Power Allocation for F/TDMA Multiuser Two-Way Relay Networks

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***Abstract*—We consider a multiuser two-way relay network where multiple pairs of users exchange information with the assistance of a relay node, using orthogonal channels per pair. For a variety of two-way relaying mechanisms, such as decode-and-forward (DF), amplify-and-forward (AF) and compress-and-forward (CF), we investigate the problem of optimally allocating relay's power among the user pairs it assists such that an arbitrary weighted sum rate of all users is maximized, and solve the problem as one or a set of convex problems for each relaying scheme. Numerical results are presented to demonstrate the performance of the optimum relay power allocation as well as the comparison among different two-way relaying schemes.**

***Index Terms*—Two-way relaying, network coding, weighted sum-rate maximization, decode/amplify/compress-and-forward, power allocation.**

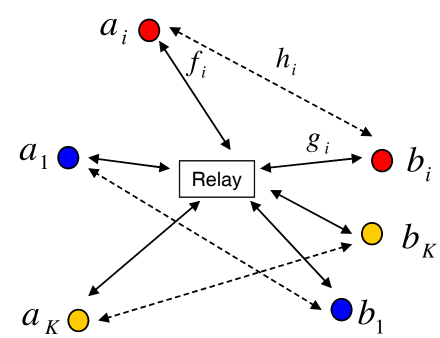

Fig. 1. System model.

I. Introduction

TWO-WAY relaying[1], where the intermediate relay(s) help communicating nodes exchange information, has recently emerged as a means to facilitate relay-assisted cooperation in ad hoc and peer-to-peer wireless networks. A variety of two-way relaying protocols have been proposed, relying on decode-and-forward (DF)[1]–[4], amplify-and-forward (AF)[3], [5], [6] and compress-and-forward relaying (CF)[7], [8], and have shown significant improvement on spectral efficiency upon one-way relaying.

Two-way relaying for multiple users via a sufficiently large number of relay nodes is considered in [3]. Earlier, we have proposed a multiuser two-way relay network where multiple user pairs are assisted by a shared relay[9]. While we have employed code division multiple access (CDMA) which results in an interference limited system in [9], using orthogonal channels by means of frequency or time division multiple access (F/TDMA) to avoid interference is a valid design choice as well. In this scenario, the relay's resources, most notably, its power, need to be appropriately shared between the pairs whose data exchange it shall aid.

In this paper, we address the problem of optimally allocating relay's power among all the user pairs it assists such that an arbitrary weighted sum rate of all users is maximized, for a variety of two-way relaying schemes including DF, AF and CF. We show that the problem for each relaying scheme is equivalent to one or a set of convex problems that can be solved via convex optimization techniques. Since the closed-form power allocation solution does not exist, we develop an iterative algorithm which can be applied to all the relaying schemes, to show how the power allocation is affected by the channel gains of different users as well as the amount of the total relay power. In Section II, we present the system model. We formulate and solve the power allocation problems for various relaying schemes and develop the iterative algorithm in Section III. Numerical results are presented in Section V. Section VI concludes the paper.

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II. System Description

We consider a multiuser two-way relay network shown in Figure 1, which consists of K pairs of users and an intermediate relay node, all half-duplex. User a_i and b_i ($i \in \{1, ..., K\}$) are a pair of pre-assigned partners who wish to communicate with each other with the help of the relay node r. We assume reciprocal channels and denote f_i, g_i and h_i the channel coefficients of the links between a_i and r, b_i and r, and a_i and b_i (if direct link exists) on the ith channel, and assume all channels stay constant for the duration of the communication. Without loss of generality, we assume i.i.d. additive white Gaussian noise with zero mean and unit variance at each receiver.

We study both three- and two-phase DF and two-phase AF and CF relaying schemes. As shown in Figure 2, each user pair is assigned an orthogonal channel in each phase of equal duration, by means of non-overlapping time/frequency slots with equal duration/bandwidth. In the following, we briefly describe the relaying schemes in consideration.

- **Decode-and-Forward**: In three- or two-phase protocols, users a_i and b_i transmit with power P_{a_i} and P_{b_i} sequentially or simultaneously. After decoding the messages m_{a_i} of a_i and m_{b_i} of b_i from the received signals, the relay can employ decode-and-superposition-forward (DSF),

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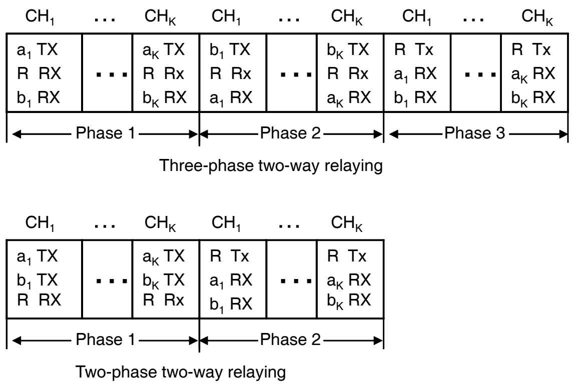

Fig. 2. Channel assignment of various multiuser two-way relaying schemes.

i.e., re-encode the messages individually and transmit $t_{r_i} = \sqrt{P_{r_{a_i}}} x_{r_{a_i}} + \sqrt{P_{r_{b_i}}} x_{r_{b_i}}$, or employ decode-and-XOR-forward (DXF)[4], i.e., encode the message $m_i = m_{a_i} \oplus m_{b_i}$ and transmit $t_{r_i} = \sqrt{P_{r_i}} x_{r_i}$ where $P_{r_{a_i}}$, $P_{r_{b_i}}$ and P_{r_i} denote the corresponding relay transmit power, and $x_{r_{a_i}}$, $x_{r_{b_i}}$ and x_{r_i} are drawn from Gaussian codebooks[1].

