Energy Harvesting Communications with Continuous Energy Arrivals

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***Abstract*—This work considers an energy harvesting transmitter that gathers a continuous flow of energy from intermittent sources, thus relaxing the modeling assumption of discrete amounts of harvested energy present in all previous work on energy harvesting communications. Tools from convex analysis are utilized to describe the optimal transmission policy as the boundary of a properly defined region based on the energy profile. The results are extended to include models where the transmitter has a finite capacity battery with various imperfections, as well as those that incorporate a processing cost (circuit power) at the transmitter whenever it is in operation.**

***Index Terms*—Energy harvesting wireless transmitter, continuous arrivals, throughput maximization, finite battery, battery imperfections.**

I. Introduction

Wireless communication networks comprised of devices that harvest the energy needed for their operation are recently introduced into wireless system design research [1], [2]. Energy harvesting devices are those which intermittently acquire energy over the course of their operation. They differ from conventional communication devices in that they are not guaranteed a constant power for their operation available at all times. Instead, energy harvesting devices receive their energy from external *intermittent* sources such as solar radiation, wind, vibration, radio waves, or body heat [3]–[7]. Although a continuous modeling better captures most of these sources, initial work on energy harvesting communications has almost exclusively assumed a packetized, i.e., discrete, model for the arriving energy. The justification of such an approach has been the need for a minimum energy amount for communication at a meaningful scale, as well as the mathematical tractability of the ensuing system optimization.

Energy harvesting has been studied in various communication models including single and multi user networks [1], [2], [8], networks with relaying [9]–[14], devices with limited capacity batteries with imperfections [15]–[17], and cooperative networks where the nodes can transfer their harvested energy to one another [18], [19]. The simplest setup with one energy harvesting transmitter and one receiver is studied in [1] with the objective of minimizing the time of a file transfer to the receiver. The problem of maximizing the total number of bits the transmitter can send by a deadline is studied in [2], [20] with a finite capacity battery at the transmitter. In particular, reference [2] has shown that the optimal transmission policy follows the shortest path in the *feasible energy tunnel* which is defined by the cumulative harvested energy curve and a minimum energy curve resulting from the finite capacity of the battery. The throughput maximization problem is considered for fading channels in [20] resulting in a directional water-filling algorithm. Energy harvesting receivers are considered in [21]–[23]. The multiple access energy harvesting channel is studied in [8] where it is shown that the allocated sum power should follow the shortest path in the feasible energy tunnel in order to maximize the sum throughput. Other multi user systems are studied in [9]–[14], [24]–[26] including the broadcast channel, interference channel, and one way or bidirectional relay channels. Battery imperfections have also been studied such as decaying battery capacity [15], [16], leakage at a constant rate [15], and proportional storage loss [17]. Reference [27] considers the energy cost for the processing circuitry at the transmitter.

In this paper, we study a single user communication system with an energy harvesting transmitter and a receiver. We model the energy harvested at the transmitter as a continuous function, and solve the throughput maximization problem to identify optimal transmission policies. That is, we remove the assumption of discrete energy arrivals in previous work. We use tools from convex analysis to obtain a geometric description of the problem and its optimal solution. We present the optimal policy with an infinite battery, and also for a finite battery; battery inefficiency in the form of a proportional loss of all drawn energy; and a constant processing cost, e.g., circuit power at the transmitter. For the last two cases, we solve the throughput maximization problem by constructing an equivalent model, and then showing that the solution for this model can be modified to yield the optimal policy for the original model.

The remainder of this paper is organized as follows. In Section II, we describe the energy harvesting communication system, and state the throughput maximization problem. In Section III, we identify the optimal transmission policy in an ideal setting with a battery that has infinite capacity and no imperfections. In Section IV, we extend our results to settings where the battery at the transmitter is of finite capacity that degrades (shrinks) in time. In Section V, we extend our solution to settings with battery inefficiency and a processing cost. In Section VI, we provide numerical results, and in

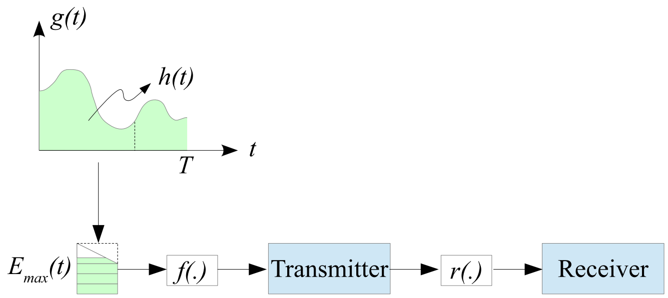

Fig. 1. The energy harvesting single user setup with continuous energy arrivals and imperfect energy storage.

Section VII, we conclude the paper.

II. System Model

Consider a wireless communication system with a transmitter receiver pair. As shown in Fig. 1, the transmitter is an energy harvesting node that receives energy from external sources in a continuous fashion. The receiver node is assumed to have a continuing source of energy which provides sufficient power for decoding at any rate the transmitter can achieve.

The objective is to maximize the total number of bits that depart the transmitter by the deadline T, as in [2], [11]–[15], [17], [20], [21], [24], [25], [27]. The transmitter node has a battery which harvests the arriving energy and stores it. We denote the size of this battery at time t by $E_{max}(t)$. Throughout the paper, we will consider various settings for the battery capacity. In Section III, we consider the battery with infinite capacity: $E_{max}(t) = \infty, \forall t \in [0, T]$. In Section IV, we consider a finite constant capacity: $E_{max}(t) = E_{max} \leq \infty, \forall t \in [0, T]$, as well as finite capacity that is decreasing in time: $E_{max}(t) \leq \infty, \forall t \in [0, T]$, $E_{max}(t_1) \leq E_{max}(t_2)$ for all $t_1 \leq t_2$. In all of these settings, we have that the battery capacity is non-increasing in time. We denote the instantaneous rate at which the intermittent and continuous energy arrives at the battery by $g(t)$ at time t, and the cumulative energy harvested by time t by $h(t)$. We denote by $h_0 \geq 0$ the energy available in the battery at the beginning of transmission. Naturally, we have

$$h(t) = h_0 + \int_0^t g(\tau) d\tau. \tag{1}$$

We consider instantaneous energy arrival rates $g(t)$ that are non-negative and integrable. It directly follows that the cumulative harvested energy curve $h(t)$ is non-decreasing in t, although it does not have to be continuous in t. We are interested in identifying offline transmission policies, and thus consider the case where non-causal knowledge of $g(t)$ is available for all $0 \leq t \leq T$.

