This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSAC.2016.2545418, IEEE Journal on Selected Areas in Communications

Delay Constrained Energy Harvesting Networks with Limited Energy and Data Storage

Burak Varan, *Student Member, IEEE,* and Aylin Yener, *Fellow, IEEE*

***Abstract*—This paper studies energy harvesting transmitters in the single user channel, the two-way channel, and the two-way relay channel with block fading. Each transmitter is equipped with a finite battery to store the harvested energy, and a finite buffer to store the data that arrive during the communication session. We consider delay sensitive applications and maximize throughput while enabling timely delivery of data with delay constraints. We show that the resulting delay limited throughput maximization problem can be solved using alternating maximization of two decoupled problems termed the energy scheduling problem and the data scheduling problem. We solve the energy scheduling problem using a modified directional waterfilling algorithm with right permeable taps, water pumps, and overflow bins and the data scheduling problem with forward induction. Additionally, we identify the online optimum policy for throughput maximization. We provide numerical results to verify our analytical findings and to demonstrate the impact of the finite data buffer capacity and the delay requirements on the throughput. We observe that larger buffer sizes become useful for more lenient delay requirements, and a data buffer size that is comparable to the throughput within one time slot accounts for the majority of the increase in throughput.**

***Index Terms*—Energy harvesting, finite energy storage, finite data storage, data delivery delay constraints, throughput maximization, two-way and two-way relay channels.**

I. Introduction

Energy harvesting wireless networks employ nodes which acquire their energy intermittently over the course of their operation [1]. The source of the harvested energy may be solar radiation, piezoelectric devices, RF signals, and other external sources [2], [3]. The intermittent nature of energy harvests requires careful scheduling of the available energy to ensure uninterrupted operation of the communication system. Physically, this entails storing the energy in a battery of finite size and drawing the energy for transmission in a manner to optimize the system performance while simultaneously ensuring energy is not wasted. The impact of energy intermittency and storage has been studied extensively in energy harvesting networks in recent years, see [1], [4]–[22] and many others. A standing assumption in most previous work is that the energy harvesting transmitters have an infinite capacity buffer to store their data until an opportune time arises for transmission. In practice, data may also be received intermittently and neither an infinite backlog of data nor an infinite capacity buffer may be available, for example in energy harvesting sensors [19]. Furthermore, quality of service constraints such as a maximum delay constraint also influence how transmission should be scheduled. These aspects necessitate and motivate the study of energy harvesting communications with these new added ingredients of limited data buffers and delay requirements, which is the focus of this paper.

Among the previous work in the area of energy harvesting communications, particularly related to this work from the perspective of modeling and analysis are references [1], [4], [5], [9], [16]. In [1], the transmission completion time minimization of an energy harvesting transmitter has been solved with intermittent data arrivals, and an infinite capacity battery and buffer at the transmitter. In [4], the throughput maximization problem has been solved for an energy harvesting transmitter with an infinite backlog of data, an infinite capacity buffer, and a finite capacity battery. In [5], directional waterfilling has been proposed and used to solve the throughput maximization problem for the single user fading channel and in [9], a generalized iterative directional waterfilling algorithm has been shown to solve the sum-throughput maximization in an interference channel. In [16], a framework for throughput maximization in a wireless network with energy harvesting transmitters and receivers, and energy storage limitations is proposed and the throughput maximization problem has been decoupled into energy efficiency and energy harvesting adaptation problems. All of these references assume knowledge of energy arrivals beforehand, the so-called offline scenario, which provides exact maximum throughput and thus a benchmark for performance as well as the comfort of using deterministic convex optimization methods.

Another line of work includes references [23]–[25] that have considered online scenarios where energy arrivals are known only causally and considered optimizing the expected long term throughput. In [23], the maximization of long term expected throughput of a sensor network with rechargeable batteries is studied and the optimal transmission policy that determines whether a message should be transmitted depending on the available energy in the battery and the reward that the transmission of the message brings is derived. In [24], the long term expected throughput maximization for an infinite battery energy harvesting broadcast channel is considered, and

Manuscript received July 31, 2015; revised December 28, 2015; accepted March 10, 2016. Date of publication June XX, 2016; date of current version June XX, 2016. This work was supported in part by the National Science Foundation Grants CNS 09-64364, CCF 14-22347, and CNS 15-26165. This work was presented in part at the Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA, November 2014, and at the IEEE Global Communications Conference Communication Theory Symposium, Austin, TX, December 2014. The conference presentations do not include the delay constraints in the formulations and are limited to offline policies.

The authors are with the department of Electrical Engineering, The Pennsylvania State University, University Park, PA 16802 USA (e-mail: varan@psu.edu; yener@ee.psu.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier:

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSAC.2016.2545418, IEEE Journal on Selected Areas in Communications

it has been shown that there exists an asymptotically optimal transmission policy that keeps the data queues stable. In [25], transmission policies that maximize the long term expected throughput or minimize the mean delay in the data queue are derived. Other previous work on online policies for throughput maximization includes [19], [26]–[28]. References [19], [27] have studied the long term average throughput maximization problem for an energy harvesting transmitter. These two works have considered a finite battery and a finite data buffer at the transmitter just like we will, but utilize queuing and network theoretic tools in order to identify asymptotically optimal and near optimal policies for long term expected throughput maximization.

In this work, we consider the short term throughput maximization problem[1] with finite energy and data buffer constraints, as well as data delivery delay constraints in both *offline* and *online* settings. We note that recent reference [29] has also considered finite data buffers, for *offline* transmission completion time minimization, which is the dual problem to the short term throughput maximization problem that we consider here. In particular, reference [29] has considered the special case where the transmitter is *required* to transmit all of the packets it receives. By contrast, we will not have this restriction and observe that, by allowing dropped packets, we can provide the system design insights for optimal operation. Furthermore, our solution methodology will extend to multi terminal networks.

We summarize the contributions of this paper as follows:

1. We solve the throughput maximization problem for energy harvesting networks in the presence of (i) energy storage constraints, (ii) data buffer constraints, and (iii) quality-of-service requirements in the form of delay constraints. We initially consider an offline setting for one energy harvesting transmitter with finite energy and data storage constraints. Our approach to solve this problem is by decomposing it into an *energy scheduling problem* and a *data scheduling problem*. Alternating maximization between these two smaller problems iteratively solves the delay limited throughput maximization problem.

2. We identify a directional waterfilling [5] interpretation for the solution of the energy scheduling problem. To do so, we add the new notions of *water pumps* and *overflow bins* to directional waterfilling. For the data scheduling problem, we show the optimality of a forward induction based solution.

3. Next, we extend the solution for the delay limited throughput maximization problem to multi terminal networks with energy harvesting. To do so, we study the energy harvesting two-way channel, the energy harvesting two-way relay channel.

4. In addition to analytically solving the offline optimization problems, we also study the online setup where energy and data arrivals are known causally. We identify various properties of the optimal online policy for simplification of the search space, and utilize dynamic programming to find the optimal policy.

[1]Short term throughput is defined as the amount of reliably communicated data in a communication session of duration T [5].

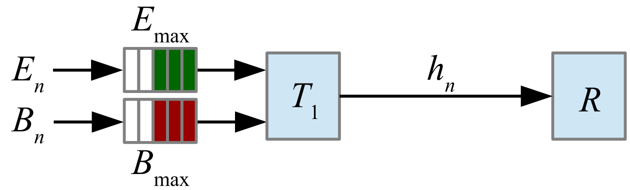

Fig. 1. The energy harvesting single user channel with a finite battery and a finite buffer at the transmitter.

We provide numerical results to assess the performance of the optimal policies, and how the data buffer capacity and the delay requirements impact the optimal throughput. We observe that data buffer sizes comparable to the throughput within one time slot on average are sufficient to obtain most of the optimal throughput and larger buffers bring in diminishing returns. In addition, stricter delay requirements result in a lower throughput, but require smaller data storage on average.

The remainder of this paper is organized as follows. In Section II, we describe the system model for the energy harvesting single user channel, and formulate the short term offline optimization problem. In Section III, we solve the problem using alternating maximization. In Section IV, we extend our results to the two-way channel. In Section V, we further extend our results to the two-way relay channel and comment on its special cases, i.e., the two-hop relay and the multiple access channels. In Section VI, we identify optimum online policies. In Section VII, we provide simulation results. In Section VIII, we conclude the paper.

II. System Model and Problem Statement

Consider an energy harvesting transmitter communicating with a receiver through a block fading channel[2]. The transmitter harvests energy and receives delay constrained data packets over the course of the communication session, which are stored in an on board battery and a data buffer. The battery and the data buffer have finite capacities $E_{\max}$ and $B_{\max}$, respectively; excess energy and data that cannot be stored are discarded. The model is depicted in Fig. 1. We will consider the two-way, two-way relay, and multiple access channels as extensions in Sections IV and V.

We consider communication with a deadline in a block fading channel. The communication session consists of N time slots of duration ℓ. We denote the fading coefficient in time slot n by h_n. Without loss of generality, we consider unit noise variance at the receiver. Transmitter T_1 allocates transmit power $p_n \geq 0$ in time slot n and discards $w_n \geq 0$ units of energy to avoid battery overflows[3]. The total number of bits communicated through the channel in time slot n is $\ell C(h_n p_n)$ where $C(x) = \frac{1}{2}\log(1+x)$. In time slot n, node T_1 harvests E_n units of energy, receives B_n units of data, and removes d_n units of data from its data buffer. We consider a delay

[2]As usual the additive noise at the receiver is assumed to be white Gaussian.

[3]We aim to find energy efficient transmission policies that maximize the throughput and spend the least amount of energy while doing so. We identify excess energy amounts w_n that cannot improve the throughput, but can be used for other purposes such as energy transfer [30].

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSAC.2016.2545418, IEEE Journal on Selected Areas in Communications

limited scenario where the data received in time slot n *expires* if not sent before the end of time slot $n+\tau$ as was in [29]. However, we do not require the successful transmission of all data packets before they expire, but instead allow node T_1 to drop some of its data packets if timely delivery of all packets is infeasible. The expired packets are immediately dropped by node T_1. d_n includes data transmitted to the receiver as well as data dropped due to limited buffer size, and data dropped due to unmet delay constraints.

Remark 1: Through d_n, the model allows node T_1 to drop some of the data in the buffer if future data arrivals are to cause an overflow. As a result, our approach proactively eliminates infeasibilities stemming from the finite buffer size, and allows us to identify jointly optimal transmission and packet dropping policies even if transmitting all data by the deadline is not feasible. This feature sets our approach apart from previous treatments. ■

The amount of data that is conveyed to the receiver in time slot n is the minimum of the amount of data scheduled to leave the buffer and the amount of data that can be transmitted by the allocated power, i.e., $\min\{\ell C(h_n p_n), d_n\}$. Therefore, the amount of data dropped due to buffer or delay constraints is given by $d_n - \min\{\ell C(h_n p_n), d_n\}$. We define our objective as the throughput of the system with a penalty $c \geq 0$ per dropped data unit, and formulate the *delay limited throughput maximization problem* with penalty c as[4]

$$\max_{\mathbf{p},\mathbf{w},\mathbf{d}\geq 0} \sum_{i=1}^{N} \min\{\ell C(h_i p_i), d_i\} - c\sum_{i=1}^{N}(d_i - \min\{\ell C(h_i p_i), d_i\}) \quad (1a)$$

$$\text{s.t.} \quad \sum_{i=1}^{n}(\ell p_i + w_i) \leq \sum_{i=1}^{n} E_i, \quad (1b)$$

$$\sum_{i=1}^{n} E_i - \sum_{i=1}^{n}(\ell p_i + w_i) \leq E_{\max}, \quad (1c)$$

$$\sum_{i=1}^{n} d_i \leq \sum_{i=1}^{n} B_i, \ \sum_{i=1}^{n} B_i - \sum_{i=1}^{n} d_i \leq B_{\max}, \quad (1d)$$

$$\sum_{i=1}^{n} d_i \geq \sum_{i=1}^{n-\tau} B_i, \ n = 1, 2, \ldots, N. \quad (1e)$$

Here, (1b) and (1c) represent the energy constraints and (1d) and (1e), the data constraints. Constraint (1b) is the energy causality constraint [1] which ensures that the total amount of energy consumed for transmission or discarded up to the end of time slot n is limited by the total amount of energy harvested by that time. Constraint (1c) is the battery capacity constraint [4] which ensures that the amount of energy stored in the battery is not greater than the battery capacity. The first constraint in (1d) is the data causality constraint which ensures that the data leaving the buffer has already arrived. The second constraint in (1d) is the buffer capacity constraint [29] which limits the amount of data stored in the buffer to the buffer capacity. Constraint (1e) is the delay constraint which ensures that no expired data remains in the buffer. We use bold face to denote vectors of decision variables, e.g., $\mathbf{p} = [p_1, \ldots, p_N]$. A finite c for the penalty models a data loss tolerant scenario, i.e., data loss is acceptable for the sake of the feasibility of the problem. Data loss is unacceptable with $c = \infty$ as in [29]. Since all constraints are linear and the objective is concave, (1) is a convex problem.