- **Amplify-and-Forward**: In phase two, the relay can simply amplify and forward the signal $y_{r_{i,1}}$ received from phase one as $t_{r_i} = \alpha_i y_{r_i,1}$[3], [5], [6], where α_i is a scalar such that the relay's transmit power for the ith user pair is P_{r_i}, i.e., $\alpha_i = \sqrt{\frac{P_{r_i}}{P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2 + 1}}$
- **Compress-and-Forward**: In phase two, the relay can compress the received signal using Wyner-Ziv coding and forward the quantized version $\hat{x}_{r_i}$ as $t_{r_i} = \sqrt{P_{r_i}} \hat{x}_{r_i}$[7], [8].

At the end of the last phase, each user can subtract its self-interference from the common signal broadcasted by the relay and decode its partner's message.

III. The Optimum Relay Power Allocation Problem

The optimum power allocation problem for the multiuser two-way relay network is posed as allocating the relay power to different user pairs such that an arbitrary weighted sum rate of all users is maximized:

$$\max_{\{P_{r_i}\}_{i=1}^K} \quad \sum_{i=1}^K (w_{ab_i} R_{ab_i} + w_{ba_i} R_{ba_i}) \tag{1}$$

$$s.t. \quad \sum_{i=1}^K P_{r_i} \le P_{r,total} \tag{2}$$

$$\{R_{ab_i}, R_{ba_i}\} \in R_* \tag{3}$$

where $P_{r,total}$ is the total relay power constraint, R_{ab_i} and R_{ba_i} denote the rate from user a_i to b_i and that from b_i to a_i, respectively, and R_* is the achievable rate region of one of the relaying schemes which will be presented in the following sections. The optimization variables are $\{P_{r_{a_i}}, P_{r_{b_i}}\}$ instead of $\{P_{r_i}\}$ for DSF schemes. The non-negative weights $\{w_{ab_i}, w_{ba_i}\}$ are to indicate the priority of the traffic amongst different directions and pairs, a larger weight indicating priority. The resulting weighted sums for all $\{w_{ab_i}, w_{ba_i}\}$ clearly allow us to trace the boundary of the achievable rate region.

[1]The communication is based on transmission and reception of codewords in terms of signal sequences, however, due to the assumption of the memoryless channel, we consider single-letter formulation for simplicity.

A. Three-phase Decode-and-Superposition-Forward (3pDSF)

We note that 3pDSF can be considered an extension of the "one-way relaying philosophy", which superposes the codewords to both directions in phase three, and hence the traffic in the two directions can be considered independently. Due to space limitations, we briefly present the achievable rate region as [10]

$$R_{ab_i} \le \begin{cases} \frac{1}{3K}\min(C(P_{a_i}|f_i|^2), C(P_{a_i}|h_i|^2) + C(P_{r_{a_i}}|g_i|^2)), & \forall i \in S_a; \\ \frac{1}{3K}C(P_{a_i}|h_i|^2), \text{otherwise} \end{cases} \tag{4}$$

$$R_{ba_i} \le \begin{cases} \frac{1}{3K}\min(C(P_{b_i}|g_i|^2), C(P_{b_i}|h_i|^2) + C(P_{r_{b_i}}|f_i|^2)), & \forall i \in S_b; \\ \frac{1}{3K}C(P_{b_i}|h_i|^2), \text{otherwise} \end{cases} \tag{5}$$

where $S_a = \{i||f_i|^2 > |h_i|^2\}$, $S_b = \{i||g_i|^2 > |h_i|^2\}$, $C(x) = \log(1+x)$. The power allocation solution is similar to the modified water-filling solution in [11] as:

$$P_{r_{a_i}} = \min\left((\tfrac{w_{ab_i}}{\mu_0} - \tfrac{1}{|g_i|^2})^+, P_{r_{a_i},3p}^{max} \right), \forall i \in S_a \tag{6}$$

$$P_{r_{b_i}} = \min\left((\tfrac{w_{ba_i}}{\mu_0} - \tfrac{1}{|f_i|^2})^+, P_{r_{b_i},3p}^{max} \right), \forall i \in S_b \tag{7}$$

$$P_{r_{a_i}} = 0, \forall i \notin S_a; \quad P_{r_{b_i}} = 0, \forall i \notin S_b \tag{8}$$

where $P_{r_{a_i},3p}^{max} = \frac{P_{a_i}(|f_i|^2 - |h_i|^2)}{|g_i|^2(1+P_{a_i}|h_i|^2)}$, $P_{r_{b_i},3p}^{max} = \frac{P_{b_i}(|g_i|^2 - |h_i|^2)}{|f_i|^2(1+P_{b_i}|h_i|^2)}$, μ_0 is the Lagrangian multiplier that satisfies the total power constraint with equality, and $(.)^+ = \max(.,0)$.

