On Performance Evaluation of Cooperative Spectrum Sensing in Cognitive Radio Networks

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***Abstract*—This paper presents the system level performance evaluation for energy-detection based cooperative spectrum sensing in cognitive radio networks. Three performance criteria are quantitively analyzed for cooperative spectrum sensing. First, the average error probability is determined given fixed amplifier gains for a fixed number of secondary users by considering all possible channel realizations. Second, the asymptotic error probability is computed in a power constrained cognitive radio network when the number of secondary user approaches infinity. Third, the outage probability is examined when instantaneous error probability is greater than a predefined threshold. In all three calculations, both additive white Gaussian noise (AWGN) and Rayleigh fading assumptions are used to capture the observation and fusion channels. Numerical results indicate that in order to maintain a desired detection performance in low and moderate fusion signal to noise ratio (SNR) regimes, fusion channels need to be as reliable as possible, while local received SNRs can be dynamic and provide spatial diversity. Moreover, it is shown that under AWGN observation channels and Rayleigh fading fusion channels, a diversity order equal to the number of secondary users can be achieved.**

I. Introduction

Cognitive radio [1] is a key technology to exploit under-utilized spectrum and enhance spectrum efficiency. In cognitive radio networks, secondary (*unlicensed*) users monitor local communication conditions and opportunistically access unoccupied spectrum when/where the primary (*licensed*) user is inactive. To enable this dynamic spectrum access, secondary users must continuously monitor local spectrum and detect spectrum holes [1]. This technique, called *spectrum sensing*, requires secondary users reliably detect the signals from primary users in order to avoid harmful interference. However, due to the detrimental nature of the wireless channel, a secondary user may not be able to reliably differentiate between a spectrum hole and a weak primary signal if it conducts spectrum sensing on its own. To improve detection reliability, multiple users can engage in cooperative spectrum sensing and take advantage of spatial diversity [2].

In [2], cooperative spectrum sensing was studied when a weighted combination of the received signals is utilized for global fusion. In [3], the performance of energy-detector-based spectrum sensing was evaluated in fading and shadowing environments using the "OR" fusion rule. In [4], the concept of estimation outage and diversity was used to evaluate the performance of distributed estimation using the best linear unbiased estimator; the results showed that a diversity order equal to the number of sensor nodes can be achieved.

Existing literature on cooperative spectrum sensing does not consider the dynamic nature of both the observation and fusion channels when evaluating system level performance. In this paper, we utilize error probability as a performance metric and consider energy-based cooperative spectrum sensing with amplify and forward (AF) relaying over parallel access fusion channels. For this system model, we address following questions that arise due to the dynamic nature of the observation and fusion channels:

1) Given fixed amplifier gains for a fixed number of secondary users and considering all possible channel realizations, what is the long term average detection performance for cooperative spectrum sensing?
2) Given a fixed global transmission power level, what is the asymptotic error probability when the number of secondary users approaches infinity?
3) In the latter case, what is the outage probability when the instantaneous error probability is greater than a predefined threshold? Can we achieve a full diversity when observation channels or/and fusion channels experience fading?

In this paper, we intend to address these issues and quantitively analyze the detection performance for cooperative spectrum sensing in cognitive radio networks. In particular, we consider both additive white Gaussian noise (AWGN) and Rayleigh fading observation and fusion channels. We demonstrate that in order to maintain desired detection performance in the low and moderate fusion signal to noise ratio (SNR) regimes, fusion channels need to be as reliable as possible, while received SNRs at secondary users can be dynamic and provide spatial diversity. Furthermore, we show that under AWGN observation channels and Rayleigh fading fusion channels, a diversity order equal to the number of secondary users can be achieved.

II. System Model

A. Local Energy Statistic

For secondary user $i,\ (1 \le i \le \mathrm{N})$, the hypothesis test for the energy of the received signal in a given spectrum band is

$$\begin{cases} \mathcal{H}_0: & x_i = (1/\kappa_i)\sum_{k=1}^{\kappa_i} |n_i(k)|^2 \\ \mathcal{H}_1: & x_i = (1/\kappa_i)\sum_{k=1}^{\kappa_i} |\tilde{h}_i s(k) + n_i(k)|^2, \end{cases} \tag{1}$$

where κ_i is the number of samples, $s(k)$ is the transmitted signal from the primary user and $n_i(k)$ is the noise received by secondary user i. We assume $s(k)$ is complex PSK modulated and independent and identically distributed (i.i.d.) with mean zero and variance σ_s^2; $\tilde{h}_i$ is the channel gain between the primary user and secondary user i and is assumed to be constant during the cooperative spectrum sensing period; and $n_i(k)$ is i.i.d. circularly symmetric complex Gaussian random variable with mean zero and variance σ_n^2 and is independent of $s(k)$. We define the local received SNR at the secondary user i as $\gamma_i = \sigma_s^2 |\tilde{h}_i|^2 / \sigma_n^2$.