Remark 2: One can envision a delay limited communication scenario with heterogeneous data where different data packets may have different delay requirements. This would necessitate the addition of data classes into the model and the reformulation of (1) with a delay constraint for each class, similar to (1e). Since we would need to track packets violating individual data constraints, the resulting problem would be more involved and is left as future work. ■

Remark 3: The penalty model can be readily extended to any penalty function ζ that is convex and nondecreasing in the total data lost, i.e., the second term in (1a) can be replaced with $-\zeta(\sum_{i=1}^{N}(d_i - \min\{\ell C(h_i p_i), d_i\}))$. While our results are valid for any convex nondecreasing ζ, we focus on the case of $\zeta(x) = cx$ in order to (i) provide a clear presentation of our results and (ii) find closed form expressions for our solution and its proof of optimality. We can envision an even more general penalty function in a heterogeneous setting where the penalty for the loss of data depends on the class of data. In this work, we focus on homogeneous data where all packets are equally significant and leave the study of the heterogeneous setting as future work. ■

We next solve (1) for jointly optimal power and data allocation policies.

III. Throughput Maximization for the Single User Channel

In this section, we solve the delay limited throughput maximization problem by decoupling it into smaller problems which we next solve individually. We begin by noting that the feasible region of (1) is separable. Namely, the energy constraints (1b) and (1c) are functions of energy variables $\mathbf{p}$ and $\mathbf{w}$ only, and the data constraints (1d) and (1e) are functions of data variable $\mathbf{d}$ only. Thus, the feasible region of (1) is the Cartesian product of the set of all $(\mathbf{p}, \mathbf{w})$ satisfying (1b) and (1c) and the set of all $\mathbf{d}$ satisfying (1d) and (1e). Therefore, we can solve the convex program (1) using alternating maximization [31, §2.7]. That is, we start with an initial feasible $\mathbf{d}$, and solve (1) for $(\mathbf{p}, \mathbf{w})$. Given these energy variables, we next solve (1) for $\mathbf{d}$. By iterating between the energy and data variables in this fashion, we can solve for one while keeping the other constant, and monotonically converge[5] to a jointly optimal power and data allocation policy for the energy harvesting single user channel [31, §2.7], [32].

We next formulate the decoupled energy and data scheduling problems for (1) where we use a superscript in square brackets to denote the iteration index. For instance, $\mathbf{p}^{[m]}$ is

[4] As is generally the case in the literature, e.g., [1], [14], [29], we too assume that the data packet sizes are sufficiently small so that we can model data as a continuous variable.

[5] Convergence to a unique optimal policy can be facilitated by regularizing the objectives as was done in [9].

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSAC.2016.2545418, IEEE Journal on Selected Areas in Communications

the power vector $\mathbf{p}$ found in the mth iteration. Suppose $\mathbf{d}^{[0]}$ is an arbitrary initial solution that satisfies (1d) and (1e). In the mth iteration, we update the energy variables $(\mathbf{p}^{[m]}, \mathbf{w}^{[m]})$ by solving the *energy scheduling problem* given by

$$\arg\max_{(\mathbf{p},\mathbf{w})\geq 0} \sum_{i=1}^{N} \min\{\ell C(h_i p_i), d_i^{[m-1]}\} \tag{2a}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} (\ell p_i + w_i) \leq \sum_{i=1}^{n} E_i, \tag{2b}$$

$$\sum_{i=1}^{n} E_i - \sum_{i=1}^{n} (\ell p_i + w_i) \leq E_{\max}, \tag{2c}$$

for $n = 1, 2, \dots, N$ where we simplify the objective by removing $-c\sum_{i=1}^{N} d_i$, which depends only on $\mathbf{d}$, and dividing by $1 + c > 0$. In the same block iteration, we update the data variables $\mathbf{d}^{[m]}$ by solving the *data scheduling problem* given by

$$\arg\max_{\mathbf{d}\geq 0} \sum_{i=1}^{N} \min\{d_i, \ell C(h_i p_i^{[m]})\} - \tilde{c}\sum_{i=1}^{N} d_i \tag{3a}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} d_i \leq \sum_{i=1}^{n} B_i, \quad \sum_{i=1}^{n} B_i - \sum_{i=1}^{n} d_i \leq B_{\max}, \tag{3b}$$

$$\sum_{i=1}^{n} d_i \geq \sum_{i=1}^{n-\tau} B_i, \tag{3c}$$

for $n = 1, 2, \dots, N$ where $\tilde{c} = c/(1+c) \geq 0$. In what follows, we identify the solutions to the energy scheduling problem (2) and the data scheduling problem (3) based on waterfilling and forward induction, respectively.

A. Solution of the Energy Scheduling Problem

In order to solve (2), we derive an equivalent problem that shows the interaction between the two problems more explicitly and admits a *modified* directional waterfilling interpretation for its solution. We first note that we can leverage the invertibility and monotonicity of $C(\cdot)$ to rewrite the objective of (2) as $\ell\sum_{i=1}^{N} C(h_i \min\{p_i, P_i\})$ where we define $P_n \triangleq C^{-1}(d_n^{[m-1]}/\ell)/h_n$ for $n = 1, 2, \dots, N$. It is now clear that the solution of the data scheduling problem in the previous iteration introduces a maximum power for each time slot in the energy scheduling problem. Note that all P_n are constant and can be computed before solving the problem.

Lemma 1: Problem (2) is equivalent to

$$\arg\max_{(\mathbf{p},\mathbf{w})\geq 0} \sum_{i=1}^{N} C(h_i p_i) \tag{4a}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} (\ell p_i + w_i) \leq \sum_{i=1}^{n} E_i, \tag{4b}$$

$$\sum_{i=1}^{n} E_i - \sum_{i=1}^{n} (\ell p_i + w_i) \leq E_{\max}, \tag{4c}$$

$$p_n \leq P_n, \ n = 1, 2, \dots, N. \tag{4d}$$

Proof: Suppose $(\mathbf{p}, \mathbf{w})$ is a feasible policy (2) with $p_n > P_n$ for some n. Define another policy $(\tilde{\mathbf{p}}, \tilde{\mathbf{w}})$ by $\tilde{p}_i = p_i$ and $\tilde{w}_i = w_i$ for all $i \neq n$, $\tilde{p}_n = P_n$, and $\tilde{w}_n = w_n + \ell p_n - \ell P_n$. $(\tilde{\mathbf{p}}, \tilde{\mathbf{w}})$ is feasible as well since $\ell p_i + w_i = \ell\tilde{p}_i + \tilde{w}_i$ for all i, and attains the same objective as $(\mathbf{p}, \mathbf{w})$ does. Thus, at least one policy that solves (2) also satisfies $p_n \leq P_n$ for all n. ■

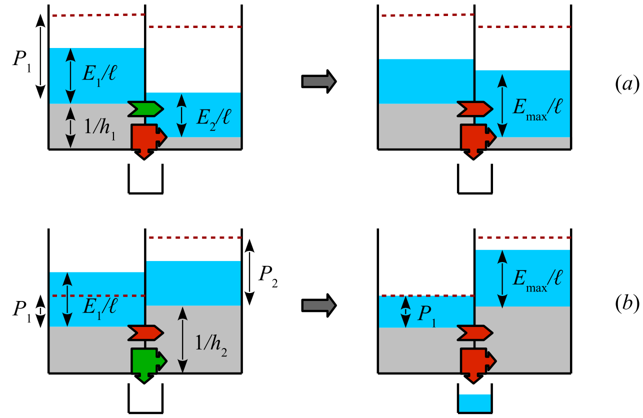

Fig. 2. Directional waterfilling with right permeable taps and pumps for (a) $p_n < P_n$ and (b) $p_n \geq P_n$. Initial allocations are shown on the left, and the optimal allocations are shown on the right. The elements between time slots are, from top to bottom, taps, water pumps, and overflow bins.

In order to obtain a waterfilling interpretation of the solution of (4), we derive the stationarity condition on p_n and w_n as

$$\left(p_n + \frac{1}{h_n}\right)^{-1} = \sum_{i=n}^{N} (\lambda_i - \mu_i) + \nu_n - \kappa_n, \tag{5}$$

$$\gamma_n = \sum_{i=n}^{N} (\lambda_i - \mu_i), \tag{6}$$

where λ_n and μ_n are the nonnegative dual variables associated with the energy causality and battery capacity constraints (4b) and (4c); and ν_n, κ_n, and γ_n are those associated with constraints $p_n \leq P_n$, $p_n \geq 0$, and $w_n \geq 0$, respectively.

The complementary slackness condition on $p_n \leq P_n$ is

$$\nu_n(p_n - P_n) = 0. \tag{7}$$

By (7), we must have $\nu_n = 0$ whenever $p_n < P_n$. In this case, there is sufficient data allocated by (3) in the previous iteration; thus, the solution is the same as the directional waterfilling solution for an infinite backlog of data and $B_{\max} = \infty$ found in [5]. That is, we model the time slots as rectangular bins (see Fig. 2) of width ℓ with base levels of height $1/h_n$, and model the energy arrivals as E_n units of water that are initially filled into the nth bin. The water level in each bin denotes the power allocated for the corresponding time slot. The taps between adjacent bins are right permeable, i.e., they allow water to flow only from left to right, and they turn off when the amount of water transferred to the next bin is $E_{\max}$ due to (1b) and (1c). An example of directional waterfilling with $p_n < P_n$ is shown in Fig. 2(a), where the initial water level is higher in the first time slot. This results in water flow to the next time slot through the directional tap. Once the battery is full in the second time slot, the tap turns off and does not allow any water flow even though the water level in

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSAC.2016.2545418, IEEE Journal on Selected Areas in Communications

the first time slot is still higher.

On the other hand, if $\nu_n > 0$, by (7) we must have $p_n = P_n$. Notice that p_n is decreasing in ν_n in (5). That is, a positive ν_n results in a decrease in p_n until $p_n = P_n$. No more than P_n units of power are allowed in each time slot, i.e., the data allocated by (3) in the previous iteration results in a maximum power of P_n for each time slot. We interpret this phenomenon by introducing *water pumps* and *overflow bins* to directional waterfilling. The water pump for the nth time slot is inactive as long as $p_n \leq P_n$. However, if this constraint is violated by the initial water levels, or the operation of the right permeable taps, then the water pump is activated. The water pump is responsible for bringing the water level down to P_n by pumping water first to the next time slot until total water flow reaches $E_{\max}$, after which it pumps water into the overflow bin. Here, the water in overflow bin represents w_n, i.e., discarded or wasted energy. An example is shown in Fig. 2(b), where the initial water level is higher in the second time slot, so the tap is off. However, since $p_n > P_n$, i.e., there is not enough data in the first time slot to utilize the transmit power to its full extent, the pump is activated. Water is pumped to the second time slot until the battery is full, at which point the excess water is discarded into the overflow bin.

Theorem 1: The directional waterfilling algorithm with water pumps and overflow bins outputs the unique optimal policy for (4).

Proof: Problem (4) is convex with a strictly concave objective. Thus, its Karush-Kuhn-Tucker (KKT) conditions, given by (5) and (6), and the respective complementary slackness and dual feasibility conditions, are satisfied by only one policy which is necessarily the unique solution to (4). Let p_n, w_n, and t_n denote the transmit power, the amount of discarded energy, and the amount of energy pumped to the next bin in the nth time slot found by the waterfilling solution. If $P_n = 0$ for some n, then $\nu_n \geq 0$ and $\kappa_n \geq 0$ can be freely chosen to satisfy (5) and (6). Hence, without loss of generality, we consider $P_n > 0$ for all n. If $p_n = 0$ for some n, then h_n must be too low for efficient transmission. $\kappa_n \geq 0$ is free and we have $\nu_n = 0$. We can set

$$\gamma_n = \begin{cases} \gamma_{n-1}, & \text{if } p_{n-1} + \frac{1}{h_{n-1}} \leq p_{n+1} + \frac{1}{h_{n+1}}, \\ \gamma_{n+1}, & \text{if } p_{n-1} + \frac{1}{h_{n-1}} > p_{n+1} + \frac{1}{h_{n+1}}, \end{cases} \tag{8}$$

and the KKT conditions are satisfied by $\kappa_n = \max\{\gamma_n - h_n, 0\}$.