B. Two-phase Decode-and-Superposition-Forward (2pDSF)

The achievable rate region using 2pDSF relaying is described as [3, III]:

$$R_{ab_i} \le \frac{1}{2K}\min(C(P_{a_i}|f_i|^2), C(P_{r_{a_i}}|g_i|^2)), \forall i \tag{9}$$

$$R_{ba_i} \le \frac{1}{2K}\min(C(P_{b_i}|g_i|^2), C(P_{r_{b_i}}|f_i|^2)), \forall i \tag{10}$$

$$R_{ab_i} + R_{ba_i} \le \frac{1}{2K}C(P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2), \forall i. \tag{11}$$

The power allocation problem is equivalent to

$$\max_{\{P_{r_{a_i}}, P_{r_{b_i}}\}_{i=1}^K} \sum_{i=1}^K \frac{w_{ab_i}}{2K}C(P_{r_{a_i}}|g_i|^2) + \frac{w_{ba_i}}{2K}C(P_{r_{b_i}}|f_i|^2) \tag{12}$$

$$s.t. \sum_{i=1}^K P_{r_{a_i}} + P_{r_{b_i}} \le P_{r,total}, P_{r_{a_i}} \ge 0, P_{r_{b_i}} \ge 0, \forall i \tag{13}$$

$$C(P_{r_{a_i}}|g_i|^2) \le C(P_{a_i}|f_i|^2), C(P_{r_{b_i}}|f_i|^2) \le C(P_{b_i}|g_i|^2), \forall i \tag{14}$$

$$C(P_{r_{a_i}}|g_i|^2) + C(P_{r_{b_i}}|f_i|^2) \le C(P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2), \forall i. \tag{15}$$

Note that constraint (15) is a nonconvex set[12] over $(P_{r_{a_i}}, P_{r_{b_i}})$. Fortunately, a simple change of variables $x_{a_i} = C(P_{r_{a_i}}|g_i|^2)$ and $x_{b_i} = C(P_{r_{b_i}}|f_i|^2)$, overcomes this hardship. For simplicity, we let $f(\mathbf{x}) = -\sum_{i=1}^K (w_{ab_i} x_{a_i} + w_{ba_i} x_{b_i})$ be the objective function where $\mathbf{x} = [x_1 \ x_2 \ \ldots \ x_K]^T$ with $x_i = [x_{a_i} \ x_{b_i}]^T$, $\mathbf{q}(\mathbf{x}) \le \mathbf{0}$ with

TABLE I
ITERATIVE ALGORITHM SOLVING POWER ALLOCATION PROBLEM FOR 2PDSF RELAYING

Initialization Step Set the maximum allowed number of iteration $n_{max} \in \mathbb{N}^+$, stop criterions $\epsilon_1, \epsilon_2, \epsilon_3 > 0$. Set the start iteration index $n = 1$. Select a start solution $\mu(1) = 0$. Let the current optimum objective value be $\theta_{best} = -\infty$, the current optimum value for $f(\mathbf{x})$ be $f_{best} = \infty$.

Main Step

Step 1): Start with $\mu(n)$, solve $\max_{\mathbf{x}\geq 0} f(\mathbf{x}) + (\mu(n))^T \mathbf{q}(\mathbf{x})$
  Specifically, for 2pDSF:
  If $n = 1$,
    If $w_{ab_i} \geq w_{ba_i}$,
      let $x_{a_i} = c_{a_i}, x_{b_i} = c_i - c_{a_i}$;
    else
      let $x_{a_i} = c_i - c_{b_i}, x_{b_i} = c_{b_i}$;
    end;
  else
    If $-w_{ab_i} + \mu_{a_i}(n) + \mu_i(n) < 0$,
      let $x_{a_i} = (\log \frac{|g_i|^2}{\mu_0(n)}(w_{ab_i} - \mu_{a_i}(n) - \mu_i(n)))^+$;
    else
      let $x_{a_i} = 0$;
    end;
    If $-w_{ba_i} + \mu_{b_i}(n) + \mu_i(n) < 0$,
      let $x_{b_i} = (\log \frac{|f_i|^2}{\mu_0(n)}(w_{ba_i} - \mu_{b_i}(n) - \mu_i(n)))^+$;
    else
      let $x_{b_i} = 0$;
    end;
  end;
  Let the solution be $\mathbf{x}(n)$; Go to Step 2;

