Multiuser Two-Way Relaying: Detection and Interference Management Strategies

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***Abstract*—We consider a *multiuser two-way relay* network where multiple pairs of users communicate with their preassigned partners, using a common intermediate relay node, in a two-phase communication scenario employing Code Division Multiple Access (CDMA). By taking advantage of the bidirectional communication structure, we first propose that each pair of partners share a common spreading signature and design a jointly demodulate-and-XOR forward (JD-XOR-F) relaying scheme, where all users transmit to the relay simultaneously followed by the relay broadcasting an estimate of the XORed symbol for each user pair. We derive the decision rules and the corresponding bit error rates (BERs) at the relay and at the users' receivers. We then investigate the joint power control and receiver optimization problem for each phase for this multiuser two-way relay network with JD-XOR-F relaying. We solve each optimization problem by constructing the iterative power control and receiver updates that converge to the corresponding unique optimum. Simulation results are presented to demonstrate the performance of the proposed multiuser two-way JD-XOR-F relaying scheme in conjunction with the joint power control and receiver optimization algorithms. Specifically, we observe significant power savings and user capacity improvement with the proposed communication scheme as compared to the designs with a "one-way" communication perspective.**

***Index Terms*—Two-way relaying, power control, MMSE-multiuser detection.**

I. INTRODUCTION

RECENT demand on wireless ad hoc and peer-to-peer networking has prompted interest towards the design of relay-assisted communication protocols. To this end, two-way relay networks, where nodes exchange information via the help of intermediate relay node(s) and make explicit use of the bi-directional nature of communication, have attracted recent attention.

A number of different protocols have been proposed for the two-way relay channel, where a relay node helps two end nodes exchange information. These protocols aim to improve upon the traditional four-phase relaying in terms of throughput, achievable rates and power efficiency [3]–[13]. In the three-phase protocols proposed in [3], [4], [7], the two end nodes transmit sequentially to a relay which broadcasts the XORed version of the two nodes' symbols in the third phase after decoding them. In contrast, in the two-phase protocols, the nodes simultaneously transmit to the relay node in the first phase, and the relay, in the second phase, can amplify and forward the received signal[8], [10], [11], or decode and forward an XORed or superposed symbol of both parties[7], [8], [12].

The two-way relay network model is extended to scenarios where nodes are equipped with multiple antennas as well as the systems with multiple pairs of communicating partners and/or multiple relay nodes. Transceiver design for MIMO two-way relaying is studied in [14]–[17]. For the networks consisting of one user pair assisted by multiple relay nodes, relay selection problem is studied in [18], and distributed space-time coding at relays is considered in [19]. Two-way relaying for multiple user pairs is considered in [8], [20], where there are a sufficiently large number of relay nodes, or antennas, such that the overall channels of all user pairs are orthogonalized by zero-forcing. Two-way amplify-and-forward relaying over OFDM is investigated in [21] where tone permutation and power allocation algorithms are found to maximize the sum rate of the two partners.

Wireless networks of the near future are most likely to consist of many nodes wishing to exchange information, potentially having to share intermediate relays. To that end, in this paper, we consider the communication scenario where a *single* intermediate relaying node assists multiple user pairs, henceforth termed, *multiuser two-way relaying*. In such a multiuser two-way communication system, a reasonable choice to support multiple users is code division multiple access (CDMA) whose merits and limitations are well understood in the context of one-way cellular communications as well as some ad hoc settings[22], [23]. It is well known that CDMA systems are interference limited, and can provide reliable communication for a desired number of users at a quality of service (QoS) level and processing gain. Multiuser two-way relaying systems employing CDMA are naturally interference limited as well: random locations and different surroundings of communicating nodes result in different radio propagation on the communication channels, affecting greatly the level of interference experienced by each user in the system, and hence the QoS of the communications. Therefore, *interference management* is a key design issue for such systems and is a focus of this paper: via careful choice of the *relaying scheme* as well as the *transmit and receive strategies*, we aim to reduce and control the interference experienced by each user such that the system QoS requirements are met with minimum power

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expenditure at all users and the relay.

In this paper, we first design the multiuser two-way relaying strategy for the system at hand. We observe that, the bi-directional nature of communications allows each pair of users to share a common spreading signature, which, when used with the appropriate detection and relaying strategy, significantly reduces the multiple access interference (MAI). The communication is carried out in two phases: all users transmit to the relay simultaneously in phase one, and the relay jointly demodulates and generates an estimate of the XORed symbol to broadcast for each user pair in phase two. This relaying scheme is referred to as *jointly demodulate-and-XOR forward* (JD-XOR-F) in the sequel.

Next, we derive the decision rules and the corresponding bit error rates (BERs) at the relay and at the end users when the JD-XOR-F relaying is employed. We further consider interference management for this multiuser two-way relay network via joint power control and receiver optimization for each phase, and construct iterative algorithms, each of which is shown to converge to the corresponding unique optimum. We validate our theoretical findings by showing numerical examples where the proposed JD-XOR-F relaying with interference management is observed to provide significant power savings and improve the user capacity over the designs with a "one-way" communication perspective. The reminder of the paper is organized as follows. In Section II, the system model and the JD-XOR-F relaying are introduced. The decision rule and the BER are derived in Section III, and the power control and receiver optimization problem is investigated in Section IV, both for phase one. Phase two is studied in Section V. Numerical results are presented in Section VI. Section VII concludes the paper.

II. System Model and JD-XOR-F Relaying

We consider the multiuser two-way relay network shown in Figure 1, which consists of $2K+1$ nodes: K pairs of users and an intermediate relay node. User i_1 and i_2 ($i \in [1, K]$) are a pair of partners who wish to communicate with each other via the relay node 0. Each node wishes to communicate with one pre-assigned partner and sees the remaining transmissions as interference. We assume all nodes are half-duplex and there is no direct link between partners. The information exchange is done in two phases. The first phase is for the transmissions from all the users to the relay, and the second phase is dedicated to the transmission from the relay to all the users.

To accommodate the communications of multiple pairs of users simultaneously, direct sequence (DS)-CDMA is employed. For clarity of exposition, we assume a synchronous[1] DS-CDMA system with coherent detection and real-valued signal processing. We assume that non-orthogonal signatures are employed and the spreading gain is N. If CDMA is employed in the traditional sense, i.e., for "one-way" communication, each user's symbol needs to be spread by a distinct signature, $2K$ signatures are needed, and the number of interfering users is $2K-1$. On the other hand, in two-way

[1]The synchronous model can be extended to asynchronous systems with time offset or phase offset when the channel state information is known at the related receivers.

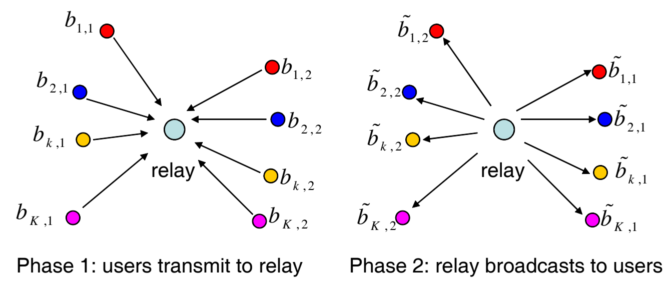

Fig. 1. System model.

communications, each user can recover its partner's symbol from a common signal broadcasted from the relay, utilizing the side information that is *its own symbol*. Therefore, we propose that in the multiuser two-way relay network, each pair of users transmit with the same signature waveform. This way, only K signatures are needed, and the multiple access interference (MAI) present in the system can be (potentially) reduced.

Considering the bi-directional communication structure, we propose the *jointly demodulate-and-XOR forward* (JD-XOR-F) relaying scheme as follows. The relay node receives the signals superimposed from all users in the first phase, employs a linear multiuser detector to jointly demodulate and decide for each pair whether the two partners have sent the same symbol. Once this decision is made, the relay can generate the XORed symbol to transmit in the second phase for that user pair, since the XOR operation of two binary symbols is equivalent to indicating whether they are the same. Receiving this from the relay, each user can employ a linear multiuser detector to suppress interference contributed by users other than its partner and itself, and then recover its partner's symbol by an XOR operation of the received XORed symbol with the symbol it transmitted in the first phase.