Now suppose without loss of generality that $P_n > 0$ and $p_n > 0$ for all n. We set the dual variables as

$$\nu_n = \begin{cases} t_n, & \text{if } w_n = 0, \\ \left(p_n + \frac{1}{h_n}\right)^{-1}, & \text{if } w_n > 0, \end{cases} \tag{9}$$

$$\gamma_n = \begin{cases} \gamma_{n+1}, & \text{if } 0 < e_n < E_{\max}, \\ \left(p_n + \frac{1}{h_n}\right)^{-1}, & \text{if } e_n = 0, \\ \left(p_n + \frac{1}{h_n}\right)^{-1} - \nu_n, & \text{if } e_n = E_{\max} \text{ and } w_n = 0, \\ 0, & \text{if } w_n > 0, \end{cases} \tag{10}$$

$$\lambda_n = \max\{\gamma_n - \gamma_{n+1}, 0\}, \tag{11}$$

$$\mu_n = \max\{\gamma_{n+1} - \gamma_n, 0\}, \tag{12}$$

$$\kappa_n = 0, \tag{13}$$

for $n = 1, 2, \ldots, N$, where $\gamma_{N+1} = 0$ and $e_n = \sum_{i=1}^{n}(E_i - l_i p_i - w_i)$. The stationarity condition in (6) and the non-negativity of λ_n, μ_n, ν_n, and κ_n are trivially satisfied. The stationarity condition in (5), the complementary slackness conditions, and the nonnegativity of γ_n can be shown to hold by induction using (10) and the water flow dynamics outlined by the directional waterfilling algorithm. ■

Having identified all optimal policies for the energy scheduling problem, we next solve the data scheduling problem.

B. Solution of the Data Scheduling Problem

The data scheduling problem determines the optimal $\mathbf{d}$, i.e., how much data departs the data buffer in each time slot. If the transmit power scheduled by (2) does not suffice to transmit d_n in its entirety for some n, the remaining data is dropped. As such, $p_n^{[m]}$ defines a maximum throughput $D_n \triangleq \ell C(h_n p_n^{[m]})$ for each time slot. Problem (3) does not have a strictly concave objective, and thus may have multiple solutions. By utilizing the linearity of the objective for $d_n \leq D_n$, we characterize the optimal policy that transmits packets as soon as possible and uses the buffer in a first-in-first-out fashion. That is, when the policy schedules d_n units of data to depart the buffer, the oldest d_n units of data are transmitted or dropped.

When $\tau = 0$, the data constraints (1d) and (1e) become a single equality constraint, and the only feasible, and hence optimal scheduling is $d_n = B_n$ for $n = 1, \ldots, N$. For $\tau > 0$, we express the solution recursively starting from the first time slot, and show its optimality by induction.

Theorem 2: Problem (3) admits an optimal policy $\mathbf{d} = [d_1, \ldots, d_N]$ which is given by

$$d_n = b_{n-1} - b_n + B_n, \quad n = 1, \ldots, N. \tag{14}$$

Here, b_n is the amount of data in the buffer at the end of time slot n, which evolves as

$$b_n = \max\left\{0, \min\left\{B_{\max}, b_{n-1} + B_n - D_n - d_n^{\text{exp}}\right\}\right\} \tag{15}$$

for $n = 1, \ldots, N$ where $b_0 = 0$, and

$$d_n^{\text{exp}} = \begin{cases} 0, & n = 1, \ldots, \tau, \\ \max\left\{0, b_{n-1} - \sum_{i=n-\tau}^{n-1} B_i - D_n\right\}, & \\ & n = \tau + 1, \ldots, N, \end{cases} \tag{16}$$

is the amount of data that will expire at the end of time slot n and needs to be dropped.

Proof: Suppose $\tilde{\mathbf{d}}$ is a policy that differs from $\mathbf{d}$ in the first time slot. We will consider three cases, namely (i) $B_1 - D_1 \leq 0$, (ii) $0 < B_1 - D_1 \leq B_{\max}$, and (iii) $B_1 - D_1 > B_{\max}$. In cases (i) and (ii), $\tilde{d}_1 < d_1$ leaves additional data in the buffer which could have been transmitted in the first time slot, whereas in case (iii), $\tilde{d}_1 < d_1$ violates (1d). On the other hand, in case (i), $\tilde{d}_1 > d_1$ is infeasible, and in cases (ii) and (iii), $\tilde{d}_1 > d_1$ drops additional data that could have been transmitted in future time slots. Hence, $\tilde{\mathbf{d}}$ cannot outperform $\mathbf{d}$ in the first

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSAC.2016.2545418, IEEE Journal on Selected Areas in Communications

Algorithm 1 Throughput maximization algorithm.

Set $m = 0$.
Initialize $\mathbf{d}^{[0]} = [B_1, B_2, \dots, B_N]$.
do
  Update $m := m + 1$.
  Initialize $\mathbf{p}^{[m]} = [E_1, E_2, \dots, E_N]/\ell$ and $\mathbf{w}^{[m]} = \mathbf{0}$.
  Find $P_n = C^{-1}(d_n^{[m-1]}/\ell)/h_n$ for $n = 1, 2, \dots, N$.
  Run the waterfilling algorithm to update $\mathbf{p}^{[m]}$ and $\mathbf{w}^{[m]}$.
  if $\tau = 0$ **then**
    Set $\mathbf{d}^{[m]} = \mathbf{d}^{[0]}$.
    continue
  end if
  Find $D_n = \ell C(h_n p_n^{[m]})$ for $n = 1, 2, \dots, N$.
  Set $b_0 = 0$.
  for $n = 1, 2, \dots, N$ **do**
    Find $d_n^{\exp}$ using (16).
    Set $b_n = \max\{0, \min\{B_{\max}, b_{n-1} + B_n - D_n - d_n^{\exp}\}\}$.
    Set $d_n^{[m]} = b_{n-1} - b_n + B_n$.
  end for
while $(\mathbf{p}^{[m]}, \mathbf{w}^{[m]}, \mathbf{d}^{[m]}) \neq (\mathbf{p}^{[m-1]}, \mathbf{w}^{[m-1]}, \mathbf{d}^{[m-1]})$
return $(\mathbf{p}^{[m]}, \mathbf{w}^{[m]}, \mathbf{d}^{[m]})$

time slot, i.e., no other policy can transmit more data or drop less data in the first time slot.

Now suppose no policy outperforms $\mathbf{d}$ in the first $n-1$ time slots, and $\tilde{\mathbf{d}}$ is a policy with $\tilde{d}_i = d_i$ for $i = 1, \dots, n-1$. Consider the cases (i) $b_{n-1} + B_n - D_n \leq 0$, (ii) $0 < b_{n-1} + B_n - D_n \leq B_{\max} + d_n^{\exp}$, and (iii) $B_1 - D_1 > B_{\max}$ in time slot n. In cases (i) and (ii), $\tilde{d}_n < d_n$ leaves additional data in the buffer which could have been transmitted in the first time slot, whereas in case (iii), $\tilde{d}_n < d_n$ violates (1d). On the other hand, in case (i), $\tilde{d}_n > d_n$ is infeasible, and in cases (ii) and (iii), $\tilde{d}_n > d_n$ drops additional data that could have been transmitted in future time slots. Hence, $\tilde{\mathbf{d}}$ cannot outperform $\mathbf{d}$ in time slot n, and by induction, $\mathbf{d}$ cannot be outperformed by any other feasible data schedule. ∎

We note that when there is sufficient transmit power scheduled by (2), delaying the transmission of some packets by leaving more data in the buffer could result in the same objective, assuming the delayed transmission is still feasible. However, we design $\mathbf{d}$ to maximize the transmitted data and minimize the dropped data on a slot-by-slot basis, assuming optimality for the past time slots. In doing so, we identify the optimal policy that never leaves packets for future time slots unless it is necessary to leave them due to D_n. Although there may be multiple solutions to (3), $\mathbf{d}$ is unique in the sense that it also minimizes individual packet delays. This completes the solution of the data scheduling problem.

We present the solution of the delay limited throughput maximization problem for the single user channel in Algorithm 1. We next extend our findings to multi terminal networks.

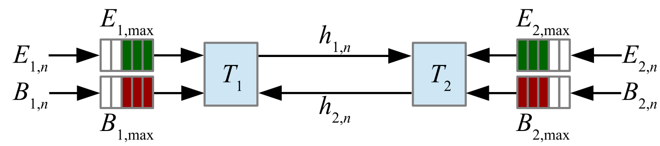

Fig. 3. The energy harvesting two-way channel with a finite battery and a finite buffer at the transmitters.

IV. Throughput Maximization for the Two-Way Channel

A. System Model and Problem Statement

We next consider a block fading two-way channel with energy harvesting transmitters T_1 and T_2 as in Fig. 3. Both transmitters receive data intermittently to send to each other which they store in their respective data buffers. Node T_j employs a finite battery of capacity $E_{j,\max}$ and a finite data buffer of capacity $B_{j,\max}$, $j = 1, 2$. We consider delay constrained communication over N time slots of duration ℓ, and block fading with fading coefficients $h_{1,n}$ from T_1 to T_2, and $h_{2,n}$ from T_2 to T_1 in time slot n. Without loss of generality, both nodes have unit variance noise.

We consider half duplex nodes and let $\Delta_n \in [0, 1]$ denote the fraction of the nth time slot that is reserved for T_1's transmission. Thus, node T_1 transmits for $\ell\Delta_n$ seconds and node T_2 transmits for $\ell(1-\Delta_n)$ seconds in time slot n. Let $p_{j,n}$ denote the average transmit power chosen by T_j in time slot n, averaged over the entire time slot. The total amount of data that node T_1 can transmit is at most $\ell\Delta_n C(h_{1,n}p_{1,n}/\Delta_n)$. Likewise, the amount of data that node T_2 can transmit is at most $\ell(1-\Delta_n)C(h_{2,n}p_{2,n}/(1-\Delta_n))$.

In time slot n, T_j harvests $E_{j,n}$ units of energy, receives $B_{j,n}$ units of data, schedules transmit power $p_{j,n}$, discards $w_{j,n}$ units of energy, and pulls $d_{j,n}$ units of data from its buffer. We denote by τ_j the maximum delay for node T_j's data. That is, packets arriving at T_j must be transmitted or dropped within τ_j time slots of their arrival. The transmission of expired packets does not contribute to the sum-throughput, thus the expired packets are immediately dropped at both nodes.

For this model, we aim to find optimal transmission policies that maximize the sum-throughput of the network while satisfying delay requirements, and simultaneously minimize the lost data at both nodes. Let $\mathcal{T}_j(p_{j,n}, d_{j,n}, \Delta_n)$ denote the amount of data that node T_j can send in time slot n. We have

$$\mathcal{T}_1(p_{1,n}, d_{1,n}, \Delta_n) = \min\left\{\ell\Delta_n C\left(\frac{h_{1,n}p_{1,n}}{\Delta_n}\right), d_{1,n}\right\}, \quad (17)$$

$$\mathcal{T}_2(p_{2,n}, d_{2,n}, \Delta_n) = \min\left\{\ell(1-\Delta_n) C\left(\frac{h_{2,n}p_{2,n}}{1-\Delta_n}\right), d_{2,n}\right\}. \quad (18)$$

We formulate the *delay limited throughput maximization problem* with penalty c for the two-way channel as

$$\max_{\substack{\mathbf{p},\mathbf{w},\mathbf{d}\geq 0,\\ 0\leq\mathbf{\Delta}\leq 1}} \sum_{j=1}^{2}\sum_{i=1}^{N}[\mathcal{T}_j(p_{j,i}, d_{j,i}, \Delta_i) - c(d_{j,i} - \mathcal{T}_j(p_{j,i}, d_{j,i}, \Delta_i))] \quad (19a)$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSAC.2016.2545418, IEEE Journal on Selected Areas in Communications

$$\text{s.t.} \quad \sum_{i=1}^{n}(\ell p_{j,i}+w_{j,i}) \le \sum_{i=1}^{n} E_{j,i}, \tag{19b}$$

$$\sum_{i=1}^{n} E_{j,i} - \sum_{i=1}^{n}(\ell p_{j,i}+w_{j,i}) \le E_{j,\max}, \tag{19c}$$

$$\sum_{i=1}^{n} d_{j,i} \le \sum_{i=1}^{n} B_{j,i}, \tag{19d}$$

$$\sum_{i=1}^{n} B_{j,i} - \sum_{i=1}^{n} d_{j,i} \le B_{j,\max}, \tag{19e}$$

$$\sum_{i=1}^{n} d_{j,i} \ge \sum_{i=1}^{n-\tau_j} B_{j,i}, \tag{19f}$$

$$j=1,2,\ n=1,2,\ldots,N, \tag{19g}$$

where we use the bold face notation to denote tuples of decision variables across all users over all time slots, e.g., $\mathbf{p}=[p_{1,1},\ldots,p_{1,N},p_{2,1},\ldots,p_{2,N}]$, and $\boldsymbol{\Delta}=[\Delta_1,\ldots,\Delta_N]$. Note that (19b)–(19f) are the multi user extensions of (1b)–(1e). We next solve (19) for jointly optimal transmission policies.