Step 2): Calculate $\theta(\mu(n)) = f(\mathbf{x}(n)) + (\mu(n))^T \mathbf{q}(\mathbf{x}(n))$;
  If $\theta(\mu(n)) > \theta_{best}$,
    let $\theta_{best} = \theta(\mu(n))$ and $\mu_{best} = \mu(n)$;
  end; Go to Step 3;

Step 3): Calculate $\mathbf{q}(\mathbf{x}(n))$;
  If $\mathbf{q}(\mathbf{x}(n)) \leq 0$,
    If $f(\mathbf{x}(n)) \leq f_{best}$,
      let $f_{best} = f(\mathbf{x}(n))$ and $x_{best} = \mathbf{x}(n)$;
    end;
  end; Go to Step 4;

Step 4): If $|\frac{f_{best} - \theta_{best}}{\theta_{best}}| < \epsilon_1$,
    Stop;
  end; Go to Step 5;

Step 5): Let subgradient $\xi(n) = \mathbf{q}(\mathbf{x}(n))$;
  If $||\xi(n)|| < \epsilon_2$,
    Stop;
  end;
  Let $\bar{\mu}(n+1) = \mu(n) + \lambda(n)\frac{\xi(n)}{||\xi(n)||}$;
  (Note: the update step size $\lambda(n)$ is set using block halving method[12, Ch. 8.9].)
  Let $\mu(n+1) = (\bar{\mu}(n+1))^+$;
  If $||\mu(n+1) - \mu(n)|| < \epsilon_3$,
    Stop;
  end; Go to Step 6;

Step 6): Replace n by $n+1$;
  If $n \leq n_{max}$,
    Go to Step 1;
  else
    Stop;
  end;

$\mathbf{q}(\mathbf{x})=[q_0(\mathbf{x})\ q_{a_1}(\mathbf{x})\ q_{b_1}(\mathbf{x})\ q_1(\mathbf{x})\ ...\ q_{a_K}(\mathbf{x})\ q_{b_K}(\mathbf{x})\ q_K(\mathbf{x})]^T$ be the vector of the inequality constraints where $q_0(\mathbf{x})=\sum_{i=1}^{K}(e^{x_{a_i}}-1)/|g_i|^2 + (e^{x_{b_i}}-1)/|f_i|^2 - P_{r,total}$, $q_{a_i}(\mathbf{x})=x_{a_i} - c_{a_i}$, $q_{b_i}(\mathbf{x})=x_{b_i} - c_{b_i}$, $q_i(\mathbf{x})=x_{a_i} + x_{b_i} - c_i$, $c_{a_i}=C(P_{a_i}|f_i|^2)$, $c_{b_i}=C(P_{b_i}|g_i|^2)$ and $c_i=C(P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2)$. Thus, the problem can be compactly rewritten as

$$\min_{\mathbf{x}\geq\mathbf{0}} \quad f(\mathbf{x}) \tag{16}$$

$$s.t. \quad \mathbf{q}(\mathbf{x}) \leq \mathbf{0}. \tag{17}$$

It can be verified that the above problem is convex[12], i.e., $\mathbf{f}$ is a convex function and the constraint $\mathbf{q} \leq \mathbf{0}$ and $\mathbf{x} \geq \mathbf{0}$ define a convex set, and hence has a unique global optimum. The optimum solution can be found via convex optimization techniques[12].

Since the closed-form solution for the power allocation problem does not exist, we next develop an iterative algorithm to show how the power allocation is affected by the channel gains of different users as well as the available relay power. Let $\mu = [\mu_0\ \mu_{a_1}\ \mu_{b_1}\ \mu_1\ ...\ \mu_{a_K}\ \mu_{b_K}\ \mu_K]^T$ be the vector of the Lagrangian multipliers corresponding to the inequality constraints in $\mathbf{q}(\mathbf{x})$. The Lagrangian dual problem is

$$\max \quad \theta(\mu) \tag{18}$$

$$s.t. \quad \mu \geq \mathbf{0} \tag{19}$$

where $\theta(\mu) = \min_{\mathbf{x}\geq\mathbf{0}} f(\mathbf{x}) + \mu^T\mathbf{q}(\mathbf{x})$. It can be verified that $f(\mathbf{x})$ and $\mathbf{q}(\mathbf{x})$ are both convex and there exists an $\hat{\mathbf{x}} \geq 0$ such that $\mathbf{q}(\hat{\mathbf{x}}) < 0$. Therefore, the primal and dual problems satisfy strong duality[12, Ch. 6.2], i.e., there is no duality gap. In particular, we can use subgradient method[12] to iteratively solve the Lagrangian dual problem, which also finds the optimum solution to the primal problem. The iterative algorithm so developed is presented in Table I.

Remark 1: In the first iteration ($n = 1$), we set $\mu_0 = 0$ which means we temporarily remove the total power constraint, and hence the problem becomes a linear programming problem.