Following in part the notation in [15], we define the minimum energy curve as the minimum cumulative amount of energy that has to be spent by time t, and denote it by $m(t)$. The minimum energy curve is not a physical constraint, but a design imposed constraint, which arises from the fact that the throughput maximizing policy should not allow battery overflows as they result in waste of energy that otherwise can be utilized for transmission of data. The minimum energy curve is a function of the cumulative harvested energy $h(t)$, and the battery capacity $E_{max}(t) \geq 0$. It can be calculated as

$$m(t) = \max\{h(t) - E_{max}(t), 0\} \tag{2}$$

for all $t \in [0, T]$. Since $h(t)$ is non-decreasing in t, and $E_{max}(t)$ is non-increasing in t, we have that the minimum energy curve is non-decreasing in t, i.e., $t_1 \leq t_2$ implies that $m(t_1) \leq m(t_2)$. With the assumption that $g(t)$ is known beforehand for all t, $h(t)$ and $m(t)$ are also known non-causally for all t. We let $s(t)$ denote the cumulative amount of energy that has been drawn from the battery by time t. We assume that $s(t)$ is continuous and differentiable, so that its first derivative is the instantaneous rate of consumption at which energy is drawn from the battery which we denote by $p(t)$. Due to potential battery imperfections and processing costs, the transmitter may not be able to fully utilize the drawn energy for transmission. Thus, we define a function $f\colon [0, \infty) \to [0, \infty)$ which maps the instantaneous rate of consumption to the instantaneous power at which the transmitter can send its messages. Function f for different assumptions on the battery and transmission circuitry at the transmitter can be defined as follows:

- $f(p) = p, \forall p$ for an ideal battery with no losses and no processing costs,
- $f(p) = \eta p, \forall p$ for some $0 \leq \eta \leq 1$ for an imperfect battery which leaks a constant percentage of all drawn energy, and no processing costs,
- $f(p) = (\eta p - c)^+, \forall p$ for some $0 \leq \eta \leq 1$ for an imperfect battery with constant proportional loss and a constant processing cost per unit time $c \geq 0$ which the transmitter incurs whenever it is in operation. Here, $(x)^+ = \max\{x, 0\}$. Using this definition of f, we can also model a battery that leaks some of the stored energy at a constant rate whenever the transmitter draws energy from it. In this case c denotes the sum of the processing cost per unit time and the constant leakage rate.

We denote the instantaneous rate achieved from the transmitter to the receiver by $r(\cdot)$, and characterize it as a function of the instantaneous transmit power $p_T(t) \triangleq f(p(t))$. We consider rate functions that satisfy the following.

1) $r(0) = 0$,
2) $r(\cdot)$ is non-decreasing in transmit power, i.e., $r(p_1) \leq r(p_2)$ if $p_1 \leq p_2$,
3) $r(p)$ is concave in p.

As an example, consider a Gaussian channel with a channel power gain of γ from the transmitter to the receiver, normalized by the variance of the Gaussian noise. Then, we have

$$r(p_T(t)) = \frac{1}{2} \log\left(1 + \gamma p_T(t)\right) \tag{3a}$$

$$= \frac{1}{2} \log\left(1 + \gamma f(p(t))\right). \tag{3b}$$

We assume during transmission that the transmitter has backlogged data so that whenever it is optimal to transmit, the transmitter will have enough data to transmit.

Our objective is to find optimal offline transmission policies which maximize the total amount of bits that depart the transmitter by deadline T, that is, we aim to solve the short term throughput maximization problem

$$\max_{p(t)} \quad \int_0^T r(f(p(t)))dt \tag{4a}$$

$$\text{s.t.} \quad m(t) \leq \int_0^t p(\tau)d\tau \leq h(t), \ \forall t \in [0,T], \tag{4b}$$

$$p(t) \geq 0, \ \forall t \in [0,T]. \tag{4c}$$

Here, (4b) ensures that the amount of energy drawn from the battery up until time t is never greater than the cumulative harvested energy by t, and it is never less than the value of the minimum energy curve at time t. It is worth reiterating that the latter, i.e., the minimum energy curve follows from the observation that whenever a transmission policy $s(t) = \int_0^t p(\tau)d\tau$ goes below $m(t)$, say at time t', some energy will have to be wasted. Hence, we can replace $s(t)$ by another policy which spends the wasted amount of energy before t' and possibly achieves a higher rate since $r(p)$ is non-decreasing in p. For a more detailed proof, see [2]. Constraint (4c) ensures that the instantaneous rate of consumption $p(t)$ is never negative, which would impractically imply that the transmitter is able to charge the battery by drawing energy at a negative rate.

In the following sections, we solve (4) under various characterizations of function f, i.e., battery imperfections, and $E_{max}(t)$, i.e., battery size characteristics. Instead of using calculus of variations in solving this continuous optimization problem, we utilize tools from convex analysis to arrive at the solution.

III. Optimal Policies for an Infinite Capacity Battery

In this section, we study an ideal setting of (4) as follows.

- We set $f(p) = p$, $\forall p \in [0,\infty)$, that is, the energy drawn from the battery can be fully utilized for transmission without any losses or processing costs. In this case, the instantaneous transmit power is the same as the instantaneous rate of consumption, i.e., $p_T(t) = p(t)$. Since this case provides the largest efficiency of the stored energy, the results we get with $f(p) = p$ will be an upperbound on results that would follow in settings with a different efficiency characterization $f(\cdot)$.
- We set $E_{max}(t) = \infty$, $\forall t \in [0,T]$. Hence, we have that

$$m(t) = 0 \tag{5}$$

  for all $0 \leq t \leq T$, resulting in the removal of the left inequality in constraint (4b).

We begin by defining the following region that depends on the cumulative harvested energy curve $h(t) = \int_0^t g(\tau)d\tau$.

$$\mathcal{H} = \{(t,e) : 0 \leq t \leq T, h(t) \leq e \leq h(T)\} \cup \{(0,0)\} \tag{6}$$

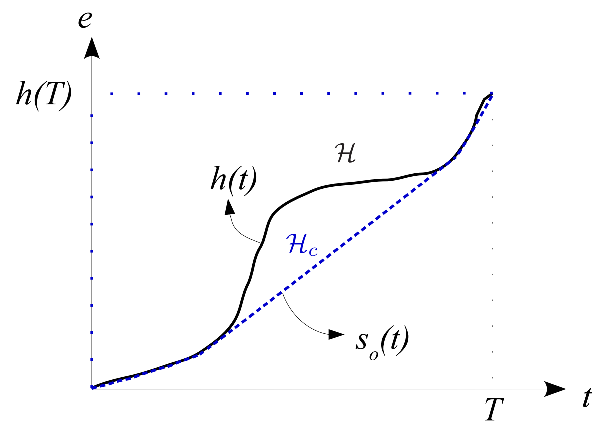

Fig. 2. An example of $\mathcal{H}$ and $\mathcal{H}_c$.

which is the area above $h(t)$ and below the $e = h(T)$ line plus the origin in case $h_0 > 0$, see Fig. 2. We denote by $\mathcal{H}_c$ the convex hull of $\mathcal{H}$:

$$\mathcal{H}_c = \text{conv}(\mathcal{H}) = \{(\lambda t_1 + (1-\lambda)t_2, \lambda e_1 + (1-\lambda)e_2) : 0 \leq \lambda \leq 1, (t_1,e_1),(t_2,e_2) \in \mathcal{H}\}. \tag{7}$$

We use the ∂ operator to denote boundaries of regions, i.e., we denote the boundary of region $\mathcal{A}$ by $\partial\mathcal{A}$:

$$\partial\mathcal{A} = \text{cl}(\mathcal{A}) \setminus \text{int}(\mathcal{A}) \tag{8}$$

where $\text{cl}(\mathcal{A})$ is the closure of $\mathcal{A}$ and $\text{int}(\mathcal{A})$ is the interior of $\mathcal{A}$. Since $\mathcal{H}$ is closed by definition, so is $\mathcal{H}_c$, and we have $\partial\mathcal{H}_c = \mathcal{H}_c \setminus \text{int}(\mathcal{H}_c)$. We define $\mathcal{H}_{inf}$ as

$$\mathcal{H}_{inf} = \text{int}(\mathcal{H}) \cup \{(0,e) : 0 < e \leq h(T)\} \cup \{(t,h(T)) : 0 \leq t < T\} \tag{9}$$

and $s_o(t)$ as the lower path that tracks $\partial\mathcal{H}_c$ from $(0,0)$ to $(T,h(T))$. We use $\mathcal{H}_{inf}$ to identify infeasible policies in the following Lemma.

