Stability of Bi-Directional Cooperative Relay Networks

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***Abstract*— We consider a pair of nodes who wish to communicate with each other via intermediate relays. In this bi-directional network with stochastic flows, we develop the throughput optimal control policy, i.e., a policy that stabilizes the network whenever the arrival rates are within the region established. We investigate the effect of implementing different practical transmission protocols and network coding. The network control policies we present offer diverse possibilities in relaying and cooperation structure depending on the channel and the queue states.**

I. Introduction

The increasing demand for wireless connectivity necessitates communication via multiple hops where intermediate nodes serve as relays that may cooperate with source(s) or other relay(s) forming wireless ad hoc networks [1]–[3]. There is recent growing interest in cross-layer solutions for wireless ad hoc networks, where decisions on physical layer are made jointly with higher layers [4]. One such prominent approach is to consider the stochastic nature of the traffic to be communicated and to determine the power levels and the rates allocated for nodes according to the queue states as well as the channel states and allowable transmission modes [5].

Throughput optimal control policies, i.e., policies that ensure bounded queues whenever arrival rates lie in the stability region of the network are of special recent interest [5]–[7]. The backpressure policy, also known as the Maximum Differential Backlog (MDB) algorithm, has the desirable property of not requiring any a priori information on the input traffic statistics [4], [5]. MDB has recently been proven throughput optimal for cooperative communication scenarios as well [6]. Multiple traffic streams are allowed with a common and unique destination node in the models analyzed in [6].

Many wireless applications with growing demand such as ad-hoc networks and peer-to-peer systems are based on two-way traffic. As a result, there is growing interest in understanding and exploiting the bi-directional nature of the information flow with intermediate relays [7]–[11]. To that end, reference [7] considers stochastic flows between two end nodes with a single relaying node. In a general ad-hoc network setting, it is likely to have multiple intermediate relays that can cooperatively assist the end nodes. It remains essential to understand the impact of stochastic arrivals in this general setting where pairs of nodes exchange information via multiple intermediate relays.

In this paper, we consider a pair of nodes with stochastic flows that communicate with each other in a bi-directional fashion, via two-hops. We find that the backpressure policy tailored to the problem at hand offers a diverse variety of transmission protocols and queueing options. In addition to a hop-by-hop scheduling mechanism, we also investigate the option of immediately forwarding the received information at the relay nodes. In both cases, we utilize physical layer enhancements such as interference cancelation and network coding, and provide the resulting stability region.

II. System Model

We consider a two-hop bi-directional network G with four nodes $N=\{1,2,3,4\}$ as shown in Figure 1. The end nodes $\{1,4\}$ are sources which aim to communicate with each other. No direct link exists between the end nodes, and, thus the relay nodes $\{2,3\}$ enable communication[1]. We assume that the intermediate nodes do not have exogenous arrivals; however the model can be extended to include arrivals for the relay nodes. Relay nodes assist communication by either direct forwarding or forming a cooperative set to exploit beamforming gains. Decode-and-forward is used by the relays [1].

As in reference [6], within a time slot, we enforce a half-duplex constraint for the cooperative set $S \triangleq \{2,3\}$, i.e., these nodes cannot transmit and receive information simultaneously. For simplicity, some operating modes which do not violate the half-duplex constraints for the individual nodes are also not allowed, such as node 1 transmitting to node 2 while node 3 transmitting to node 4. Rate allocation decisions are made in each slot using the maximum differential backlog algorithm [6], tailored to the problem at hand.

Both utilizing only one of the relay nodes, and relaying by transmitting common information to both of the relays are allowed. The bi-directional information flow is carried out in two phases. In the first phase, information is transmitted to the relay node(s). While transmitting from the end nodes 1 and 4 to the relay nodes, we allow three different modes: (i) both end nodes can transmit information to node 2, (ii) both

[1] We note that the use of two relays is for clarity of exposition; our model can be generalized to N relay nodes.

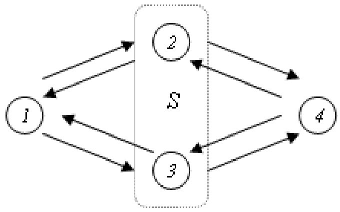

Fig. 1. Four-node network topology.

can transmit information to node 3, and (iii) both can transmit information to the cooperative set $\{2,3\}$. In the second phase, again three modes are allowed: (i) node 2 transmits to the end nodes, (ii) node 3 transmits to the end nodes, and (iii) the cooperative set transmits to the end nodes[2]. Note that for each of these cases, the relay node(s) must transmit information destined to both 1 and 4.