III. Phase One: Decision Rule and the BER

In phase one, all users transmit their symbols to the relay simultaneously. The ith pair, users i_1 and i_2, use their common signature waveform $s_i(t)$ to spread their symbols $b_{i,1}$ and $b_{i,2}$, with transmit power $p_{i,1}$ and $p_{i,2}$, respectively. The index t indicates time. The channel gain from user i_m to the relay is denoted by $h_{i,m}$ for $i = 1, ..., K$ and $m = 1, 2$. Reciprocal channels are assumed, i.e., the channel gain from the relay to user i_m is $h_{i,m}$ as well. The channel gains stay constant for the duration of the communication. The received signal at the relay is given by

$$r_0(t) = \sum_{i=1}^{K} (\sqrt{p_{i,1} h_{i,1}} b_{i,1} + \sqrt{p_{i,2} h_{i,2}} b_{i,2}) s_i(t) + n_0(t) \quad (1)$$

where $n_0(t)$ denotes the additive white Gaussian noise (AWGN) at the relay. We assume that $b_{i,1}, b_{i,2} \in \{-1, +1\}$ with equal probability. The discrete-time equivalent received signal at the output of the chip matched filter is

$$\mathbf{r}_0 = \sum_{i=1}^{K} (\sqrt{p_{i,1} h_{i,1}} b_{i,1} + \sqrt{p_{i,2} h_{i,2}} b_{i,2}) \mathbf{s}_i + \mathbf{n}_0 \quad (2)$$

where $\mathbf{s}_i$ denotes the unit norm spreading sequence, $\mathbf{n}_0$ is the zero-mean Gaussian random vector with $E[\mathbf{n}_0 \mathbf{n}_0^\intercal] = \sigma_{n_0}^2 \mathbf{I}_N$,

$(\cdot)^{\mathsf{T}}$ denotes the transpose operation, and $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. In the sequel, we will use this discrete-time representation.

Upon receiving $\mathbf{r}_0$, the relay employs linear filter $\mathbf{c}_i$ to obtain $y_i = \mathbf{c}_i^{\mathsf{T}}\mathbf{r}_0$ as

$$y_i = (\sqrt{p_{i,1}h_{i,1}}b_{i,1} + \sqrt{p_{i,2}h_{i,2}}b_{i,2})\mathbf{c}_i^{\mathsf{T}}\mathbf{s}_i + \underbrace{\sum_{j\neq i}^{K}(\sqrt{p_{j,1}h_{j,1}}b_{j,1} + \sqrt{p_{j,2}h_{j,2}}b_{j,2})\mathbf{c}_i^{\mathsf{T}}\mathbf{s}_j + \mathbf{c}_i^{\mathsf{T}}\mathbf{n}_0}_{N_i} \quad (3)$$

$$= \sqrt{q_{i,1}}b_{i,1} + \sqrt{q_{i,2}}b_{i,2} + N_i \quad (4)$$

where $q_{i,1} = p_{i,1}h_{i,1}(\mathbf{c}_i^{\mathsf{T}}\mathbf{s}_i)^2$ and $q_{i,2} = p_{i,2}h_{i,2}(\mathbf{c}_i^{\mathsf{T}}\mathbf{s}_i)^2$ denote the received power of user i_1 and i_2 at the output of the filter, and N_i denotes the interference plus noise term. Let $\sigma_i^2 = \sum_{j\neq i}(p_{j,1}h_{j,1} + p_{j,2}h_{j,2})(\mathbf{c}_i^{\mathsf{T}}\mathbf{s}_j)^2 + \sigma_{n0}^2\mathbf{c}_i^{\mathsf{T}}\mathbf{c}_i$ be the variance of N_i. We approximate N_i with a Gaussian. This makes the detection for each user pair and the BER analysis tractable. As we will observe in Section VI, the analytical results we obtain in the following match with the simulation results.

The relay uses y_i to make a decision in favor of one of two hypotheses:

$$H_{i0} : b_{i,1} = b_{i,2} \quad (5)$$

$$H_{i1} : b_{i,1} \neq b_{i,2}. \quad (6)$$

Notice that this is tantamount to determining $\hat{b}_i$, the estimate of $b_i = b_{i,1} \oplus b_{i,2}$, where $\oplus$ represents XOR, the bitwise exclusive operation[2]. Specifically, we have $\hat{b}_i = -1$ when the relay decides $b_{i,1}$ and $b_{i,2}$ are same (H_{i0}) and $\hat{b}_i = 1$ if they have the opposite sign (H_{i1}). The maximum aposteriori probability (MAP) decision rule can be expressed as

$$\frac{\Pr(y_i|b_{i,1} = b_{i,2})\Pr(b_{i,1} = b_{i,2})}{\Pr(y_i|b_{i,1} \neq b_{i,2})\Pr(b_{i,1} \neq b_{i,2})} \underset{\hat{b}_i=1}{\overset{\hat{b}_i=-1}{\gtrless}} 1. \quad (7)$$

With the Gaussian assumption on N_i, this is equivalent to the following:

$$e^{-\frac{(y_i-(-\sqrt{q_{i,1}}-\sqrt{q_{i,2}}))^2}{2\sigma_i^2}} + e^{-\frac{(y_i-(\sqrt{q_{i,1}}+\sqrt{q_{i,2}}))^2}{2\sigma_i^2}} \underset{\hat{b}_i=1}{\overset{\hat{b}_i=-1}{\gtrless}} e^{-\frac{(y_i-(-\sqrt{q_{i,1}}+\sqrt{q_{i,2}}))^2}{2\sigma_i^2}} + e^{-\frac{(y_i-(\sqrt{q_{i,1}}-\sqrt{q_{i,2}}))^2}{2\sigma_i^2}}. \quad (8)$$

Based on (8), we find that the optimum decision rule is:

$$\hat{b}_i = \begin{cases} 1, & \text{when} \quad y_i \in R^{opt} = \{y_i| -y_{th}^{opt} < y_i < y_{th}^{opt}\}; \\ -1, & \text{otherwise.} \end{cases} \quad (9)$$

where the decision threshold y_{th}^{opt} is the positive root of the following equation:

$$1+e^{\frac{2y_{th}^{opt}(\sqrt{q_{i,1}}+\sqrt{q_{i,2}})}{\sigma_i^2}} = \left(e^{\frac{2y_{th}^{opt}\sqrt{q_{i,1}}}{\sigma_i^2}} + e^{\frac{2y_{th}^{opt}\sqrt{q_{i,2}}}{\sigma_i^2}}\right)e^{\frac{2\sqrt{q_{i,1}q_{i,2}}}{\sigma_i^2}} \quad (10)$$

which can be shown to have two real roots with identical absolute values. Under this decision rule, $Pe1_i^{opt}$, the probability that the relay makes an incorrect decision on $\hat{b}_i$, is obtained as

$$Pe1_i^{opt} = \frac{1}{2}\Big(Q\Big(\frac{-y_{th}^{opt}+\sqrt{q_{i,1}}+\sqrt{q_{i,2}}}{\sigma_i}\Big) + Q\Big(\frac{-y_{th}^{opt}-\sqrt{q_{i,1}}-\sqrt{q_{i,2}}}{\sigma_i}\Big) + Q\Big(\frac{y_{th}^{opt}-\sqrt{q_{i,1}}+\sqrt{q_{i,2}}}{\sigma_i}\Big) + Q\Big(\frac{y_{th}^{opt}+\sqrt{q_{i,1}}-\sqrt{q_{i,2}}}{\sigma_i}\Big) - 1\Big) \quad (11)$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}}e^{-t^2/2}dt$. Since the above decision rule does not have a closed-form expression for the decision threshold in general[3], it may bring implementation difficulties in practice and the evaluation of the BER may become intractable.