When κ_i is large, x_i can be approximated as Gaussian random variable [2][5], i.e.,

$$\begin{cases} \mathcal{H}_0: & x_i \sim \mathcal{N}(\sigma_n^2,\ \sigma_n^4/\kappa_i) \\ \mathcal{H}_1: & x_i \sim \mathcal{N}((1+\gamma_i)\sigma_n^2,\ (1+2\gamma_i)\sigma_n^4/\kappa_i). \end{cases} \tag{2}$$

In this paper, we assume the local received SNR γ_i is known at the secondary user i. For instance, in IEEE 802.22, this value could be obtained through estimation of pilot signals periodically transmitted from TV stations [6].

B. Amplify and Forward Transmission Strategy

During the cooperation period, each secondary user transmits its local energy statistic to the fusion center using AF on parallel access channels (PAC). The received signal at the fusion center is shown in Fig. 1, i.e.,

$$y_i = g_i h_i x_i + v_i, \tag{3}$$

where g_i is the amplifier gain for the secondary user i, h_i is channel gain between secondary user i and and the fusion center and v_i is i.i.d. Gaussian noise, i.e., $v_i \sim \mathcal{N}(0, \sigma_v^2)$ and is independent of x_i. We assume that h_i is known at the fusion center (e.g., via channel estimation) and remains constant during the sensing period.

We can then rewrite (3) in a matrix form as

$$\boldsymbol{y} = \mathbf{H}\boldsymbol{x} + \boldsymbol{v}, \tag{4}$$

where $\mathbf{H} = \mathrm{diag}\{g_1 h_1, g_2 h_2, \cdots, g_{\mathrm{N}} h_{\mathrm{N}}\}$.

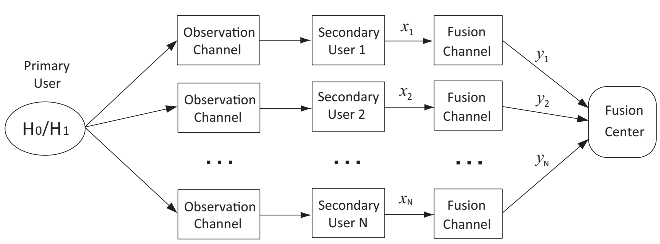

Fig. 1. Cooperative spectrum sensing in cognitive radio networks.

Given this system model, we see that

$$\xi_i \stackrel{\text{def}}{=} \mathbb{E}\{x_i^2\} = \left[1 + 1/\kappa_i + \pi_1\left(\gamma_i + 2\left(1 + 1/\kappa_i\right)\right)\gamma_i\right]\sigma_n^4,$$

where $\pi_0 = \mathrm{P}(\mathcal{H}_0)$ and $\pi_1 = \mathrm{P}(\mathcal{H}_1)$ are the probabilities that spectrum is idle and occupied, respectively.

In the cognitive radio networks, the received primary user power measured by the secondary user can be very small [7], i.e., $\gamma_i \ll 1$. Additionally, the number of samples can be large, i.e., $\kappa_i \gg 1$. Then, we can approximate the transmitted power for the secondary user i as $\mathcal{P}_i = \xi_i g_i^2 \simeq g_i^2(1 + 2\pi_1\gamma_i)\sigma_n^4$.

C. Optimal Fusion Rule

Under hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1$, the received signal $\boldsymbol{y}$ has a Gaussian distribution, i.e.,

$$\begin{cases} \mathcal{H}_0: & \boldsymbol{y} \sim \mathcal{N}\left(\mathbf{H}\mathbf{1}\sigma_n^2,\ \boldsymbol{\Sigma}_0\right) \\ \mathcal{H}_1: & \boldsymbol{y} \sim \mathcal{N}\left(\mathbf{H}(\mathbf{1}+\boldsymbol{\gamma})\sigma_n^2,\ \boldsymbol{\Sigma}_1\right), \end{cases} \tag{5}$$

where $\mathbf{1} = [1, 1, \cdots, 1]^{\mathsf{T}}$, $\boldsymbol{\Sigma}_0 = \mathbf{H}\mathbf{H}^{\dagger}\sigma_n^4/\kappa_i + \sigma_v^2\mathbf{I}$ and $\boldsymbol{\Sigma}_1 = \mathbf{H}(\mathbf{I} + 2\boldsymbol{\Gamma})\mathbf{H}^{\dagger}\sigma_n^4/\kappa_i + \sigma_v^2\mathbf{I}$, here, $\boldsymbol{\Gamma} = \mathrm{diag}\{\gamma_1, \gamma_2, \cdots, \gamma_{\mathrm{N}}\}$.

Without loss of generality, we assume that $\pi_0 = \pi_1 = 0.5$. Then, optimal likelihood ratio test (LRT) is given as:

$$\log \frac{p(\boldsymbol{y}|\mathcal{H}_1)}{p(\boldsymbol{y}|\mathcal{H}_0)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} 0. \tag{6}$$

