Cost Constrained Spectrum Sensing in Cognitive Radio Networks

Gang Xiong and Shalinee Kishore
Department of Electrical and Computer Engineering
Lehigh University, Bethlehem, PA 18015
Email: {gxiong, skishore}@lehigh.edu

Aylin Yener
Department of Electrical Engineering
Pennsylvania State University, PA 16802
Email: yener@ee.psu.edu

***Abstract*—This paper addresses optimal spectrum sensing in cognitive radio networks considering its system level cost that accounts for the local processing cost of sensing (sample collection and energy calculation at each secondary user) as well as the transmission cost (forwarding energy statistic from secondary users to fusion center). The optimization problem solves for the appropriate number of samples to be collected and amplifier gains at each secondary user to minimize the global error probability subject to a total cost constraint. In particular, closed-form expressions for optimal solutions are derived and a generalized water-filling algorithm is proposed when number of samples or amplifier gains are fixed and additional constraints are imposed. Furthermore, when jointly designing the number of samples and amplifier gains, optimal solution indicates that only one secondary user needs to be active, i.e., collecting samples for local energy calculation and transmitting energy statistic to fusion center.**

I. Introduction

Cognitive radio [1] is a key technology to exploit under-utilized spectrum and enhance spectrum efficiency. In cognitive radio networks, secondary (*unlicensed*) users monitor local communication conditions and opportunistically access unoccupied spectrum when/where the primary (*licensed*) user is inactive. To enable this dynamic spectrum access, secondary users must continuously monitor local spectrum and detect spectrum holes [1]. This technique, called *spectrum sensing*, requires secondary users reliably detect the signals from primary users in order to avoid harmful interference. However, due to the detrimental nature of the wireless channel, a secondary user may not be able to reliably differentiate between a spectrum hole and a weak primary signal if it conducts spectrum sensing on its own. To improve detection reliability, multiple users can engage in cooperative spectrum sensing and take advantage of spatial diversity [2][3].

In [4], a logic "OR" fusion rule for hard-decision combining was presented to cooperatively detect the primary user. Reference [5] introduced an amplify and forward cooperation strategy into spectrum sensing and claimed overall agility can be substantially improved by exploiting spatial diversity. An optimal linear detector for cooperative spectrum sensing was proposed in [2], where the received signals at the fusion center were assigned different weights for global fusion and a convex optimization was formulated to find the linear weights. In [6], detection problems are formulated that account for constraints on expected cost due to transmission and measurement. Here we will take a more inclusive approach and account for various factors that contribute to the cost incurred by spectrum sensing.

In this paper, we study energy-based cooperative spectrum sensing in which local statistics are forwarded to the fusion center using amplify and forward (AF) over parallel access channels. We aim to minimize the global error probability of this cooperative spectrum sensing scheme given that the cost associated with local processing (sample collection and energy calculation) and transmission (forwarding energy statistic to the future center) is constrained. The goal of the minimization is to select the appropriate number of samples and amplifier gains for each secondary user. To this end, we 1) derive closed-form expressions for optimal solutions; and 2) propose a generalized water-filling algorithm when number of samples or amplifier gains are fixed and additional constraints are imposed. Furthermore, when jointly designing the number of samples and amplifier gains, we demonstrate that only one secondary user needs be active, i.e., collecting samples for local energy calculation and transmitting energy statistic to the fusion center. That is, in this case, having one secondary user perform spectrum sensing is sufficient to achieve optimal performance.

II. System Model

A. Local Energy Statistic

For secondary user i, $(1 \leq i \leq \mathrm{N})$, the hypothesis test for the energy of the received signal in a given spectrum band is

$$\left\{ \begin{array}{ll} \mathcal{H}_0: & x_i = (1/\kappa_i)\sum_{k=1}^{\kappa_i} |n_i(k)|^2 \\ \mathcal{H}_1: & x_i = (1/\kappa_i)\sum_{k=1}^{\kappa_i} |\tilde{h}_i s(k) + n_i(k)|^2, \end{array} \right. \tag{1}$$

where κ_i is the number of samples, $s(k)$ is the transmitted signal from the primary user and $n_i(k)$ is the noise received by secondary user i. We assume $s(k)$ is complex PSK modulated and independent and identically distributed (i.i.d.) with mean zero and variance σ_s^2; $\tilde{h}_i$ is the channel gain between the primary user and secondary user i and is assumed to be constant during the cooperative spectrum sensing period; and $n_i(k)$ is i.i.d. circularly symmetric complex Gaussian random variable with mean zero and variance σ_n^2 and is independent of $s(k)$. We define the local received SNR at the secondary user i as $\gamma_i = \sigma_s^2|\tilde{h}_i|^2/\sigma_n^2$.

When κ_i is large, x_i can be approximated as Gaussian random variable [2], i.e.,

$$\left\{ \begin{array}{ll} \mathcal{H}_0: & x_i \sim \mathcal{N}(\sigma_n^2,\ \sigma_n^4/\kappa_i) \\ \mathcal{H}_1: & x_i \sim \mathcal{N}((1+\gamma_i)\sigma_n^2,\ (1+2\gamma_i)\sigma_n^4/\kappa_i). \end{array} \right. \tag{2}$$

In this paper, we assume the local received SNR γ_i is known at the secondary user i. For instance, in IEEE 802.22, this value could be obtained through estimation of pilot signals periodically transmitted from TV stations [7].

B. Amplify and Forward Transmission Strategy

During the cooperation period, the secondary user transmits its local energy statistic to the fusion center using AF on parallel access channels (PAC). The received signal at the fusion center is shown in Fig. 1, i.e.,

$$y_i = g_i h_i x_i + v_i, \tag{3}$$

where g_i is the amplifier gain for the secondary user i, h_i is is the channel gain between secondary user i and the fusion center and v_i is i.i.d. Gaussian noise, i.e., $v_i \sim \mathcal{N}(0, \sigma_v^2)$ and is independent of x_i. We assume that h_i is known at the fusion center (e.g., via channel estimation) and remains constant during the sensing period.

We can then rewrite (3) in a matrix form as

$$\boldsymbol{y} = \mathbf{H}\boldsymbol{x} + \boldsymbol{v}, \tag{4}$$

where $\mathbf{H} = \text{diag}\{g_1 h_1, g_2 h_2, \cdots, g_{\text{N}} h_{\text{N}}\}$.