Lemma 1: A transmission policy $s(t)$ is infeasible if there exists $t' \in [0,T]$ such that $(t',s(t')) \in \mathcal{H}_{inf}$.

Proof: We begin the proof with the observation that $\mathcal{H}_{inf}$ can also be characterized as

$$\mathcal{H}_{inf} = \mathcal{H} \setminus \{(t,h(t)) : 0 \leq t \leq T\}. \tag{10}$$

Hence, any point in $\mathcal{H}_{inf}$ is strictly above the cumulative harvested energy curve $h(t)$. This means that any transmission policy that lies in region $\mathcal{H}_{inf}$ violates energy causality, i.e., the second inequality in (4b), and thus is infeasible. ∎

Jensen's inequality, stated in the next Lemma, will be useful in characterizing the optimal transmission policy [15], [28].

Lemma 2 (Jensen's inequality [29]): Let $\rho,\pi\colon [a,b] \to \mathbb{R}$ be two functions such that $\alpha \leq \rho(x) \leq \beta$ and $\pi(x) > 0$ for all $x \in [a,b]$. Let $\phi\colon [\alpha,\beta] \to \mathbb{R}$ be concave. Then,

$$\phi\left(\frac{\int_a^b \rho(x)\pi(x)dx}{\int_a^b \pi(x)dx}\right) \geq \frac{\int_a^b \phi(\rho(x))\pi(x)dx}{\int_a^b \pi(x)dx} \tag{11}$$

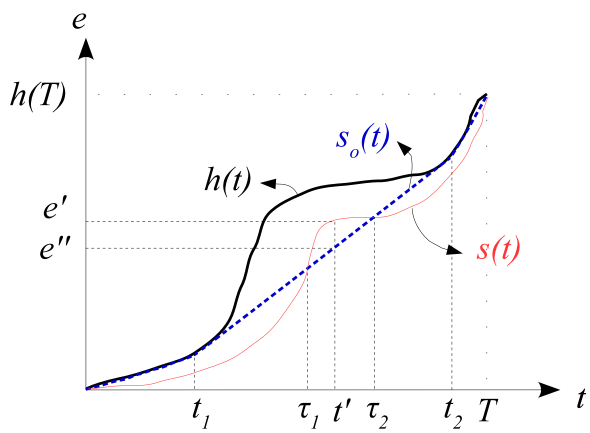

Fig. 3. A transmission policy $s(t)$ that exceeds $s_o(t)$ at time t'.

with strict inequality if ϕ is strictly concave, $a \neq b$, and $\alpha \neq \beta$.

We next present the following Lemma.

Lemma 3: A transmission policy $s(t)$ for which there exists $t' \in [0, T]$ such that $(t', s(t')) \in \text{int}(\mathcal{H}_c) \setminus \mathcal{H}_{inf}$ can be replaced with another transmission policy which does not lie in $\text{int}(\mathcal{H}_c) \setminus \mathcal{H}_{inf}$ and performs at least as well as $s(t)$.

Proof: Let t' be as given in the statement of the Lemma and $e' = s(t')$. Consider the line segment that connects $(t', s(t'))$ to $(t', 0)$. Since $(t', e') \in \text{int}(\mathcal{H}_c)$ and $\mathcal{H}_c$ is convex, the line segment will intersect $\partial\mathcal{H}_c$ at only one point, (t', e'') where $e'' = s_o(t')$. See Fig. 3 for an example.

(t', e'') is a point on the boundary of $\mathcal{H}_c$, which is the convex hull of $\mathcal{H}$. Then, by Caratheodory's theorem [30], there exist two points $(t_1, e_1), (t_2, e_2) \in \partial\mathcal{H}_c \cap \mathcal{H}$ such that $(t', e'') = \lambda(t_1, e_1) + (1 - \lambda)(t_2, e_2)$ for some $\lambda \in [0, 1]$. Without loss of generality, suppose that $t_1 < t_2$, which implies that $e_1 \leq e_2$. Let $l(t)$ be the line segment that connects (t_1, e_1) and (t_2, e_2), i.e.,

$$l(t) = \frac{e_2 - e_1}{t_2 - t_1}(t - t_1) + e_1. \tag{12}$$

Since $s(t)$ and $l(t)$ are both continuous, so is $s(t) - l(t)$. We also know that $s(t') - l(t') > 0$. Following from the feasibility of $s(t)$, we have that $s(t_1) \leq h(t_1)$ and $s(t_2) \leq h(t_2)$. Then, by the intermediate value theorem [31], there exist τ_1 and τ_2 such that $t_1 \leq \tau_1 < t' < \tau_2 \leq t_2$, $s(\tau_1) = l(\tau_1)$, and $s(\tau_2) = l(\tau_2)$. Define a new feasible policy $\bar{s}(t)$ as

$$\bar{s}(t) = \begin{cases} l(t), & \text{if } t \in [\tau_1, \tau_2], \\ s(t), & \text{if } t \in [0, T] \setminus [\tau_1, \tau_2]. \end{cases} \tag{13}$$

$\bar{s}(t)$ is continuous and we have

$$\frac{d}{dt}l(t) = \frac{e_2 - e_1}{t_2 - t_1} = \frac{l(\tau_2) - l(\tau_1)}{\tau_2 - \tau_1}. \tag{14}$$

The difference between the throughput values achieved by $s(t)$ and $\bar{s}(t)$ can be computed as

$$\int_0^T r\left(\frac{d}{dt}\bar{s}(t)\right) dt - \int_0^T r\left(\frac{d}{dt}s(t)\right) dt \tag{15a}$$

$$= \int_{\tau_1}^{\tau_2} r\left(\frac{d}{dt}\bar{s}(t)\right) dt - \int_{\tau_1}^{\tau_2} r\left(\frac{d}{dt}s(t)\right) dt \tag{15b}$$

$$= \int_{\tau_1}^{\tau_2} r\left(\frac{l(\tau_2) - l(\tau_1)}{\tau_2 - \tau_1}\right) dt - \int_{\tau_1}^{\tau_2} r\left(\frac{d}{dt}s(t)\right) dt, \tag{15c}$$

$$= (\tau_2 - \tau_1) r\left(\frac{l(\tau_2) - l(\tau_1)}{\tau_2 - \tau_1}\right) - \int_{\tau_1}^{\tau_2} r\left(\frac{d}{dt}s(t)\right) dt. \tag{15d}$$

Now we invoke Lemma 2 with $\phi = r$, $\pi \equiv 1$, and $\rho(t) = \frac{d}{dt}s(t)$ for all $t \in [\tau_1, \tau_2]$. We get

$$r\left(\frac{\int_{\tau_1}^{\tau_2}\left(\frac{d}{dt}s(t)\right) dt}{\tau_2 - \tau_1}\right) \geq \frac{\int_{\tau_1}^{\tau_2} r\left(\frac{d}{dt}s(t)\right) dt}{\tau_2 - \tau_1}, \tag{16a}$$

$$r\left(\frac{\int_{\tau_1}^{\tau_2}\left(\frac{l(\tau_2) - l(\tau_1)}{\tau_2 - \tau_1}\right) dt}{\tau_2 - \tau_1}\right) \geq \frac{\int_{\tau_1}^{\tau_2} r\left(\frac{d}{dt}s(t)\right) dt}{\tau_2 - \tau_1}, \tag{16b}$$

$$(\tau_2 - \tau_1) r\left(\frac{l(\tau_2) - l(\tau_1)}{\tau_2 - \tau_1}\right) \geq \int_{\tau_1}^{\tau_2} r\left(\frac{d}{dt}s(t)\right) dt, \tag{16c}$$

which implies that the right hand side of (15d) is non-negative, that is, the new policy $\bar{s}(t)$ performs at least as well as the original policy $s(t)$. By repeating this procedure, we can get a feasible policy which does not lie in $\text{int}(\mathcal{H}_c) \setminus \mathcal{H}_{inf}$ and performs at least as well as $s(t)$. ■