B. Solution of the Delay Limited Throughput Maximization Problem

The feasible region of (19) is separable between block variable $(\mathbf{p},\mathbf{w},\boldsymbol{\Delta})$ and data variable $\mathbf{d}$, admitting a solution using alternating maximization. Starting with an arbitrary feasible $\mathbf{d}^{[0]}$, we decompose (19) into an *energy scheduling problem* where we find $(\mathbf{p}^{[m]},\mathbf{w}^{[m]},\boldsymbol{\Delta}^{[m]})$ by solving

$$\underset{(\mathbf{p},\mathbf{w},\boldsymbol{\Delta})\ge 0}{\arg\max} \quad \sum_{j=1}^{2}\sum_{i=1}^{N} \mathcal{T}_j(p_{j,i}, d_{j,i}^{[m-1]}, \Delta_i) \tag{20a}$$

$$\text{s.t.} \quad \sum_{i=1}^{n}(\ell p_{j,i}+w_{j,i}) \le \sum_{i=1}^{n} E_{j,i}, \tag{20b}$$

$$\sum_{i=1}^{n} E_{j,i} - \sum_{i=1}^{n}(\ell p_{j,i}+w_{j,i}) \le E_{j,\max}, \tag{20c}$$

$$\boldsymbol{\Delta} \le 1,\ j=1,2,\ n=1,2,\ldots,N, \tag{20d}$$

and a *data scheduling problem* where we find $\mathbf{d}^{[m]}$ by solving

$$\underset{\mathbf{d}\ge 0}{\arg\max} \quad \sum_{j=1}^{2}\sum_{i=1}^{N}[\mathcal{T}_j(p_{j,i}^{[m]}, d_{j,i}, \Delta_i^{[m]}) - \tilde{c} d_{j,i}] \tag{21a}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} d_{j,i} \le \sum_{i=1}^{n} B_{j,i}, \tag{21b}$$

$$\sum_{i=1}^{n} B_{j,i} - \sum_{i=1}^{n} d_{j,i} \le B_{j,\max}, \tag{21c}$$

$$\sum_{i=1}^{n} d_{j,i} \ge \sum_{i=1}^{n-\tau_j} B_{j,i}, \tag{21d}$$

$$j=1,2,\ n=1,2,\ldots,N. \tag{21e}$$

We start with the energy scheduling problem (20). We first observe that the Δ_n variables are not constrained by the energy constraints (20b) and (20c), and they are not coupled, i.e., we can optimize Δ_n for each time slot given $(\mathbf{p},\mathbf{w})$. We rewrite the energy scheduling problem as

$$\underset{(\mathbf{p},\mathbf{w})\ge 0}{\arg\max} \quad \sum_{j=1}^{2}\sum_{i=1}^{N} \mathcal{T}_j(p_{j,i}, d_{j,i}^{[m-1]}, \Delta_i^*) \tag{22a}$$

$$\text{s.t.} \quad \sum_{i=1}^{n}(\ell p_{j,i}+w_{j,i}) \le \sum_{i=1}^{n} E_{j,i}, \tag{22b}$$

$$\sum_{i=1}^{n} E_{j,i} - \sum_{i=1}^{n}(\ell p_{j,i}+w_{j,i}) \le E_{j,\max}, \tag{22c}$$

for $j=1,2$ and $n=1,2,\ldots,N$ where Δ_n^* is found as

$$\underset{0\le\Delta_n\le 1}{\arg\max}\ [\mathcal{T}_1(p_{1,n}, d_{1,n}^{[m-1]}, \Delta_n) + \mathcal{T}_2(p_{2,n}, d_{2,n}^{[m-1]}, \Delta_n)] \tag{23}$$

for $n=1,2,\ldots,N$. In other words, Δ_n^* is the optimal time allocation for the two nodes' transmission given their transmit powers and data available for transmission. Suppose that the data scheduling problem allocates sufficiently large amounts of data in the previous time slot, i.e., we have $d_{1,n}^{[m-1]} = d_{2,n}^{[m-1]} = \infty$. In this case, we can use the concavity of $C(\cdot)$ to bound the objective of (23) as

$$\mathcal{T}_1(p_{1,n}, d_{1,n}^{[m-1]}, \Delta_n) + \mathcal{T}_2(p_{2,n}, d_{2,n}^{[m-1]}, \Delta_n)$$
$$= \ell\Delta_n C\left(h_{1,n}\frac{p_{1,n}}{\Delta_n}\right) + \ell(1-\Delta_n) C\left(h_{2,n}\frac{p_{2,n}}{1-\Delta_n}\right) \tag{24a}$$
$$\le \ell C(h_{1,n}p_{1,n} + h_{2,n}p_{2,n}) \tag{24b}$$

where (24b) is an equality when $\Delta_n = \frac{h_{1,n}p_{1,n}}{h_{1,n}p_{1,n}+h_{2,n}p_{2,n}}$. However, the data scheduling problem may not necessarily schedule as much data as can be transmitted for this choice of Δ_n. Thus, the data scheduling problem results in upper and lower bounds on Δ_n, i.e., the greatest $\Delta_n = \overline{\Delta_n}$ and the smallest $\Delta_n = \underline{\Delta_n}$ such that nodes T_1 and T_2 are given just enough time to transmit their available data, respectively. These limits are found as the solutions to

$$\overline{\Delta_n}\ell C\left(h_{1,n}\frac{p_{1,n}}{\overline{\Delta_n}}\right) = d_{1,n}^{[m-1]}, \tag{25}$$

$$(1-\underline{\Delta_n})\ell C\left(h_{2,n}\frac{p_{2,n}}{1-\underline{\Delta_n}}\right) = d_{2,n}^{[m-1]}, \tag{26}$$

which are nonlinear equations with unique solutions that can be obtained using the bisection method [33, §2.1] or the Lambert W function [34]. The optimal time allocation Δ_n^* for epoch n is then found as

$$\Delta_n^* = \begin{cases} \min\left\{\max\left\{\frac{h_{1,n}p_{1,n}}{h_{1,n}p_{1,n}+h_{2,n}p_{2,n}}, \underline{\Delta_n}\right\}, \overline{\Delta_n}\right\}, & \text{if } \overline{\Delta_n} \ge \underline{\Delta_n}, \\ \frac{1}{2}(\overline{\Delta_n}+\underline{\Delta_n}), & \text{if } \overline{\Delta_n} < \underline{\Delta_n}, \end{cases} \tag{27}$$

for $n=1,2,\ldots,N$. Note that in the second case with $\overline{\Delta_n} < \underline{\Delta_n}$, any Δ_n such that $\overline{\Delta_n} \le \Delta_n \le \underline{\Delta_n}$ is optimal and both nodes have enough time to transmit all their data.

Having found the optimal $\boldsymbol{\Delta}$ given any $(\mathbf{p},\mathbf{w})$, we go back to solving (22). Note that by expressing $\boldsymbol{\Delta}$ in terms

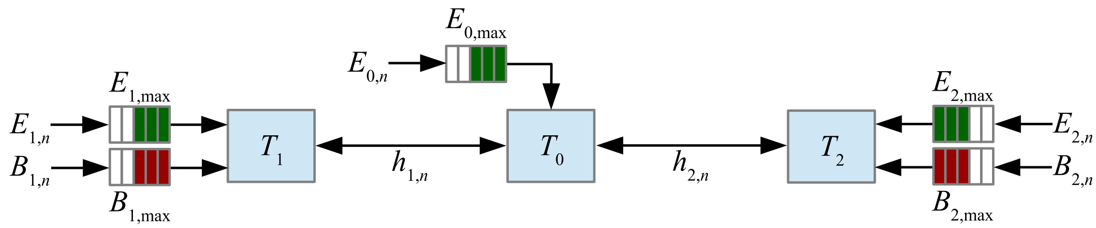

Fig. 4. The energy harvesting two-way relay channel with a finite battery at all nodes and a finite buffer at the transmitters.

of $\mathbf{p}$ and $\mathbf{w}$, we have decoupled the energy scheduling problem between nodes T_1 and T_2. Therefore, we can further separate (22) between the two transmitters and identify $\mathbf{p}_j^{[m]} = [p_{j,1}^{[m]}, \dots, p_{j,N}^{[m]}]$ and $\mathbf{w}_j^{[m]} = [w_{j,1}^{[m]}, \dots, w_{j,N}^{[m]}]$ for each $j = 1, 2$. This approach decomposes (22) into two single user energy scheduling problems which are equivalent to (2) for each node. Thus, the waterfilling solution found in Section III-A applies immediately in each iteration. The data scheduling problem (21) can be reduced to single user data scheduling problems as well. In order to accomplish this, we first compute $\Delta_n^{[m]}$ using (27). We next solve (21) in $\mathbf{d}_j^{[m]} = [d_{j,1}^{[m]}, \dots, d_{j,N}^{[m]}]$ for each $j = 1, 2$ separately, noting that the objective and the constraints are separable between the two transmitters. The optimization of each $\mathbf{d}_j^{[m]}$ is equivalent to (3), and therefore the solution in Section III-B applies. This concludes the solution of (19).

V. Throughput Maximization for the Two-Way Relay Channel

A. System Model and Problem Statement

We next consider a more general channel model, namely the block fading two-way relay channel, for delay limited throughput maximization. This channel consists of three transmitters, namely two energy harvesting source nodes T_1 and T_2 and an energy harvesting relay T_0, as shown in Fig. 4. Node T_j, $j = 0, 1, 2$, employs a finite battery of capacity $E_{j,\max}$ to store its harvested energy. The source nodes receive data intermittently over the course of the communication session. The two source nodes cannot communicate directly, requiring relay T_0 for their communication. Source node T_k, $k = 1, 2$, employs a finite data buffer of capacity $B_{k,\max}$, whereas the relay node T_0 does not employ a data buffer and must forward received packets immediately. The communication session is composed of N time slots of duration ℓ. Without loss of generality, we consider channel reciprocity, i.e., the block fading coefficient is $h_{k,n}$ from T_k to T_0 and from T_0 to T_k, $k = 1, 2$, in time slot n. All nodes experience unit variance noise.

We focus on half duplex nodes and multiple access-broadcast decode-and-forward relaying [35]. That is, each time slot is divided into two phases: (i) the multiple access phase in which the two transmitters send their data to the relay simultaneously, and the relay decodes all incoming data, and (ii) the broadcast phase in which the relay forwards nodes T_1 and T_2's data simultaneously, utilizing each transmitter's knowledge of its own data, see [35] for details on the relaying scheme. We denote by $\Delta_n \in [0, 1]$ the fraction of the nth time slot that is reserved for the multiple access phase, and by Δ_n the fraction for the broadcast phase.

For $j = 0, 1, 2$, let $E_{j,n}$ denote the amount of energy harvested by T_j, $w_{j,n}$ the amount of energy discarded by T_j in time slot n, and $p_{j,n}$ the average transmit power of T_j averaged over the entire time slot. Likewise, for $k = 1, 2$, let $B_{k,n}$ denote the amount of data that transmitter T_k receives, $d_{k,n}$ the amount of data that transmitter T_k removes from its buffer in time slot n, and τ_k the maximum number of time slots for which the delivery of the packets arriving at T_k can be delayed. Similar to the previous models, all packets must depart the buffers before they expire, and if they do expire, their transmission does not contribute to the sum-throughput.

