Matching Games for Wireless Networks with Energy Cooperation

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Abstract—We consider a wireless ad hoc network composed of N transmitters and M receivers which are all selfish in the sense that they wish to optimize their individual utilities rather than a network wide utility. Each node can acquire energy from a supplier at a price to power the transmission or reception of data. For such a network, we consider a matching game played between the transmitters and the receivers. The transmitters compute the optimal rate for them and propose this to a receiver. The receivers determine the best proposal they have received to maximize their utilities. We identify the optimal decisions for all nodes and the resulting utilities. We next consider a Vickrey auction between transmitters which have proposed to the same receiver. We show that the transmitters can compete with each other by offering energy transfer to the receiver. The energy transfer reduces the processing costs of the receiver and influences its decision, thereby pointing to the merit of energy cooperation. We observe that populating the network with additional nodes generally results in more options for all nodes to choose from, and larger rates for the entire network, which are improved even further by energy cooperation.

I. INTRODUCTION

Wireless networks with nodes that can benefit from establishing cooperation pairs arise in many practical communication scenarios. Among these are cloud radio access networks [1] where a base station can send its data to a cloud for computing, sensor networks [2] where the sensors can pair up with relays for the delivery of their measurements, and vehicular networks [3] where the transmitter-receiver pairs may change during the communication session due to the dynamic network topology. The majority of previous work on cooperation in wireless networks has assumed altruistic behavior for all nodes where they follow the directions of a network operator and collectively improve a network wide utility, e.g., the sum throughput of the network. It remains interesting to study the selfish behavior of wireless nodes that would rather improve their individual utilities than work together for the sake of the entire network, which is the focus of this paper.

In this paper, we consider a general model for such networks whose nodes are capable of energy cooperation, using the framework of matching games [4]. In particular, we consider a wireless ad hoc network of N transmitters and M receivers which are selfish, but are willing to form transmitter-receiver

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pairs if such pairing improves both sides' utilities. We consider that the expenditure of energy at each node comes at a price and results in a decrease in the node's utility. We formulate a matching game between the transmitters and receivers where the transmitters propose to the receivers with the optimal communication rate for the transmitters' utilities. The receivers choose one among all proposals they have received to improve their own utilities. We find the optimal decisions for all nodes and derive the resulting utilities. We next provide the transmitters with the knowledge of the utility functions of the receivers so that they can take into account the needs of the receivers when they determine their proposals. In addition, we let the transmitters offer to transfer energy to their favorite receiver, i.e., energy cooperation. This allows the transmitters to assist the receivers with their processing costs to increase their chances of forming a beneficial cooperation pair. We modify the well known Deferred Acceptance Algorithm [4] to solve these games. We observe that the competition between the nodes facilitated by the matching framework becomes more intense with the addition of energy cooperation and results in improved rates for the whole network. In addition, we observe that our modified approach yields larger rates and requires a smaller number of proposals before it can identify the solution as compared to the Deferred Acceptance Algorithm.

Related work: Matching games are a suitable model for communities of individuals with conflicting interests that may cooperate in pairs for mutual benefit [4], [5] and have previously been employed for resource allocation in wireless networks [6]-[8]. Energy cooperation has been proposed as a way of increasing the energy efficiency of wireless networks by means of a transfer of energy from energy rich nodes to energy deficient nodes [9]-[11]. References [9], [10] have studied the sum throughput maximization problem for energy harvesting multi terminal networks with energy transfer. Reference [11] has proposed energy transfer over radio frequencies (RF) performed simultaneously with the transfer of data. RF energy harvesting has been considered in a number of models including cognitive radio networks [12] which has studied cognitive radio networks with primary users whose radio transmission can be used as a source of energy by the secondary users, and in non-cooperative or leaderfollower game theoretic settings [13] where we have modeled cooperation between selfish nodes as noncooperative games and Stackelberg games.