Alternatively, we can consider a four-hypothesis testing model with $H_{i(0,0)}$, $H_{i(0,1)}$, $H_{i(1,0)}$, $H_{i(1,1)}$ corresponding to $(b_{i,1}, b_{i,2})$ =$(-1,-1)$, $(-1,1)$, $(1,-1)$ and $(1,1)$, respectively. The relay first uses the MAP rule to detect $(\hat{b}_{i,1}, \hat{b}_{i,2})$, i.e.,

$$(\hat{b}_{i,1}, \hat{b}_{i,2}) = \underset{\substack{(\hat{b}_{i,1},\hat{b}_{i,2})\in \\ \{(-1,-1),(1,-1),(-1,1),(1,1)\}}}{\arg\max} \Pr\left(y_i|(\hat{b}_{i,1}, \hat{b}_{i,2})\right)\Pr\left(\hat{b}_{i,1}, \hat{b}_{i,2}\right) \quad (12)$$

and then generates $\hat{b}_i$ as $\hat{b}_i = \hat{b}_{i,1} \oplus \hat{b}_{i,2}$. While this decision rule has a potential performance loss[4] as compared to the one based on the two-hypothesis model as our aim is to estimate the XORed quantity rather than the individual bits, the performance loss is negligible as we will observe in Section VI, and it greatly simplifies the decision rule to be:

$$\hat{b}_i = \begin{cases} 1, & \text{when} \quad y_i \in R = \{y_i| -y_{th} < y_i < y_{th}\}; \\ -1, & \text{otherwise.} \end{cases} \quad (13)$$

$$\text{with} \quad y_{th} = \begin{cases} \sqrt{q_{i,1}}, & \text{when} \quad q_{i,1} \geq q_{i,2}; \\ \sqrt{q_{i,2}}, & \text{otherwise.} \end{cases} \quad (14)$$

Under this decision rule, $Pe1_i$, the probability of incorrectly determining $\hat{b}_i$ is

$$Pe1_i = \begin{cases} Q(\sqrt{\eta_{i,2}}) + \frac{1}{2}Q(2\sqrt{\eta_{i,1}} - \sqrt{\eta_{i,2}}) - \frac{1}{2}Q(2\sqrt{\eta_{i,1}} + \sqrt{\eta_{i,2}}), & \text{when} \quad \eta_{i,1} \geq \eta_{i,2} \\ Q(\sqrt{\eta_{i,1}}) + \frac{1}{2}Q(2\sqrt{\eta_{i,2}} - \sqrt{\eta_{i,1}}) - \frac{1}{2}Q(2\sqrt{\eta_{i,2}} + \sqrt{\eta_{i,1}}), & \text{when} \quad \eta_{i,1} < \eta_{i,2} \end{cases} \quad (15)$$

where $\eta_{i,1} = q_{i,1}/\sigma_i^2$ and $\eta_{i,2} = q_{i,2}/\sigma_i^2$ are the received SIR of user i_1 and i_2 at the relay.

Note that having $\eta_{i,1}$=0 or $\eta_{i,2}$=0 leads to $Pe1_i$=0.5, which is not a desired situation in communication systems. In the sequel, we will exclude this case. We then have the following lemma.

Lemma 1: The error probability $Pe1_i(\eta_{i,1}, \eta_{i,2})$ is a quasiconvex function of $(\eta_{i,1}, \eta_{i,2})$ on both R_1 and R_2, where $R_1 = \{(\eta_{i,1}, \eta_{i,2})|\eta_{i,1} > 0,\ \eta_{i,2} > 0,\ \text{and}\ \eta_{i,1} \geq \eta_{i,2}\}$, and $R_2 = \{(\eta_{i,1}, \eta_{i,2})|\eta_{i,1} > 0,\ \eta_{i,2} > 0,\ \text{and}\ \eta_{i,1} < \eta_{i,2}\}$.

Proof: See Appendix A. ■

IV. Phase One: Power Control and Receiver Optimization

Having determined the BER performance for given transmit power levels of all users and the receivers at the relay, let us now turn to optimum interference management in phase

[2]"0" is mapped to "$\hat{b}_i = -1$" and "1" to "$\hat{b}_i = 1$".

[3]When a pair of users have equal received power level, the analytical expression for the decision threshold y_{th}^{opt} is found as given in [2], [12].

[4]The four-hypothesis approach has the same performance as the two-hypothesis one in the absence of noise.

one. Specifically, we will seek to expend the minimum total transmit power while satisfying the BER requirement on estimating each XORed symbol by jointly optimizing the powers and the receivers.

$$\min_{\{p_{i,1},p_{i,2},\mathbf{c}_i\}} \quad \textstyle\sum_{i=1}^{K}(p_{i,1}+p_{i,2}) \tag{16}$$
$$s.t. \quad Pe1_i \leq \rho 1_i \tag{17}$$
$$p_{i,1}\geq 0, \quad p_{i,2}\geq 0, \quad \mathbf{c}_i\in\mathbb{R}^N, \quad \forall i \tag{18}$$

where $\rho 1_i$ is the corresponding system QoS requirement which is given.

At the outset, one might be tempted to think that a direct application of the existing power control algorithms for (one-way) CDMA systems [24]–[26] would work for the two-way system. In reality, the very advantage that the bi-directional communication presents, i.e., the use of a common signature per pair in conjunction with JD-XOR-F relaying, is the reason why a direct application does not work. In particular, the probability that the relay estimates an incorrect XORed symbol of a pair of users becomes a function of the received signal-to-interference ratio (SIR) of both partners, and we no longer have a one-to-one mapping between the error probability and the partners' received SIRs. To tackle this difficulty, employing the four-hypothesis decision rule found in Section III and the property of the corresponding BER in Lemma 1, we first convert each BER constraint in (17) to its equivalent received SIR constraint by solving a convex problem, and then implement a suitable iterative power control and receiver optimization algorithm whose convergence to the optimum is guaranteed.

First, let us fix the power levels of all users but the ith pair, $\{p_{j,1}\}_{j\neq i}$ and $\{p_{j,2}\}_{j\neq i}$, and the linear filter $\mathbf{c}_i$. The optimization problem in (16)-(18) then reduces to the following optimization problem over the transmit power of the ith user pair only with fixed $\sigma_i^2/(\mathbf{c}_i^{\mathsf T}\mathbf{s}_i)^2$,

$$\min_{(\eta_{i,1},\eta_{i,2})} \quad g(\eta_{i,1},\eta_{i,2}) = \left(\tfrac{\eta_{i,1}}{h_{i,1}}+\tfrac{\eta_{i,2}}{h_{i,2}}\right)\cdot\tfrac{\sigma_i^2}{(\mathbf{c}_i^{\mathsf T}\mathbf{s}_i)^2} \tag{19}$$
$$s.t. \quad Pe1_i\leq\rho 1_i \tag{20}$$
$$\eta_{i,1}>0, \quad \eta_{i,2}>0. \tag{21}$$

Note that since $Pe1_i$ is expressed as a function of $(\eta_{i,1},\eta_{i,2})$ in (15), we replace the variables $p_{i,1}$ and $p_{i,2}$ in (16) by $\eta_{i,1}$ and $\eta_{i,2}$, using the relationship $p_{i,1}=\frac{\eta_{i,1}\sigma_i^2}{h_{i,1}(\mathbf{c}_i^{\mathsf T}\mathbf{s}_i)^2}$ and $p_{i,2}=\frac{\eta_{i,2}\sigma_i^2}{h_{i,2}(\mathbf{c}_i^{\mathsf T}\mathbf{s}_i)^2}$, and let $g(\eta_{i,1},\eta_{i,2})$ denote the objective function.