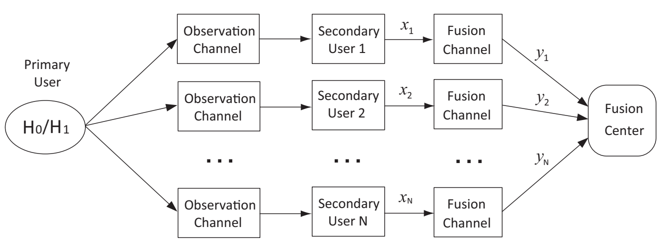

Fig. 1. Cooperative spectrum sensing in cognitive radio networks.

Given this system model, we see that

$$\xi_i \stackrel{\text{def}}{=} \mathbb{E}\{x_i^2\} = \left[1 + 1/\kappa_i + \pi_1 \left(\gamma_i + 2\left(1 + 1/\kappa_i\right)\right)\gamma_i\right] \sigma_n^4,$$

where $\pi_0 = \text{P}(\mathcal{H}_0)$ and $\pi_1 = \text{P}(\mathcal{H}_1)$ are the probabilities that spectrum is idle and occupied, respectively.

In the cognitive radio networks, the received primary user power measured by the secondary user can be very small [8], i.e., $\gamma_i \ll 1$. Additionally, the number of samples can be large, i.e., $\kappa_i \gg 1$. Then, we can approximate the transmitted power for the secondary user i as $\mathcal{P}_i = \xi_i g_i^2 \simeq g_i^2(1 + 2\pi_1\gamma_i)\sigma_n^4$.

C. Optimal Fusion Rule

Under hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1$, the received signal $\boldsymbol{y}$ has a Gaussian distribution, i.e.,

$$\begin{cases} \mathcal{H}_0: & \boldsymbol{y} \sim \mathcal{N}\left(\mathbf{H}\mathbf{1}\sigma_n^2,\ \boldsymbol{\Sigma}_0\right) \\ \mathcal{H}_1: & \boldsymbol{y} \sim \mathcal{N}\left(\mathbf{H}(\mathbf{1} + \boldsymbol{\gamma})\sigma_n^2,\ \boldsymbol{\Sigma}_1\right), \end{cases} \tag{5}$$

where $\mathbf{1} = [1, 1, \cdots, 1]^{\mathsf{T}}$, $\boldsymbol{\Sigma}_0 = \mathbf{H}\mathbf{H}^{\dagger}\sigma_n^4/\kappa_i + \sigma_v^2\mathbf{I}$ and $\boldsymbol{\Sigma}_1 = \mathbf{H}(\mathbf{I} + 2\boldsymbol{\Gamma})\mathbf{H}^{\dagger}\sigma_n^4/\kappa_i + \sigma_v^2\mathbf{I}$, here, $\boldsymbol{\Gamma} = \text{diag}\{\gamma_1, \gamma_2, \cdots, \gamma_{\text{N}}\}$.

Without loss of generality, we assume that $\pi_0 = \pi_1 = 0.5$. Then, optimal likelihood ratio test (LRT) is given as:

$$\log \frac{p(\boldsymbol{y}|\mathcal{H}_1)}{p(\boldsymbol{y}|\mathcal{H}_0)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} 0. \tag{6}$$

Since $\gamma_i \ll 1$ and $\kappa_i \gg 1$, then, $\gamma_i/\kappa_i \approx 0$ and we have $\boldsymbol{\Sigma}_0 \approx \boldsymbol{\Sigma}_1$. Thus, the optimal LRT can be approximated as

$$\mathcal{T}(\boldsymbol{y}) = (\mathbf{H}\boldsymbol{\gamma})^{\dagger}\boldsymbol{\Sigma}_0^{-1}\boldsymbol{y} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \tau, \tag{7}$$

where $\tau = (\mathbf{H}\boldsymbol{\gamma})^{\dagger}\boldsymbol{\Sigma}_0^{-1}\mathbf{H}(\mathbf{1} + 0.5\boldsymbol{\gamma})\sigma_n^2$. It is easy to see that the error probability is given as

$$\text{P}_e \ = \ Q\left(\frac{1}{2}\sqrt{\mathcal{F}(\boldsymbol{\kappa}, \boldsymbol{g})}\right), \tag{8}$$

where $Q(x)$ is the complementary distribution function of the standard Gaussian, i.e., $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} \exp(-t^2/2)\text{d}t$; and

$$\mathcal{F}(\boldsymbol{\kappa}, \boldsymbol{g}) = \sum_{i=1}^{\text{N}} \frac{g_i^2\kappa_i\gamma_i^2|h_i|^2}{g_i^2|h_i|^2 + \kappa_i\tilde{\sigma}_v^2},$$

where $\tilde{\sigma}_v^2 = \sigma_v^2/\sigma_n^4$.

It is also easy to see that the asymptotic error probability expressions when the number of samples or amplifier gains approach infinity are given by

$$\text{P}_e(\kappa_\infty) \stackrel{\text{def}}{=} \lim_{\kappa_i \to \infty} \text{P}_e = Q\left(\tfrac{1}{2\tilde{\sigma}_v}\left(\textstyle\sum_{i=1}^{\text{N}} g_i^2\gamma_i^2|h_i|^2\right)^{1/2}\right), \tag{9}$$

and

$$\text{P}_e(g_\infty) \stackrel{\text{def}}{=} \lim_{g_i \to \infty} \text{P}_e = Q\left(\tfrac{1}{2}\left(\textstyle\sum_{i=1}^{\text{N}} \kappa_i\gamma_i^2\right)^{1/2}\right), \tag{10}$$

respectively.

D. System Level Cost for Cooperative Spectrum Sensing

In this paper, we consider system level cost for cooperative spectrum sensing in cognitive radio networks. The system level cost consists of three parts: local processing cost, transmission cost, reporting and broadcasting cost.

