b}_i - \mathbf{c}_{im}^{\intercal} \mathbf{r}_{im})^2] \quad (43)$$

The optimization problem in (27)-(29) is hence equivalent to

$$\min_{\mathbf{p}_0} \quad \textstyle\sum_{i=1}^{K} p_{0i} \quad (44)$$

$$s.t. \quad p_{0i} \geq T2_i(\mathbf{p}_0) = \max(T2_{i1}(\mathbf{p}_0), T2_{i2}(\mathbf{p}_0)), \quad \forall i \quad (45)$$

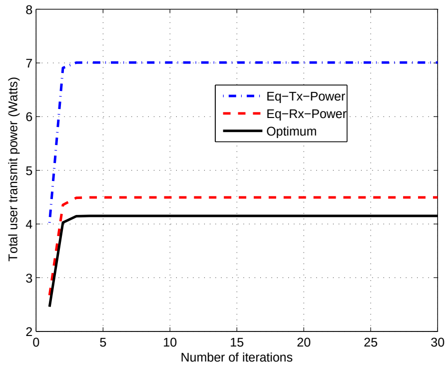

Fig. 2. Comparison of the total user transmit power among different power control algorithms in phase one of two-way JD-XOR-F relaying.

It can be shown that $T2_i(\mathbf{p}_0)$ is a standard interference function [13], [14]. Therefore, we define the power control algorithm for the second phase as

$$\mathbf{p}_0(n+1) = \mathbf{T}2(\mathbf{p}_0(n)) \tag{46}$$

where $\mathbf{T}2(\mathbf{p}_0)$=$[T2_1(\mathbf{p}_0), ..., T2_K(\mathbf{p}_0)]^\intercal$. The standard interference function $\mathbf{T}2(\mathbf{p}_0(n))$ guarantees that the power control algorithm in (46) converges to the minimum total transmit power solution of the optimization problem in (11)-(13). Note that the power control and receiver optimization in phase two can be implemented in a distributed fashion as well.

V. Numerical Results

Numerical results are presented in this section to demonstrate the performance gain of the proposed power control and receiver optimization algorithm for the two-way JD-XOR-F relaying system, compared with the one-way CDMA system, where all users transmit to the relay with distinct signatures in the first phase, and the relay obtains the estimated symbol for each user, and spreads and forwards it to its corresponding partner with the partner's signature in the second phase.

In the simulations, all users are randomly distributed in a disk with the relay at the center, and the distance between relay and users are between 100m-1000m. All channel gains follow the path-loss model, i.e., h_{im}=d_{im}^{-a} where d_{im} is the distance between user im and the relay, for i=$1, ..., K, m$=$1, 2$. The path-loss exponent a=4 is used in the simulation. The spreading signatures are randomly generated. Both the system topology and the signatures are generated once and then fixed for the simulations presented in Fig. 2-Fig. 5, while the system topology is randomly generated for each realization for the simulation in Fig. 6. The AWGN power is 10^{-13}. The system BER requirement for the two-way JD-XOR-F relaying scheme is $\rho 1_i$=$\rho 2_{i1}$=$\rho 2_{i2}$=0.0189. The BER requirement of receiving each symbol at the relay in phase one and at the users in phase two in the one-way CDMA system are set as 0.0189 as well. This way, the two systems achieve the same end-to-end BER for a fair comparison. The spreading gain is N=20, and the number of user pairs is K=11 in Fig. 2-Fig. 4 and K=13 in Fig. 5, i.e., the number of users is 22 and 26 respectively.

In Fig. 2, we compare the total user transmit power of three different power control algorithms in the first phase of the two-way JD-XOR-F relaying: the optimum power control (Optimum), the power control with equal received power between each pair of partners (Eq-RX-Power), the power control with equal transmit power between each pair partners (Eq-TX-Power). As expected, the optimum power control achieves the minimum total user power consumption. While both Eq-TX-Power and Eq-RX-Power consume more power, we observe that Eq-RX-Power algorithm requires only a slightly higher power level compared with the optimum one.

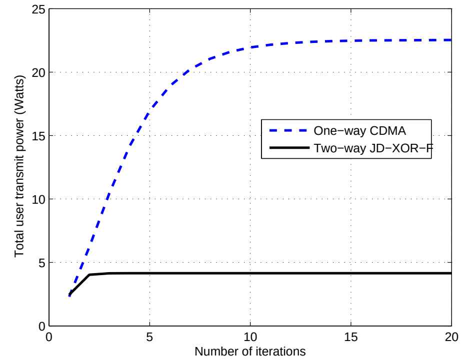

Fig. 3. Comparison of the total user transmit power between two-way JD-XOR-F relaying and one-way CDMA in phase one.