Since $\gamma_i \ll 1$ and $\kappa_i \gg 1$, then, $\gamma_i/\kappa_i \approx 0$ and we have $\boldsymbol{\Sigma}_0 \approx \boldsymbol{\Sigma}_1$. Thus, the optimal LRT can be approximated as

$$\mathcal{T}(\boldsymbol{y}) = (\mathbf{H}\boldsymbol{\gamma})^{\dagger}\boldsymbol{\Sigma}_0^{-1}\boldsymbol{y} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \tau, \tag{7}$$

where $\tau = (\mathbf{H}\boldsymbol{\gamma})^{\dagger}\boldsymbol{\Sigma}_0^{-1}\mathbf{H}(\mathbf{1} + 0.5\boldsymbol{\gamma})\sigma_n^2$. After some steps, the error probability conditioned on the local received SNRs $\boldsymbol{\gamma}$ and the fusion channel gains $\boldsymbol{h}$ is given by

$$\begin{aligned} \mathrm{P}_{e|\boldsymbol{\gamma},\boldsymbol{h}} &= \pi_0 \mathrm{P}_{f|\boldsymbol{\gamma},\boldsymbol{h}} + \pi_1 \mathrm{P}_{m|\boldsymbol{\gamma},\boldsymbol{h}} \\ &= Q\left(\frac{\sigma_n^2}{2}\left[(\mathbf{H}\boldsymbol{\gamma})^{\dagger}\boldsymbol{\Sigma}_0^{-1}\mathbf{H}\boldsymbol{\gamma}\right]^{1/2}\right) \\ &= Q\left(\frac{1}{2}\sqrt{\mathcal{F}(\boldsymbol{\gamma}, \boldsymbol{h})}\right), \end{aligned} \tag{8}$$

where $Q(x)$ is the complementary distribution function of the standard Gaussian, i.e., $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} \exp(-t^2/2)\mathrm{d}t$; and

$$\mathcal{F}(\boldsymbol{\gamma}, \boldsymbol{h}) = \sum_{i=1}^{\mathrm{N}} \frac{g_i^2 \kappa_i \gamma_i^2 |h_i|^2}{g_i^2 |h_i|^2 + \kappa_i \tilde{\sigma}_v^2},$$

where $\tilde{\sigma}_v^2 = \sigma_v^2/\sigma_n^4$.

D. Channel Scenarios

In this paper, we consider three channel scenarios when evaluating the system level performance of cooperative spectrum sensing, as shown in Table I. In particular, for AWGN channels, we assume $\gamma_i = \bar{\gamma}$ and $h_i = 1$, $\forall i$. For Rayleigh fading channels, we see that the local received SNRs γ_i and fusion channel gains $|h_i|^2$ follow an exponential distribution. Here, we assume that γ_i and $|h_i|^2$ are i.i.d. with mean $\bar{\gamma}$ and 1, respectively, i.e., $p_{\gamma_i}(x) = \frac{1}{\bar{\gamma}}\exp(-\frac{x}{\bar{\gamma}})$ and $p_{|h_i|^2}(x) = \exp(-x)$, where $\bar{\gamma} = \sigma_s^2/\sigma_n^2$.

TABLE I
THREE SCENARIOS FOR PERFORMANCE EVALUATION

	Observation channels	Fusion channels
Case I	AWGN	Rayleigh fading
Case II	Rayleigh fading	AWGN
Case III	Rayleigh fading	Rayleigh fading

III. Average Error Probability

In this section, we first analyze the long term average error probability for our system model. We then derive an upper bound that provides more insight and enables performance evaluation. We assume in this section that the amplifier gains and the number of samples collected at each secondary user are fixed and not adjusted according to the observation channel gains.

A. Exact Average Error Probability

From (8), we see that the long term average error probability can be calculated as

$$\mathrm{P}_{e,\mathrm{avg}} \;=\; \mathbb{E}_{\boldsymbol{\gamma},\boldsymbol{h}}\left\{\mathrm{P}_{e|\boldsymbol{\gamma},\boldsymbol{h}}\right\}. \tag{9}$$

To simplify the calculation of the average error probability, we consider the following alternate expression for the Q function [8]

$$Q(x)=\frac{1}{\pi}\int_0^{\pi/2}\exp\Big(-\frac{x^2}{2\sin^2\phi}\Big)\mathrm{d}\phi, \qquad x\geq 0.$$

When the local received SNRs γ_i and fusion channel gains $|h_i|^2$ are independent, respectively, we can simplify the average error probability in (9) as

$$\mathrm{P}_{e,\mathrm{avg}} \;=\; \frac{1}{\pi}\int_0^{\pi/2}\prod_{i=1}^{\mathrm{N}}\mathcal{B}_i(\phi)\mathrm{d}\phi, \tag{10}$$

where

$$\mathcal{B}_i(\phi)=\int_0^\infty\int_0^\infty\exp\left(\frac{\mathcal{A}_i(s,t)}{\sin^2\phi}\right)p_{\gamma_i}(s)p_{|h_i|^2}(t)\mathrm{d}s\mathrm{d}t$$

and

$$\mathcal{A}_i(s,t)=-\frac{1}{8}\cdot\frac{g_i^2\kappa_i s^2 t}{g_i^2 t+\kappa_i\tilde{\sigma}_v^2}.$$

Here, $p_{\gamma_i}(s)$ and $p_{|h_i|^2}(t)$ are PDFs of γ_i and $|h_i|^2$, respectively. If we further assume that $g_i=g$, $\kappa_i=\kappa$, γ_i and $|h_i|^2$ are i.i.d., respectively, i.e., $p_{\gamma_i}(s)=p_\gamma(s)$ and $p_{|h_i|^2}(t)=p_{|h|^2}(t)$, we have $\mathcal{B}_i(\phi)=\mathcal{B}(\phi),\ \forall i$. In this case, the average error probability in (9) reduces to

$$\mathrm{P}_{e,\mathrm{avg}} \;=\; \frac{1}{\pi}\int_0^{\pi/2}\left[\mathcal{B}(\phi)\right]^{\mathrm{N}}\mathrm{d}\phi. \tag{11}$$