Traffic arriving at node *i* is assumed to be an ergodic process. Packet lengths $\{L_i\}$ of traffic at node *i* are assumed to be i.i.d. with $E[L_i] < \infty$ and $E[L_i^2] < \infty$. We assume infinite buffers. Due to the bi-directional nature of the model, we differentiate queues at relays according to their final destination. We thus define "forward" and "reverse" queues associated with transmission to node 4 (from 1) and 1 (from 4), respectively. Furthermore, as in [6], the queues at the relay nodes are differentiated as "direct" and "cooperative" queues, corresponding to data that is to be relayed by one node directly or to be relayed by both (*S*) cooperatively. The contents of the cooperative queues are identical for both relays, since packets to be forwarded cooperatively are received by both relay nodes as common information. To summarize, four queues are present at each relay node with two of these queues with identical content for both nodes in *S*. All nodes know all channel coefficients and queue states.

III. Rate Regions

A. Phase I

In the first phase, where data is transmitted to the relay nodes, all three cases of allowable transmission modes correspond to a multiple access communication model, with different receivers in each case resulting in different capacity regions. In the sequel, we assume that the power constraint is *P* for all nodes, the noise variance and bandwidth are normalized to one. For instance, for the multiple access mode to the cooperative set, the capacity region is

$$\begin{aligned} R_{1S} &\leq \min(\log(1+h_{12}P), \log(1+h_{13}P)) \\ R_{4S} &\leq \min(\log(1+h_{24}P), \log(1+h_{34}P)) \\ R_{1S}+R_{4S} &\leq \min(\log(1+(h_{12}+h_{24})P), \\ &\qquad \log(1+(h_{13}+h_{34})P)), \end{aligned} \tag{1}$$

where $\sqrt{h_{ij}}$ denotes the channel gain from node *i* to node *j*. The first inequality denotes the maximum common information that can be transmitted from node 1 to the cooperative set. Likewise, the second inequality follows from the maximum information that can be transmitted from node 4 to the cooperative set. The first term in the right-hand-side of the last inequality is due to the multiple access capacity region of node 2, and the second term is due to that of node 3. On the other hand, the multiple access capacity region for relaying via node i only $(i = 2, 3)$ is given by:

$$\begin{aligned} R_{1i} &\leq \log(1+h_{1i}P) \\ R_{4i} &\leq \log(1+h_{i4}P) \\ R_{1i}+R_{4i} &\leq \log(1+(h_{1i}+h_{i4})P). \end{aligned} \tag{2}$$

B. Phase II

In this phase, relays broadcast to the end receivers. Note that, due to the bi-directional nature of communication, the end nodes 1 and 4 can subtract their own information from the received broadcast message [7]. Thus, we can assume that the operation is interference-free since all packets originate from the end nodes. In the case of both relays cooperatively transmitting to both end nodes, we aim to obtain coherent beamforming gain[3] by splitting the total power of each relay to each direction and superposing the transmissions beamformed by the two relays towards each direction. That is, for data to be transmitted to node 1 with power αP, where $\alpha \in$[0,1] is the power splitting parameter for the two codewords destined for nodes 1 and 4 respectively, relays coherently beamform by adjusting signal phases for node 1. Similarly, for the data destined for node 4 with the remaining power $(1-\alpha)P$, relays coherently beamform by adjusting signal phases for node 4. The signals Y_i received at node $i = 1, 4$ are given as:

$$\begin{aligned} Y_1 &= ((\sqrt{h_{12}}+\sqrt{h_{13}})\sqrt{\alpha})X_1 \\ &\quad +((\sqrt{h_{12}}e^{-j\theta_{2f}}+\sqrt{h_{13}}e^{-j\theta_{3f}})\sqrt{1-\alpha})X_4+Z_1, \\ Y_4 &= ((\sqrt{h_{24}}e^{-j\theta_{2r}}+\sqrt{h_{34}}e^{-j\theta_{3r}})\sqrt{\alpha})X_1 \\ &\quad +((\sqrt{h_{24}}+\sqrt{h_{34}})\sqrt{1-\alpha})X_4+Z_4, \end{aligned} \tag{3}$$