Remark 2: Updating x_{a_i}, x_{b_i} in the nth iteration for $n > 1$ is equivalent to updating the relay power allocation as

$$P_{r_{a_i}}(n) = (\frac{1}{\mu_0(n)}(w_{ab_i} - \mu_{a_i}(n) - \mu_i(n)) - \frac{1}{|g_i|^2})^+ \tag{20}$$

$$P_{r_{b_i}}(n) = (\frac{1}{\mu_0(n)}(w_{ba_i} - \mu_{b_i}(n) - \mu_i(n)) - \frac{1}{|f_i|^2})^+. \tag{21}$$

We observe that, the power allocation $P_{r_{a_i}}$ ($P_{r_{b_i}}$) is a modified water-filling solution, with a base level at $\frac{1}{|g_i|^2}$ ($\frac{1}{|f_i|^2}$). The water level depends on μ_0, μ_{a_i} (μ_{b_i}) and μ_i, and we use the subgradient method to update μ_0 and all $\{\mu_i, \mu_{a_i}, \mu_{b_i}\}$.

Note that the iterative algorithm can be used to solve other convex problems emerging in the following sections by properly replacing the corresponding objective function and constraints[10].

C. Three-phase Decode-and-XOR-Forward (3pDXF)

Unlike the DSF scheme, in DXF relaying the relay forwards a single XORed message with power P_{r_i} for the ith pair of users, and consequently, P_{r_i} simultaneously controls the rates on both directions. The achievable rate region of the 3pDXF relaying can be obtained by replacing both $P_{r_{a_i}}$ and $P_{r_{b_i}}$ by P_{r_i} in (4)-(5)[4, III]. For 3pDXF relaying, we have the following observations:

- Assigning more relay power to the ith user pair in $S_{ab} = S_a \cap S_b$ increases R_{ab_i} until $P_{r_i} \geq P^{max}_{r_{a_i},3p}$, and increases R_{ba_i} until $P_{r_i} \geq P^{max}_{r_{b_i},3p}$, where S_a, S_b, $P^{max}_{r_{a_i},3p}$ and $P^{max}_{r_{b_i},3p}$ are given in Section III-A. Therefore, we further partition S_{ab} as $S_{ab1} = \{i | i \in S_{ab} \text{ and } P^{max}_{r_{a_i},3p} \geq P^{max}_{r_{b_i},3p}\}$ and $S_{ab2} = S_{ab} \backslash S_{ab1}$. For $i \in S_{ab1}$, increasing P_{r_i} beyond $P^{max}_{r_{b_i},3p}$ but below $P^{max}_{r_{a_i},3p}$ will increase the data rate R_{ab_i} but not R_{ba_i} since it has reached the upper bound. Similarly, for $i \in S_{ab2}$, increasing P_{r_i} beyond $P^{max}_{r_{a_i},3p}$ but below $P^{max}_{r_{b_i},3p}$ will increase R_{ba_i} but not R_{ab_i}. Therefore, we can introduce new variables $\{\hat{P}_{r_i}\}$ in the problem formulation given below, which are not the actual power allocation but to ensure that the upper bound of R_{ba_i} for $i \in S_{ab1}$ and that of R_{ab_i} for $i \in S_{ab2}$ are not violated in the problem formulation.
- We define sets $S'_a = S_a \backslash S_{ab}$ and $S'_b = S_b \backslash S_{ab}$. The relay can only increase R_{ab_i} for $i \in S'_a$ and only increase R_{ba_i} for $i \in S'_b$.

Thus, the relay power allocation problem is equivalent to [10]

$$\max_{\{P_{r_i},\hat{P}_{r_i}\}_{i=1}^K} \sum_{i\in S_{ab1}} \Big(\frac{w_{ab_i}}{3K}C(P_{r_i}|g_i|^2)+\frac{w_{ba_i}}{3K}C(\hat{P}_{r_i}|f_i|^2)\Big) + \sum_{i\in S_{ab2}} \Big(\frac{w_{ab_i}}{3K}C(\hat{P}_{r_i}|g_i|^2)+\frac{w_{ba_i}}{3K}C(P_{r_i}|f_i|^2)\Big) + \sum_{i\in S'_a}\frac{w_{ab_i}}{3K}C(P_{r_i}|g_i|^2)+\sum_{i\in S'_b}\frac{w_{ba_i}}{3K}C(P_{r_i}|f_i|^2) \quad (22)$$

$$s.t. \sum_{i=1}^K P_{r_i} \leq P_{r,total};\ \hat{P}_{r_i} \geq 0, P_{r_i} \geq 0, \forall i; \quad (23)$$

$$\hat{P}_{r_i} \leq P_{r_i}, \hat{P}_{r_i} \leq P^{max}_{r_{b_i},3p}, P_{r_i} \leq P^{max}_{r_{a_i},3p}, \text{ for } i \in S_{ab1} \quad (24)$$

$$\hat{P}_{r_i} \leq P_{r_i}, \hat{P}_{r_i} \leq P^{max}_{r_{a_i},3p}, P_{r_i} \leq P^{max}_{r_{b_i},3p}, \text{ for } i \in S_{ab2} \quad (25)$$

$$P_{r_i} \leq P^{max}_{r_{a_i},3p}, \text{ for } i \in S'_a; P_{r_i} \leq P^{max}_{r_{b_i},3p}, \text{ for } i \in S'_b \quad (26)$$

$$P_{r_i} = 0, \text{for } i \in \overline{S_a \cup S_b}; \hat{P}_{r_i} = 0, \text{for } i \in S'_a \cup S'_b \cup \overline{S_a \cup S_b}. \quad (27)$$

For simplicity, we have removed all the additive constant terms in the objective function. The problem is convex, consisting of concave objective function and linear constraints, and the global optimum can be found. Note that $\hat{P}_{r_i} = \min(P_{r_i}, P^{max}_{r_{b_i},3p})$ for $i \in S_{ab1}$ and $\hat{P}_{r_i} = \min(P_{r_i}, P^{max}_{r_{a_i},3p})$ for $i \in S_{ab2}$ in the optimum solution.