The amount of data each transmitter can send in each time slot depends on the transmit powers for all nodes, Δ_n, and the data available at the transmitters, i.e., the amount of data they have pulled from their buffers. In order to express the sum-throughput for the two-way relay channel, we define [35]

$$g_1(p_{0,n}, p_{1,n}, d_{1,n}, \Delta_n) = \min \left\{ \ell \Delta_n C\left(\frac{h_{1,n}p_{1,n}}{\Delta_n}\right), \right.$$
$$\left. \ell(1-\Delta_n) C\left(\frac{h_{2,n}p_{0,n}}{1-\Delta_n}\right), d_{1,n} \right\}, \quad \text{(28a)}$$
$$g_2(p_{0,n}, p_{2,n}, d_{2,n}, \Delta_n) = \min \left\{ \ell \Delta_n C\left(\frac{h_{2,n}p_{2,n}}{\Delta_n}\right), \right.$$
$$\left. \ell(1-\Delta_n) C\left(\frac{h_{1,n}p_{0,n}}{1-\Delta_n}\right), d_{2,n} \right\}, \quad \text{(28b)}$$
$$g_{\text{sum}}(p_{1,n}, p_{2,n}, \Delta_n) = \ell \Delta_n C\left(\frac{h_{1,n}p_{1,n} + h_{2,n}p_{2,n}}{\Delta_n}\right). \quad \text{(28c)}$$

Here, $g_k(p_{0,n}, p_{1,n}, p_{2,n}, \Delta_n)$ constrains the amount of data that node T_k can send in time slot n for $k = 1, 2$, and models three factors that determine this amount: the individual rate constraint for the multiple access phase for T_k, the individual rate constraint for the broadcast phase for T_k, and the data availability at node T_k. Due to the sum rate constraint for the multiple access phase, we need to also define $g_{\text{sum}}(p_{0,n}, p_{1,n}, p_{2,n}, \Delta_n)$ which constrains the total amount of data that the two transmitters can send in time slot n. Since we are interested in the sum-throughput for each epoch, we define

$$\mathcal{T}(\{p_{j,n}\}, \{d_{k,n}\}, \Delta_n) = \min\{g_1(p_{0,n}, p_{1,n}, d_{1,n}, \Delta_n)$$
$$+ g_2(p_{0,n}, p_{2,n}, d_{2,n}, \Delta_n), g_{\text{sum}}(p_{1,n}, p_{2,n}, \Delta_n)\}, \quad (29)$$

where $\{p_{j,n}\} = (p_{0,n}, p_{1,n}, p_{2,n})$ and $\{d_{k,n}\} = (d_{1,n}, d_{2,n})$.

Here, $\mathcal{T}(\{p_{j,n}\},\{d_{k,n}\},\Delta_n)$ is the sum-throughput for time slot n given a choice of transmit powers, data availability, and Δ_n.

For the channel model and the communication scheme described above, we will identify optimal energy and data allocation policies that maximize the sum-throughput of the network, minimize lost data, and satisfy delay requirements for both transmitters' data. We formulate the *delay limited throughput maximization problem* with penalty c for the two-way relay channel as

$$\max_{\substack{\mathbf{p},\mathbf{w},\mathbf{d}\geq 0,\\ 0\leq\mathbf{\Delta}\leq 1}} \sum_{i=1}^{N}\mathcal{T}(\{p_{j,n}\},\{d_{k,n}\},\Delta_n) - c\sum_{i=1}^{N}(d_{1,i}+d_{2,i}-\mathcal{T}(\{p_{j,n}\},\{d_{k,n}\},\Delta_n)) \quad (30a)$$

$$\text{s.t.} \quad \sum_{i=1}^{n}(\ell p_{j,i}+w_{j,i}) \leq \sum_{i=1}^{n}E_{j,i}, \quad (30b)$$

$$\sum_{i=1}^{n}E_{j,i} - \sum_{i=1}^{n}(\ell p_{j,i}+w_{j,i}) \leq E_{j,\max}, \quad (30c)$$

$$\sum_{i=1}^{n}d_{k,i} \leq \sum_{i=1}^{n}B_{k,i}, \quad (30d)$$

$$\sum_{i=1}^{n}B_{k,i} - \sum_{i=1}^{n}d_{k,i} \leq B_{k,\max}, \quad (30e)$$

$$\sum_{i=1}^{n}d_{k,i} \geq \sum_{i=1}^{n-\tau_k}B_{k,i}, \quad (30f)$$

$$j=0,1,2,\ k=1,2,\ n=1,2,\ldots,N, \quad (30g)$$

where $\mathbf{p} = [p_{0,1},\ldots,p_{0,N},p_{1,1},\ldots,p_{1,N},p_{2,1},\ldots,p_{2,N}]$, $\mathbf{w} = [w_{0,1},\ldots,w_{1,1},\ldots,w_{2,1},\ldots,w_{2,N}]$, $\mathbf{d} = [d_{1,1},\ldots, d_{2,1},\ldots,d_{2,N}]$, and $\mathbf{\Delta} = [\Delta_1,\ldots,\Delta_N]$. We next solve (30) for jointly optimal energy and data allocation policies.

B. Solution of the Delay Limited Throughput Maximization Problem

We solve (30) by alternating maximization, noting that it can be separated into an energy scheduling problem over $(\mathbf{p},\mathbf{w},\mathbf{\Delta})$ and a data scheduling problem over $\mathbf{d}$. We start with a feasible initial $\mathbf{d}^{[0]}$, and update the solution in the mth iteration $(\mathbf{p}^{[m]},\mathbf{w}^{[m]},\mathbf{\Delta}^{[m]})$ by solving the *energy scheduling problem* given by

$$\arg\max_{(\mathbf{p},\mathbf{w},\mathbf{\Delta})\geq 0} \sum_{i=1}^{N}\mathcal{T}(\{p_{j,n}\},\{d_{k,n}^{[m-1]}\},\Delta_i) \quad (31a)$$

$$\text{s.t.} \quad \sum_{i=1}^{n}(\ell p_{j,i}+w_{j,i}) \leq \sum_{i=1}^{n}E_{j,i}, \quad (31b)$$

$$\sum_{i=1}^{n}E_{j,i} - \sum_{i=1}^{n}(\ell p_{j,i}+w_{j,i}) \leq E_{j,\max}, \quad (31c)$$

$$\mathbf{\Delta}\leq 1,\ j=0,1,2,\ n=1,2,\ldots,N, \quad (31d)$$

and $\mathbf{d}^{[m]}$ by solving the *data scheduling problem* given by

$$\arg\max_{\mathbf{d}\geq 0} \sum_{i=1}^{N}[\mathcal{T}(\{p_{j,n}^{[m]}\},\{d_{k,n}\},\Delta_i^{[m]}) - \tilde{c}(d_{1,i}+d_{2,i})] \quad (32a)$$

$$\text{s.t.} \quad \sum_{i=1}^{n}d_{k,i} \leq \sum_{i=1}^{n}B_{k,i}, \quad (32b)$$

$$\sum_{i=1}^{n}B_{k,i} - \sum_{i=1}^{n}d_{k,i} \leq B_{k,\max}, \quad (32c)$$

$$\sum_{i=1}^{n}d_{k,i} \geq \sum_{i=1}^{n-\tau_k}B_{k,i}, \quad (32d)$$

$$k=1,2,\ n=1,2,\ldots,N. \quad (32e)$$

We solve the energy scheduling problem (31) by decomposing it into single user energy scheduling problems. We observe that we can first find the optimal Δ_n for a given $\mathbf{p}$ and $\mathbf{d}$ by solving

$$\Delta_n^* = \arg\max_{0\leq\Delta_n\leq 1} \mathcal{T}(\{p_{j,n}\},\{d_{k,n}^{[m-1]}\},\Delta_n) \quad (33)$$

for $n = 1,2,\ldots,N$. Consequently, the maximum sum-throughput for time slot n is $\mathcal{T}(\{p_{j,n}\},\{d_{k,n}^{[m-1]}\},\Delta_n^*)$. Problem (33) represents sum-throughput maximization for a given time slot in the two-way relay channel with maximum data amounts $d_{1,n}^{[m-1]}$ and $d_{2,n}^{[m-1]}$. We know from reference [11] that there exists at least one optimal policy for (31) which operates on the front face of the rate region for the multiple access phase. That is, there exists a solution with sum-throughput equal to $g_{\text{sum}}(p_{1,n},p_{2,n},\Delta_n)$ in every time slot. Hence, to solve (33), we consider the three cases shown in Fig. 5. The intersection of the multiple access region and the broadcast region, shown in Fig. 5 in blue and red, respectively, determine which one of these three cases will be valid for given transmit powers. Since the broadcast region and the data availability region are both rectangular, we lump these two constraints and represent them by a single rectangle in Fig. 5.

Instead of solving three different versions of (33), we express $\mathcal{T}(\{p_{j,n}\},\{d_{k,n}^{[m-1]}\},\Delta_n^*)$ as the minimum of three sum rates. Note that in all three cases, the maximum sum-throughput is equal to $\bar{g}_{\text{sum}}(p_{1,n},p_{2,n},\Delta_n)$ evaluated at different Δ_n values. However, only one of these Δ_n values will yield a feasible sum-throughput, depending on the given transmit powers. Hence, out of the three Δ_n values, we need to choose the one with the lowest sum-throughput, i.e.,

$$\Delta_n^* = \arg\min_{\alpha=1,2,3} g_{\text{sum}}(p_{1,n},p_{2,n},\Delta_{n,\alpha}^*) \quad (34)$$

where $\Delta_{n,\alpha}^*$, $\alpha=1,2,3$ are found by solving

$$\Delta_{n,1}^* C\left(\frac{h_{1,n}p_{1,n}+h_{2,n}p_{2,n}}{\Delta_{n,1}^*}\right) = (1-\Delta_{n,1}^*)\left(C\left(\frac{h_{1,n}p_{0,n}}{1-\Delta_{n,1}^*}\right)+C\left(\frac{h_{2,n}p_{0,n}}{1-\Delta_{n,1}^*}\right)\right), \quad (35a)$$

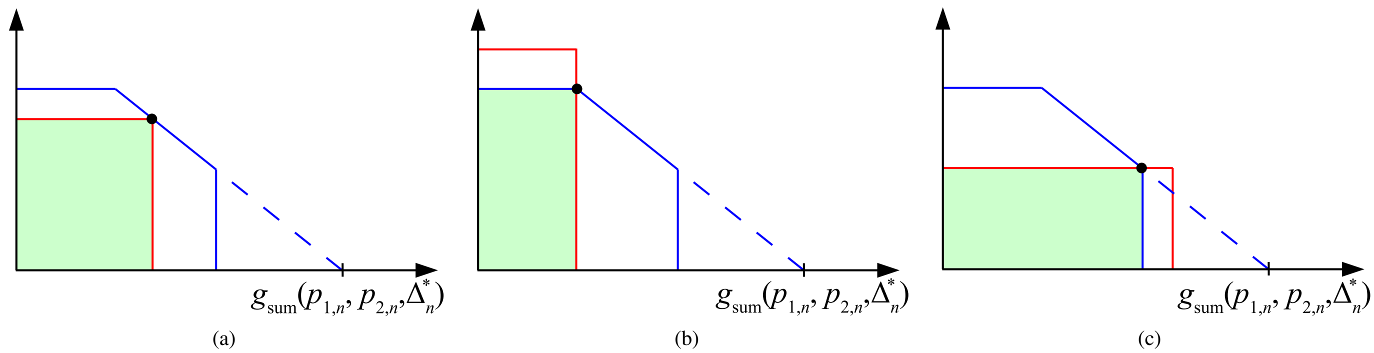

Fig. 5. The three cases for the sum-throughput $\mathcal{T}(\{p_{j,n}\}, \{d_{k,n}^{[m-1]}\}, \Delta_n^*)$. The set of all throughput pairs that can be obtained in the multiple access phase is shown with blue borders. The set of all throughput pairs smaller than $d_{1,n}^{[m-1]}$ and $d_{2,n}^{[m-1]}$, respectively, that can be obtained in the broadcast phase is shown with red borders. The shaded region is the intersection of these two regions, and the throughput pair in this region that has the largest sum-throughput is marked with a dot.

$$\Delta_{n,2}^* C\left(\frac{h_{1,n}p_{1,n}}{h_{2,n}p_{2,n}+\Delta_{n,2}^*}\right) = \min\left\{(1-\Delta_{n,2}^*)C\left(\frac{h_{2,n}p_{0,n}}{1-\Delta_{n,2}^*}\right), \frac{d_{1,n}^{[m-1]}}{\ell}\right\}, \quad (35b)$$

$$\Delta_{n,3}^* C\left(\frac{h_{2,n}p_{2,n}}{h_{1,n}p_{1,n}+\Delta_{n,3}^*}\right) = \min\left\{(1-\Delta_{n,3}^*)C\left(\frac{h_{1,n}p_{0,n}}{1-\Delta_{n,3}^*}\right), \frac{d_{2,n}^{[m-1]}}{\ell}\right\}. \quad (35c)$$

The left hand sides of equations (35) increase from 0 to their respective maxima as $\Delta_{n,\alpha}^*$ is increased from 0 to 1. Conversely, the right hand sides decrease from their respective maxima to 0 as $\Delta_{n,\alpha}^*$ is increased from 0 to 1. Therefore, all three equations have a unique solution which can be found using simple root finding algorithms such as the bisection method.