Corollary 1: Lemmas 1 and 3 imply that any policy $s(t)$ that, for some $t' \in [0, T]$, satisfies $s(t') > s_o(t')$ can be replaced by another feasible policy which never exceeds $s_o(t')$ and performs at least as well as $s(t)$. Therefore, we can limit our search for the optimal transmission policy to the following set of transmission policies.

$$\mathcal{R} \triangleq \{s(t) : s(t) \leq s_o(t), 0 \leq t \leq 1\} \tag{17}$$

Another way of interpreting this result is that there exists at least one optimal policy which intersects $\mathcal{H}_c$ only at points that are on the $s_o(t)$ curve.

In the following Lemma, we reduce the search space of transmission policies further.

Lemma 4: A transmission policy $s(t) \in \mathcal{R}$ for which there exists $t' \in [0, T]$ such that $s(t') < s_o(t')$ can be replaced with another transmission policy $\bar{s}(t) \in \mathcal{R}$ which satisfies $\bar{s}(t') = s_o(t')$ and performs at least as well as $s(t)$.

Proof: Define $\mathcal{S}$ as the region above the $s(t)$ curve in the same way as we defined $\mathcal{H}$, i.e.,

$$\mathcal{S} = \{(t, e) : 0 \leq t \leq T, s(t) \leq e \leq s(T)\}. \tag{18}$$

First, suppose that $\mathcal{S}$ is not convex. In this case, we can think of $s(t)$ as the cumulative harvested energy curve for another system. Then, we know from Lemma 3 that the lower path that tracks $\partial\,\text{conv}(\mathcal{S})$ from $(0, 0)$ to $(T, s(T))$ performs at least as well as $s(t)$, and thus can replace $s(t)$ without decreasing the achieved throughput.

Now, let $\mathcal{S}$ be a convex region. We have $\mathcal{S} \supset \mathcal{H}_c$ since $s(t) \leq s_o(t)$ for all $t \in [0, T]$. Let $e' = s(t')$ and $e'' = s_o(t)$. Then, $(t', e'') \in \partial\mathcal{H}_c$, and since $\mathcal{H}_c$ is convex, there exists a line that is tangent to $\mathcal{H}_c$ at (t', e''). Since $\mathcal{S}$ is convex, and

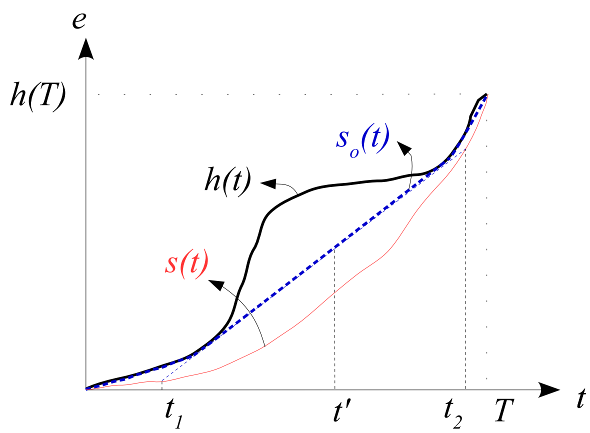

Fig. 4. A transmission policy $s(t)$ that falls below $s_o(t)$ at time t'.

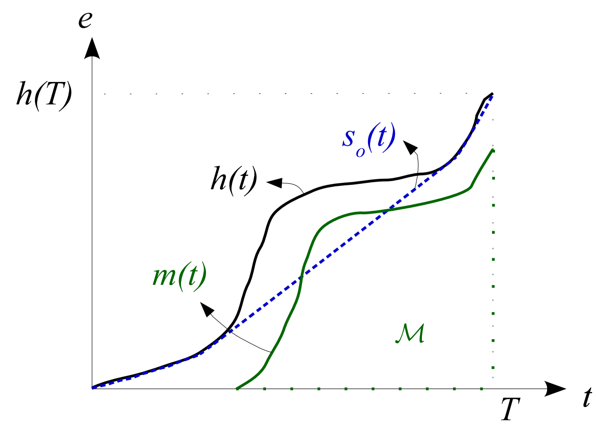

Fig. 5. An example for region $\mathcal{M}$.

$\mathcal{S} \supset \mathcal{H}_c$, this line must intersect $s(t)$ at two points, say (t_1, e_1) and (t_2, e_2) (see Fig. 4). Define a new policy $\bar{s}(t)$ as

$$\bar{s}(t) = \begin{cases} \frac{e_2-e_1}{t_2-t_1}(t-t_1)+e_1, & \text{if } t \in [t_1, t_2], \\ s(t), & \text{if } t \in [0,T] \setminus [t_1, t_2]. \end{cases} \quad (19)$$

The continuity of $\bar{s}(t)$ follows from the fact that the line intersects $s(t)$ at (t_1, e_1) and (t_2, e_2). Also, since the line is tangent to convex set $\mathcal{H}_c$, it cannot intersect $\text{int}(\mathcal{H}_c)$, and therefore, $\bar{s}(t) \in \mathcal{R}$. We can compute the difference in the throughput values achieved by $\bar{s}(t)$ and $s(t)$, and use Lemma 2 in the same way as we did in Lemma 3 to show that $\bar{s}(t)$ does not perform worse than $s(t)$. This completes the proof. ■

We now describe an optimal transmission policy using Corollary 1 and Lemma 4.

Theorem 1: An optimal solution of (4) with $f(p) = p$, $\forall p \in [0, \infty)$ and $E_{max}(t) = \infty$, $\forall t \in [0, T]$ can be given as

$$s_o(t) = \min\{e : (t,e) \in \partial\mathcal{H}_c\} \quad (20)$$

where $\mathcal{H}_c$ is as defined in (7). Moreover, (20) is the unique optimal policy if $r(p)$ is strictly concave in p.

Proof: We already know from Corollary 1 that there exists at least one optimal policy in $\mathcal{R}$. We also know from Lemma 4 that any policy that goes below $s_o(t)$ at some time can be replaced with another policy which matches $s_o(t)$ at that time, and does not perform worse than $s(t)$. Then, by repeating the procedure in Lemma 4, we can conclude that $s_o(t)$ performs at least as well as any other policy in $\mathcal{R}$; and thus, is optimal.

The uniqueness of $s_o(t)$ with a strictly concave rate function $r(p)$ follows from the fact that with a strictly concave rate function, we can use Jensen's inequality (Lemma 2) with strict inequality and show that any policy that goes above (Lemma 3) or below (Lemma 4) $s_o(t)$ is not only replaceable by $s_o(t)$, but also performs worse than $s_o(t)$. Thus, with a strictly concave rate function, the optimal transmission policy is unique and equal to $s_o(t)$. ■

In this section, we have provided the solution for $E_{max}(t) = \infty, \forall t \in [0, T]$. In the next section, we remove the infinite battery assumption and solve (4) for any finite valued $E_{max}(t)$.