where $E[X_i^2] \leq P$ are signals destined to nodes $i = 1, 4$ and Z_i are additive noise at receivers $i = 1, 4$. θ_{if}, θ_{ir} denote phase shifts encountered at receivers 1 and 4 respectively for the undesired transmission from relay $i = 2, 3$ adjusted coherently for receivers 4 and 1. Nodes 1 and 4 know the phase shifts of the undesired signals and apply interference cancelation, so $X_4(X_1)$ is canceled at $Y_1(Y_4)$. The resulting rate region for the cooperative set are:

$$\begin{aligned} R_{S1} &\leq \log(1+(\sqrt{h_{12}}+\sqrt{h_{13}})^2\alpha P) \\ R_{S4} &\leq \log(1+(\sqrt{h_{24}}+\sqrt{h_{34}})^2(1-\alpha)P), \end{aligned} \tag{4}$$

In contrast to the case where both relays cooperatively transmit in the second phase, the capacity region for the case of either node at $i = 2, 3$ transmitting is given as:

$$\begin{aligned} R_{i1} &\leq \log(1+h_{1i}\alpha P) \\ R_{i4} &\leq \log(1+h_{i4}(1-\alpha)P). \end{aligned} \tag{5}$$

In the bi-directional network, we can exploit network coding following a similar approach to reference [9], by having

[2]These three modes do not exhaust all possible ways that the network could be operated. However, we limit ourselves to these choices to keep the problem tractable.

[3]Perfect synchronization is assumed between the cooperating relay nodes for ease of exposition.

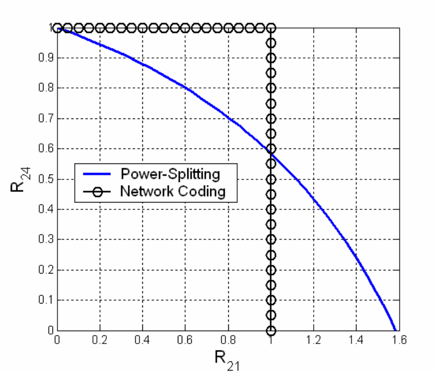

Fig. 2. Rate regions for power splitting and network coding for relay 2 with $h_{21}P = 2$ and $h_{24}P = 1$.

the relay nodes transmit the exclusive-OR (XOR) of the information destined to nodes 1 and 4. In this case, the overall rate must be selected as the minimum of rates achievable between the relay and both end nodes. Both end nodes would be able to decode the overall codeword, obtaining the desired information by an XOR operation. If the queue for data to be combined for one of the end nodes empties before the end of the allocated transmission duration, zero padding is applied. In such a scenario, the remaining part of the codeword essentially consists of data for one end. Hence, for the direct relaying case, the effective rate region for Phase II using relay $i = 2, 3$ is given by

$$\begin{aligned} R_{i1} &\leq min(\log(1 + h_{1i}P), \log(1 + h_{i4}P)). \\ R_{i4} &\leq min(\log(1 + h_{1i}P), \log(1 + h_{i4}P)). \end{aligned} \quad (6)$$

Note that we have the choice of splitting powers and using superposition coding, or allocating all relay power to communicate to both ends using network coding. Clearly, the impact of the relative channel gains is significant for the network coding scenario: A significant difference between h_{i1} and h_{i4} for relay $i = 2, 3$ would cause network coding not to be able to achieve points in the rate region achievable by power splitting (see Figure 2).

We also note that, when the XORed information is to be transmitted, applying coherent beamforming simultaneously to both directions becomes challenging. Accordingly, we choose to adopt the network coding option only for direct relaying.

C. Immediate Forwarding

Assuming there is no exogenous traffic associated with the relay nodes, an alternative transmission scheme for bi-directional communication is to divide the time slot into two phases of equal duration and immediately forward the traffic received in the first phase by the relay nodes to the end nodes in the second phase. An improvement to equal time duration is to optimize the durations for the successive phases similar to references [8], [9] which consider three-node networks with immediate forwarding and a single relay. In our model, transmission options of the previous subsections can be applied for each of the phases. For instance, when multiple cooperating relays exist, the end-to-end rate region for the cooperative model is given as:

$$\begin{aligned} \mathrm{R}_1 &\leq \min\{\Delta_1 \min(\log(1 + h_{12}P), \log(1 + h_{13}P)), \\ &\qquad \Delta_2 \log(1 + (\sqrt{h_{24}} + \sqrt{h_{34}})^2 \alpha P)\} \\ \mathrm{R}_4 &\leq \min\{\Delta_1 \min(\log(1 + h_{24}P), \log(1 + h_{34}P)), \\ &\qquad \Delta_2 \log(1 + (\sqrt{h_{12}} + \sqrt{h_{13}})^2 (1-\alpha) P)\} \\ \mathrm{R}_1 + \mathrm{R}_4 &\leq \Delta_1 \min(\log(1 + (h_{12} + h_{24})P), \\ &\qquad \log(1 + (h_{13} + h_{34})P)) \\ \Delta_1 + \Delta_2 &= 1. \end{aligned} \quad (7)$$

where Δ_1,Δ_2 are the relative proportions of the relay receive and transmit phases respectively.

Having discussed the achievable rate regions for different transmission modes and stages, we next present the throughput optimal rate allocation policy.

IV. Throughput Optimal Policy for the Bi-directional Cooperative Network

Our aim is to ensure the bi-directional network to operate according to a policy, where the queue backlogs remain bounded for any rate arrival vector that lies within the stability region of the network. The stability region of a network is defined as the closure of the set of all arrival rate vectors such that there exists some feasible joint rate allocation and routing policy in the network that guarantees that all queues in the network are stable. For the bi-directional network, at each time slot, active link selection and the corresponding rate allocation is done in accordance with the cooperative maximum differential backlog (CMDB) policy [6]. The most significant difference of the policy from previous throughput optimal maximum differential backlog policies is the queue coupling effect. While previous policies accounted for direct relaying, the CMDB policy models cooperative communications and the effect of identical queues in multiple relay nodes of the same cooperative set are considered jointly, i.e., the queues at the cooperating nodes are coupled. The optimal rate allocation at each time slot is given by the solution of the following optimization problem:

$$\max_{R \in C} \sum_{(i,j)\in L} w^*_{ij} R_{ij} + \sum_{(i,T)\in T} w^*_{iT} R_{iT} + \sum_{(S,i)\in S} w^*_{Si} R_{Si}, \quad (8)$$

where *L* denotes the set of direct links, *T* denotes the set of one-to-many links, *S* denotes the set of many-to-one links, and *w** are the weights associated with each link given by

$$\begin{aligned} w^*_{ij} &= \max_{k\in K}\ q^k_i - q^k_j \\ w^*_{iT} &= \max_{k\in K}\ q^k_i - |T| q^k_T \\ w^*_{Si} &= \max_{k\in K}\ |S| q^k_S - q^k_i, \end{aligned} \quad (9)$$

with q^k_i denoting the queue length associated with destination *k* at node *i* in bits. The queue coupling effect has intuitively pleasing interpretations: for beamforming, packets will be released from multiple queues which will reduce the overall queue backlog in the network. Similarly, while transmitting to a cooperative set multiple queues will grow, justifying the negative impact of queue coupling on the corresponding weight. In our problem, due to the half-duplex constraints, one of the two activation sets defines the allowable rates for the capacity regions depending on whether the relays are receiving or transmitting:

$$C_M \rightarrow \{R_{1S}, R_{12}, R_{13}, 0, 0, 0, R_{4S}, R_{42}, R_{43}, 0, 0, 0\}$$
$$C_B \rightarrow \{0, 0, 0, R_{S4}, R_{24}, R_{34}, 0, 0, 0, R_{S1}, R_{21}, R_{31}\}. \quad (10)$$

The overall capacity region is defined as $C = C_M \bigcup C_B$. As defined in Section II, out of the 6 rates that can be nonzero in a time slot, pairs of two can be simultaneously active and time sharing can be applied between these three pairs. The weight vector is given by $\vec{w^*}=(q_1 - 2q_{cf}, q_1 - q_{2f}, q_1 - q_{3f}, 2q_{cf}, q_{2f}, q_{3f}, q_4 - 2q_{cr}, q_4 - q_{2r}, q_4 - q_{3r}, 2q_{cr}, q_{2r}, q_{3r})$, where subscripts "$f$" denote "forward", "$r$" denote "reverse", and "c" denote "cooperative" for relay queues. Accordingly, the resulting rate allocation is given by the configuration maximizing either

$$\bigcup_{\substack{R \in C_M \\ \sum \Delta_i = 1}} \left\{ \begin{array}{l} \Delta_1(R_{1S}(q_1 - 2q_{cf}) + R_{4S}(q_4 - 2q_{cr})) \\ +\Delta_2(R_{12}(q_1 - q_{2f}) + R_{42}(q_4 - q_{2r})) \\ +\Delta_3(R_{13}(q_1 - q_{3f}) + R_{43}(q_4 - q_{3r})) \end{array} \right\} \quad (11)$$