We next solve (31) for $\mathbf{p}^{[m]}$ and $\mathbf{w}^{[m]}$. Note that the coupling between the three nodes is now removed with the separate optimization of Δ_n. Thus, the feasible region of (31) is separable between the three nodes. For $j = 0, 1, 2$, we solve (31) for node T_j using the waterfilling solution given in Section III-A since (31) is equivalent to (2) when the optimization is restricted to only one node's energy variables. Likewise, the data scheduling problem (32) reduces to single user data scheduling problems that are equivalent to (3) and can be solved using the optimal data allocation policy found in Section III-A. As such, we obtain the solution to the delay limited sum-throughput maximization problem for the half duplex two-way relay channel.

Remark 4: The two-hop relay channel is a special case of the two-way relay channel with data flow in only one direction, e.g., from T_1 to T_2. The optimal policy can thus be found by setting the energy and data profile for T_2 to have no arrivals. In this case, the multiple access phase corresponds to the duration in which T_1 sends its data to the relay, i.e., the first hop, and the broadcast phase corresponds to the second hop in which the relay forwards T_1's data to T_2. The multiple access channel is also a special case of the two-way relay channel without the broadcast phase. Hence, there is no need for Δ_n optimization, and the sum-throughput is constrained by the multiple access phase constraints only. The energy and data scheduling problems can be separated between the two transmitters, and the resulting single user problems can be solved using the solutions found in Section III. ■

Remark 5: Our results can be readily extended to the full duplex two-way and two-way relay energy harvesting channels with perfect self-interference cancellation. This is done by eliminating Δ_n from the analysis, leading to two separate single user channels for the two-way channel, and rendering the Δ_n optimization in (34) unnecessary for the two-way relay channel. In practice, full duplex systems have residual self-interference. Studying these models with realistic self-interference constraints is an interesting future direction. ■

Remark 6: In this paper, we address delay limited communication and therefore consider a relay which forwards data immediately without buffering it. While the identified policies can certainly be used in the setting where the relay is capable of buffering data, they may or may not yield the optimum policy, since data buffering at the relay can potentially improve throughput under certain system parameters.

Remark 7: There has been limited contribution in energy harvesting relay channels with relay buffers. For instance, references [13], [36] have studied a half duplex energy harvesting two-hop channel with backlogged data at the source and an infinite data buffer at the relay. We have studied the impact of the size of the relay's data buffer in [37] and concluded that it can be done away with under certain conditions. All of these are for single-way relay communications. Sum-throughput maximization for an energy harvesting two-way relay channel with relay data buffer remains an open problem and is an interesting future direction. ■

Remark 8: The computational complexity of the solution given in Section III-A for the energy scheduling problem is $\mathcal{O}(N^2)$ since the p_n, w_n, and t_n values for each epoch can be found using a binary search which necessitates the recalculation of p_n, w_n, and t_n for at most $N-1$ epochs. The computational complexity of the solution for the data scheduling problem in Section III-B is $\mathcal{O}(N)$. The computa-

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSAC.2016.2545418, IEEE Journal on Selected Areas in Communications

tional complexity of Algorithm 1 is therefore $\mathcal{O}(N^2)$ for the single user channel, the two-way channel, and the two-way relay channel since alternating maximization is guaranteed to converge in a finite number of iterations. ■

VI. Online Policies

So far we considered the offline setting where the complete energy and data arrival profiles are available to design the transmission policies offline. This approach is applicable to settings with predictable arrivals, and also serves as a benchmark for the setting where arrivals are revealed causally to the transmitters. In this section, we focus on this latter setting, and provide *online policies* which only require causal knowledge of energy and data arrivals. In doing so, we utilize properties of the problem at hand for tractability. We consider the single user channel in this section, i.e., the online counterpart of the problem in Section III, and note that the extension to multi terminal networks is straightforward.

In time slot n, the transmitter has knowledge of harvested energy amounts $E^n = (E_1, \dots, E_n)$, data arrival amounts $B^n = (B_1, \dots, B_n)$, block fading coefficients h_n, and current energy and data buffer states before arrivals, denoted by $x_{n-1} = (e_{n-1}, b_{n-1})$ with $e_0 = b_0 = 0$. Using this information, the transmitter chooses the variables p_n, w_n, and d_n based on an action function

$$\phi_n \colon (x_{n-1}, E^n, B^n, h_n) \mapsto (p_n, w_n, d_n). \tag{36}$$

We consider the sum-throughput maximization problem, and thus define $R_n(\phi_n(x_{n-1}, E^n, B^n, h_n))$, the reward function for the nth epoch as the sum-throughput corresponding to the variables p_n, w_n, and d_n given by $\phi_n(x_{n-1}, E^n, B^n, h_n)$. The expected sum-throughput of the system from time slot n to the end of the transmission is therefore the value function $J_n(x_{n-1}, E^n, B^n, h_n)$. In each time slot, the optimal action ϕ_n is the one that maximizes the value J_n. Notice that the value function can be defined recursively, as the sum of the throughput of the current time slot and the expected value function for the next time slot, which yields the Bellman equations

$$J_N(x_{N-1}, E^N, B^N, h_N) = \max_{\phi_N} R_N(\phi_N(x_{N-1}, E^N, B^N, h_N)), \tag{37}$$

$$J_n(x_{n-1}, E^n, B^n, h_n) = \max_{\phi_n} \big(R_n(\phi_n(x_{n-1}, E^n, B^n, h_n)) + \mathbb{E}_{E_{n+1}, B_{n+1}, h_{n+1} | E^n, B^n}\left[J_{n+1}(x_{n+1}, E^{n+1}, B^{n+1}, h_{n+1})\right]\big), \tag{38}$$

for $n = 1, \dots, N-1$. In its current state, the dynamic program solving (37) and (38) may be intractable since the value function and the action function have an exponentially large parameter space. Therefore, before we solve this problem, we consider possible simplifications. We first note that there exists an optimal action which chooses only p_n and assigns deterministic values to w_n and d_n.

Lemma 2: There exists an optimal action function in the form of

$$\phi_n^* \colon (x_{n-1}, E^n, B^n, h_n) \mapsto (p_n, w_n^*, d_n^*). \tag{39}$$

where $w_n^* = \max\{0, e_{n-1} + E_n - \ell p_n - E_{\max}\}$ and d_n^* is computed using (14).

Proof: We show that for any action function $\phi_n \colon (x_{n-1}, E^n, B^n, h_n) \mapsto (p_n, w_n, d_n)$, there exists an action function ϕ_n^* in the form of (39) which yields a value function at least as good as that of ϕ_n. The choice of w_n^* and d_n^* in (39) implies only discarding energy and data that will overflow if not discarded. Therefore, the action ϕ_n^* always results in energy and data buffer states that are greater than or equal to those of an arbitrary feasible ϕ_n. Since $J_N(x_{N-1}, E^N, B^N, h_N)$ is nondecreasing in buffer states x_{N-1}, so is $J_n(x_{n-1}, E^n, B^n, h_n)$ in x_{n-1}, and therefore ϕ_n^* performs no worse than any ϕ_n with the same p_n. ■

As a consequence of Lemma 2, we can restrict our attention to actions of the form (39) without loss of generality, and only choose the optimal p_n while calculating w_n and d_n accordingly. Next, we consider the case where energy and data arrivals are i.i.d. or first order Markov processes. This implies that given E_n and B_n, E_{n+1} and B_{n+1} are independent of past arrivals E^{n-1} and B^{n-1}, and hence the expectation in (38) can be rewritten to be independent of E^{n-1} and B^{n-1}. In this case, choosing a different action for different (E^{n-1}, B^{n-1}) values is also redundant, and (37) and (38) can be revised as

$$J_N(x_{N-1}, E_N, B_N, h_N) = \max_{\phi_N} R_N(\phi_N(x_{N-1}, E_N, B_N, h_N)), \tag{40}$$

$$J_n(x_{n-1}, E_n, B_n, h_n) = \max_{\phi_n} \big(R_n(\phi_n(x_{n-1}, E_n, B_n, h_n)) + \mathbb{E}_{E_{n+1}, B_{n+1}, h_{n+1} | E_n, B_n}\left[J_{n+1}(x_n, E_{n+1}, B_{n+1}, h_{n+1})\right]\big), \tag{41}$$

for $n = 1, \dots, N-1$. Note that in (40) and (41), the dimension of the action and value functions is significantly decreased. At this point, the optimal online transmission policy can be found by solving (40) for the last epoch, and then solving (38) for $n = N-1, N-2, \dots, 1$.

A simpler alternative online policy can be found by solving

$$J(x, E, B, h) = \max_{\phi}(R(\phi(x, E, B, h)) + \beta \mathbb{E}_{x', E', B', h' | x, E, B}\left[J(x', E', B', h')\right]) \tag{42}$$

iteratively, where the states and the actions no longer depend on the epoch index, and x' denotes the next state. This is the infinite horizon characterization of the online throughput maximization problem with a discount factor $\beta < 1$. Problem (42) can be solved by value iteration, i.e., by starting from an arbitrary action ϕ and iterating until ϕ converges. This approach provides a single action ϕ for all time slots, as opposed to n actions $\phi_1, \dots, \phi_n$ for the case in (40) and (41). As a result, this action is easier to store and implement in practice. We implement this numerically in Section VII, along with the original finite horizon problem, and assess the resulting performance as compared to the offline optimal policy found in Section III.

We note that the implementation entails quantizing the input, state, and action variables for Q quantization levels. The value iterations in (37) and (38) with the original definition of actions in (36) require Q^3 value updates, leading

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSAC.2016.2545418, IEEE Journal on Selected Areas in Communications

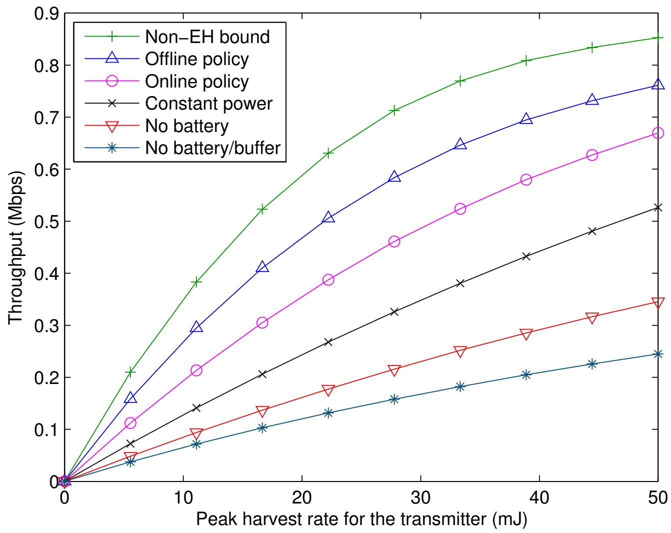

Fig. 6. Throughput for varying peak harvest rate E_{peak} resulting from various transmission policies in a single user channel with $B_{\text{peak}} = 2$ Mbit, $B_{\max} = 1$ Mbit, $E_{\max} = 50$ mJ, and $\tau = 9$ s.