Based on this, we see that $\mathrm{P}_{e,\mathrm{avg}}$ is a decreasing function of N, which indicates that in a power unconstrained cognitive radio network, global error performance can be improved by increasing the number of secondary users. This statement is valid because $\mathcal{A}(s,t)\leq 0$ and $\mathcal{B}(\phi)\leq\int_0^\infty\int_0^\infty p_\gamma(s)p_{|h|^2}(t)\mathrm{d}s\mathrm{d}t=1$.

In general, we see that closed-form expression of $\mathrm{P}_{e,\mathrm{avg}}$ is extremely difficult to obtain. However, as we will show next, only elementary functions, such as exponential and Q functions, are involved in integral calculation, thus the average error probability can be readily solved numerically.

B. Upper Bound for Average Error Probability

To gain more insight on the performance of cooperative spectrum sensing, we investigate here an upper bound for average error probability. Since $Q(x)\leq\frac{1}{2}\exp(-x^2/2)$, the upper bound can be obtained as

$$\tilde{\mathrm{P}}_{e,\mathrm{avg}} \;=\; \frac{1}{2}\prod_{i=1}^{\mathrm{N}}\mathcal{M}_i, \tag{12}$$

where

$$\mathcal{M}_i=\int_0^\infty\int_0^\infty\exp[\mathcal{A}_i(s,t)]p_{\gamma_i}(s)p_{|h_i|^2}(t)\mathrm{d}s\mathrm{d}t.$$

Similarly, we assume $g_i=g,\ \kappa_i=\kappa$, γ_i and $|h_i|^2$ are i.i.d., respectively. Then, we have $\mathcal{M}_i=\mathcal{M},\ \forall i$, and

$$\tilde{\mathrm{P}}_{e,\mathrm{avg}}=\frac{1}{2}\mathcal{M}^{\mathrm{N}}. \tag{13}$$

It is worth mentioning that when $g\to 0$, we have $\tilde{\mathrm{P}}_{e,\mathrm{avg}}\to\frac{1}{2}$. This is not surprising since when the amplifier gains are low, the fusion center will not be able to make a global decision due to the lack of local energy statistic.

C. Closed-Form Expressions for Three Scenarios

To simplify our analysis, in this section we assume $g_i=g,\ \kappa_i=\kappa,\ \forall i$. Then, we can use (11) and (13) to evaluate the average error probability for cooperative spectrum sensing for the three channel scenarios in Table I.

1) Case I: In this case, $\gamma_i=\bar{\gamma}$ and $p_{|h_i|^2}(t)=\exp(-t)$. After some manipulations, we have

$$\mathcal{B}(\phi)=\exp\left(-\frac{\kappa\bar{\gamma}^2}{8\sin^2\phi}\right)\Psi_1\left(\frac{\kappa\bar{\gamma}^2}{8\sin^2\phi},\frac{\kappa\tilde{\sigma}_v^2}{g^2}\right),$$

where $\Psi_1(a,b)=\int_0^\infty\exp\big(-x+\frac{ab}{x+b}\big)\mathrm{d}x,\ (a,b>0)$. After calculating $\mathcal{B}(\phi)$, we plug it in (11) to obtain the average error probability. It is interesting to note that a similar definition of $\Psi_1(\phi,a,b)$ can be found in [9].

Furthermore, the upper bound is given as

$$\tilde{\mathrm{P}}_{e,\mathrm{avg}}^{(1)}=\frac{1}{2}\exp\left(-\frac{\mathrm{N}\kappa\bar{\gamma}^2}{8}\right)\left[\Psi_1\Big(\frac{\kappa\bar{\gamma}^2}{8},\frac{\kappa\tilde{\sigma}_v^2}{g^2}\Big)\right]^{\mathrm{N}}.$$

When $g\to\infty$, we see that $\tilde{\mathrm{P}}_{e,\mathrm{avg}}^{(1)}(g_\infty)=\frac{1}{2}\exp\left(-\frac{\mathrm{N}\kappa\bar{\gamma}^2}{8}\right)$. This indicates that when the fusion channel is perfect, average error performance is limited by local observed energy statistic.