or

$$\bigcup_{\substack{R \in C_B \\ \sum \Delta_i = 1}} \left\{ \begin{array}{l} \Delta_4(R_{S4}(2q_{cf}) + R_{S1}(2q_{cr})) \\ +\Delta_5(R_{24}(q_{2f}) + R_{21}(q_{2r})) \\ +\Delta_6(R_{34}(q_{3f}) + R_{31}(q_{3r})) \end{array} \right\}, \quad (12)$$

where Δ_i are the time sharing coefficients.

We first observe that the solution of the optimization problems given in (11) and (12) is that Δ_{i^*}=1 for the rate pair which maximizes the multiplier of the time sharing coefficient and Δ_i=0 for all other rate pairs. That is, the policy selects only one of the multiple access or broadcast configurations which maximizes (8) for the bi-directional network.

Similar to [6], for either the multi-access or the broadcast case, the rate allocation is determined in order to maximize the weighted sum of the two rate terms. The solution is the point along the capacity region boundary whose normal vector of supporting hyperplane is equal to the direction of the corresponding queue weight vector. For the network coding scenario, the optimal rate allocation is just the maximum rate that can be transmitted to both users simultaneously. The optimal operating points are found for the three multi-access and three broadcast cases, and the policy selects the optimal rate allocation as the operating point yielding the maximum weighted rate. The rate allocation for the MAC phase defines the decoding order to be followed by the relays. In particular, data of the end node with the higher weight is decoded after the data of the other node. That is, the overall network controller gives priority and the highest rate possible to the traffic originating from the end with higher backlog size, provided that the relay queues are equally congested. In a similar fashion, for the broadcast case, the policy gives a higher priority to the direction which has more traffic to be delivered from the relay node(s). Note that in the case of immediate forwarding, the CMDB policy reduces to the classical backpressure approach due to the absence of cooperative and relay queues, and the backlogs reduce to the queue lengths for the two end nodes since the opposite edge of the link is assumed to be the other end node. The optimization problem reduces into the maximization of the inner product of two end-to-end rate terms and two queue terms at 1 and 4. The end-to-end rates depend on whether direct relaying or cooperative communication is selected before the data transmission.

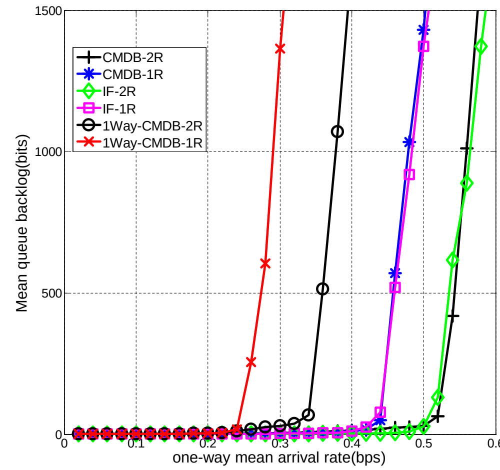

Fig. 3. Mean queue backlog vs load for two-way CMDB, Immediate Forwarding and one-way CMDB with one or two relays.

V. Results and Conclusion

As we discussed in Sections III and IV, the two possible options for operation are immediately forwarding data from the relay nodes or storing the data received from the end nodes in the relay buffers, and scheduling the next transmission. Obeying the backpressure policy guarantees that the system will be stable for any arrival vector within the stability regions for both configurations.

In order to evaluate the performance of various transmission strategies, we next present numerical simulation results. Input traffic to the two nodes are independent Poisson processes. Figures 3 and 4 demonstrate queue evolution for a symmetric network with normalized channel gains and power levels. Figure 3 shows the mean queue backlog for the case where the mean arrival rate for both input traffic streams are identical. Figure 4 demonstrates the empirical stability regions for various strategies. The advantage of using multiple relays is seen for both hop-by-hop scheduling and immediate forwarding, with the system being able to support a higher load as compared to one relay. For comparison purposes, we also present results of transmission schemes where only one-way traffic is allowed at each time slot.