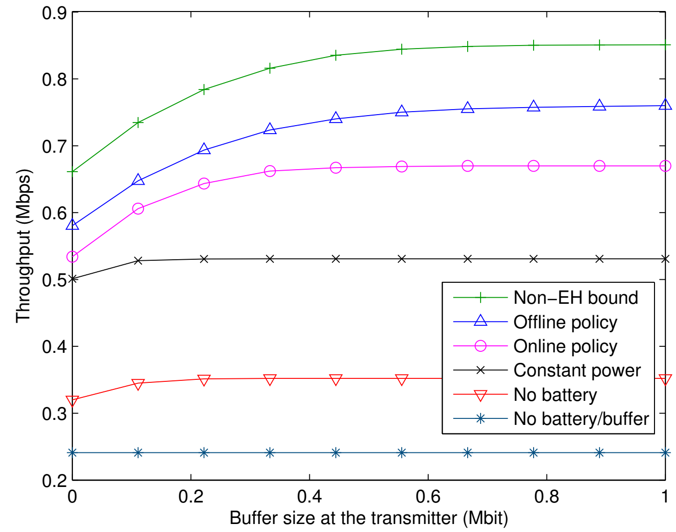

Fig. 7. Throughput for varying buffer sizes $B_{\max}$ at the transmitter resulting from various transmission policies in a single user channel with $B_{\text{peak}} = 2$ Mbit, $E_{\text{peak}} = 50$ mJ, $E_{\max} = 50$ mJ, and $\tau = 9$ s.

to the computational complexity of $\mathcal{O}(NQ^3)$, $\mathcal{O}(NQ^6)$, and $\mathcal{O}(NQ^8)$ for the single user, two-way, and two-way relay channels, respectively. By restricting the actions to the form in (39) without loss of optimality, we reduce the complexity to $\mathcal{O}(NQ)$, $\mathcal{O}(NQ^2)$, and $\mathcal{O}(NQ^3)$, respectively. We further reduce the complexity by eliminating the time dependence of the actions and propose the time-invariant infinite horizon simplification in (42). The infinite horizon approach has computational complexity $\mathcal{O}(Q)$, $\mathcal{O}(Q^2)$, and $\mathcal{O}(Q^3)$ for the single user, two-way, and two-way relay channels, respectively.

VII. Numerical Results

For our simulations, we first consider a block fading single user channel with Rayleigh power gains with mean -110 dB, receiver noise spectral density 10^{-19} W/Hz, bandwidth 1 MHz, and a communication session of $N = 10$ time slots each of duration $\ell = 1$ s. The energy harvests and data arrivals are uniform in $[0, E_{\text{peak}}]$ and $[0, B_{\text{peak}}]$, respectively. We consider a delay constraint, i.e., a data loss tolerant scenario unless otherwise stated. We specify the remaining parameters for each setup in the captions of Figs. 6–12.

The performance of the policies identified in this paper is compared with others in Fig. 6 for the single user channel. Here, we plot the throughput of the offline and online solutions (labeled "Offline policy" and "Online policy"), and upper and lower bounds versus the peak harvested energy E_{peak}. For the upper bound (Non-EH bound), we consider a classical, i.e., non-energy harvesting, transmitter where the total energy to be harvested throughout the session is available at the start of the session and thus only the data scheduling problem needs to be solved. The "constant power" policy attempts to transmit with transmit power $E_{\text{peak}}/(2\ell)$ whenever possible. The "no battery" policy does not utilize the battery, and consumes all harvested energy for transmission as soon as it is harvested. The "no battery/buffer" policy does not utilize the battery or the buffer, and discards unused energy or data at the end

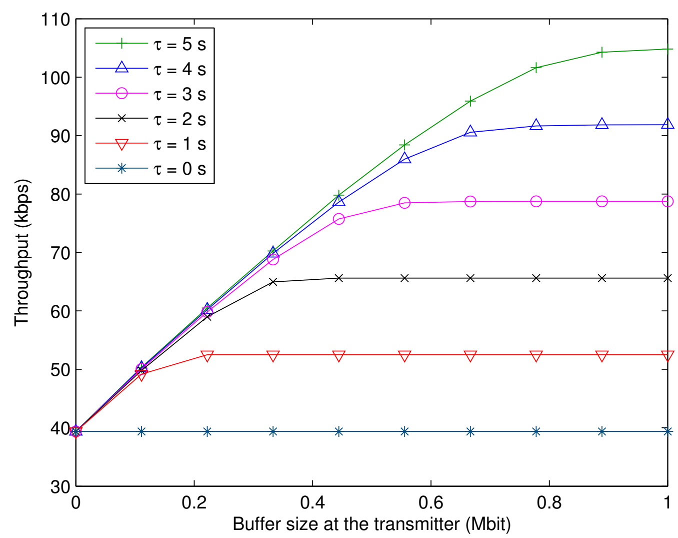

Fig. 8. Optimal throughput for varying buffer sizes $B_{\max}$ and maximum packet delays τ in a single user channel with $E_{\text{peak}} = 5$ mJ, $E_{\max} = 5$ mJ, $B_1 = B_2 = B_3 = 1$ Mbit, and $B_4 = \cdots = B_{10} = 0$ Mbit.

of each time slot. As seen in Fig. 6, the offline and online policies perform considerably better than the lower bounds. The advantage of having data buffers is evident even for simple transmission policies since the difference between "no battery" and "no battery/buffer" policies is increasing in E_{peak}.

Fig. 7 shows the throughput resulting from the six transmission policies for varying data buffer size $B_{\max}$. For the offline policy in the setting of Fig. 7, we observe that on average, a data buffer size roughly equal to the throughput within one time slot accounts for most of the increase in performance. We thus infer that larger buffers do not improve throughput indefinitely. This saturation phenomenon arises for other policies as well, except for the "no battery/buffer" policy which does not use the buffer.

Fig. 8 demonstrates the impact of the packet delay constraint

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSAC.2016.2545418, IEEE Journal on Selected Areas in Communications

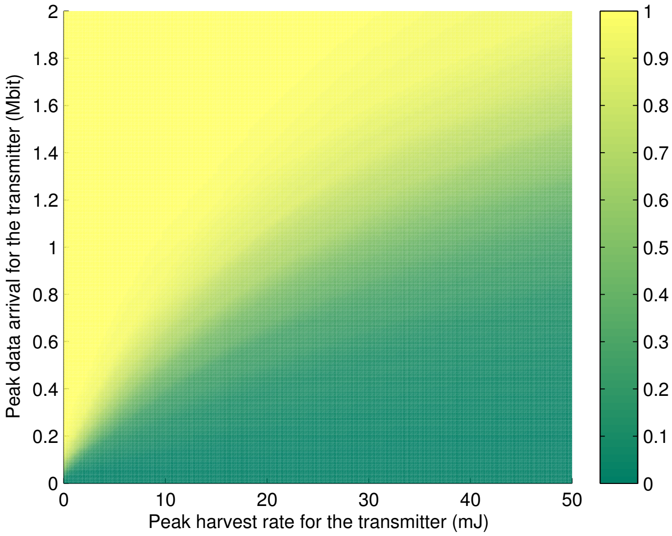

Fig. 9. The probability of infeasibility for varying peak harvest rate E_{peak} and peak data arrival B_{peak} in a single user channel with $E_{\max} = 50$ mJ, $B_{\max} = 1$ Mbit, $\tau = 9$ s, and $c = \infty$.

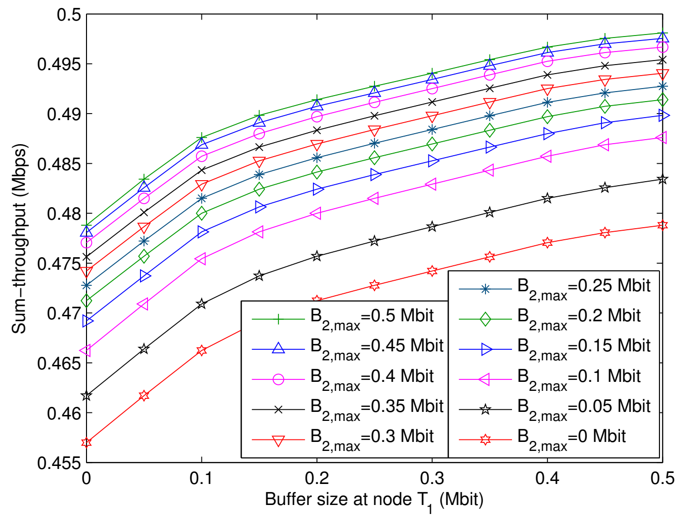

Fig. 10. Optimal sum-throughput for varying buffer sizes $B_{1,\max}$ and $B_{2,\max}$ in a two-way channel with $E_{\text{peak}} = 25$ mJ and $E_{j,\max} = 50$ mJ for all nodes, $B_{\text{peak}} = 0.5$ Mbit for both transmitters, and $\tau_1 = \tau_2 = 9$ s.

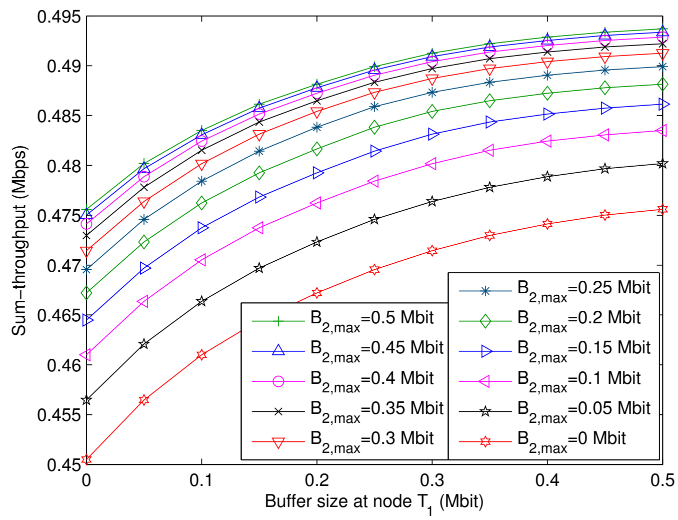

Fig. 11. Optimal sum-throughput for varying buffer sizes $B_{1,\max}$ and $B_{2,\max}$ in a two-way relay channel with $E_{\text{peak}} = 25$ mJ and $E_{j,\max} = 50$ mJ for all nodes, $B_{\text{peak}} = 0.5$ Mbit for both transmitters, and $\tau_1 = \tau_2 = 9$ s.

on the optimal throughput and the required buffer size. For clarity of exposition, we consider a bursty data arrival scenario where 3 Mbits of data arrive at the transmitter within the first three time slots, and there are no other data arrivals. As can be seen, for low values of τ, the data packets need to be departed quickly, and thus the throughput saturates at relatively small buffer sizes. As τ is increased, the transmitter can utilize more of its data buffer to avail the transmission of additional data packets which would expire with low τ values. Hence, the minimum buffer size required for optimality is increasing in τ. This is because the use of larger data buffers implicitly delays the transmission of some data packets that can contribute to the throughput only if τ is sufficiently large.

In the next simulation setup, we set $c = \infty$, i.e., we do not allow data loss. This is the requirement in reference [29] and no solution exists unless *all* data packets can be sent to the receiver. Fig. 9 shows the empirical probability of this infeasibility, based on 10,000 trials over channel gains, energy and data arrivals. As can be seen, lossless transmission is feasible only when a sufficiently large amount of energy is harvested or the incoming data rate is low. As the system becomes energy deprived with smaller energy harvests or larger data arrivals, the transmitter is compelled to drop some data packets, and the probability of infeasibility increases. By contrast, our formulation returns a non-zero throughput in all of these scenarios (including those indicated with yellow) by allowing packet loss, see for example Fig. 6.

We next simulate the two-way channel and the two-way relay channel to investigate the interaction between the sizes of the two data buffers at nodes T_1 and T_2, and their collective impact on the throughput. Figs. 10 and 11 show the sum-throughput for varying $B_{1,\max}$ and $B_{2,\max}$. Similar to the single user case, we observe that near optimal operation of the network is attained with buffers that can store the sum-throughput within one time slot on average. We also remark the diminishing returns of larger data buffer sizes for both nodes, in line with the single user case in Fig. 7.

Finally, Fig. 12 shows the sum-throughput resulting from the offline solution in Section V and the finite and infinite horizon online solutions in Section VI extended to the two-way relay channel. As can be seen, the online solutions perform closely to the offline solution in the case of the two-way relay channel as well. Potentially larger energy arrivals at node T_1 result in a larger sum-throughput until the data arrivals become the bottleneck of the system. As can be seen, the infinite horizon simplification results in little loss compared to finite horizon in exchange for lower complexity.