D. Two-phase Decode-and-XOR-Forward (2pDXF)

The achievable rate region of the 2pDXF relaying scheme can be obtained by replacing both $P_{r_{a_i}}$ and $P_{r_{b_i}}$ by P_{r_i} in (9)-(11)[4, III]. Defining the thresholds $P^{max}_{r_{a_i},2p} = P_{a_i}|f_i|^2/|g_i|^2$ and $P^{max}_{r_{b_i},2p} = P_{b_i}|g_i|^2/|f_i|^2$, we note that the relay-assisted transmission can potentially increase R_{ab_i} when $P_{r_i} \leq P^{max}_{r_{a_i},2p}$, and increase R_{ba_i} when $P_{r_i} \leq P^{max}_{r_{b_i},2p}$, under the sum rate constraint $R_{ab_i} + R_{ba_i} \leq C(P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2)/2K$. This sum rate constraint is equivalent to $P_{r_i} \leq P^{max}_{r_i,2p}$ with

$$P^{max}_{r_i,2p} = \begin{cases} P^{max1}_{r_i,2p} = \frac{P_{a_i}|f_i|^2}{|g_i|^2(1+P_{b_i}|g_i|^2)}, \text{ if } P^{max}_{r_{b_i},2p} \leq P_{r_i} \leq P^{max}_{r_{a_i},2p} \\ P^{max2}_{r_i,2p} = \frac{P_{b_i}|g_i|^2}{|f_i|^2(1+P_{a_i}|f_i|^2)}, \text{ if } P^{max}_{r_{a_i},2p} \leq P_{r_i} \leq P^{max}_{r_{b_i},2p} \\ P^{max3}_{r_i,2p}, \text{ if } P_{r_i} < \min(P^{max}_{r_{a_i},2p}, P^{max}_{r_{b_i},2p}) \end{cases} \quad (28)$$

where $P^{max3}_{r_i,2p} = \frac{-(|g_i|^2+|f_i|^2)+\sqrt{(|g_i|^2+|f_i|^2)^2+4|g_i|^2|f_i|^2(P_{a_i}|f_i|^2+P_{b_i}|g_i|^2)}}{2|g_i|^2|f_i|^2}$.

Next, we partition all user pairs as sets $\tilde{S}_{ab}$, $\tilde{S}_a$ and $\tilde{S}_b$, where $\tilde{S}_{ab} = \{i | P^{max3}_{r_i,2p} \leq \min(P^{max}_{r_{a_i},2p}, P^{max}_{r_{b_i},2p})\}$, $\tilde{S}_a = \{i | P^{max3}_{r_i,2p} > \min(P^{max}_{r_{a_i},2p}, P^{max}_{r_{b_i},2p}) = P^{max}_{r_{b_i},2p}\}$, and $\tilde{S}_b = \{i | P^{max3}_{r_i,2p} > \min(P^{max}_{r_{a_i},2p}, P^{max}_{r_{b_i},2p}) = P^{max}_{r_{a_i},2p}\}$. Let us assume arbitrary subsets $\tilde{S}_{a1} \subseteq \tilde{S}_a$ and $\tilde{S}_{b1} \subseteq \tilde{S}_b$, and define $\tilde{S}_{a2} = \tilde{S}_a \backslash \tilde{S}_{a1}$ and $\tilde{S}_{b2} = \tilde{S}_b \backslash \tilde{S}_{b1}$. The relay power allocation problem is equivalent to[10]

$$\max_{\{P_{r_i}\},\tilde{S}_{a1},\tilde{S}_{b1}} \sum_{i\in\tilde{S}_{ab}\cup\tilde{S}_{a1}\cup\tilde{S}_{b1}} \frac{w_{ab_i}}{2K}C(P_{r_i}|g_i|^2)+\frac{w_{ba_i}}{2K}C(P_{r_i}|f_i|^2) + \sum_{i\in\tilde{S}_{a2}}\frac{w_{ab_i}}{2K}C(P_{r_i}|g_i|^2)+\sum_{i\in\tilde{S}_{b2}}\frac{w_{ba_i}}{2K}C(P_{r_i}|f_i|^2) \quad (29)$$

$$s.t. \quad \sum_{i=1}^K P_{r_i} \leq P_{r,total} \quad (30)$$

$$0 \leq P_{r_i} \leq P^{max3}_{r_i,2p}, \forall i \in \tilde{S}_{ab}; \quad (31)$$