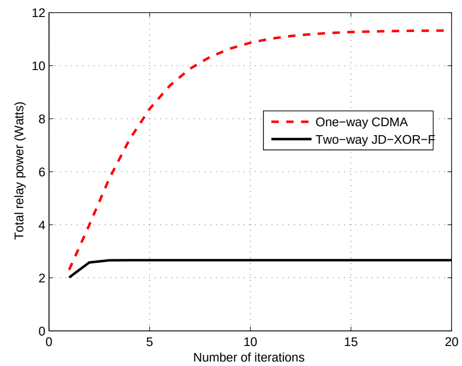

Fig. 4. Comparison of the total relay transmit power between two-way JD-XOR-F relaying and one-way CDMA in phase two.

Fig. 3 and Fig. 4 present the comparison between the two-way JD-XOR-F relaying and the one-way CDMA scheme, on the total user power expended in phase one, and on the total relay power in phase two, respectively. A large power savings of the two-way JD-XOR-F relaying upon the one-way CDMA is presented in both phases. This is due to the fact that in the two-way JD-XOR-F relaying scheme, the relay jointly demodulates and generates the estimate of the XORed symbol for each user pair in the first phase, and transmits one binary symbol for each user pair in the second phase, and hence in both phases the interference is significantly reduced.

In Fig. 5, an overloaded system is considered. For the one-way CDMA system, the user capacity, i.e., the maximum number of users that can be supported with optimum signature assignment and without power constraint, is 24 [18]. As expected, we observe in Fig. 5 that the power control problem with the one-way CDMA scheme is infeasible in this setting since the number of users is 26 $>$ 24, i.e., there are no

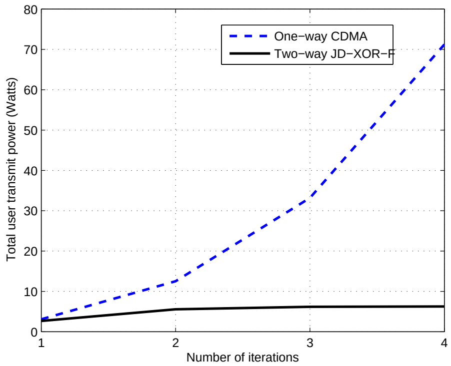

Fig. 5. Comparison of the total user transmit power between two-way JD-XOR-F relaying and one-way CDMA in phase one in an overloaded system.

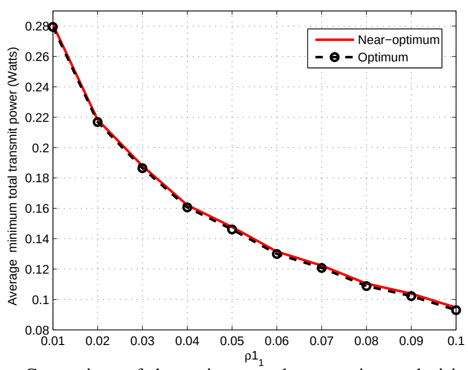

Fig. 6. Comparison of the optimum and near-optimum decision rules for JD-XOR-F scheme at the relay in phase one.

transmit power values that satisfy the QoS requirements; while the two-way JD-XOR-F relaying scheme still has a feasible power control solution since it only needs half number of the signatures used in one-way CDMA. This shows the benefit of considering the two-way communication structure, especially for a heavy loaded interference limited multiuser system.

Note that, all the numerical results presented above employ the near-optimum decision rule in (16) at the relay to estimate the XORed symbol for each user pair in phase one. In Fig. 6, we present the performance comparison between the optimum [1] and the near-optimum decision rule. Specifically, we consider a single pair of users present in the system. For different $\rho 1_1$ in phase one, ranging from 0.01 to 0.1, we search the minimum total transmit power of the pair of users for each network topology realization, with the optimum and near-optimum decision rules, according to their respective BER expressions, and average the power over 1000 realizations. We observe in Fig. 6 that, the performance loss of the near-optimum decision rule compared with the optimum one on the average total transmit power is negligible. Therefore, by employing the near-optimum decision rule with little extra power expenditure, we can solve the joint power control and receiver optimization problem for the multiuser two-way relay system with distributed algorithms, which appears intractable with the optimum decision rule.

VI. Conclusion

In this paper, we consider a multiuser two-way relay network where multiple user pairs exchange information sharing the same intermediate relay. For this two-phase communication scenario where each pair shares a CDMA signature in the uplink, and the relay performs digital network coding per pair in the downlink, we have constructed distributed iterative power control and multiuser detection algorithms that converge to their optimum solution, and showed that the design choices made considering the bi-directional nature of communication lead to significant system-wide power savings.

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