2) Case II: In this case, $p_{\gamma_i}(s)=\frac{1}{\bar{\gamma}}\exp(-\frac{s}{\bar{\gamma}})$ and $h_i=1$. After some manipulations (using eq.(3.322.2) in [10]), we have

$$\mathcal{B}(\phi)=\sqrt{8\pi c}\sin\phi\exp\left(2c\sin^2\phi\right)Q\left(2\sqrt{c}\sin\phi\right),$$

where $c=\frac{1}{\bar{\gamma}^2}\left(\frac{1}{\kappa}+\frac{\tilde{\sigma}_v^2}{g^2}\right)$. Furthermore, the upper bound is

$$\tilde{\mathrm{P}}_{e,\mathrm{avg}}^{(2)}=\frac{1}{2}\left(8\pi c\right)^{\mathrm{N}/2}\exp\left(2\mathrm{N}c\right)\left[Q\left(2\sqrt{c}\right)\right]^{\mathrm{N}}.$$

When $g\to\infty$, we see that $c\to 1/(\kappa\bar{\gamma}^2)$ and

$$\tilde{\mathrm{P}}_{e,\mathrm{avg}}^{(2)}(g_\infty)=\frac{1}{2}\left(\frac{8\pi}{\kappa\bar{\gamma}^2}\right)^{\mathrm{N}/2}\exp\left(\frac{2\mathrm{N}}{\kappa\bar{\gamma}^2}\right)\left[Q\left(\frac{2}{\bar{\gamma}\sqrt{\kappa}}\right)\right]^{\mathrm{N}}.$$

3) Case III: In this case, $p_{\gamma_i}(s) = \frac{1}{\bar{\gamma}}\exp(-\frac{s}{\bar{\gamma}})$ and $p_{|h_i|^2}(t) = \exp(-t)$. After some manipulations, we have

$$\mathcal{B}(\phi) = \sqrt{8\pi}\exp\left(\frac{2\sin^2\phi}{\kappa\bar{\gamma}^2}\right)\Psi_2\left(\frac{\sin^2\phi}{\kappa\bar{\gamma}^2}, \frac{\tilde{\sigma}_v^2\sin^2\phi}{g^2\bar{\gamma}^2}\right),$$

where

$$\Psi_2(a,b) = \int_0^\infty \Big(a + \frac{b}{x}\Big)^{1/2}\exp\Big(-x + \frac{2b}{x}\Big) \cdot Q\Big(2\big(a+\frac{b}{x}\big)^{1/2}\Big)\mathrm{d}x,\ \ (a, b > 0).$$

Furthermore, the upper bound is

$$\tilde{\mathrm{P}}_{e,\mathrm{avg}}^{(3)} = \frac{1}{2}\big(8\pi\big)^{\mathrm{N}/2}\exp\left(\frac{2\mathrm{N}}{\kappa\bar{\gamma}^2}\right)\left[\Psi_2\Big(\frac{1}{\kappa\bar{\gamma}^2}, \frac{\tilde{\sigma}_v^2}{g^2\bar{\gamma}^2}\Big)\right]^{\mathrm{N}}.$$

When $g \to \infty$, we see that $\tilde{\mathrm{P}}_{e,\mathrm{avg}}^{(3)}(g_\infty) = \tilde{\mathrm{P}}_{e,\mathrm{avg}}^{(2)}(g_\infty)$. This is primarily due to the fact that when $g \to \infty$, the fusion channel no longer impacts the average error performance.

IV. Outage Analysis in Power Constrained Cognitive Radio Networks

In this section, we investigate the global detection performance in power constrained cognitive radio networks. Specifically, under the assumption of equal transmission power and equal number of samples, we present asymptotic error probability for cooperative spectrum sensing when the number of secondary users approaches infinity. Additionally, we analyze the detection outage when the instantaneous error probability is greater than a certain threshold.

A. Asymptotic Error Probability

In a power constrained cognitive radio network, the transmitted power of the secondary users satisfies a global power constraint, i.e., $\mathbf{1}^\mathsf{T}\boldsymbol{\mathcal{P}} \le \mathcal{P}_{\text{tot}}$. Without loss of generality, we assume an equal transmission power scheme, i.e., $\mathcal{P}_i = \mathcal{P}_{\text{tot}}/\mathrm{N}$ and equal number of samples, $\kappa_i = \kappa$. Then, we have

$$g_i^2 = \frac{\mathcal{P}_{\text{tot}}}{\mathrm{N}\xi_i} = \frac{\mathcal{P}_{\text{tot}}}{\mathrm{N}(1+\gamma_i)\sigma_n^4}.$$

Here we are interested in evaluating the asymptotic error probability when the number of secondary users approaches infinity. Let us define the asymptotic error probability as

$$\mathrm{P}_e(\mathrm{N}_\infty) \overset{\text{def}}{=} \lim_{\mathrm{N}\to\infty}\mathrm{P}_{e|\boldsymbol{\gamma},\boldsymbol{h}}.$$

Then we have the following theorem.

Theorem IV.1. *Consider cooperative spectrum sensing in power constrained cognitive radio networks where the local received SNRs γ_i and fusion channel gains $|h_i|^2$ are i.i.d. random variables, respectively. Let us define*

$$\zeta_i = \frac{\gamma_i^2|h_i|^2}{1+\gamma_i}.$$

When $\mathbb{E}\{\zeta_i\}$ and $\mathbb{E}\{|h_i|^4\}$ are finite, then

$$\mathrm{P}_e(\mathrm{N}_\infty) = Q\left(\frac{1}{2}\sqrt{\mathsf{SNR}\,\mathbb{E}\{\zeta_i\}}\right), \tag{14}$$

where SNR is the global fusion SNR, i.e., $\mathsf{SNR} = \mathcal{P}_{\text{tot}}/\sigma_v^2$.