IV. Optimal Policies for a Degrading Battery of Finite Capacity

In this section, we solve (4) for any battery size that may be decaying in time. The result is trivially applicable when the battery size is constant, i.e., $E_{max}(t) = E_{max}, \ \forall t \in [0, T]$. We still keep the assumption that $f(p) = p, \forall p \in [0, \infty)$. With a general $E_{max}(t)$ that is non-increasing in t, the minimum energy curve $m(t)$ no longer satisfies (5). Instead, we have (2) and since $h(t)$ is non-decreasing in t, and $E_{max}(t)$ is non-increasing in t, $m(t)$ is non-decreasing in t. Denote the region below $m(t)$ by $\mathcal{M}$ which can be expressed as

$$\mathcal{M} = \{(t,e) : 0 \le t \le T, 0 \le e \le m(t)\}. \quad (21)$$

See Fig. 5 for an example.

In what follows, we describe the solution for various instances of $\mathcal{M}$, and finally, arrive at a solution for any $\mathcal{M}$.

Lemma 5: If $\mathcal{M} \cap \mathcal{H}$ is not empty, then the problem can be divided into sub-problems, each of which either does not have such a non-empty intersection, or has a trivial solution.

Proof: The definition of $m(t)$ in (2) implies that $m(t) \le h(t)$ for all $t \in [0, T]$. Then,

$$\mathcal{M} \cap \mathcal{H} = \{(t, h(t)) : \forall t \in [0,T] : h(t) = m(t)\}, \quad (22)$$

that is, $\mathcal{M} \cap \mathcal{H}$ is the intersection of curves $h(t)$ and $m(t)$. Whenever $h(t) = m(t)$, the optimal transmission policy has to follow $h(t)$ since it would otherwise go below $m(t)$ which would be infeasible. For the remaining parts of the transmission, we have $h(t) > m(t)$ which means that we can consider these parts as subproblems for which this intersection will be empty. ■

As a result of Lemma 5, we can assume without loss of generality that $\mathcal{M} \cap \mathcal{H} = \emptyset$.

Lemma 6: If $\mathcal{M} \cap \mathcal{H}_c \subset \partial\mathcal{H}_c$, then there exists an optimal solution which is the same as the optimal solution found in the previous section with $m(t) = 0$.

Proof: We already know that the problem considered in the previous section is relaxed compared to the problem we have in this section. Hence, if $s_o(t)$ is feasible, it will

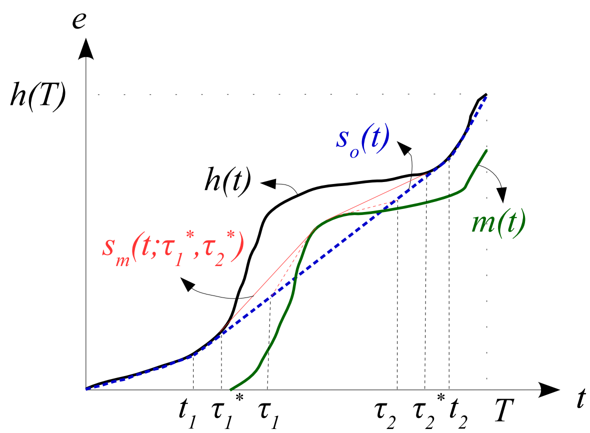

Fig. 6. Optimal transmission policy with a finite battery and a $\mathcal{M} \cap \mathcal{H}_c$ region that satisfies Property 1.

also be optimal. Suppose there exists $t' \in [0, T]$ such that $s_o(t') < m(t')$. Then, the point $(t', m(t'))$ is above $s_o(t')$, and thus, in $\operatorname{int}(\mathcal{H}_c)$. It is also in $\mathcal{M}$ since it is on the $m(t)$ curve. This implies that $\mathcal{M} \cap \mathcal{H}_c \not\subset \partial\mathcal{H}_c$ which is a contradiction. Therefore, $s_o(t) \geq m(t)$ for all $t \in [0, T]$, i.e., $s_o(t)$ is feasible, and thus optimal. ■

In light of Lemmas 5 and 6, we conclude that it only remains to find the optimal transmission policy for $\mathcal{M}$ such that $\mathcal{M} \cap \mathcal{H}_c \subset (\mathcal{H}_c \setminus \mathcal{H})$.

Any point $(t, e) \in \mathcal{M} \cap \mathcal{H}_c$ can be projected onto the $s_o(t)$ curve as $(t, s_o(t))$ where $s_o(t) \leq e$. Since all these projections are on $\partial\mathcal{H}_c$, each of them can be written as convex combination of two points in $\partial\mathcal{H}_c \cap \mathcal{H}$. First, suppose that $\mathcal{M} \cap \mathcal{H}_c$ satisfies the following property.

Property 1: There exist two points $(t_1, e_1), (t_2, e_2) \in \partial\mathcal{H}_c \cap \mathcal{H}$ such that for all $(t, e) \in \mathcal{M} \cap \mathcal{H}_c$, we have $(t, s_o(t)) = \lambda(t_1, e_1) + (1 - \lambda)(t_2, e_2)$ for some $\lambda \in [0, T]$. In other words, the projection of any point in $\mathcal{M} \cap \mathcal{H}_c$ can be written as a convex combination of the same two points in $\partial\mathcal{H}_c \cap \mathcal{H}$, possibly with different coefficients $\lambda \in [0, 1]$.

See Fig. 6 for an example of $\mathcal{M} \cap \mathcal{H}_c$ that satisfies Property 1. Without loss of generality, suppose that $t_1 < t_2$, and therefore $e_1 \leq e_2$. Consider the following transmission policy for $t_1 \leq t \leq t_2$, which is parametrized by τ_1 and τ_2 such that $t_1 \leq \tau_1 < \tau_2 \leq t_2$.

$$s_m(t; \tau_1, \tau_2) = \begin{cases} s_\partial(t), & \text{if } t \in [\tau_1, \tau_2] \\ \frac{e_2 - e_1}{t_2 - t_1}(t - t_1) + e_1, & \text{if } t \in [t_1, t_2] \setminus [\tau_1, \tau_2] \end{cases} \tag{23}$$

where $s_\partial(t)$ is used to denote the upper path that tracks $\partial\mathcal{C}$ from $(\tau_1, s_o(\tau_1))$ to $(\tau_2, s_o(\tau_2))$ where

$$\mathcal{C} \triangleq \operatorname{conv}\left((\mathcal{M} \cap \mathcal{H}_c) \cup \{(\tau_1, s_o(\tau_1)), (\tau_2, s_o(\tau_2))\}\right). \tag{24}$$

If $\mathcal{C} \cap \mathcal{H} \neq \emptyset$, then we can separate the problem into subproblems as described in Lemma 5. Without loss of generality, suppose $\mathcal{C} \cap \mathcal{H} = \emptyset$. The continuity of $s_m(t; \tau_1, \tau_2)$ follows from the fact that $s_o(t) = \frac{e_2 - e_1}{t_2 - t_1}(t - t_1) + e_1$ for all $t \in [t_1, t_2]$. By invoking Lemma 3, we conclude that the throughput achieved by $s_m(t; \tau_1, \tau_2)$ never decreases as τ_1 and τ_2 move closer to t_1 and t_2. Then, we need to minimize τ_1 and maximize τ_2 while keeping $t_1 \leq \tau_1$ and $\tau_2 \leq t_2$ and making sure that $s_m(t; \tau_1, \tau_2)$ does not enter $\operatorname{int}(\mathcal{H})$. We can decrease τ_1 until $\mathcal{C}$ is tangent to $\mathcal{H}$, or $\tau_1 = t_1$. Let τ_1^* denote the minimum such τ_1. Similarly, we can increase τ_2 until $\mathcal{C}$ is tangent to $\mathcal{H}$, or $\tau_2 = t_2$. Let τ_2^* denote the maximum such τ_2. It follows that $s_m(t; \tau_1^*, \tau_2^*)$ hits $\partial\mathcal{H}$, or $h(t)$ at $t = \tau_1^*$ and $t = \tau_2^*$. In other words, the optimal policy must deplete the battery at $t = \tau_1^*$ and $t = \tau_2^*$. Therefore, we can separate the problem into three subproblems as follows.