$$0 \leq P_{r_i} \leq P^{max}_{r_{b_i},2p}, \forall i \in \tilde{S}_{a1}; \quad (32)$$

$$0 \leq P_{r_i} \leq P^{max}_{r_{a_i},2p}, \forall i \in \tilde{S}_{b1}; \quad (33)$$

$$P^{max}_{r_{b_i},2p} < P_{r_i} \leq P^{max1}_{r_i,2p}, \forall i \in \tilde{S}_{a2}; \quad (34)$$

$$P^{max}_{r_{a_i},2p} < P_{r_i} \leq P^{max2}_{r_i,2p}, \forall i \in \tilde{S}_{b2}; \quad (35)$$

$$\tilde{S}_{a1} \subseteq \tilde{S}_a, \quad \tilde{S}_{b1} \subseteq \tilde{S}_b. \quad (36)$$

The above problem is convex for fixed $\tilde{S}_{a1}$ and $\tilde{S}_{b1}$, and the optimum solution can be found by comparing the solutions corresponding to different $(\tilde{S}_{a1}, \tilde{S}_{b1})$.

IV. Amplify-and-Forward (AF) and Compress-and-Forward (CF) Relaying

The achievable rate region for AF [3, III] and CF relaying[7, IV][8, IV] are given as:

$$AF: \quad R_{ab_i} \leq \frac{1}{2K}C(\frac{P_{r_i}|f_i|^2|g_i|^2P_{a_i}}{P_{r_i}|g_i|^2+P_{a_i}|f_i|^2+P_{b_i}|g_i|^2+1}), \forall i \quad (37)$$

$$R_{ba_i} \leq \frac{1}{2K}C(\frac{P_{r_i}|f_i|^2|g_i|^2P_{b_i}}{P_{r_i}|f_i|^2+P_{a_i}|f_i|^2+P_{b_i}|g_i|^2+1}), \forall i \quad (38)$$

$$CF: \quad R_{ab_i} \leq \frac{1}{2K}C(\frac{P_{r_i}|f_i|^2|g_i|^2P_{a_i}}{P_{r_i}|g_i|^2+P_{a_i}|f_i|^2+1}), \forall i \quad (39)$$

$$R_{ba_i} \leq \frac{1}{2K}C(\frac{P_{r_i}|f_i|^2|g_i|^2P_{b_i}}{P_{r_i}|f_i|^2+P_{b_i}|g_i|^2+1}), \forall i. \quad (40)$$

The relay power allocation problem for AF and CF relaying can thus be expressed as:

$$\max_{\{P_{r_i}\}} \quad f_{AF/CF}(P_{r_1}, P_{r_2}, ...P_{r_K}) \quad (41)$$

$$s.t. \quad \sum_{i=1}^K P_{r_i} \leq P_{r,total}, \quad P_{r_i} \geq 0, \forall i \quad (42)$$

where $f_{AF/CF} = \sum_{i=1}^K \frac{w_{ab_i}}{2K}C(\frac{P_{r_i}|f_i|^2|g_i|^2P_{a_i}}{P_{r_i}|g_i|^2+P_{a_i}|f_i|^2+P_{b_i}|g_i|^2+1}) + \frac{w_{ba_i}}{2K}C(\frac{P_{r_i}|f_i|^2|g_i|^2P_{b_i}}{P_{r_i}|f_i|^2+P_{a_i}|f_i|^2+P_{b_i}|g_i|^2+1})$ for AF relaying

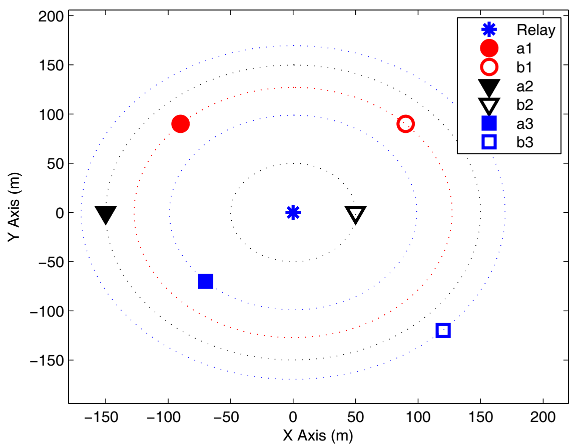

Fig. 3. A multiuser two-way relay network.

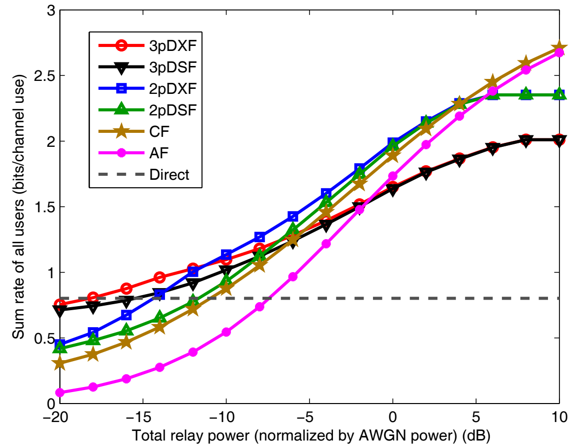

Fig. 4. Comparison of various two-way relaying schemes with optimum power allocation.