Due to space limitations, we omit the proof and refer the reader to [11]. According to Theorem IV.1, we see that in a power constrained cognitive radio network, when the number of secondary users approaches infinity, the asymptotic error probability is bounded away from zero. In particular, $\mathrm{P}_e(\mathrm{N}_\infty) = Q(\nu\sqrt{\mathsf{SNR}})$, where ν is a constant. This is similar to the error probability of detection of BPSK signal in AWGN channels. When the global fusion SNR approaches infinity, we see that the asymptotic error probability $\mathrm{P}_e(\mathrm{N}_\infty) \to 0$.

Furthermore, from Theorem IV.1, we see that when the number of secondary users is large, the error probability does not continue to decrease and converges to a non-zero value. Thus, we may question if there is any advantage when more secondary users cooperate in power constrained cognitive radio networks? The answer is yes. Similar to the analysis in [4], we introduce the concept of detection outage in the next subsection and illustrate the benefits of increasing the number of secondary users.

As in Section III, we now consider the three scenarios in Table I for evaluating the asymptotic error probability in power constrained cognitive radio networks. Let us denote the asymptotic error probability for these three scenarios as $\mathrm{P}_e^{(i)}(\mathrm{N}_\infty),\ i = 1, 2, 3$, respectively.

1) Case I: In this case, we have

$$\mathrm{P}_e^{(1)}(\mathrm{N}_\infty) \ = \ Q\left(\frac{\bar{\gamma}}{2}\sqrt{\frac{\mathsf{SNR}}{1+\bar{\gamma}}}\right).$$

2) Case II and III: Using eq. (3.353.5) in [10], we can show that

$$\begin{aligned}\mathrm{P}_e^{(2)}(\mathrm{N}_\infty) \ &= \ \mathrm{P}_e^{(3)}(\mathrm{N}_\infty) \\ &= \ Q\left(\sqrt{\frac{\mathsf{SNR}}{4\bar{\gamma}}\Big[\bar{\gamma}^2 - \bar{\gamma} - e^{1/\bar{\gamma}}\mathrm{Ei}\left(-1/\bar{\gamma}\right)\Big]}\right),\end{aligned}$$

where $\mathrm{Ei}(x)$ is the exponential integral function [12], i.e., $\mathrm{Ei}(x) = -\int_{-x}^{\infty} e^{-t}/t\,\mathrm{d}t,\ x < 0$.

Given the asymptotic error probability for these three scenarios, we note that

Lemma IV.2. $\mathrm{P}_e^{(1)}(\mathrm{N}_\infty) > \mathrm{P}_e^{(2)}(\mathrm{N}_\infty) = \mathrm{P}_e^{(3)}(\mathrm{N}_\infty)$.

Due to space limitations, we omit the proof. For detailed proof, please refer to [11]. From Lemma IV.2, we see that case II and III have lower asymptotic error probability than case I. However, asymptotic error probability might not be a good performance metric in practice. We next discuss more feasible outage probability to compare these three scenarios for cooperative spectrum sensing.

B. Detection Outage Analysis

Let us define the outage event as the instantaneous error probability is greater than a predefined threshold $\bar{\mathrm{P}}_e$, i.e,

$$\mathrm{P}_{out} \overset{\text{def}}{=} \mathrm{P}\big(\mathrm{P}_{e|\boldsymbol{\gamma},\boldsymbol{h}} \ge \bar{\mathrm{P}}_e\big).$$

Based on this definition, we note that

Theorem IV.3. *Consider the cooperative spectrum sensing in power constrained cognitive radio networks where the local received SNRs γ_i and fusion channel gains $|h_i|^2$ are i.i.d. random variables, respectively. When $\mathbb{E}\{\zeta_i\}$ is finite, with a sufficiently large* N *and $\bar{\mathrm{P}}_e > \mathrm{P}_e(\mathrm{N}_\infty)$, the outage probability is*

$$\mathrm{P}_{out} \sim \exp(-\mathrm{N}\mathcal{R}_\zeta(\epsilon)) \quad \text{or} \quad -\log \mathrm{P}_{out} \sim \mathrm{N}\mathcal{R}_\zeta(\epsilon),$$

where $\epsilon = 4\bar{\alpha}^2/\mathsf{SNR}$ with $\bar{\alpha} = Q^{-1}(\bar{\mathrm{P}}_e)$, and $\mathcal{R}_\zeta(\epsilon)$ is the rate function of ζ_i, i.e.,

$$\mathcal{R}_\zeta(\epsilon) = \sup_{\theta \in \mathbb{R}} \{\theta\epsilon - \lambda(\theta)\},$$

here $\lambda(\theta) = \log \mathbb{E}_\zeta\{\exp(\theta x)\}$ is the cumulant generating function.

Due to space limitations, we omit the proof. For detailed proof, please refer to [11]. Now, we provide the outage analysis for cooperative spectrum sensing assuming the three channel conditions in Table I.