1) Throughput maximization for $0 \leq t \leq \tau_1^*$. This subproblem can be solved using the solution found in Section III since it does not have a non-empty $\mathcal{M} \cap \mathcal{H}_c$ region.
2) Throughput maximization for $\tau_1 \leq t \leq \tau_2^*$. For this problem, we know that $s_m(t; \tau_1^*, \tau_2^*)$ is optimal.
3) Throughput maximization for $\tau_2 \leq t \leq T^*$. This subproblem can also be solved using the solution found in Section III since it does not have a non-empty $\mathcal{M} \cap \mathcal{H}_c$ region.

To finalize our optimal solution, we remove the assumption that $\mathcal{M} \cap \mathcal{H}_c$ satisfies Property 1, and observe that any $\mathcal{M} \cap \mathcal{H}_c$ that may or may not satisfy Property 1, can be written as a disjoint union of regions that do satisfy Property 1. Let these regions be indexed by $i \in I$. We can calculate $\tau_{i,1}^*$, $\tau_{i,2}^*$, and the optimal transmission policy for $\tau_{i,1}^* \leq t \leq \tau_{i,2}^*$ using the methodology described above for each $i \in I$. We know that the battery is empty at $t = \tau_{i,1}^*$ and $t = \tau_{i,2}^*$ for all $i \in I$. Hence, we can start a new subproblem at $t = \tau_{i,1}^*$ and $t = \tau_{i,2}^*$ for all $i \in I$. We already have the optimal transmission policies for $t \in \bigcup_{i \in I} [\tau_{i,1}^*, \tau_{i,2}^*]$. For the subproblems in $[0, T] \setminus \bigcup_{i \in I} [\tau_{i,1}^*, \tau_{i,2}^*]$, we know that the optimal policy found in Section III can be used since they do not have non-empty $\mathcal{M} \cap \mathcal{H}_c$ regions. This concludes the description of our solution for any non-increasing non-negative $E_{max}(t)$.

Remark 1: Although we found the solution with the assumption that $E_{max}(t)$ is non-increasing in t, the validity of our solution only requires that $E_{max}(t) \geq 0$ for all $t \in [0, T]$. With an $E_{max}(t)$ function that is only non-negative valued, but not necessarily non-increasing, the resulting $m(t)$ curve will not be non-decreasing. In that case, we can define

$$\bar{m}(t) = \max_{0 \leq \tau \leq t} m(\tau) \tag{25}$$

which is non-decreasing and also satisfies $\bar{m}(t) \geq m(t)$ for all $t \in [0, T]$. Since we know that all feasible transmission policies are non-decreasing, we can find the optimal policy with $\bar{m}(t)$ as the minimum energy curve, and this policy will also be optimal with $m(t)$ as the minimum energy curve. Therefore, $E_{max}(t)$ does not actually have to be non-increasing. That said, a battery whose capacity increases over time is not very realistic. □

In the next section, we relax our assumption on $f(p)$ to a more general case, and solve (4).

V. Optimal Policies with a Processing Cost

In this section, we solve (4) with the following characterizations of $E_{max}(t)$ and $f(p)$.

- We set $f(p) = (\eta p - c)^+, \forall p \in [0,\infty)$ for some $0 \le \eta \le 1$ for an imperfect battery with constant proportional loss and a constant processing cost per unit time $c \ge 0$. Note that when the transmitter draws energy from the battery at a rate $0 \le p \le c/\eta$, the drawn power will be wasted since it is not sufficient for both processing and transmission. Then, it follows that the transmitter should either draw energy from the battery at a rate higher than c/η, or should not draw any energy and remain silent. In this case, the instantaneous transmit power and the instantaneous rate of consumption will be related as

$$p_T(t) = f(p(t)) = (\eta p(t) - c)^+, \forall t \in [0,T]. \quad (26)$$

- We study a generic battery capacity function $E_{max}(t)$ that is non-negative valued. As observed in Remark 1, if the minimum energy curve $m(t) = \max\{h(t) - E_{max}(t), 0\}$ that results from any non-negative $E_{max}(t)$ is not non-decreasing, we can solve the equivalent problem with $\bar{m}(t)$, defined in (25), as the minimum energy curve, and the optimal transmission policy that we find for the equivalent problem is also optimal for the original problem. Therefore, without loss of generality, we assume that $m(t)$ is non-decreasing.

Let us begin by studying the rate function. We know, that $r(p_T)$ is non-decreasing and concave in *transmit* power $p_T = f(p)$, and $r(0) = 0$. The rate can also be written as a function of the instantaneous rate of consumption p as

$$\begin{aligned}\tilde{r}(p) &\triangleq r(f(p)), & (27a)\\ &= r((\eta p - c)^+), & (27b)\\ &= \begin{cases} 0, & \text{if } 0 \le p \le c/\eta \\ r(\eta p - c), & \text{if } p \ge c/\eta \end{cases} & (27c)\end{aligned}$$

It is clear that $\tilde{r}(p)$ is not concave in p. However, it is concave on $[c/\eta, \infty)$ and equal to 0 everywhere else. We can concavify $\tilde{r}(p)$ by time sharing between achievable points on it, as done in [21], [25], [32], i.e.,

$$r_c(p) = \begin{cases} (p/p^*) r(\eta p^* - c), & \text{if } 0 \le p \le p^* \\ r(\eta p - c), & \text{if } p \ge p^* \end{cases} \quad (28)$$

where $p^* \in [c/\eta, \infty)$ can be found as

$$p^* \in \arg\max_{c/\eta \le p} \frac{r(\eta p - c)}{p}. \quad (29)$$

It is shown in [15] that p^* is unique if $r(p_T)$ is strictly concave in p_T. With a concave rate function, the uniqueness of p^* is not guaranteed. We can relax the constraint on p in (29) since $r(p_T)$ is non-negative valued and equal to 0 for all $p_T \le c/\eta$. Then, a necessary and sufficient condition of optimality can

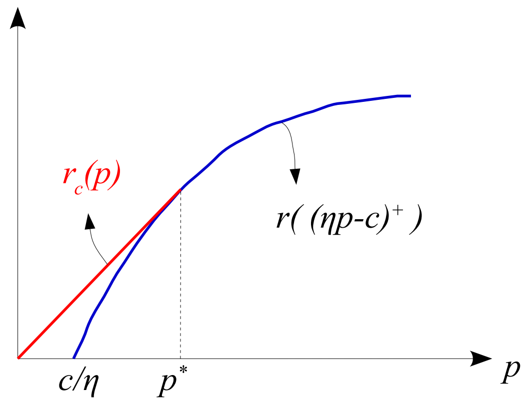

Fig. 7. An example for the concavification of $\tilde{r}(p)$.