While the majority of work on energy management in wireless networks has been for transmission energy, the receivers' processing costs have recently gained attention [14]–[16]. Reference [14] has studied an energy harvesting network with sampling and decoding costs at the receiver and shown that when the battery at the receiver is the bottleneck of the system, it is optimal for the receiver to sample data packets at every opportunity and decode them only to avoid battery overflows. Reference [15] has proposed a framework for utility maximization in wireless networks with energy harvesting transmitters and receivers. Reference [16] has studied decoding costs at the receivers in energy harvesting networks with energy harvesting receivers. Reference [16] has considered a decoding cost that is convex in the rate and in particular, an exponential cost model as we will in the sequel.

II. SYSTEM MODEL

Consider an ad hoc network with transmitters T_n , $n \in \mathcal{N} \triangleq \{1, 2, \ldots, N\}$, and receivers R_m , $m \in \mathcal{M} \triangleq \{1, 2, \ldots, M\}$ with block fading as shown in Fig. 1. The transmitters have data which they can transmit to the receivers over orthogonal links. Thus, without loss of generality, the noise at each receiver is zero-mean and unit-variance. For clarity of exposure, we consider a time slotted scenario with slots of equal duration. For a given time slot, the block fading coefficient from T_n to R_m , which we denote by $h_{n,m}$, is drawn from a continuous distribution.

Each node has access to an energy supplier that can provide any desired amount of energy at a price. T_n can purchase energy from its supplier at a price of σ_n , and likewise, R_m can purchase energy at a price of $\bar{\sigma}_m$. The prices lead to a reduction of the total reward that is due to the expended energy. The unit for the price is bits/Joule, leading to the total reward in bits as the total bits transmitted or received minus the energy cost.

Some of the game formulations considered in this work allow the transmitters to transfer energy to the receivers. For this case, we consider that R_m has a harvesting efficiency of $\eta_m \in [0,1], m \in \mathcal{M}$, i.e., if R_m receives E units of energy from an energy cooperating transmitter, it will be able to utilize ηE units while the remaining $(1 - \eta)E$ units will be lost. The nodes do not have access to any other source of energy for transmission or decoding, i.e., they must either acquire energy from the supplier or harvest energy from an energy cooperating node's transmission.

During a given time slot, each receiver is interested in receiving data from one transmitter only, and likewise, each transmitter wishes to send data to one receiver only. We focus on one-to-one matchings in this work and leave the study of many-to-one matchings as future work. At the beginning of each slot, transmitter-receiver pairs are formed which will communicate over the orthogonal link reserved for the transmitter for the duration of the time slot.



Fig. 1. The N-by-M ad hoc network with energy cooperation. For clarity of exposition, only one energy transfer is shown as a dotted line with the harvesting efficiency of the corresponding receiver.

Suppose for a given time slot, nodes T_n and R_m , for some $n \in \mathcal{N}$ and $m \in \mathcal{M}$, are matched with each other and agree on a data rate of $r_{n,m}$. We begin with a general definition of utilities for all transmitters and receivers which are given as

$$\iota_{n|m}(r_{n,m}) = \rho_n(r_{n,m}) - \sigma_n \kappa_n(r_{n,m}) \tag{1}$$

for T_n given it is matched to R_m , and

$$\bar{u}_{m|n}(r_{n,m}) = \bar{\rho}_m(r_{n,m}) - \bar{\sigma}_m \bar{\kappa}_m(r_{n,m}) \tag{2}$$

for R_m given it is matched to T_n . Here, $\rho_n(r_{n,m})$ and $\bar{\rho}_m(r_{n,m})$ are concave and non-decreasing in $r_{n,m}$, and represent the reward that nodes T_n and R_m obtain for transmitting or receiving data at rate $r_{n,m}$, respectively. Conversely, $\kappa_n(r_{n,m})$ and $\bar{\kappa}_m(r_{n,m})$ are convex and non-decreasing in $r_{n,m}$, and represent the energy cost of nodes T_n and R_m for transmitting or receiving data at rate $r_{n,m}$, respectively. Note that the reward and cost functions are averaged over the duration of the time slot.