Let the feasible set of the problem in (19)-(21) be S, which can be partitioned into two sets, $S_1=\{(\eta_{i,1},\eta_{i,2})|(\eta_{i,1},\eta_{i,2})\in S, \text{ and } \eta_{i,1}\geq\eta_{i,2}\}$ and $S_2=\{(\eta_{i,1},\eta_{i,2})|(\eta_{i,1},\eta_{i,2})\in S, \text{ and } \eta_{i,1}<\eta_{i,2}\}$. Since it is proved in Lemma 1 that $Pe1_i$ is quasiconvex on R_1 and R_2, the corresponding lower level sets S_1 and S_2 are both convex sets [27, Ch. 3]. Therefore, replacing S by S_1 and S_2 respectively in problem (19)-(21) leads to two convex problems, each having a unique optimum solution, $(\eta_{i,1}^{\dagger},\eta_{i,2}^{\dagger})$ and $(\eta_{i,1}^{\dagger\dagger},\eta_{i,2}^{\dagger\dagger})$, respectively, due to the fact that each problem minimizes a linear objective function g over a convex feasible set. Thus, the optimum solution of problem (19)-(21), $(\eta_{i,1}^{\star},\eta_{i,2}^{\star})$, can be obtained as

$$\begin{cases}\eta_{i,1}^{\star}=\eta_{i,1}^{\dagger},\eta_{i,2}^{\star}=\eta_{i,2}^{\dagger}, & \text{when } g(\eta_{i,1}^{\dagger},\eta_{i,2}^{\dagger})\leq g(\eta_{i,1}^{\dagger\dagger},\eta_{i,2}^{\dagger\dagger})\\ \eta_{i,1}^{\star}=\eta_{i,1}^{\dagger\dagger},\eta_{i,2}^{\star}=\eta_{i,2}^{\dagger\dagger}, & \text{otherwise.}\end{cases} \tag{22}$$

We note that minimizing g is equivalent to minimizing $\frac{\eta_{i,1}}{h_{i,1}}+\frac{\eta_{i,2}}{h_{i,2}}$ since $\sigma_i^2/(\mathbf{c}_i^{\mathsf T}\mathbf{s}_i)^2$ is fixed. Therefore, $(\eta_{i,1}^{\star},\eta_{i,2}^{\star})$ depends only on $\{\rho 1_i,h_{i,1},h_{i,2}\}$.

Now, we define two quantities γ_i and α_i as

$$\gamma_i=\frac{\eta_{i,1}^{\star}}{h_{i,1}}+\frac{\eta_{i,2}^{\star}}{h_{i,2}} \tag{23}$$
$$\alpha_i=\frac{p_{i,1}^{\star}}{p_{i,2}^{\star}}=\frac{\eta_{i,1}^{\star}}{h_{i,1}}\cdot\frac{h_{i,2}}{\eta_{i,2}^{\star}} \tag{24}$$

where $(\eta_{i,1}^{\star},\eta_{i,2}^{\star})$ is the optimum solution of the problem in (19)-(21). Note that γ_i and α_i depend only on $\{\rho 1_i,h_{i,1},h_{i,2}\}$. We have the following proposition.

Proposition 1: The constraint (17) is satisfied by the following conditions on the transmit power pair $(p_{i,1},p_{i,2})$:

$$p_{i,1}+p_{i,2}\geq\gamma_i\frac{\sigma_i^2}{(\mathbf{c}_i^{\mathsf T}\mathbf{s}_i)^2} \quad\text{and}\quad \frac{p_{i,1}}{p_{i,2}}=\alpha_i. \tag{25}$$

Equivalently, (25) is a sufficient condition for $Pe1_i\leq\rho 1_i$, with the minimum total transmit power requirement on the ith user pair.

Proof: See Appendix B. ■

Thus, after we solve the optimization problem (19)-(21) for the ith user pair, we can replace (17) by (25) and rewrite the optimization problem in (16)-(18) as

$$\min_{\{p_{i,1},p_{i,2},\mathbf{c}_i\}} \quad \textstyle\sum_{i=1}^{K}(p_{i,1}+p_{i,2}) \tag{26}$$
$$s.t. \quad p_{i,1}+p_{i,2}\geq\gamma_i\tfrac{\sigma_i^2}{(\mathbf{c}_i^{\mathsf T}\mathbf{s}_i)^2} \quad\text{and}\quad \tfrac{p_{i,1}}{p_{i,2}}=\alpha_i \tag{27}$$
$$p_{i,1}>0, \quad p_{i,2}>0, \quad \mathbf{c}_i\in\mathbb{R}^N, \quad \forall i. \tag{28}$$

Letting $p_{i,1}=\alpha_i p_{i,2}$, $h_i'=\alpha_i h_{i,1}+h_{i,2}$, $\gamma_i'=\frac{\gamma_i}{1+\alpha_i}$, and recalling that $\sigma_i^2=\sum_{j\neq i}(p_{j,1}h_{j,1}+p_{j,2}h_{j,2})(\mathbf{c}_i^{\mathsf T}\mathbf{s}_j)^2+\sigma_{n_0}^2\mathbf{c}_i^{\mathsf T}\mathbf{c}_i$, the above optimization problem is equivalent to

$$\min_{\{p_{i,2}\}} \quad \textstyle\sum_{i=1}^{K}(1+\alpha_i)p_{i,2} \tag{29}$$
$$s.t. \quad p_{i,2}\geq\gamma_i'\min_{\mathbf{c}_i\in\mathbb{R}^N}\frac{\sum_{j\neq i}h_j'p_{j,2}(\mathbf{c}_i^{\mathsf T}\mathbf{s}_j)^2+\sigma_{n_0}^2\mathbf{c}_i^{\mathsf T}\mathbf{c}_i}{(\mathbf{c}_i^{\mathsf T}\mathbf{s}_i)^2} \tag{30}$$
$$p_{i,2}>0 \quad \forall i. \tag{31}$$

Note that, we have moved the receiver optimization to the constraint (30)[25], which optimizes the receiver of each user pair for a fixed power vector, and its solution is the well-known maximum SIR (and minimum mean-squared error (MMSE)) filter given by

$$\mathbf{c}_i^{\star}=\left(\sum_{j=1}^{K}h_j'p_{j,2}\mathbf{s}_j\mathbf{s}_j^{\mathsf T}+\sigma_{n_0}^2\mathbf{I}\right)^{-1}h_i'p_{i,2}\mathbf{s}_i \tag{32}$$
$$=\left(\sum_{j=1}^{K}(p_{j,1}h_{j,1}+p_{j,2}h_{j,2})\mathbf{s}_j\mathbf{s}_j^{\mathsf T}+\sigma_{n_0}^2\mathbf{I}\right)^{-1}(p_{i,1}h_{i,1}+p_{i,2}h_{i,2})\mathbf{s}_i. \tag{33}$$

We can now define

$$I_i(\mathbf{p}_2, \mathbf{c}_i) = \gamma_i' \frac{\sum_{j\neq i} h_j' p_{j,2} (\mathbf{c}_i^\mathsf{T} \mathbf{s}_j)^2 + \sigma_{n_0}^2 \mathbf{c}_i^\mathsf{T} \mathbf{c}_i}{(\mathbf{c}_i^\mathsf{T} \mathbf{s}_i)^2} \quad (34)$$

$$U_i(\mathbf{p}_2) = \min_{\mathbf{c}_i \in \mathbb{R}^N} I_i(\mathbf{p}_2, \mathbf{c}_i) \quad (35)$$

and the iterative power control algorithm

$$\mathbf{p}_2(n+1) = \mathbf{U}(\mathbf{p}_2(n)) \quad (36)$$

where $\mathbf{p}_2 = [p_{1,2}, ..., p_{K,2}]^\mathsf{T}$, $\mathbf{U}(\mathbf{p_2}) = [U_1(\mathbf{p_2}), ..., U_K(\mathbf{p_2})]^\mathsf{T}$, and n is the iteration index. It is worth emphasizing that h_i' and γ_i' are calculated prior to the iterative updates, using γ_i and α_i in (23) and (24), which are obtained by solving the problem in (19)-(21) for the ith user pair. As shown in [25], [26], $\mathbf{U}(\mathbf{p}_2)$ is a standard interference function for all $\mathbf{p}_2 \geq 0$, i.e., it satisfies the three properties[24]:

- Positivity: $\mathbf{U}(\mathbf{p}_2) > 0$
- Monotonicity: If $\mathbf{p}_2 \geq \mathbf{p}_2'$ then $\mathbf{U}(\mathbf{p}_2) \geq \mathbf{U}(\mathbf{p}_2')$
- Scalability: For all $\mu > 1$, $\mu \mathbf{U}(\mathbf{p}_2) > \mathbf{U}(\mu \mathbf{p}_2)$

Therefore, the power control algorithm (36), which includes an optimization of the linear receivers, converges to the minimum total transmit power solution of the optimization problem in (16)-(18), where the linear receivers converge to the corresponding MMSE receivers at the optimum. Recall that $p_{i,1} = \alpha_i p_{i,2}$. Note that the implementation of this algorithm requires that the pair of partners know each other's channel gain, but not those of the interferers.