- Local processing cost includes the receiver RF scanning and local energy calculation. For simplicity, we assume that the local processing cost $\mathcal{C}_{pi}(\cdot)$ for secondary user i is a linear function of the number of samples, i.e.,

$$\mathcal{C}_{pi}(\kappa_i) = c_0\kappa_i,$$

  where c_0 is the local processing cost per sample.
- Transmission cost is the transmit power required from a secondary user to transmit the local calculated energy to the fusion center. Here, we assume that this cost for secondary user i is given as

$$\mathcal{C}_{ti}(g_i) = \mathcal{P}_i = \xi_i g_i^2.$$

- For optimal system design, the fusion center needs to know the local received SNR for each secondary user. In practice, this means that secondary users will report their local received SNRs to the fusion center. The fusion center then determines the resource allocated to each secondary user, and broadcasts this to all secondary users. In this paper, we assume that total reporting and broadcasting cost $\mathcal{C}_{rb}$ is fixed, and thus do not consider it in the optimization problem.

The total cost during the cooperative spectrum sensing is given as

$$\begin{aligned}\mathcal{C}(\boldsymbol{\kappa},\boldsymbol{g}) &= \sum_{i=1}^{\mathrm{N}}\mathcal{C}_{pi}(\kappa_i)+\sum_{i=1}^{\mathrm{N}}\mathcal{C}_{ti}(g_i)\\ &= \sum_{i=1}^{\mathrm{N}}\left(c_0\kappa_i+\xi_i g_i^2\right).\end{aligned}$$

III. Minimization of Error Probability

In this section, we aim to minimize the error probability for the system model in Fig. 1 subject to a total cost constraint. Specifically, we determine the appropriate number of samples and amplifier gains for each secondary user and consider the following two scenarios for this optimization problem:

1) **Scenario A:** First, we consider the total cost constraint. Hence, the optimization problem is formulated as:

$$\begin{aligned}\min_{\boldsymbol{\kappa},\boldsymbol{g}}\quad & \mathrm{P}_e(\boldsymbol{\kappa},\boldsymbol{g})\\ \text{s.t.}\quad & \mathcal{C}(\boldsymbol{\kappa},\boldsymbol{g})\le\bar{\mathcal{C}}\\ & \boldsymbol{\kappa}\succeq\mathbf{0},\ \boldsymbol{g}\succeq\mathbf{0},\end{aligned}\qquad (11)$$

where $\mathbf{0}=[0,0,\cdots,0]^{\mathsf{T}}$ and $\bar{\mathcal{C}}$ is the total cost threshold.

2) **Scenario B:** In some applications, local sample collection for each secondary user may be scheduled in a fixed time slot. In other words, the number of samples is upper bounded by a maximum value $\kappa_{\max}$. Furthermore, the transmission power for each secondary user may be required to be below a predefined power limit $\mathcal{P}_{\max}$. By incorporating these additional individual constraints imposed on each secondary user, we can model the optimization problem as

$$\begin{aligned}\min_{\boldsymbol{\kappa},\boldsymbol{g}}\quad & \mathrm{P}_e(\boldsymbol{\kappa},\boldsymbol{g})\\ \text{s.t.}\quad & \mathcal{C}(\boldsymbol{\kappa},\boldsymbol{g})\le\bar{\mathcal{C}}\\ & \mathbf{0}\preceq\boldsymbol{\kappa}\preceq\kappa_{\max}\mathbf{1}\\ & \boldsymbol{g}\succeq\mathbf{0},\ \xi_i g_i^2\le\mathcal{P}_{\max}.\end{aligned}\qquad (12)$$

To better understand the optimal resource allocation for cooperative spectrum sensing, we consider the following three cases for this optimization problem: 1) when $\boldsymbol{g}$ is fixed; 2) when $\boldsymbol{\kappa}$ is fixed; and 3) when $\boldsymbol{\kappa}$ and $\boldsymbol{g}$ are unknown.

A. Case I: When $\boldsymbol{g}$ is Fixed

In this case, we assume fixed amplifier gains, i.e., $\boldsymbol{g}=\tilde{\boldsymbol{g}}$; thus we need to minimize the error probability by choosing appropriate number of samples. Let us define the total number of samples as $\kappa_{\text{tot}}=\lfloor(\bar{\mathcal{C}}-\sum_{i=1}^{\mathrm{N}}\xi_i\tilde{g}_i^2)/c_0\rfloor$.

1) Scenario A: In this scenario, the optimization problem in (11) reduces to

$$\begin{aligned}\max_{\boldsymbol{\kappa}}\quad & \mathcal{F}(\boldsymbol{\kappa},\tilde{\boldsymbol{g}})\\ \text{s.t.}\quad & \mathbf{1}^{\mathsf{T}}\boldsymbol{\kappa}\le\kappa_{\text{tot}},\ \boldsymbol{\kappa}\succeq\mathbf{0}.\end{aligned}\qquad (13)$$

Let us define $a_i=\tilde{g}_i^2\gamma_i^2|h_i|^2/\tilde{\sigma}_v^2$ and $b_i=\tilde{g}_i^2|h_i|^2/\tilde{\sigma}_v^2$. Then, the optimization problem in (13) is equivalent to

$$\begin{aligned}\min_{\boldsymbol{\kappa}}\quad & \sum_{i=1}^{\mathrm{N}}\frac{a_ib_i}{\kappa_i+b_i}\\ \text{s.t.}\quad & \mathbf{1}^{\mathsf{T}}\boldsymbol{\kappa}\le\kappa_{\text{tot}},\ \boldsymbol{\kappa}\succeq\mathbf{0}.\end{aligned}\qquad (14)$$

It is straightforward to see that optimization problem in (14) is convex. The Karush-Kuhn-Tucker (KKT) conditions can be given as

$$\frac{a_ib_i}{(\kappa_i+b_i)^2}+u_i-\lambda_0 = 0 \qquad (15)$$

$$\lambda_0(\mathbf{1}^{\mathsf{T}}\boldsymbol{\kappa}-\kappa_{\text{tot}}) = 0 \qquad (16)$$

$$u_i\kappa_i = 0. \qquad (17)$$

where $\lambda_0\ge0$ and $u_i\ge0$ are Lagrangian multipliers.