We observe that hop-by-hop forwarding and immediate forwarding result in similar performance. To achieve this performance, immediate forwarding involves time-slot optimization and a more complex scheduling of two hops jointly, whereas

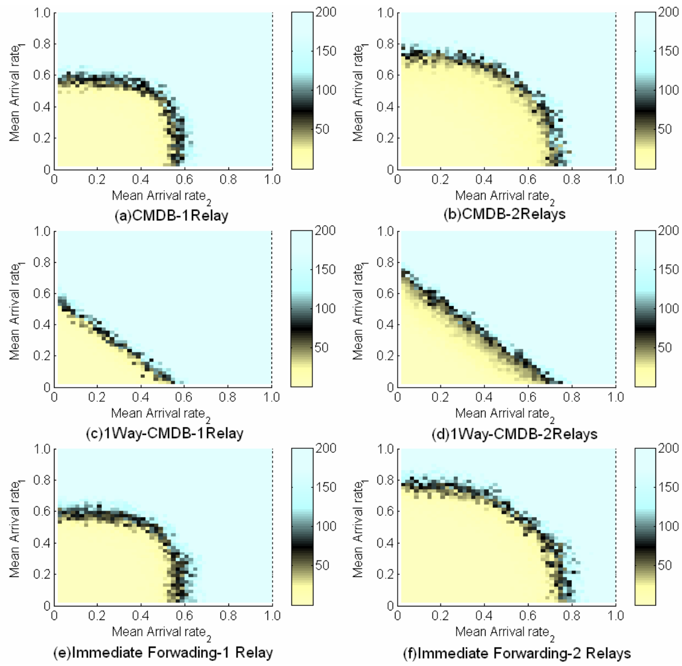

Fig. 4. Mean queue backlogs with symmetric channel conditions.

hop-by-hop forwarding necessitates employing buffers at the relays.

Figure 5 demonstrates results with $h_{i1}P = 1$ and $h_{i4}P = 2$ for $i = 2, 3$, which models a scenario where the relays are closer to node 4. Note that if the channel conditions of one of the hops are significantly worse than the other, this effects both components of the end-to-end rates since the effect is seen in one rate for the MAC region and the other rate for the BC region, bounding the overall end-to-end rate region from both sides. On the other hand, the introduction of the cooperative communication, i.e., the transmission by the cooperating relays provides a larger stability region. If the channel conditions between one of the end nodes and the cooperative nodes is significantly worse than the other, than this effect is seen for only one of the end-to-end rates with multiple relays, since the introduction of beamforming typically leads to a BC rate region that includes the MAC rate region. This effect is observed in Figure 5, where with one relay both rates are affected, but the beamforming effect overcomes the deficiency in the second phase, and as a result, the better first phase results in a higher rate for the closer relay. Figure 5 also demonstrates the fact that using two separate relays with beamforming outperforms the case with a single relay with twice the power.

The option of employing XOR network coding offers further possibilities. In particular, we can operate with a hybrid strategy where either power splitting (4), or single relay network coding (6) is employed whenever beneficial with respect to relative channel gains and queue backlogs. For instance, if there is a significant difference in channel gains as in Figure 2, and the side of the network with the better channel gain has also a larger queue weighting term, power splitting would be the choice. In particular, we observe that network

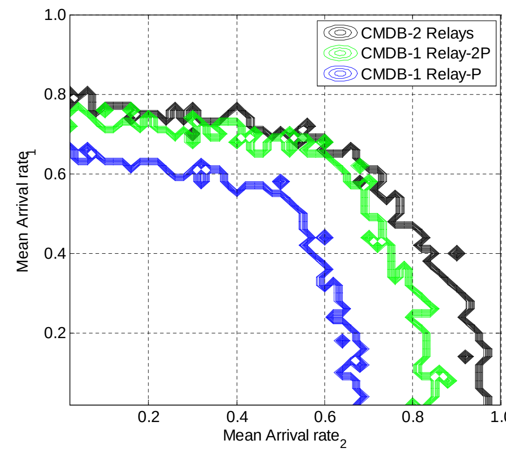

Fig. 5. Mean queue backlog contours with relays closer to one end.

coding proves to be more beneficial for scenarios where the channels conditions are symmetric and traffic is evenly balanced, enlarging the stability region for such situations for single relay networks.

In this paper, we considered the stability region for two-hop bi-directional communication between a pair of nodes with stochastic flows. We assumed decode and forward relaying and perfect global channel state information (CSI). Understanding the impact of imperfect CSI as well as other relaying strategies on the stability region is of future interest.

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