V. Phase Two

A. Communications, Decision Rule and the BER in Phase Two

In phase two, the relay spreads $\hat{b}_i$ with $\mathbf{s}_i$, for $i = 1, ..., K$, and broadcasts

$$x_0 = \sum_{i=1}^{K} \sqrt{p_{0,i}} \hat{b}_i \mathbf{s}_i \quad (37)$$

to all users, where $p_{0,i}$ is the transmit power to broadcast $\hat{b}_i$ at the relay. The received signal at user i_m in the second phase is

$$\mathbf{r}_{i,m} = \sqrt{h_{i,m}} \left(\sum_{j=1}^{K} \sqrt{p_{0,j}} \hat{b}_j \mathbf{s}_j \right) + \mathbf{n}_{i,m}, \quad i = 1, ..., K, \ m = 1, 2 \quad (38)$$

where $\mathbf{n}_{i,m}$ is the AWGN vector with covariance matrix $\sigma_{n_{i,m}}^2 \mathbf{I}_N$. User i_m employs $\mathbf{c}_{i,m}$:

$$\begin{aligned} y_{i,m} &= \mathbf{c}_{i,m}^\mathsf{T} \mathbf{r}_{i,m} \\ &= \sqrt{p_{0,i} h_{i,m}} (\mathbf{c}_{i,m}^\mathsf{T} \mathbf{s}_i) \hat{b}_i + \sum_{j \neq i} \sqrt{p_{0,j} h_{i,m}} (\mathbf{c}_{i,m}^\mathsf{T} \mathbf{s}_j) \hat{b}_j + \mathbf{c}_{i,m}^\mathsf{T} \mathbf{n}_{i,m}. \end{aligned} \quad (39)$$

The received SIR is given by

$$SIR_{i,m} = \frac{p_{0,i} h_{i,m} (\mathbf{c}_{i,m}^\mathsf{T} \mathbf{s}_i)^2}{\sum_{j \neq i} p_{0,j} h_{i,m} (\mathbf{c}_{i,m}^\mathsf{T} \mathbf{s}_j)^2 + \sigma_{n_{i,m}}^2 \mathbf{c}_{i,m}^\mathsf{T} \mathbf{c}_{i,m}}. \quad (40)$$

Since $\hat{b}_i$ is binary with equal probability, user i_m can obtain $\hat{b}_{i,m}$, the hard decision estimate of $\hat{b}_i$, as $\hat{b}_{i,m} = 1$ if $y_{i,m} > 0$ and $\hat{b}_{i,m} = -1$ otherwise. Similar as in phase one, we approximate the interference plus noise term with a Gaussian. This way, the error probability of recovering $\hat{b}_i$ at user i_m can be approximated as

$$Pe2_{i,m} \approx Q(\sqrt{SIR_{i,m}}). \quad (41)$$

Next, user i_m performs an XOR operation on $\hat{b}_{i,m}$ and its own symbol $b_{i,m}$ to recover its partner's symbol.

B. Power Control and Receiver Optimization in Phase Two

The joint power control and receiver optimization problem in phase two can be formulated as

$$\min_{\{p_{0,i}, \mathbf{c}_{i,1}, \mathbf{c}_{i,2}\}} \quad \textstyle\sum_{i=1}^{K} p_{0,i} \quad (42)$$

$$s.t. \quad Pe2_{i,1} \leq \rho 2_{i,1}, \quad Pe2_{i,2} \leq \rho 2_{i,2} \quad (43)$$

$$p_{0,i} \geq 0, \quad \mathbf{c}_{i,1} \in \mathbb{R}^N, \quad \mathbf{c}_{i,2} \in \mathbb{R}^N, \ \forall i \quad (44)$$

where $\rho 2_{i,1}$ and $\rho 2_{i,2}$ are the corresponding system QoS requirements at the ith pair of users, which are given.

From the one-to-one mapping in (41), we note that the error probability requirement $Pe2_{i,m} \leq \rho 2_{i,m}$ is equivalent to the SIR requirement $SIR_{i,m} \geq \gamma_{i,m}$ with $\gamma_{i,m}$ satisfying $\rho 2_{i,m} = Q(\sqrt{\gamma_{i,m}})$. Therefore, we have

$$p_{0,i} \geq V_{i,m}(\mathbf{p}_0) = \min_{\mathbf{c}_{i,m} \in \mathbb{R}^N} I_{i,m}, \ i = 1, ..., K, \ m = 1, 2 \quad (45)$$

where $\mathbf{p}_0 = [p_{0,1}, ..., p_{0,K}]^\mathsf{T}$ and

$$I_{i,m} = \frac{\gamma_{i,m}}{h_{i,m}} \cdot \frac{\sum_{j \neq i} p_{0,j} h_{i,m} (\mathbf{c}_{i,m}^\mathsf{T} \mathbf{s}_j)^2 + \sigma_{n_{i,m}}^2 \mathbf{c}_{i,m}^\mathsf{T} \mathbf{c}_{i,m}}{(\mathbf{c}_{i,m}^\mathsf{T} \mathbf{s}_i)^2}. \quad (46)$$

The solution of the optimization problem on the right hand side of (45) for fixed power levels is, once again, the MMSE filter that is given by:

$$\mathbf{c}_{i,m}^\star = \left(\sum_{j=1}^{K} h_{j,m} p_{0,j} \mathbf{s}_j \mathbf{s}_j^\mathsf{T} + \sigma_{n_{i,m}}^2 \mathbf{I} \right)^{-1} \sqrt{h_{i,m} p_{0,i}} \mathbf{s}_i. \quad (47)$$

The optimization problem in (42)-(44) is equivalent to

$$\min_{\mathbf{p}_0} \quad \textstyle\sum_{i=1}^{K} p_{0,i} \quad (48)$$

$$s.t. \quad p_{0,i} \geq V_i(\mathbf{p}_0) = \max(V_{i,1}(\mathbf{p}_0), V_{i,2}(\mathbf{p}_0)), \ \forall i. \quad (49)$$

We note that $V_{i,1}(\mathbf{p}_0)$ and $V_{i,2}(\mathbf{p}_0)$ are in the same form as the interference function defined in [25], which is proved to be standard. Also, we know by[24, Theorem 5] that the maximum of two standard interference functions is standard. Thus, $V_i(\mathbf{p}_0)$ is standard. We define the power control algorithm for the second phase as

$$\mathbf{p}_0(n+1) = \mathbf{V}(\mathbf{p}_0(n)) \quad (50)$$

where $\mathbf{V}(\mathbf{p}_0) = [V_1(\mathbf{p}_0), ..., V_K(\mathbf{p}_0)]^\mathsf{T}$. The standard interference function $\mathbf{V}(\mathbf{p}_0(n))$ guarantees that the power control algorithm in (50) converges to the minimum total transmit power solution of the optimization problem in (42)-(44). Similar as in phase one, updating the relay power and the receivers for the ith pair in phase two only requires the information of the channel gains of that pair, but none of the interferers.

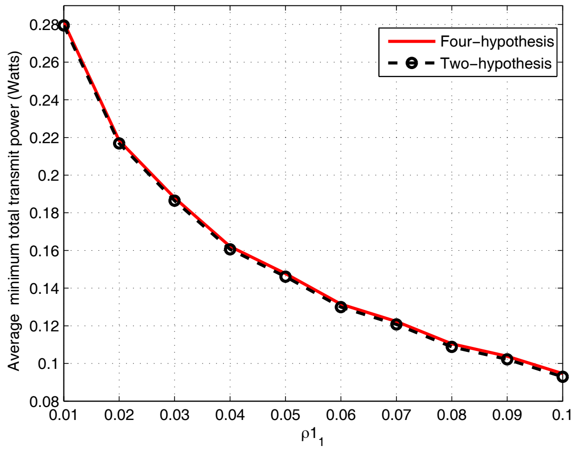

Fig. 2. Comparison of the two- and four-hypothesis decision rules for JD-XOR-F scheme at the relay in phase one.