First we assume that $\lambda_0>0$ and $u_i=0$, then from (15), we see that $\kappa_i=\left[\sqrt{a_ib_i/\lambda_0}-b_i\right]^+$, where $[x]^+=\max\{0,x\}$. Plugging this into (16), we have $\sqrt{\lambda_0}=\frac{\sum_{i\in\mathcal{S}_0}\sqrt{a_ib_i}}{\kappa_{\text{tot}}+\sum_{i\in\mathcal{S}_0}b_i}$, where $\mathcal{S}_0=\{i|\kappa_i>0\}$. Then, we need to determine the set $\mathcal{S}_0$ to obtain the closed-form solution for $\boldsymbol{\kappa}$. To do this, let us define $\alpha_i=\sqrt{b_i/a_i}$. Without loss of generality, we assume $\alpha_1\le\alpha_2\le\cdots\le\alpha_{\mathrm{N}}$ and can show that (please see [9] for details)

$$\mathcal{S}_0=\begin{cases}\{1,\cdots,i_{\mathcal{S}}|f(i_{\mathcal{S}})<1,f(i_{\mathcal{S}}+1)\ge1\}, & f(\mathrm{N})\ge1\\ \{1,\cdots,\mathrm{N}\}, & \text{otherwise},\end{cases}\qquad (18)$$

where $f(i)=\frac{\alpha_i\sum_{j=1}^{i}\sqrt{a_jb_j}}{\kappa_{\text{tot}}+\sum_{j=1}^{i}b_j}$. Thus, the optimal number of samples can be obtained as

$$\kappa_{p,i}^{(\text{opt})}=\begin{cases}\left[\frac{\tilde{g}_i^2|h_i|^2}{\tilde{\sigma}_v^2}(\gamma_i\mu-1)\right]^{\sharp}, & i\in\mathcal{S}_0\\ 0, & i\notin\mathcal{S}_0,\end{cases}\qquad (19)$$

where $\mu=\frac{\sum_{i\in\mathcal{S}_0}\tilde{g}_i^2|h_i|^2+\kappa_{\text{tot}}\tilde{\sigma}_v^2}{\sum_{i\in\mathcal{S}_0}\tilde{g}_i^2\gamma_i|h_i|^2}$ and $[\cdot]^{\sharp}$ denotes the integer operation. It is worth noting that this operation should guarantee $\mathbf{1}^{\mathsf{T}}\boldsymbol{\kappa}=\kappa_{\text{tot}}$. A simple strategy for the integer operation can be given as

$$[\kappa_i]^{\sharp}=\begin{cases}\lceil\kappa_i\rceil, & i\le\lfloor i_{\mathcal{S}}/2\rfloor\\ \lfloor\kappa_i\rfloor, & \text{otherwise},\end{cases}$$

where $\lceil\cdot\rceil$ and $\lfloor\cdot\rfloor$ denote the ceiling and floor operations, respectively.

Remark: We note that the optimal number of samples for each secondary user follows a water-filling strategy, i.e., with larger α_i, the chance for the secondary user to be inactive is higher. Note that $\alpha_i=1/\gamma_i$. Hence, when the local received SNR is low, the secondary user tends not to collect the samples for local energy calculation.

For comparison, we consider two suboptimal solutions for this optimization problem as follows:

- A simple solution is to choose equal number of samples, i.e., $\kappa_{p,i}^{(\text{equ})}=\lfloor\kappa_{\text{tot}}/\mathrm{N}\rfloor$.
- Using Cauchy-Schwarz inequality, we see that $\mathrm{P}_e(g_\infty)$ in (10) can be minimized when $\kappa_i=c\gamma_i^2$, where c is a constant. Based on this, we propose a suboptimal solution

for the number of samples[1], i.e., $\kappa_{p,i}^{(\text{sub})} = \left[\frac{\gamma_i^2}{\sum_{i=1}^{\mathrm{N}} \gamma_i^2}\kappa_{\text{tot}}\right]^{\sharp}$. In this case, $i_{\mathcal{S}} = \mathrm{N}$ and secondary users only need to know the norm square of observation channel gains to calculate the number of samples.

Let us denote the asymptotic error probabilities when $\tilde{g}_i \to \infty$ for these three solutions of number of samples as $\mathrm{P}_e^{(\text{equ})}(g_\infty)$, $\mathrm{P}_e^{(\text{sub})}(g_\infty)$ and $\mathrm{P}_e^{(\text{opt})}(g_\infty)$, respectively. Then, we note that

Lemma III.1. *When* $\alpha_2 > \alpha_1$ *and* $\alpha_2 - \alpha_1$ *is finite,* $\mathrm{P}_e^{(\text{equ})}(g_\infty) \geq \mathrm{P}_e^{(\text{sub})}(g_\infty) \geq \mathrm{P}_e^{(\text{opt})}(g_\infty)$.

Due to space limitations, we omit the proof. For detailed proof, please refer to [9].

2) Scenario B: In this scenario, the optimization problem in (12) becomes

$$\max_{\boldsymbol{\kappa}} \quad \mathcal{F}(\boldsymbol{\kappa}, \tilde{\boldsymbol{g}}) \qquad \text{s.t.} \quad \mathbf{1}^{\mathsf{T}}\boldsymbol{\kappa} \leq \kappa_{\text{tot}},\ \mathbf{0} \preceq \boldsymbol{\kappa} \preceq \kappa_{\max}\mathbf{1}. \tag{20}$$

With the additional constraint, updated KKT conditions are

$$\frac{a_i b_i}{(\kappa_i + b_i)^2} + u_i - v_i - \lambda_0 = 0 \tag{21}$$

$$v_i(\kappa_i - \kappa_{\max}) = 0, \tag{22}$$

where $v_i \geq 0$ are Lagrangian multipliers. First we assume that $\lambda_0 > 0$ and $u_i = v_i = 0$, then from (21), we see that $\kappa_i = \sqrt{a_i b_i/\lambda_0} - b_i$. Thus, based on the value of $\sqrt{\lambda_0}$, we can determine the optimal solution of κ_i as

$$\kappa_i = \begin{cases} 0, & \text{if } \sqrt{\lambda_0} > \sqrt{a_i/b_i} \\ \kappa_{\max}, & \text{if } 0 < \sqrt{\lambda_0} < \sqrt{a_i b_i}/(\kappa_{\max} + b_i) \\ \sqrt{a_i b_i/\lambda_0} - b_i, & \text{otherwise.} \end{cases}$$

Let us define two disjoint sets for secondary users as $\mathcal{S}_1 = \{i | \kappa_i = \kappa_{\max}\}$ and $\mathcal{S}_2 = \{i | 0 < \kappa_i < \kappa_{\max}\}$. Plugging κ_i into (16), we have

$$|\mathcal{S}_1|\kappa_{\max} + \left(1/\sqrt{\lambda_0}\right) \sum_{i\in\mathcal{S}_2} \sqrt{a_i b_i} - \sum_{i\in\mathcal{S}_2} b_i = \kappa_{\text{tot}},$$

which implies that $\sqrt{\lambda_0} = \frac{\sum_{i\in\mathcal{S}_2}\sqrt{a_i b_i}}{\kappa_{\text{tot}} - |\mathcal{S}_1|\kappa_{\max} + \sum_{i\in\mathcal{S}_2} b_i}$.