1) Case I: In this case, we have $\zeta_i = \bar{\gamma}^2|h_i|^2/(1+\bar{\gamma})$. Let us define $\zeta_i' = |h_i|^2$ and $\epsilon' = (1+\bar{\gamma})\epsilon/\bar{\gamma}^2$. Similar to the analysis in [4], we have

$$\mathcal{R}_{\zeta'}(\epsilon') = \frac{4(1+\bar{\gamma})\bar{\alpha}^2}{\bar{\gamma}^2\mathsf{SNR}} - \log \frac{4(1+\bar{\gamma})\bar{\alpha}^2}{\bar{\gamma}^2\mathsf{SNR}} - 1.$$

Furthermore, we are interested in high global fusion SNR regime, then $4(1+\bar{\gamma})\bar{\alpha}^2/(\bar{\gamma}^2\mathsf{SNR}) \ll 1$, and

$$\mathcal{R}_{\zeta'}(\epsilon') \sim \log \mathsf{SNR},$$

According to Theorem IV.3, this implies

$$-\log \mathrm{P}_{out} \sim \mathrm{N} \cdot \mathsf{SNR}(\mathrm{dB}).$$

Hence, we see that under AWGN observation channels and Rayleigh fading fusion channels, a diversity order equal to N can be achieved.

2) Case II: In this case, we have $\zeta_i = \gamma_i^2/(1+\gamma_i)$. Since $\gamma_i \ll 1$, we can approximate $\zeta_i \approx \gamma_i^2$. Then, the PDF of ζ_i can be obtained as

$$p_\zeta(x) = \frac{1}{2\bar{\gamma}\sqrt{x}} \exp\left(-\frac{\sqrt{x}}{\bar{\gamma}}\right).$$

Let us define $\beta = -1/(4\bar{\gamma}^2\theta)$, $(\theta < 0)$, then cumulant generating function can be obtained as $\lambda(\beta) = 2\sqrt{\pi\beta}\exp(\beta)Q\left(\sqrt{2\beta}\right)$. After some manipulations, we have

$$\mathcal{R}_\zeta(\epsilon) = -\frac{\epsilon}{4\bar{\gamma}^2 x^*} - x^* - \log\left[2\sqrt{\pi x^*}Q\left(\sqrt{2x^*}\right)\right], \quad (15)$$

where x^* is the positive solution of the equation $g(x) = \epsilon/\bar{\gamma}^2$ with

$$g(x) = 4x^2 + 2x - \frac{2x^{3/2}\exp(-x)}{\sqrt{\pi}Q\left(\sqrt{2x}\right)}.$$

When $\mathsf{SNR} \gg 1$, $\epsilon \ll 1$, then we can expect a sufficiently small value of x^*. Using the fact that $\exp(-x) \approx 1 - x$, $Q(x) \approx \frac{1}{2} - \frac{1}{\sqrt{2\pi}}x$ and $\frac{1}{1-x} \approx 1 + x$ for $x \ll 1$, $g(x)$ can be approximated as

$$g(x) \approx \tfrac{8}{\pi}x^3 + \tfrac{4}{\sqrt{\pi}}x^{5/2} + \left(4 - \tfrac{8}{\pi}\right)x^2 - \tfrac{4}{\sqrt{\pi}}x^{3/2} + 2x.$$

We can further eliminate the high order of x and approximate the solution as $x^* \approx \epsilon/(2\bar{\gamma}^2)$. Plugging this into (15) and noting that $\log(1-x) \approx -x$ for $x \ll 1$, after some manipulations, we have

$$\mathcal{R}_\zeta(\epsilon) \approx \frac{\bar{\alpha}}{\bar{\gamma}}\sqrt{\frac{8}{\pi\mathsf{SNR}}} - \frac{2\bar{\alpha}^2}{\bar{\gamma}^2\mathsf{SNR}} - \frac{1}{2}\log\frac{2\bar{\alpha}^2}{\bar{\gamma}^2\mathsf{SNR}} + \tilde{c},$$

where $\tilde{c}$ is a constant. Then, in the high global fusion SNR regime, we have $\mathcal{R}_\zeta(\epsilon) \sim \frac{1}{2}\log\mathsf{SNR}$, which implies

$$-\log \mathrm{P}_{out} \sim \frac{\mathrm{N}}{2} \cdot \mathsf{SNR}(\mathrm{dB}).$$

Hence, we see that under fading observation channels and AWGN fusion channels, only a diversity order equal to $\mathrm{N}/2$ can be approximately achieved.

3) Case III: In this case, we see that closed-form expression for the rate of function of ξ_i is extremely difficult to obtain. However, we use simulation results in Section V to demonstrate the diversity order for this scenario.

V. Simulation Results

In this section, we present the numerical results for system level performance evaluation of cooperative spectrum sensing. The simulation parameters are described as follows: $\kappa = 100$, $\sigma_n^2 = \sigma_v^2 = 1$ and $\bar{\gamma} = -8$dB.

A. Average Error Probability

In Fig. 2, we plot the average error probability versus the (equal) amplifier gain for all three channel scenarios from Table I. We see that in the low and moderate fusion SNR regimes, case II (Rayleigh fading observation channels and AWGN fusion channels) provides the lowest average error probability among all three scenarios. Thus, to maintain a desired detection performance, the fusion channels need to be as reliable as possible, while the local received SNRs can be dynamic and be used to exploit spatial diversity. However, in the high SNR regime, we observe that case I (AWGN observation channels and Rayleigh fading fusion channels) has the best performance. This is because in the high global SNR regime, fusion channel gain does not impact error probability and reliable observation channels are needed to improve detection performance.