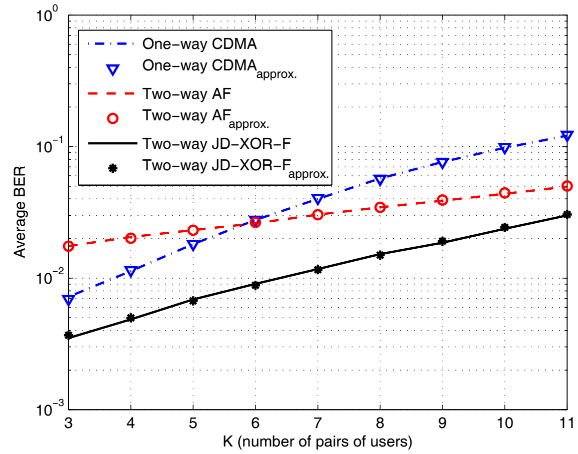

Fig. 3. End-to-end BER performance.

VI. Numerical Results

In this section, we present numerical results related to the performance of the proposed multiuser two-way relay system with JD-XOR-F scheme, and compare it with the one-way CDMA relaying, where all the users transmit to the relay with distinct signatures in the first phase, and the relay estimates the symbol for each user, and spreads and forwards it to its corresponding partner with the partner's signature in the second phase.

In Figure 2, we first present the performance comparison between the two-hypothesis and the four-hypothesis decision rule at the relay in (9) and (13), respectively, for one pair of users. For different QoS requirement of the first phase, $\rho 1_1$, ranging from 0.01 to 0.1, we search the minimum total transmit power of the users for 1000 network topology realizations using the two- and four-hypothesis decision rules, according to (11) and (15), respectively, and average the total transmit power over all realizations. In each realization, the users are randomly distributed on a disk with the relay at the center, and the distances between the relay and users are between 100m-1000m following a uniform distribution. All channel gains follow the path-loss model, i.e., $h_{1,m} = d_{1,m}^{-a}$ where $d_{1,m}$ is the distance between user 1_m and the relay, for $m = 1, 2$. The path-loss exponent $a = 4$ and the AWGN variance 10^{-13} are used in the simulations[5]. In Figure 2, we observe that the performance loss of the four-hypothesis decision rule compared with the two-hypothesis one on the average total transmit power is negligible.

Next, we demonstrate the performance gain on end-to-end BER and user capacity of the proposed two-way JD-XOR-F relaying scheme, when all users have equal received power. In Figure 3, we plot the end-to-end BER of a user receiving its partner's symbol in the multiuser two-way relay system using different relaying schemes: JD-XOR-F relaying, one-way CDMA relaying, and amplify-and-forward (AF) relaying. With the two-way AF relaying [2], upon receiving the signals from users in phase one, the relay amplifies and broadcasts the received signal in phase two, where the amplifying scalar is chosen to satisfy the relay transmit power constraint. In phase two, each user first subtracts its self-interference from the signal received from the relay, and then recovers its partner's symbol with a linear MMSE receiver. We consider a system with K pairs of users where $K \in [3, 11]$ and spreading gain $N = 20$. We set the distance between each user and the relay to 500m, and use the same path-loss model and AWGN variance 10^{-13} as in Figure 2. The transmit power of each user in phase one is set to 0.0625 Watts for all schemes, so that the received SNR of each user symbol at the relay is 10dB. To ensure a fair comparison between the different schemes, the relay transmit power in phase two is set to 0.0625 Watts for each XORed symbol in JD-XOR-F scheme, 0.0625 Watts for each user symbol in one-way CDMA, and $2K \times 0.0625$ Watts as the total relay power in the AF scheme. The BER is averaged over 100 sets of randomly generated sequences, and the BER of each set of sequences is averaged over 1000 realizations of the AWGN channels. We also calculate the approximate BER using the corresponding Q-function expressions, denoted by the subscript "approx." in Figure 3. We observe that the BER approximations match with the simulation results.

We observe in Figure 3 that, the proposed JD-XOR-F scheme provides a remarkable BER performance gain upon one-way CDMA scheme as a pair of users share a common signature. The AF scheme outperforms one-way CDMA scheme as K increases beyond 6, since the AF scheme acts as one-hop communication with an enlarged noise term which no longer dominates the BER when MAI becomes larger. The JD-XOR-F scheme outperforms the AF scheme due to the fact that the former one reduces the MAI by transmitting an XORed symbol from the relay for each user pair in the second phase.

In Figure 4, we compare the maximum number of pairs, K_{max}, that can be simultaneously supported in the multiuser two-way relay system with JD-XOR-F relaying and one-way CDMA relaying. We set the received SNR levels large enough (40dB) so that the maximum number of supported users of the one-way CDMA scheme approaches the user capacity achieved by optimal sequences and power control without power constraint [28]. For processing gain $N \in \{24, 28, 32, 36, 40\}$, we find the K_{max} that achieves an end-to-

[5]With these settings, a user located 500m away from the relay with transmit power 0.0625 Watts has received SNR 10dB at the relay.

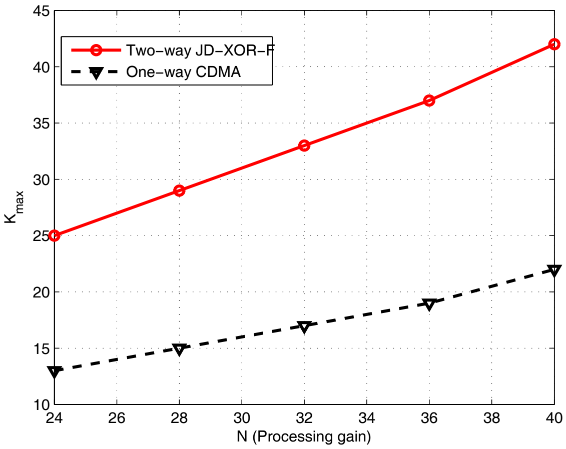

Fig. 4. Maximum number of pairs of users that can be supported with end-to-end BER ≤ 0.01.

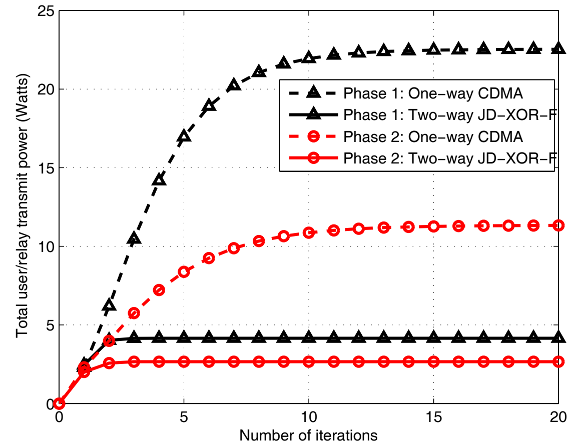

Fig. 6. Comparison of the total user/relay transmit power between two-way JD-XOR-F relaying and one-way CDMA relaying in phase one/two.

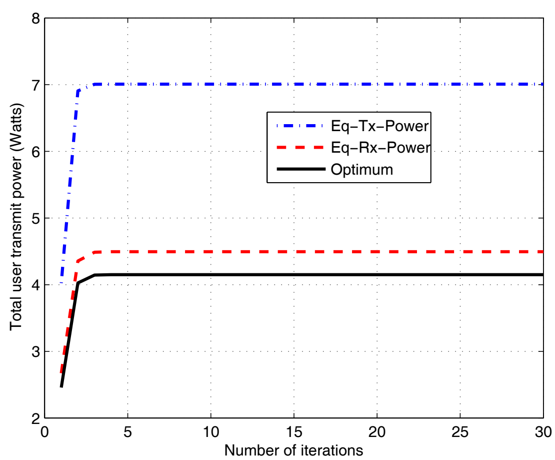

Fig. 5. Comparison of the total user transmit power among different power control algorithms in phase one of the two-way JD-XOR-F relaying.