be stated as

$$\frac{d}{dp}\left(\frac{r(\eta p - c)}{p}\right)\bigg|_{p=p^*} = 0 \quad (30)$$

which implies

$$p^* \eta r'(\eta p^* - c) - r(\eta p^* - c) = 0 \quad (31)$$

where $r'(\cdot)$ denotes the first derivative of $r(\cdot)$. Rearranging the terms, we get

$$\frac{r(\eta p^* - c)}{p^*} = r'(\eta p^* - c), \quad (32)$$

that is, we are searching for a point $p^* \in [c/\eta, \infty)$ such that the line that connects the origin to $(p^*, r(\eta p^* - c))$ is tangent to $r(\eta p^* - c)$ at p^*, see Fig. 7. As an example, consider a Gaussian channel, for which $r(p_T)$ can be given as

$$r(p_T) = \frac{1}{2}\log(1 + \gamma p_T). \quad (33)$$

Then, the optimality condition in (32) becomes

$$\ln(1 + \gamma(\eta p^* - c)) = \frac{\gamma \eta p^*}{1 + \gamma(\eta p^* - c)} \quad (34)$$

which can be solved numerically, or using the Lambert W function.

Having found a p^* that satisfies (32), we can compute $r_c(p)$ which we know is concave in p. We observe that with a processing cost per unit time, c, the efficient transmission strategy is to never transmit at a power value that is less than p^*. Instead of transmitting at $p < p^*$ for τ seconds, expending $p\tau$ amount of energy, the transmitter should transmit at p^* for $\tau p/p^*$ seconds, and remain silent for $\tau(1-p/p^*)$ seconds. This is a convex combination of two feasible strategies: silence with weight $1-p/p^*$, and transmission at p^* with weight p/p^*; and thus, explains the linear structure of $r_c(p)$ for $0 \le p \le p^*$.

We can now solve (4) in the setting described at the beginning of this section using $r_c(p)$ and an alternative channel setting. Consider another instance of the system model described in Section II where $f(p) = p, \forall p$, and $r_c(p)$ is the

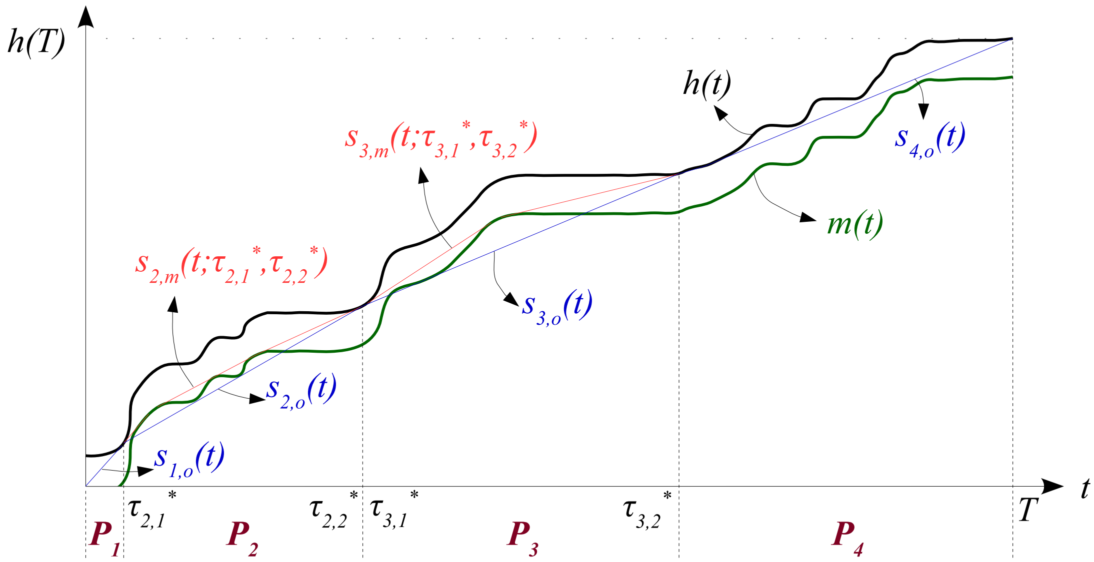

Fig. 8. An example for the optimal policy found with our methodology.

instantaneous rate that can be achieved with transmit power or instantaneous rate of consumption equal to p. The optimal transmission policy for this setup can be found by using our findings in Sections V and IV, let it be denoted by $\hat{s}(t)$, and let $\hat{p}(t)$ denote its first derivative, i.e., the optimal power policy. If the rate $r_c(p)$ is not achieved at instantaneous power p for $p < p^*$, we would be done. However, it can achieve the linear portion of the curve $r_c(p)$ for $p < p^*$ only through time sharing. Then, we must modify $\hat{p}(t)$ without changing the energy expended and the amount of bits departed in a given in time. We state and prove the Lemma which we later use to find optimal policies for the setting of interest.

Lemma 7: Let $\tau \in [0, T)$, and $\epsilon > 0$ be arbitrarily chosen such that $\hat{p}(t) \le p^*$ for all $t \in [\tau, \tau+\epsilon)$. There exists a power policy $p_m(t)$ such that

1) $p_m(t) \in \{0, p^*\}, \forall t \in [\tau, \tau+\epsilon)$,
2) $\int_\tau^{\tau+\epsilon} p_m(t)dt = \int_\tau^{\tau+\epsilon} \hat{p}(t)dt$,
3) $\int_\tau^{\tau+\epsilon} r_c(p_m(t))dt = \int_\tau^{\tau+\epsilon} r_c(\hat{p}(t))dt$.

Proof: Define λ as

$$\lambda = \frac{1}{p^*\epsilon}\int_\tau^{\tau+\epsilon} \hat{p}(t)dt. \tag{35}$$

Since $\hat{p}(t) \le p^*$ for all $t \in [\tau, \tau+\epsilon)$, we have that

$$\int_\tau^{\tau+\epsilon} \hat{p}(t)dt \le \int_\tau^{\tau+\epsilon} p^*(t)dt = p^*\epsilon, \tag{36}$$

which implies that $\lambda \in [0, 1]$. Let

$$p_m(t) = \begin{cases} p^*, & \text{if } \tau \le t \le \tau+\lambda\epsilon \\ 0, & \text{if } \tau+\lambda\epsilon \le t \le \tau+\epsilon \end{cases}. \tag{37}$$

Then,

$$\int_\tau^{\tau+\epsilon} p_m(t)dt = \int_\tau^{\tau+\lambda\epsilon} p^* dt, \tag{38a}$$

$$= \lambda p^*\epsilon, \tag{38b}$$

$$= \int_\tau^{\tau+\epsilon} \hat{p}(t)dt. \tag{38c}$$

$r_c(p)$ is linear on $[0, p^*]$, thus

$$\int_\tau^{\tau+\epsilon} r_c(\hat{p}(t))dt = r_c\left(\int_\tau^{\tau+\epsilon} \hat{p}(t)dt\right), \tag{39a}$$

$$= r_c(\lambda p^*\epsilon), \tag{39b}$$

$$= \lambda\epsilon r_c(p^*), \tag{39c}$$

and

$$\int_\tau^{\tau+\epsilon} r_c(p_m(t))dt = \int_\tau^{\tau+\lambda\epsilon} r_c(p^*)dt, \tag{40a}$$

$$= \lambda\epsilon r_c(p^*). \tag{40b}$$

This completes the proof. ■

Finally, we can construct the optimal power policy $p_m(t)$ for the original setting. First, we set $p_m(t) = \hat{p}(t)$ whenever $\hat{p}(t) \ge p^*$. Let $U = \{t : \hat{p}(t) < p^*\}$. We can partition U into intervals of sufficiently small length so that when Lemma 7 is applied on each of these partitions, the constraints in (4b) are not violated. The power policy $p_m(t)$ we get at the end of this procedure is an optimal solution of (4) in the general case with any concave rate, non-negative valued $E_{max}(t)$, any processing cost $c \ge 0$, and any proportional loss coefficient $0 \le \eta \le 1$.