B. Asymptotic Error Probability in Power Constrained Cognitive Radio Networks

Fig. 3 plots the error probability versus global fusion SNR for all three scenarios in power constrained cognitive radio networks when $\mathrm{N} = 1, 10$ and $\mathrm{N} \to \infty$. As expected, the error probability decreases when the number of secondary users increases due to cooperation diversity. As expected, we see that case II and III have lower asymptotic error probability than case I as stated in Lemma IV.2.

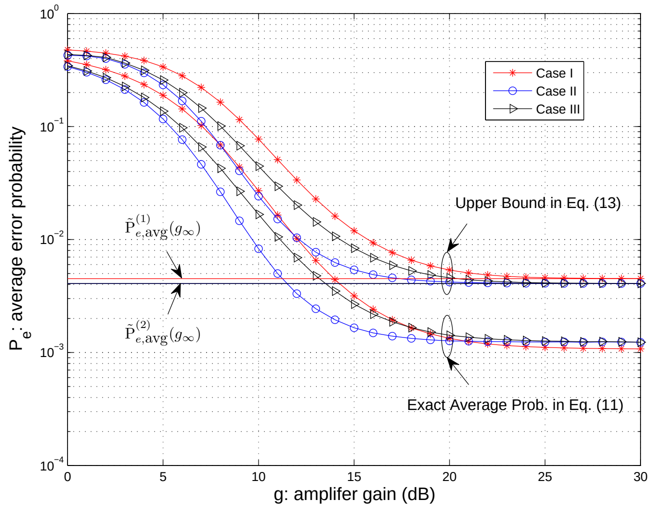

Fig. 2. Average error probability for cooperative spectrum sensing. In the simulation, we choose $N = 15$.

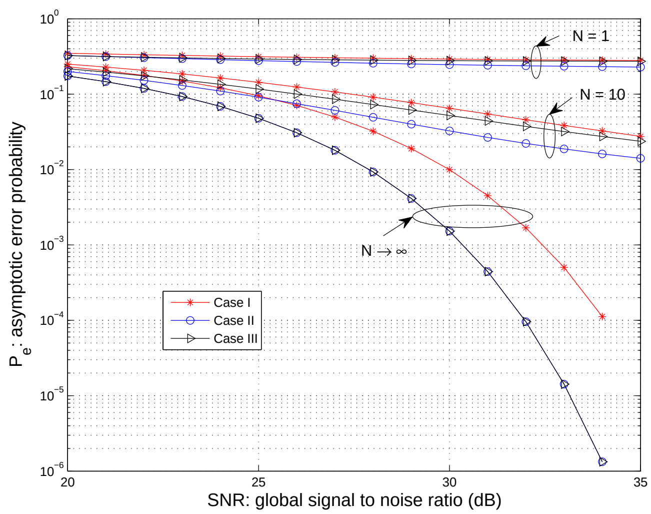

Fig. 3. Asymptotic error probability in power constrained cognitive radio networks. In the simulation, we average results over 10^6 channel realizations for $N = 1$ and 10, and use closed-form expressions in Section IV-A for $N \to \infty$.

C. Outage Analysis in Power Constrained Cognitive Radio Networks

Fig. 4 compares the detection outage probability for the three channel scenarios in power constrained cognitive radio networks. In our simulation, we choose $\bar{P}_e = 0.05$, which corresponds to 5% false alarm probability and 95% detection probability. From the plots, we see that in case I, a diversity order equal to N can be achieved, while in case II and III, a diversity order equal to $N/2$ can be approximately achieved. Similar to the average error probability, we also observe that in the moderate global fusion SNR regime, case II performs the best among three scenarios, while in the high global fusion SNR regime, case I has the best performance.

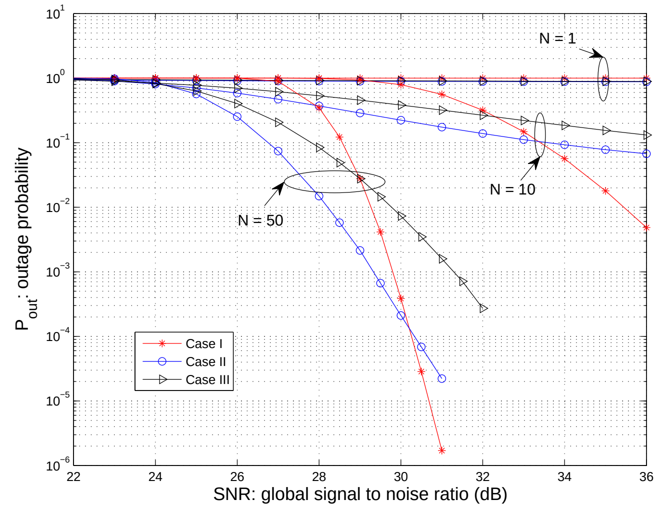

Fig. 4. Outage probability in power constrained cognitive radio networks. In the simulation, we average results over 10^8 channel realizations.

VI. Conclusions

In this paper, we have investigated the system level performance evaluation for cooperative spectrum sensing to cognitive radio networks. In particular, we quantitively analyze three performance criteria for cooperative spectrum sensing: average error probability, asymptotic error probability and outage probability in three channel scenarios.

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