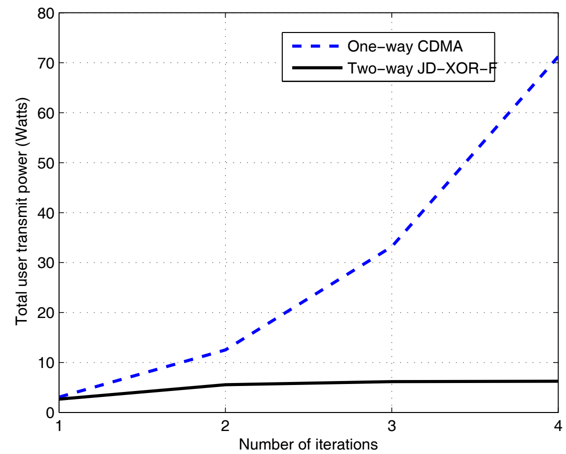

Fig. 7. Comparison of the total user transmit power between two-way JD-XOR-F relaying and one-way CDMA relaying in phase one in an overloaded system.

end BER≤ 0.01. We observe that the multiuser two-way relay system with JD-XOR-F scheme can support almost *twice* of the number of user pairs that can be supported with the one-way CDMA scheme. This is expected as the two-way relaying utilizes the bi-directional communication structure and needs only half number of the signatures as compared to the one-way CDMA relaying. It significantly reduces the MAI experienced in the system by jointly demodulating and XOR combining the symbols of each user pair at the relay.

Next, we demonstrate the performance gain of the proposed joint power control and receiver optimization algorithm for the multiuser two-way relay system. Specifically, we plot the updated total power with respect to the number of iterations for different iterative power control algorithms for the JD-XOR-F relaying and the one-way CDMA relaying. In the results presented in Figures 5-7, all users are randomly distributed in the same area and follow the same path-loss model as in the settings of Figure 2. Both the system topology and the signatures are generated once and then fixed for all the simulations. The variance of all AWGN terms is 10^{-13}. The system BER requirements for the multiuser two-way JD-XOR-F relaying scheme in both the first and the second phases are $\rho 1_i = \rho 2_{i,1} = \rho 2_{i,2} = 0.0189$. The BER requirements of receiving each user's symbol at the relay in phase one and at the users in phase two in the one-way CDMA relaying are set to 0.0189 as well. This way, the two systems achieve the same end-to-end BER at 0.037 for a fair comparison. The spreading gain is $N = 20$, and the number of user pairs is $K = 11$ in Figures 5-6 and $K = 13$ in Figure 7, i.e., the number of users is 22 and 26 respectively.

In Figure 5, we compare the total user transmit power of three different power control algorithms in the first phase of the multiuser two-way JD-XOR-F relaying: the optimum power control (Optimum), the power control with equal received power (Eq-RX-Power) and with equal transmit power (Eq-TX-Power) of each of the two partners. As expected, the optimum power control achieves the minimum total user power consumption. While both Eq-TX-Power and Eq-RX-Power consume more power than the optimum power control, we observe that Eq-RX-Power algorithm may not be too far off as compared to the optimum one. When the ith pair of partners have equal received power at the relay, the BER expression in (15) can be simplified to $Pe1_i = \frac{3}{2}Q(\sqrt{\eta_i}) - \frac{1}{2}Q(3\sqrt{\eta_i})$, where $\eta_i = \eta_{i,1} = \eta_{i,2}$ is the equal received SIR of both partners. It can be verified that $Pe1_i$ is a strictly decreasing function of η_i for $\eta_i > 0$. Therefore, the BER constraint in (17) can be replaced by the equivalent SIR constraint $\eta_i \geq \eta_i^\star$ where $\eta_i^\star$ satisfies $\frac{3}{2}Q(\sqrt{\eta_i^\star}) - \frac{1}{2}Q(3\sqrt{\eta_i^\star}) = \rho 1_i$, and the power control problem for phase one in (16)-(18) can be solved directly by the iterative power control algorithm. Therefore, by applying the equal received power constraint, the implementation of the algorithm can be simplified with a moderate sacrifice in total power expenditure.

Figure 6 presents the comparison between the multiuser two-way JD-XOR-F relaying and the one-way CDMA relaying, on the total user power expended for phase one, and on the total relay power for phase two, respectively. A large power savings of the JD-XOR-F relaying upon the one-way CDMA relaying is presented in both phases. This is due to the fact that the relay jointly demodulates and generates the estimate of the XORed symbol for each user pair in the first phase, and transmits one binary symbol for each user pair in the second phase, and hence in both phases the interference is significantly reduced.

In Figure 7, the first phase power control of an overloaded system is considered, i.e., $N = 20$ and $K = 13$. For the one-way CDMA relaying scheme, the user capacity[28], i.e., the maximum number of users that can be supported by optimal interference management, is 24. As expected, we observe in Figure 7 that the power control problem with the one-way CDMA scheme is infeasible since the number of users is $26 > 24$, i.e., there are no transmit power values that satisfy the QoS requirements. On the other hand, the multiuser two-way JD-XOR-F relaying scheme has a feasible power control solution in phase one since it only needs half number of the signatures needed in the one-way CDMA scheme. This shows the considerable benefit on user capacity of considering the two-way communication structure.

VII. Conclusion

In this paper, we have considered a multiuser two-way relay network where multiple pairs of partners exchange information via a shared intermediate relay node. We have proposed a two-phase communication scenario where each pair wishing to exchange information shares a CDMA signature in the uplink, and the relay performs digital network coding per pair in the downlink. We have proposed the jointly demodulate-and-XOR forward relaying scheme, constructed iterative power control and linear multiuser detection algorithms that converge to their optimum solution, and showed that the design choices made considering the bi-directional nature of communication lead to significant power savings.

We remark that interesting and challenging issues remain to be discovered in multiuser two-way relay networks including employing nonlinear multiuser detection and channel coding, CDMA signature design, partner selection and channel estimation. Finally, we remark that in this paper, we have dealt with optimum interference management in the two phases of communication separately as the relay was required to have a pre-defined QoS level for reliable communication. The system-wide optimization problem, which jointly optimizes the transmit power levels and the receivers at all users and the relay with end-to-end BER constraints for all users is discussed in [29]. We omit the details of this discussion due to space limitations, and refer the reader to [29] where we show that the optimum solution to the overall optimization problem requires an exhaustive search over multiple dimensions of BER values. Numerical results in[29], however, ensure that for reasonable range of system parameters, having similar BER requirements per phase and solving the decoupled interference management problems as done in this paper does not lead to more than negligible performance loss.

Appendix A
Proof of Lemma 1

First, we prove that $Pe1_i(\eta_{i,1}, \eta_{i,2})$ is quasiconvex on R_1. Let $f = Pe1_i$, $x_1 = \eta_{i,1}$, $x_2 = \eta_{i,2}$ and $\mathbf{x} = [x_1, x_2]^\mathsf{T}$ for simplicity. A sufficient condition for f to be quasiconvex on R_1 is that for each $\mathbf{x} \in R_1$, $\det(B_n(\mathbf{x})) < 0$ for $n = 1, 2$[27], where $\det(\cdot)$ is the determinant of a matrix, and B_n denotes the nth submatrix of the bordered Hessian of f, i.e.,

$$B_1 = \begin{bmatrix} f_{11} & f_1 \\ f_1 & 0 \end{bmatrix} \quad \text{and} \quad B_2 = \begin{bmatrix} f_{11} & f_{12} & f_1 \\ f_{21} & f_{22} & f_2 \\ f_1 & f_2 & 0 \end{bmatrix} \tag{51}$$

where $f_m = \frac{\partial f}{\partial x_m}$ and $f_{mn} = \frac{\partial^2 f}{\partial x_m \partial x_n}$ with $n, m \in \{1, 2\}$. From (15), f can be expressed as

$$\begin{aligned} f(\mathbf{x}) = & \int_{\sqrt{x_2}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt + \frac{1}{2} \int_{2\sqrt{x_1}-\sqrt{x_2}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt \\ & - \frac{1}{2} \int_{2\sqrt{x_1}+\sqrt{x_2}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt. \end{aligned} \tag{52}$$

The first and second order derivatives in B_1 and B_2 are found as follows. Note that all the following inequalities are held only for $R_1 = \{\mathbf{x} | x_1 > 0,\ x_2 > 0,\ \text{and}\ x_1 \geq x_2\}$.