In order to determine $\mathcal{S}_1$, $\mathcal{S}_2$ and $\sqrt{\lambda_0}$ and thus obtain the closed-form solution for κ_i, we propose here a two-stage generalized water-filling algorithm as follows:

1) In the first stage, we aim to determine the set $\mathcal{S}_1$. To do this, let us define $\tilde{\alpha}_i = \frac{\kappa_{\max} + b_i}{\sqrt{a_i b_i}}$. Without loss of generality, we assume $\tilde{\alpha}_1 \leq \tilde{\alpha}_2 \leq \cdots \leq \tilde{\alpha}_{\mathrm{N}}$. Then, similar to Scenario A, $\mathcal{S}_1$ can be obtained by (18) with (please see [9] for details)

$$\tilde{f}(i) = \frac{\tilde{\alpha}_i \sum_{m\in\tilde{\mathcal{S}}_i}\sqrt{a_m b_m}}{\kappa_{\text{tot}} - i\kappa_{\max} + \sum_{m\in\tilde{\mathcal{S}}_i} b_m}, \quad i \leq \left\lfloor \frac{\kappa_{\text{tot}}}{\kappa_{\max}} \right\rfloor, \tag{23}$$

where $\tilde{\mathcal{S}}_i = \{m | \alpha_m < \tilde{\alpha}_i,\ i < m \leq \mathrm{N}\}$. After $\mathcal{S}_1$ is determined, we have $\kappa_i = \kappa_{\max},\ \forall i \in \mathcal{S}_1$.

2) In the second stage, we follow a procedure similar to Scenario A to obtain $\mathcal{S}_2$ and κ_i for $i \notin \mathcal{S}_1$. The solution is given in (19), except that κ_{tot} and N are replaced by $\kappa_{\text{tot}} - |\mathcal{S}_1|\kappa_{\max}$ and $\mathrm{N} - |\mathcal{S}_1|$, respectively.

[1]It is worth mentioning that a similar discussion can be found in [10].

B. Case II: When $\boldsymbol{\kappa}$ is Fixed

In this case, we assume fixed number of samples, i.e., $\boldsymbol{\kappa} = \tilde{\boldsymbol{\kappa}}$. Let us define global transmission power constraint as $\mathcal{P}_{\text{tot}} = \bar{\mathcal{C}} - c_0 \mathbf{1}^{\mathsf{T}}\tilde{\boldsymbol{\kappa}}$ and $z_i = g_i^2$.

1) Scenario A: Here we follow a similar derivation to that in Section III-A1. Let us define $\beta_i = \frac{\tilde{\sigma}_v\sqrt{\xi_i}}{\gamma_i|h_i|}$. Without loss of generality, assume $\beta_1 \leq \beta_2 \leq \cdots \leq \beta_{\mathrm{N}}$. Then, define a set $\mathcal{I}_0$ from (18) with $f(i) = \frac{\beta_i\tilde{\sigma}_v\sum_{j=1}^{i}\tilde{\kappa}_j\sqrt{\xi_j}\gamma_j/|h_j|}{\mathcal{P}_{\text{tot}} + \tilde{\sigma}_v^2\sum_{j=1}^{i}\tilde{\kappa}_j\xi_j/|h_j|^2}$. The optimal amplifier gains can be obtained as

$$g_{p,i}^{(\text{opt})} = \begin{cases} \left[\frac{\tilde{\kappa}_i\tilde{\sigma}_v^2}{|h_i|^2}\left(\frac{\gamma_i|h_i|}{\sqrt{\xi_i}}\eta - 1\right)\right]^{1/2}, & i \in \mathcal{I}_0 \\ 0, & i \notin \mathcal{I}_0, \end{cases} \tag{24}$$

where $\eta = \frac{\sum_{i\in\mathcal{I}_0}\tilde{\kappa}_i\xi_i/|h_i|^2 + \mathcal{P}_{\text{tot}}/\tilde{\sigma}_v^2}{\sum_{i\in\mathcal{I}_0}\tilde{\kappa}_i\sqrt{\xi_i}\gamma_i/|h_i|}$.

Remark: Again we see that the optimal amplifier gains follow a water-filling strategy, i.e., with larger β_i, the chance for the secondary user to be inactive is higher. Furthermore, we note that $\beta_i \propto 1/(\gamma_i|h_i|)$. Hence, when the local received SNR is low or the fusion channel quality is poor, the secondary user tends not to transmit the local calculated energy to the fusion center.

For comparison, we consider two suboptimal solutions for this optimization problem as follows:

- A simple solution is to choose equal transmission power for each secondary user, i.e., $g_{p,i}^{(\text{equ})} = \sqrt{\mathcal{P}_{\text{tot}}/(\mathrm{N}\xi_i)}$.
- Furthermore, similar to Section III-A1, we propose a suboptimal solution for amplifier gains based on (9) using the Cauchy-Schwarz inequality, i.e.,

$$g_{p,i}^{(\text{sub})} = \left(\frac{\gamma_i^2|h_i|^2/\xi_i^2}{\sum_{i=1}^{\mathrm{N}}\gamma_i^2|h_i|^2/\xi_i}\mathcal{P}_{\text{tot}}\right)^{1/2}.$$

Let us denote the asymptotic error probabilities when $\tilde{\kappa}_i \to \infty$ for these three solutions of amplifier gains as $\mathrm{P}_e^{(\text{equ})}(\kappa_\infty)$, $\mathrm{P}_e^{(\text{sub})}(\kappa_\infty)$ and $\mathrm{P}_e^{(\text{opt})}(\kappa_\infty)$, respectively. Similar to Lemma III.1, we note that

Lemma III.2. *When* $\beta_2 > \beta_1$ *and* $\beta_2 - \beta_1$ *is finite,* $\mathrm{P}_e^{(\text{equ})}(\kappa_\infty) \geq \mathrm{P}_e^{(\text{sub})}(\kappa_\infty) \geq \mathrm{P}_e^{(\text{opt})}(\kappa_\infty)$.