VI. Example and Numerical Results

In this section, we provide an example of the optimal policy found using our methodology, and compare its performance with that of optimal policies with discrete energy arrivals.

Fig. 8 shows an example of the optimal policy found with our methodology. The finite capacity of the battery results in the separation of the problem into four subproblems, marked

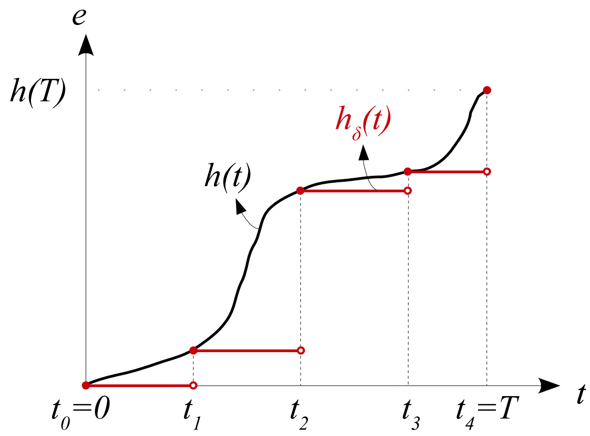

Fig. 9. An example for the generation of $h_\delta(t)$ using $h(t)$.

P_1, P_2, P_3, and P_4 in the figure. The procedure to identify these subproblems is as follows. We first compute $s_o(t)$ which intersects $\mathcal{M}$. Thus, the solution method in Section IV applies. The resulting $\mathcal{M} \cap \mathcal{H}_c$ region is a disjoint union of two regions each of which satisfies Property 1. We can calculate $(\tau_{i,1}^*, \tau_{i,2}^*)$ as the maximizer of the throughput achieved by (23) for both of these two regions, i.e., $i = 2, 3$. Then, the divisions will be at $\tau_{2,1}^*$, $\tau_{2,2}^*$, $\tau_{3,1}^*$, and $\tau_{3,2}^*$. If $\tau_{2,2}^* \neq \tau_{3,1}^*$, then there will be another subproblem for $\tau_{2,2}^* \leq t \leq \tau_{3,1}^*$ which can be trivially optimized by following $h(t)$. Thus, without loss of generality, suppose $\tau_{2,2}^* = \tau_{3,1}^*$. Let $s_{i,o}(t)$ be the lower boundary of the $\mathcal{H}_c$ region found for the ith subproblem, for all $i = 1, 2, 3, 4$. We have $\mathcal{M} \cap \mathcal{H}_c = \emptyset$ for the first and fourth subproblems, that is, the minimum energy curve $m(t)$ does not affect the optimal policy. Thus, Lemmas 3 and 4 apply, and the optimal policies for these two subproblems are $s_{1,o}(t)$ and $s_{4,o}(t)$, respectively. However, $\mathcal{M} \cap \mathcal{H}_c \neq \emptyset$ for the second and third subproblems. Therefore, we have to calculate $s_{2,m}(t; \tau_{2,1}^*, \tau_{2,2}^*)$ and $s_{3,m}(t; \tau_{3,1}^*, \tau_{3,2}^*)$ as explained in Section IV to find the optimal policies for these two subproblems.

In order to quantify the impact of discretizing the energy arrivals on optimum throughput, we let $0 < \delta \leq T$, and define

$$t_i = \begin{cases} 0, & \text{if } i = 0 \\ i\delta, & \text{if } i = 1, 2, \ldots, \lceil \frac{T}{\delta} \rceil - 1 \\ T, & \text{if } i = \lceil \frac{T}{\delta} \rceil \end{cases} \quad (41)$$

and a discretized cumulative harvested energy curve $h_\delta(t)$ as

$$h_\delta(t) = \begin{cases} h(t_{i-1}), & \text{if } t_{i-1} \leq t < t_i \\ h(T), & \text{if } t = T \end{cases} \quad (42)$$

for all $i = 1, 2, \ldots, \lceil \frac{T}{\delta} \rceil$. See Fig. 9 for an example. Similarly, we can discretize $m(t)$ by replacing it with $m_\delta(t)$. Fig. 10 shows average rates achieved by the optimal solution of the discretized problem for varying δ and E_{max} for a Gaussian channel with available bandwidth 1 MHz, channel gain -110 dB, noise density 10^{-19} W/Hz, and $g(t) \in [0, 200\text{ mW}]$. $\delta = 0$

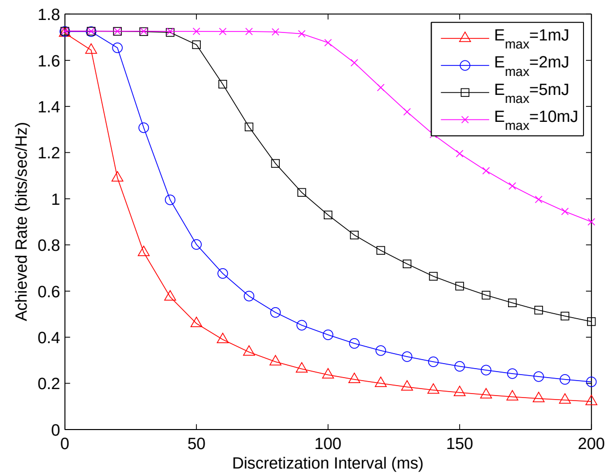

Fig. 10. Average rates achieved by the optimal solution of the discretized problem for varying δ and $E_{max}(t) = E_{max}$.

represents the original problem with continuous arrivals. We observe that as the discretization interval gets coarser, the optimal achievable average rate decreases, i.e., the assumption of discrete energy arrivals at the transmitter limits the transmitter's throughput. This limitation becomes more severe as E_{max} decreases. This is because, in the discretized problem, the transmitter receives a large amount of energy at once which it would otherwise receive over an interval of duration δ. When this amount is larger than E_{max}, the transmitter loses the difference which, in the continuous case, is utilized.

VII. Conclusion

In this paper, we have studied an energy harvesting transmitter which receives a continuous flow of energy, rather than discrete packets of energy. We used tools from convex analysis to solve the throughput maximization problem with a finite capacity battery that may be degraded, and a processing cost. We have shown that the optimal solution with an infinite capacity battery can be characterized as the boundary of a well defined region, which is the convex hull of the area above the cumulative harvested energy curve, confirming that the shortest path interpretation of the optimal policy remains valid. For finite capacity batteries or processing costs, we have shown that the solution can also be computed using this region. Through simulations, we observed that discretization of the throughput maximization problem yields a reduced performance, especially with coarse quantization of the harvested energy.

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