$$f_1 = -\frac{\sqrt{2}}{4\sqrt{\pi x_1}} \left(e^{-\frac{1}{2}(2\sqrt{x_1}-\sqrt{x_2})^2} - e^{-\frac{1}{2}(2\sqrt{x_1}+\sqrt{x_2})^2}\right) < 0 \tag{53}$$

$$f_2 = \frac{\sqrt{2}}{8\sqrt{\pi x_2}} e^{-\frac{x_2}{2}} \left(e^{-2x_1+2\sqrt{x_1 x_2}} + e^{-2x_1-2\sqrt{x_1 x_2}} - 2\right) < 0. \tag{54}$$

Inequality (53) is because $(2\sqrt{x_1} - \sqrt{x_2})^2 < (2\sqrt{x_1} + \sqrt{x_2})^2$. Inequality (54) is because $e^{-2x_1+2\sqrt{x_1 x_2}} < 1$ and $e^{-2x_1-2\sqrt{x_1 x_2}} < 1$.

$$\begin{aligned} f_{21} &= f_{12} \\ &= -\frac{\sqrt{2}}{8} \frac{(2\sqrt{x_1} - \sqrt{x_2})}{\sqrt{\pi x_1 x_2}} e^{-\frac{1}{2}(2\sqrt{x_1}-\sqrt{x_2})^2} \\ &\quad - \frac{\sqrt{2}}{8} \frac{(2\sqrt{x_1} + \sqrt{x_2})}{\sqrt{\pi x_1 x_2}} e^{-\frac{1}{2}(2\sqrt{x_1}+\sqrt{x_2})^2} < 0 \end{aligned} \tag{55}$$

$$\begin{aligned} f_{22} = & \underbrace{\frac{\sqrt{2}}{8\sqrt{\pi x_2}} e^{-\frac{x_2}{2}}}_{A} + \underbrace{\frac{\sqrt{2}}{8\sqrt{\pi}(x_2)^{3/2}} e^{-\frac{x_2}{2}}}_{B} \\ & \underbrace{+ \frac{\sqrt{2}}{16\sqrt{\pi} x_2} (2\sqrt{x_1} - \sqrt{x_2}) e^{-\frac{1}{2}(2\sqrt{x_1}-\sqrt{x_2})^2}}_{C} \end{aligned}$$

$$\underbrace{-\frac{\sqrt{2}}{16\sqrt{\pi}(x_2)^{3/2}}e^{-\frac{1}{2}(2\sqrt{x_1}-\sqrt{x_2})^2}}_{D}$$
$$\underbrace{-\frac{\sqrt{2}}{16\sqrt{\pi}x_2}(2\sqrt{x_1}+\sqrt{x_2})e^{-\frac{1}{2}(2\sqrt{x_1}+\sqrt{x_2})^2}}_{E}$$
$$\underbrace{-\frac{\sqrt{2}}{16\sqrt{\pi}(x_2)^{3/2}}e^{-\frac{1}{2}(2\sqrt{x_1}+\sqrt{x_2})^2}}_{F}. \qquad (56)$$

In the above we find that $A+C+E>0$ and $B+D+F>0$, leading to $f_{22}>0$.

$$f_{11} = \frac{\sqrt{2}e^{-2x_1-\frac{x_2}{2}}}{8\sqrt{\pi}(x_1)^{3/2}} \cdot \underbrace{\left((4x_1-2\sqrt{x_1x_2}+1)e^{2\sqrt{x_1x_2}}-(4x_1+2\sqrt{x_1x_2}+1)e^{-2\sqrt{x_1x_2}}\right)}_{G} \qquad (57)$$

$$\frac{\partial G}{\partial x_1} = (4+4\sqrt{x_1x_2}-2x_2)\left(e^{2\sqrt{x_1x_2}}-e^{-2\sqrt{x_1x_2}}\right) +8\sqrt{x_1x_2}e^{-2\sqrt{x_1x_2}}>0 \qquad (58)$$
$$\frac{\partial G}{\partial x_2} = \left(4x_1\frac{\sqrt{x_1}}{\sqrt{x_2}}-2x_1\right)e^{2\sqrt{x_1x_2}} +\left(4x_1\frac{\sqrt{x_1}}{\sqrt{x_2}}+2x_1\right)e^{-2\sqrt{x_1x_2}}>0. \qquad (59)$$

Inequalities (58) and (59) are due to $x_1 \geq x_2$, $x_1>0$ and $x_2>0$. Therefore, $G(x_1,x_2)$ is a strictly increasing function of x_1 for fixed x_2, and a strictly increasing function of x_2 for fixed x_1 as well. Hence, $G(x_1,x_2) > \lim_{x_1\to 0, x_2\to 0} G(x_1,x_2) = 0$. Thus, $f_{11}>0$.

Since $f_1<0$, $f_2<0$, $f_{11}>0$, $f_{22}>0$, and $f_{12}=f_{21}<0$, the determinants of the submatrices of the bordered Hessian are both negative, i.e.,

$$\det(B_1) = -(f_{11})^2 < 0; \qquad (60)$$
$$\det(B_2) = -(f_1)^2 f_{22} + 2f_1f_2f_{12} - (f_2)^2 f_{11} < 0. \qquad (61)$$

Therefore, $f(\mathbf{x})$, equivalently, $Pe1_i(\eta_{i,1},\eta_{i,2})$, is quasiconvex on R_1.

We note that for the case when $\eta_{i,1}<\eta_{i,2}$, $Pe1_i$ can be obtained by switching $\eta_{i,1}$ and $\eta_{i,2}$ in the case of $\eta_{i,1}\geq\eta_{i,2}$. Therefore, by switching x_1 and x_2 in the above proof for R_1, we can obtain the first and second order derivatives and show that the determinants of the submatrices of the bordered Hessian are negative. Thus, $Pe1_i(\eta_{i,1},\eta_{i,2})$, is quasiconvex on R_2 as well.

APPENDIX B
PROOF OF PROPOSITION 1

First, we convert the transmit power condition in (25) to its equivalent received SIR condition

$$\frac{\eta_{i,1}}{h_{i,1}} + \frac{\eta_{i,2}}{h_{i,2}} \geq \gamma_i \quad \text{and} \quad \frac{\eta_{i,1}}{\eta_{i,2}} = \frac{\eta^\star_{i,1}}{\eta^\star_{i,2}}. \qquad (62)$$

Next, we show that (62) is sufficient for $Pe1_i \leq \rho 1_i$, with the minimum total transmit power requirement on the ith user pair. Let $\eta_{i,1} = \eta_{i,2}\frac{\eta^\star_{i,1}}{\eta^\star_{i,2}}$, from (62) we have

$$\frac{\eta_{i,2}}{\eta^\star_{i,2}}\left(\frac{\eta^\star_{i,1}}{h_{i,1}}+\frac{\eta^\star_{i,2}}{h_{i,2}}\right) \geq \gamma_i. \qquad (63)$$

Since $\frac{\eta^\star_{i,1}}{h_{i,1}}+\frac{\eta^\star_{i,2}}{h_{i,2}} = \gamma_i$ as defined in (23), we have $\eta_{i,2}\geq\eta^\star_{i,2}$ and $\eta_{i,1}\geq\eta^\star_{i,1}$. As we show in Appendix A that $\frac{\partial Pe1_i}{\partial\eta_{i,1}}<0$ and $\frac{\partial Pe1_i}{\partial\eta_{i,2}}<0$, $Pe(\eta_{i,1},\eta_{i,2})$ is a decreasing function of $\eta_{i,1}$ or $\eta_{i,2}$. Therefore, $Pe1_i(\eta_{i,1},\eta_{i,2}) \leq Pe1_i(\eta^\star_{i,1},\eta^\star_{i,2}) \leq \rho 1_i$. Thus, (62) is a sufficient condition for $Pe1_i \leq \rho 1_i$. When $\eta_{i,1}=\eta^\star_{i,1}$ and $\eta_{i,2}=\eta^\star_{i,2}$, it achieves the minimum total transmit power $\gamma_i\frac{\sigma_i^2}{(\mathbf{c}_i^\top\mathbf{s}_i)^2}$ that satisfies the BER condition, as it is the optimum solution to the problem in (19)-(21).

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