2) Scenario B: Let us define two sets as $\mathcal{I}_1 = \{i | z_i = \mathcal{P}_{\max}/\xi_i\}$ and $\mathcal{I}_2 = \{i | 0 < z_i < \mathcal{P}_{\max}/\xi_i\}$. Then, similar to Section III-A2, we propose a two-stage generalized water-filling algorithm as follows:

1) In the first stage, we aim to determine the set $\mathcal{I}_1$. Let us define $\tilde{\beta}_i = \frac{\mathcal{P}_{\max}|h_i|^2 + \tilde{\sigma}_v^2\tilde{\kappa}_i\xi_i}{\tilde{\sigma}_v\tilde{\kappa}_i\gamma_i|h_i|\sqrt{\xi_i}}$. Without loss of generality, we assume $\tilde{\beta}_1 \leq \tilde{\beta}_2 \leq \cdots \leq \tilde{\beta}_{\mathrm{N}}$. Then, similar to Section III-A2, $\mathcal{I}_1$ can be obtained by (18) with

$$\tilde{f}(i) = \frac{\tilde{\beta}_i\tilde{\sigma}_v\sum_{m\in\tilde{\mathcal{I}}_i}\tilde{\kappa}_m\sqrt{\xi_m}\gamma_m/|h_m|}{\mathcal{P}_{\text{tot}} - i\mathcal{P}_{\max} + \tilde{\sigma}_v^2\sum_{m\in\tilde{\mathcal{I}}_i}\tilde{\kappa}_m\xi_m/|h_m|^2},\ i \leq \left\lfloor \frac{\mathcal{P}_{\text{tot}}}{\mathcal{P}_{\max}} \right\rfloor,$$

where $\tilde{\mathcal{I}}_i = \{m | \beta_m < \tilde{\beta}_i,\ i < m \leq \mathrm{N}\}$. After $\mathcal{I}_1$ is determined, we have $z_i = \mathcal{P}_{\max}/\xi_i,\ \forall i \in \mathcal{I}_1$.

2) In the second stage, we follow a procedure similar to Scenario A to obtain $\mathcal{I}_2$ and z_i for $i \notin \mathcal{I}_1$. The solution is given in (24), except that $\mathcal{P}_{\text{tot}}$ and N are replaced by $\mathcal{P}_{\text{tot}} - |\mathcal{I}_1|\mathcal{P}_{\max}$ and $\mathrm{N} - |\mathcal{I}_1|$, respectively.

C. Case III: When $\boldsymbol{\kappa}$ and $\boldsymbol{g}$ are Unknown

1) Scenario A: In this case, let us define $p_i = \tilde{\sigma}_v^2/(\gamma_i^2|h_i|^2)$ and $q_i = 1/\gamma_i^2$. To simplify our analysis, when $\kappa_i = z_i = 0$, we assume $\kappa_i z_i/(p_i\kappa_i + q_i z_i) = 0$. In practice, this assumption can be alleviated by adding a sufficiently small constant in the denominator. Then, the optimization problem becomes

$$\begin{aligned} \max_{\boldsymbol{\kappa},\boldsymbol{z}} \quad & \sum_{i=1}^{\mathrm{N}} \frac{\kappa_i z_i}{p_i\kappa_i + q_i z_i} \\ \text{s.t.} \quad & c_0\mathbf{1}^{\mathsf{T}}\boldsymbol{\kappa} + \boldsymbol{\xi}^{\mathsf{T}}\boldsymbol{z} \le \bar{\mathcal{C}} \\ & \boldsymbol{\kappa} \succeq \mathbf{0},\ \boldsymbol{z} \succeq \mathbf{0}. \end{aligned} \tag{25}$$

It is easy to see that (25) is a convex optimization problem (please see [9] for details). Thus it can be solved efficiently using interior-point methods or other iterative methods [11]. To obtain insight into the closed-form solution, first we introduce the following lemma.

Lemma III.3. *Optimal solution of $(\boldsymbol{\kappa}, \boldsymbol{z})$ in (25) should satisfy either 1) $\kappa_i > 0$ and $z_i > 0$, or 2) $\kappa_i = 0$ and $z_i = 0$ for secondary user i.*

Due to space limitations, we omit the proof. For detailed proof, please refer to [9]. This lemma is not surprising because when one secondary user does not collect the energy samples, it will not waste the transmission power to transmit the null data to the fusion center. On the other hand, when one secondary user decides not to transmit the data to the fusion center, it is reasonable that this secondary user remains inactive and does not collect the samples for local energy calculation.

Using Lemma III.3, the optimal solution of $(\boldsymbol{\kappa}, \boldsymbol{g})$ can be found as stated in the following theorem.

Theorem III.4. *Consider the optimization problem in (25), let us define $\rho_i = \frac{\gamma_i^2|h_i|^2}{(\tilde{\sigma}_v\sqrt{\xi_i}+|h_i|\sqrt{c_0})^2}$ and assume $\rho_1 \ge \rho_2 \ge \cdots \ge \rho_{\mathrm{N}}$. Then, the optimal solution of $(\boldsymbol{\kappa}, \boldsymbol{g})$ is*

$$\begin{aligned} \kappa_{p,i}^{(\mathrm{opt})} &= \begin{cases} \left\lfloor \frac{|h_i|\bar{\mathcal{C}}}{\tilde{\sigma}_v\sqrt{\xi_i c_0}+|h_i|c_0} \right\rfloor, & i = 1 \\ 0, & i > 1, \end{cases} \\ g_{p,i}^{(\mathrm{opt})} &= \begin{cases} \left(\frac{\tilde{\sigma}_v\bar{\mathcal{C}}}{\tilde{\sigma}_v\xi_i+|h_i|\sqrt{\xi_i c_0}}\right)^{1/2}, & i = 1 \\ 0, & i > 1. \end{cases} \end{aligned} \tag{26}$$