VIII. Conclusion

In this paper, we have studied (sum-)throughput maximization for multiple energy harvesting setups including the single

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSAC.2016.2545418, IEEE Journal on Selected Areas in Communications

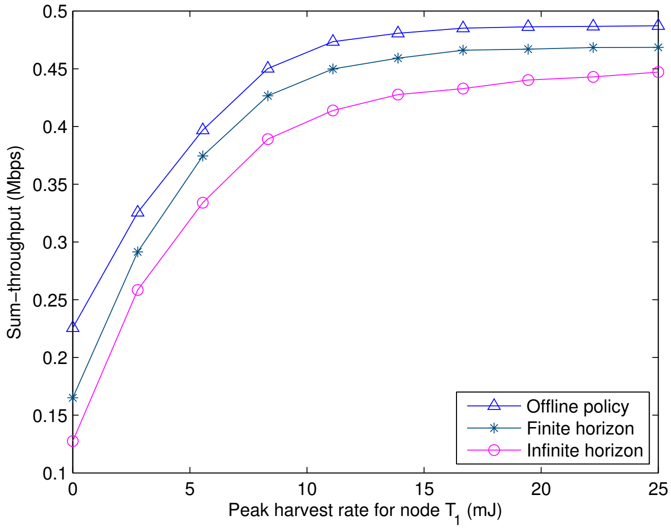

Fig. 12. Optimal sum-throughput for varying peak harvest rates at node T_1 resulting from the offline and online solutions for the two-way relay channel with $E_{\rm peak} = 25$ mJ for nodes T_2 and T_0, $E_{j,\max} = 50$ mJ for all nodes, $B_{j,\max} = B_{\rm peak} = 0.5$ Mbit for both transmitters, and $\tau_1 = \tau_2 = 9$ s.

user channel, the two-way channel, and the two-way relay channel, with finite capacity data buffers at all nodes, in addition to finite energy storage. We have considered a delay limited block fading scenario which also penalizes data loss at the source nodes. We have shown that the delay limited throughput maximization problems for each model can be decomposed into an energy scheduling problem and a data scheduling problem, and can subsequently be solved using alternating maximization. We have shown that the energy scheduling problem admits a directional waterfilling solution [5] with the addition of the new notions of *water pumps* and *overflow bins*. We have solved the data scheduling problem using forward induction where we identify optimal data amounts to transmit on a slot-by-slot basis. We have also provided an online solution to the throughput maximization problem using dynamic programming. We have assessed the improvement provided by our optimal policy over simpler solutions through numerical results, and verified our analytical findings. In particular, we have observed that for lenient delay requirements, data buffers that can store the data transmitted within one time slot on average provide the majority of the increase in throughput. However, with more strict delay requirements, even smaller buffer sizes suffice since transmitters cannot delay data packets any further by utilizing a larger buffer. Future directions include examining energy harvesting receivers as well as transmitters in the same setting for these and other channel models, and considering priorities on individual data packets.

References

[1] J. Yang and S. Ulukus, "Optimal packet scheduling in an energy harvesting communication system," *IEEE Trans. Commun.*, vol. 60, no. 1, pp. 220–230, Jan. 2012.

[2] C. Alippi and C. Galperti, "An adaptive system for optimal solar energy harvesting in wireless sensor network nodes," *IEEE Trans. Circuits and Syst. I: Regular Papers*, vol. 55, no. 6, pp. 1742–1750, Jul. 2008.

[3] G. Ottman, H. Hofmann, A. Bhatt, and G. Lesieutre, "Adaptive piezoelectric energy harvesting circuit for wireless remote power supply," *IEEE Trans. Power Electron.*, vol. 17, no. 5, pp. 669–676, Sep. 2002.

[4] K. Tutuncuoglu and A. Yener, "Optimum transmission policies for battery limited energy harvesting nodes," *IEEE Trans. Wireless Commun.*, vol. 11, no. 3, pp. 1180–1189, Mar. 2012.

[5] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, "Transmission with energy harvesting nodes in fading wireless channels: Optimal policies," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp. 1732 –1743, Sep. 2011.

[6] C. K. Ho and R. Zhang, "Optimal energy allocation for wireless communications with energy harvesting constraints," *IEEE Trans. Signal Process.*, vol. 60, no. 9, pp. 4808–4818, Sep. 2012.

[7] B. Varan, K. Tutuncuoglu, and A. Yener, "Energy harvesting communications with continuous energy arrivals," in *Proc. IEEE Inf. Theory Appl. (ITA)*, Feb. 2014.

[8] R. Yates and H. Mahdavi-Doost, "Energy harvesting receivers: Packet sampling and decoding policies," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 3, pp. 558–570, Mar. 2015.

[9] K. Tutuncuoglu and A. Yener, "Sum-rate optimal power policies for energy harvesting transmitters in an interference channel," *J. Commun. Netw.*, vol. 14, no. 2, pp. 151–161, Apr. 2012.

[10] O. Ozel, J. Yang, and S. Ulukus, "Optimal broadcast scheduling for an energy harvesting rechargeable transmitter with a finite capacity battery," *IEEE Trans. Wireless Commun.*, vol. 11, no. 6, pp. 2193–2203, Jun. 2012.

[11] K. Tutuncuoglu, B. Varan, and A. Yener, "Throughput maximization for two-way relay channels with energy harvesting nodes: The impact of relaying strategies," *IEEE Trans. Commun.*, vol. 63, no. 6, pp. 2081–2093, Jun. 2015.

[12] C. Huang, R. Zhang, and S. Cui, "Throughput maximization for the Gaussian relay channel with energy harvesting constraints," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 8, pp. 1469–1479, Aug. 2013.

[13] D. Gunduz and B. Devillers, "Two-hop communication with energy harvesting," in *Proc. IEEE Int. Workshop Comput. Adv. Multi-Sensor Adapt. Process. (CAMSAP)*, Dec. 2011.

[14] B. Varan and A. Yener, "The energy harvesting multi-way relay channel with intermittent data: The impact of buffer sizes," *EURASIP Journal on Wireless Communications and Networking Special Issue on Energy Harvesting Wireless Communications*, vol. 2015, no. 63, pp. 1–15, Mar. 2015.

[15] J. Yang and S. Ulukus, "Optimal packet scheduling in a multiple access channel with energy harvesting transmitters," *J. Commun. Netw.*, vol. 14, no. 2, pp. 140–150, Apr. 2012.

[16] K. Tutuncuoglu and A. Yener, "Communicating with energy harvesting transmitters and receivers," in *Proc. IEEE Inf. Theory Appl. (ITA)*, Feb. 2012.

[17] O. Orhan, D. Gunduz, and E. Erkip, "Energy harvesting broadband communication systems with processing energy cost," *IEEE Trans. Wireless Commun.*, vol. 13, no. 11, pp. 6095–6107, Nov. 2014.

[18] S. Ulukus, A. Yener, E. Erkip, O. Simeone, M. Zorzi, P. Grover, and K. Huang, "Energy harvesting wireless communications: A review of recent advances," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 3, pp. 360–381, Mar. 2015.

[19] R. Srivastava and C. Koksal, "Basic performance limits and tradeoffs in energy-harvesting sensor nodes with finite data and energy storage," *IEEE/ACM Trans. Netw.*, vol. 21, no. 4, pp. 1049–1062, Aug. 2013.

[20] V. Jog and V. Anantharam, "A geometric analysis of the AWGN channel with a (σ, ρ)-power constraint," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Jun. 2015.

[21] D. Shaviv and A. Ozgur, "Capacity of the AWGN channel with random battery recharges," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Jun. 2015.

[22] O. Ozel, K. Tutuncuoglu, S. Ulukus, and A. Yener, "Fundamental limits of energy harvesting communications," *IEEE Commun. Mag.*, vol. 53, no. 4, pp. 126–132, Apr. 2015.

[23] J. Lei, R. Yates, and L. Greenstein, "A generic model for optimizing single-hop transmission policy of replenishable sensors," *IEEE Trans. Wireless Commun.*, vol. 8, no. 2, pp. 547–551, Feb. 2009.

[24] M. Gatzianas, L. Georgiadis, and L. Tassiulas, "Control of wireless networks with rechargeable batteries," *IEEE Trans. Wireless Commun.*, vol. 9, no. 2, pp. 581–593, Feb. 2010.

[25] V. Sharma, U. Mukherji, V. Joseph, and S. Gupta, "Optimal energy management policies for energy harvesting sensor nodes," *IEEE Trans. Wireless Commun.*, vol. 9, no. 4, pp. 1326–1336, Apr. 2010.

[26] C. Huang, R. Zhang, and S. Cui, "Optimal power allocation for outage probability minimization in fading channels with energy harvesting constraints," *IEEE Trans. Wireless Commun.*, vol. 13, no. 2, pp. 1074–1087, Feb. 2014.

[27] Z. Mao, C. Koksal, and N. Shroff, "Near optimal power and rate control of multi-hop sensor networks with energy replenishment: Basic limitations with finite energy and data storage," *IEEE Trans. Autom. Control*, vol. 57, no. 4, pp. 815–829, Apr. 2012.

[28] Y. Dong, F. Farnia, and A. Ozgur, "Near optimal energy control and approximate capacity of energy harvesting communication," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 3, pp. 540–557, Mar. 2015.

[29] M. Gregori and M. Payaro, "Energy-efficient transmission for wireless energy harvesting nodes," *IEEE Trans. Wireless Commun.*, vol. 12, no. 3, pp. 1244–1254, Mar. 2013.

[30] B. Gurakan, O. Ozel, J. Yang, and S. Ulukus, "Energy cooperation in energy harvesting communications," *IEEE Trans. Commun.*, vol. 61, no. 12, pp. 4884–4898, Dec. 2013.

[31] D. P. Bertsekas, *Nonlinear programming*. Belmont, MA: Athena Scientific, 1999.

[32] L. Grippo and M. Sciandrone, "On the convergence of the block nonlinear Gauss–Seidel method under convex constraints," *Operations Research Letters*, vol. 26, no. 3, pp. 127–136, 2000.

[33] R. L. Burden and J. D. Faires, *Numerical Analysis, Third Ed.* Boston, MA: Prindle, Weber and Schmidt, 1985.

[34] F. Olver, D. Lozier, R. Boisvert, and C. Clark, *NIST handbook of mathematical functions*. Cambridge University Press, 2010.

[35] S. Kim, N. Devroye, P. Mitran, and V. Tarokh, "Achievable rate regions and performance comparison of half duplex bi-directional relaying protocols," *IEEE Trans. Inf. Theory*, vol. 57, no. 10, pp. 6405–6418, Oct. 2011.

[36] O. Orhan and E. Erkip, "Throughput maximization for energy harvesting two-hop networks," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Jul. 2013.

[37] B. Varan and A. Yener, "Two-hop networks with energy harvesting: The (non-)impact of buffer size," in *Proc. IEEE Global Conf. Signal Inf. Process. (GlobalSIP)*, Dec. 2013.

Burak Varan (S'13) received the B.S. degree in electrical and electronics engineering from Bogazici University, Istanbul, Turkey, in 2011. He is currently pursuing the Ph.D. degree at The Pennsylvania State University, University Park, PA, USA. He has been a Graduate Research Assistant with the Wireless Communications and Networking Laboratory (WCAN), The Pennsylvania State University, since 2011. His research interests include green communications and optimal resource allocation in energy harvesting networks under competitive and altruistic communication scenarios. He received the AT&T Graduate Fellowship Award at The Pennsylvania State University in 2016.

Aylin Yener (S'91–M'00–SM'13–F'14) received the B.Sc. degree in electrical and electronics engineering and the B.Sc. degree in physics from Bogazici University, Istanbul, Turkey, and the M.S. and Ph.D. degrees in electrical and computer engineering from Wireless Information Network Laboratory (WINLAB), Rutgers University, New Brunswick, NJ, USA. She is a Professor of Electrical Engineering at The Pennsylvania State University, University Park, PA, USA, since 2010, where she joined the Faculty as an Assistant Professor in 2002. During the academic year 2008 to 2009, she was a Visiting Associate Professor with the Department of Electrical Engineering, Stanford University, Stanford, CA, USA. Her research interests include information theory, communication theory, and network science, with recent emphasis on green communications and information security. She received the NSF CAREER award in 2003, the Best Paper Award in Communication Theory in the IEEE International Conference on Communications in 2010, the Penn State Engineering Alumni Society (PSEAS) Outstanding Research Award in 2010, the IEEE Marconi Prize Paper Award in 2014, the PSEAS Premier Research Award in 2014, and the Leonard A. Doggett Award for Outstanding Writing in Electrical Engineering at Penn State in 2014.

Dr. Yener is currently a member of the Board of Governors of the IEEE Information Theory Society, where she was previously the treasurer (2012–2014). She served as the Student Committee Chair for the IEEE Information Theory Society 2007–2011, and was the Co-founder of the Annual School of Information Theory in North America co-organizing the school in 2008, 2009, and 2010. She was a Technical (Co)-Chair for various symposia/tracks at the IEEE ICC, PIMRC, VTC, WCNC, and Asilomar (2005–2014). She served as an Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS (2009–2012), an Editor and an Editorial Advisory Board Member for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS (2001–2012), and a Guest Editor for the IEEE TRANSACTIONS ON INFORMATION FORENSICS AND SECURITY (2011) and the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS (2015).