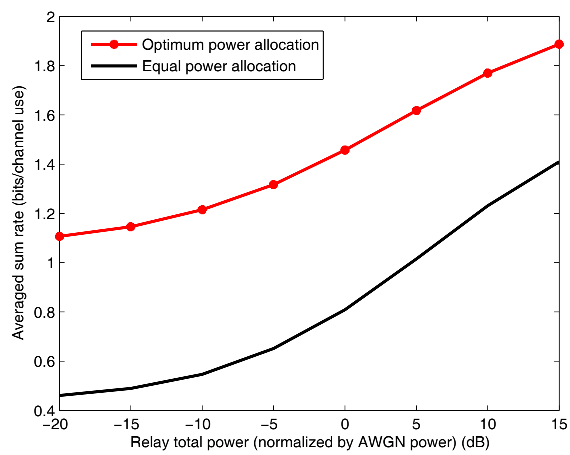

Fig. 5. Comparison of optimum power allocation and equal power allocation.

and $f_{AF/CF} = \sum_{i=1}^{K} \frac{w_{ab_i}}{2K} C(\frac{P_{r_i}|f_i|^2|g_i|^2 P_{a_i}}{P_{r_i}|g_i|^2+P_{a_i}|f_i|^2+1}) + \frac{w_{ba_i}}{2K} C(\frac{P_{r_i}|f_i|^2|g_i|^2 P_{b_i}}{P_{r_i}|f_i|^2+P_{b_i}|g_i|^2+1})$ for CF relaying. Again, the problem is convex for both AF and CF relaying, and the global optimum solution can be found. Note that for both cases the objective function is an increasing function of P_{r_i} and approaches to the upper bound $\sum_{i=1}^{K} \frac{w_{ab_i}}{2K} C(P_{a_i}|f_i|^2) + \frac{w_{ab_i}}{2K} C(P_{b_i}|g_i|^2)$ as P_{r_i} goes to infinity.

V. Numerical Results

In this section, we present the numerical results to demonstrate the performance of the optimum power allocation for various multiuser two-way relaying schemes. More numerical results are included in[10]. We first consider a network shown in Figure 3 with 3 user pairs and one relay node. We assume path-loss model where the channel gains $\{|f_i|^2, |g_i|^2, |h_i|^2\}$ are inversely proportional to the fourth power of the corresponding distances between the nodes. We set the users' transmit power to $P_{a_i}{=}P_{b_i}{=}20dBm$, and assume all AWGN terms having variance $-70dBm$.

In Figure 4, we present the users' sum rate (with $w_{ab_i}{=}w_{ba_i}{=}1$ for all i) achieved by various two-way relaying strategies with optimum power allocation, for a range of total relay power levels. The sum rate achieved by direct transmission is also included in the figure. We observe that different relaying schemes outperform one another for different range of relay power. When the relay has a low power budget, three-phase schemes outperform two-phase ones due to the dominating contribution from the direct links. As the relay power increases, two-phase schemes may become better when the relay-assisted transmissions dominate the rates since the pre-log factor is $1/2$ for two-phase schemes while it is $1/3$ for three-phase schemes. As the relay power keeps increasing, all schemes eventually reach (DSF/DXF) or approach (AF/CF) their upper bounds. We also note that the DXF schemes outperform the corresponding DSF schemes until they reach their upper bounds, because forwarding an XORed message to both directions is more power efficient than forwarding individual messages.

Next, we compare the average performance of the optimum and equal power allocation, with the latter equally distributing the relay power among all the assisted users. We generate 100 networks with the relay at the origin and 3 user pairs randomly distributed in the area of $[-250m, 250m]^2$. For each network, we generate 1000 Rayleigh fading realizations, i.e., the channel coefficients follow the Rayleigh distribution with the variances following the path-loss model. For every realization, at different relay power, we find the highest sum rate achieved by one of the considered relaying schemes with the optimum and the equal power allocation, respectively. The results are averaged over all fading and network topology realizations and presented in Figure 5. We observe that the optimum power allocation achieves a significant sum rate performance gain upon the equal power allocation, especially when the relay power is low.

VI. Conclusion

In this paper, we have investigated the optimum relay power allocation problem for a multiuser two-way relay network with a variety of two-way relaying protocols. The obtained relay power allocation solutions, which maximize an arbitrary weighted sum of rates in the network, allow us to trace the boundary of the achievable rate region for each relaying scheme. By comparing the performance of different two-way relaying schemes with optimum power allocation, we conclude that, given a relay power budget, we can always choose the relaying scheme (DXF/CF) and the corresponding optimum power allocation algorithm to obtain the highest weighted sum rate. Finally, we remark that, while this paper considers the centralized approach, the distributed relay power allocation remains interesting and to be discovered.

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