Due to space limitations, we omit the proof. For detailed proof, we refer the readers to [9]. Given the optimal solution of $(\boldsymbol{\kappa}, \boldsymbol{g})$, we see that the optimal error probability can be obtained as[2]

$$\mathrm{P}_e^{(\mathrm{opt})} = Q\left(\frac{\sqrt{\bar{\mathcal{C}}}}{2}\max\left\{\frac{\gamma_i|h_i|}{\tilde{\sigma}_v\sqrt{\xi_i}+|h_i|\sqrt{c_0}}\right\}\right).$$

Remark: When we jointly design the number of samples and amplifier gains subject to the total cost constraint, only one secondary user needs to be active in the cognitive radio network, i.e., collecting the samples for local energy calculation and transmitting the energy statistic to the fusion center. It is interesting to note that this optimal strategy is similar to multiuser diversity where base station picks the user with best channel to achieve maximum sum rate capacity [12]. In this case, fusion center will select the secondary user with best ρ_i to perform local spectrum sensing and data forwarding.

[2]For simplicity, we neglect the rounding effect of $\boldsymbol{\kappa}$ throughout the paper.

For comparison, we propose three suboptimal solutions for this optimization problem as shown in Table I.

TABLE I
SUBOPTIMAL SOLUTIONS FOR OPTIMIZATION PROBLEM IN CASE III

	Number of samples $\boldsymbol{\kappa}$	Amplifier gains $\boldsymbol{g}$
Sub I	Given $g_{p,i}^{(\mathrm{sub1})}$, obtain $\kappa_{p,i}^{(\mathrm{sub1})}$ from (19)	$g_{p,i}^{(\mathrm{sub1})} = \left(\frac{\bar{\mathcal{C}}}{2\mathrm{N}\xi_i}\right)^{1/2}$
Sub II	$\kappa_{p,i}^{(\mathrm{sub2})} = \left\lfloor \frac{\bar{\mathcal{C}}}{2\mathrm{N}c_0} \right\rfloor$	Given $\kappa_{p,i}^{(\mathrm{sub2})}$, obtain $g_{p,i}^{(\mathrm{sub2})}$ from (24)
Equal	$\kappa_{p,i}^{(\mathrm{equ})} = \left\lfloor \frac{\bar{\mathcal{C}}}{\mathrm{N}c_0+\mathbf{1}^{\mathsf{T}}\boldsymbol{\xi}} \right\rfloor$	$g_{p,i}^{(\mathrm{equ})} = \left(\frac{\bar{\mathcal{C}}}{\mathrm{N}c_0+\mathbf{1}^{\mathsf{T}}\boldsymbol{\xi}}\right)^{1/2}$

2) Scenario B: Here, the optimization problem becomes

$$\begin{aligned} \max_{\boldsymbol{\kappa},\boldsymbol{z}} \quad & \sum_{i=1}^{\mathrm{N}} \frac{\kappa_i z_i}{p_i\kappa_i + q_i z_i} \\ \text{s.t.} \quad & c_0\mathbf{1}^{\mathsf{T}}\boldsymbol{\kappa} + \boldsymbol{\xi}^{\mathsf{T}}\boldsymbol{z} \le \bar{\mathcal{C}} \\ & \mathbf{0} \preceq \boldsymbol{\kappa} \preceq \kappa_{\max}\mathbf{1},\ \boldsymbol{z} \succeq \mathbf{0},\ \xi_i z_i \le \mathcal{P}_{\max}. \end{aligned} \tag{27}$$

Again, we see that this is a convex optimization problem and can be solved by standard methods. Let us denote the optimal solution as $\left(\kappa_{p,i}^{(\mathrm{opt})}, g_{p,i}^{(\mathrm{opt})}\right)$. Similarly, we note that

Lemma III.5. *Optimal solution of $(\boldsymbol{\kappa}, \boldsymbol{z})$ in (27) should satisfy either 1) $\kappa_i > 0$ and $z_i > 0$, or 2) $\kappa_i = 0$ and $z_i = 0$ for secondary user i.*

The proof is similar to that of Lemma III.3 and thus omitted. With the additional constraints imposed on $\boldsymbol{\kappa}$ and $\boldsymbol{z}$, in general we see that it is difficult to obtain the closed-form solution for $(\boldsymbol{\kappa}, \boldsymbol{z})$. By noting that the optimal solution of $(\boldsymbol{\kappa}, \boldsymbol{z})$ needs to be equal to 0 or greater than 0 simultaneously, we propose a heuristic suboptimal algorithm for Scenario B. Specifically, first we assign $\kappa_{\max}$ and $\mathcal{P}_{\max}$ to the secondary user with largest ρ_i. If enough resource is left, we assign $\kappa_{\max}$ and $\mathcal{P}_{\max}$ to the secondary user with second largest ρ_i and so on until $\kappa_{\max}$ and $\mathcal{P}_{\max}$ cannot be assigned to any one secondary user. In this case, we merely utilize the optimal solution in (26) to allocate (κ_i, g_i) to the secondary user with the next largest ρ_i and $\kappa_i = 0,\ g_i = 0$ to the rest of the secondary users.

IV. SIMULATION RESULTS

In the simulations, we assume $\mathrm{N} = 6$, $\sigma_n^2 = \sigma_v^2 = 1$, $c_0 = 1$, $\boldsymbol{\gamma} = [-8.86, -15.23, -7.21, -5.09, -10.00, -10.97]^{\mathsf{T}}$(dB), $\boldsymbol{h} = [1.56, 1.99, 0.37, 1.52, 0.39, 1.98]^{\mathsf{T}}$. We define the global fusion SNR as $\mathsf{SNR} = \mathcal{P}_{\text{tot}}/\sigma_v^2$.

Fig. 2 and 3 show the error probability versus global fusion SNR in Case I and total number of samples in Case II, respectively. As expected, we see that the optimal solution provides superior performance to suboptimal solutions. From the plots, we also observe that with additional individual constraints, the optimal solution for Scenario B performs worse than that of Scenario A. Furthermore, when the global fusion SNR or total number of samples increases, we see that the error probability approaches the asymptotic bound. In particular, in Fig. 2, $\mathrm{P}_e^{(\mathrm{equ})}(g_\infty) \geq \mathrm{P}_e^{(\mathrm{sub})}(g_\infty) \geq \mathrm{P}_e^{(\mathrm{opt})}(g_\infty)$ as discussed in Lemma III.1 and in Fig. 3, $\mathrm{P}_e^{(\mathrm{equ})}(\kappa_\infty) \geq \mathrm{P}_e^{(\mathrm{sub})}(\kappa_\infty) \geq \mathrm{P}_e^{(\mathrm{opt})}(\kappa_\infty)$ as mentioned in Lemma III.2.

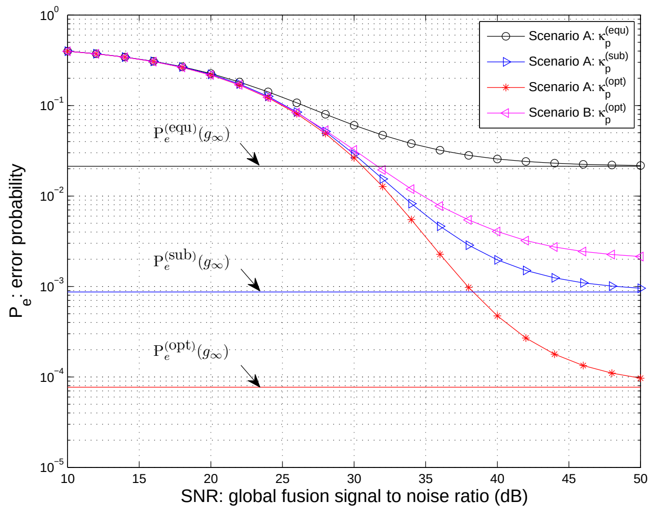

Fig. 2. Case I: error probability for different solutions of $\boldsymbol{\kappa}$. In the simulation, we choose $\kappa_{\mathrm{tot}} = 600$ and fixed amplifier gains $\tilde{g}_i = \sqrt{\mathcal{P}_{\mathrm{tot}}/(\mathrm{N}\xi_i)}$. In Scenario B, we choose $\kappa_{\max} = 0.4\kappa_{\mathrm{tot}}$.

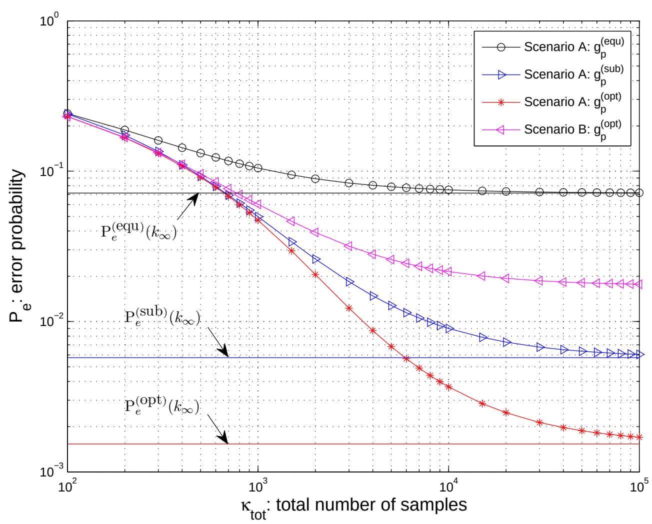

Fig. 3. Case II: error probability for different solutions of $\boldsymbol{g}$. In the simulation, we choose $\mathsf{SNR} = 25$dB and fixed number of samples $\tilde{\kappa}_i = \lfloor \kappa_{\mathrm{tot}}/\mathrm{N} \rfloor$. In Scenario B, we choose $\mathcal{P}_{\max} = 0.4\mathcal{P}_{\mathrm{tot}}$.

In Fig. 4, we plot the error probability versus total cost constraint in Case III for different solutions of $(\boldsymbol{\kappa}, \boldsymbol{g})$. In this simulation, we utilize interior-point method to solve the optimization problem in Scenario B. As expected, we see that the optimal solution significantly improves the error probability compared to three suboptimal solutions in Scenario A. Furthermore, we observe that the error performance is degraded with the additional constraints in Scenario B. Additionally, we note that our proposed suboptimal algorithm in Scenario B has negligible performance loss compared to the optimal solution.

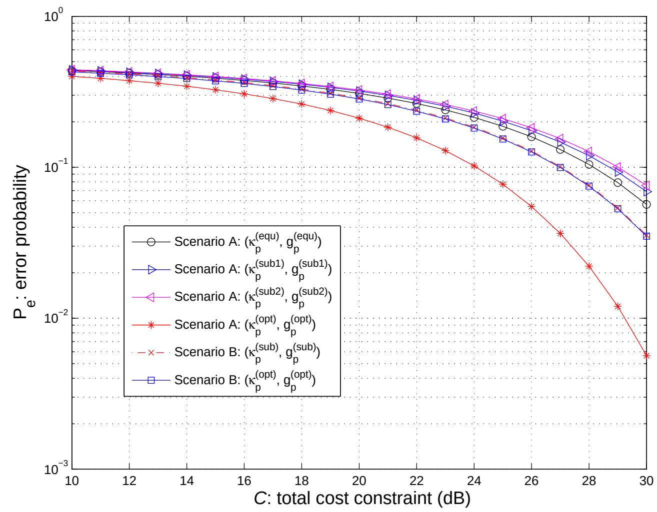

Fig. 4. Case III: error probability for different solutions of $(\boldsymbol{\kappa}, \boldsymbol{g})$. In Scenario B, we choose $\kappa_{\max} = 0.2\lfloor \bar{\mathcal{C}}/c_0 \rfloor$ and $\mathcal{P}_{\max} = 0.2\bar{\mathcal{C}}$.

V. Conclusions

In this paper, we present the optimal design for cooperative spectrum sensing in cognitive radio networks. In particular, we derive closed-form expressions for optimal solutions and propose a generalized water-filling algorithm when number of samples or amplifier gains are fixed and additional constraints are imposed. Furthermore, when jointly designing the number of samples and amplifier gains, we demonstrate that only one secondary user needs be active, i.e., collecting samples for local energy calculation and transmitting energy statistic to fusion center.

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