For clarity of exposition, we focus on the following selection of reward and cost functions, recalling that our results are valid for any concave reward and convex cost selection:

$$\rho_n(r_{n,m}) = \lambda_n r_{n,m},\tag{3}$$

$$\bar{\rho}_m(r_{n,m}) = \lambda_m r_{n,m},\tag{4}$$

$$\kappa_n(r_{n,m}) = \frac{1}{h_{n,m}} \left(2^{2r_{n,m}} - 1 \right), \tag{5}$$

$$\bar{\kappa}_m(r_{n,m}) = c_m 2^{\alpha_m r_{n,m}} + \beta_m r_{n,m} + \gamma_m, \qquad (6)$$

for some $\lambda_n, \bar{\lambda}_m, c_m \alpha_m, \beta_m \ge 0$ and $\gamma_m \in \mathbb{R}$. In other words, we consider linear rewards (3) and (4) for both nodes, additive white Gaussian noise at the receivers leading to the energy cost

for T_n given in (5), and a general processing cost for R_m given in (6) which addresses exponential and linear processing costs and activation costs. The resulting utilities for nodes T_n and R_m are expressed as

$$u_{n|m}(r_{n,m}) = \lambda_n r_{n,m} - \frac{\sigma_n}{h_{n,m}} \left(2^{2r_{n,m}} - 1 \right), \tag{7}$$

$$\bar{u}_{m|n}(r_{n,m}) = \bar{\lambda}_m r_{n,m} - \bar{\sigma}_m \left(c_m 2^{\alpha_m r_{n,m}} + \beta_m r_{n,m} + \gamma_m \right).$$
(8)

Lastly, we define $\mathcal{T} \triangleq \{T_n, n \in \mathcal{N}\}\)$ and $\mathcal{R} \triangleq \{R_m, m \in \mathcal{M}\}\)$ as the set of all transmitters and the set of all receivers, respectively. Sets \mathcal{N} and \mathcal{M} index sets \mathcal{T} and \mathcal{R} , respectively. In the sequel, we consider two matching game formulations for our model where each transmitter proposes to the receivers. Each transmitter aims to maximize its utility that results from a rate value which the transmitter and the matched receiver can agree upon.

III. MATCHING GAMES

A. Preliminaries

We begin by defining fundamental concepts from matching theory [4], [5].

Definition 1: A matching is a function $\mu: \mathcal{T} \cup \mathcal{R} \to \mathcal{T} \cup \mathcal{R}$ satisfying

- 1) $\mu(T_n) = R_m$ if and only if $\mu(R_m) = T_n$ for all $n \in \mathcal{N}$, $m \in \mathcal{M}$,
- 2) $\mu(T_n) \in \mathcal{R}$ or $\mu(T_n) = T_n$ for all $n \in \mathcal{N}$,
- 3) $\mu(R_m) \in \mathcal{T}$ or $\mu(R_m) = R_m$ for all $m \in \mathcal{M}$.

The definition of matchings requires that μ be a bijection, i.e., each node in the network can either be matched to only one other node or to itself, and it must be equal to its inverse, i.e., $\mu(\mu(K)) = K$ for any node $K \in \mathcal{T} \cup \mathcal{R}$.

Definition 2: Preference relations \succ_n on \mathcal{R} and $\overline{\succ}_m$ on \mathcal{T} for all $n \in \mathcal{N}, m \in \mathcal{M}$ are strict and complete partial orders.

Here, the preference relations symbolize each node's preference over all nodes on the other side of the network. That is, $R_m \succ_n R_{m'}$ means that T_n prefers R_m over $R_{m'}$, and likewise, $T_n \succ_m T_{n'}$ means that R_m prefers T_n over $T_{n'}$. We assume that there are no ties, i.e., the preference relations are strict. This is in line with our selection of block fading coefficients which are drawn from continuous distributions, resulting in strict preferences with probability 1. The completeness of the preference relations means that each node has a favorite among any collection of nodes from the other side of the network, i.e., for all $n \in \mathcal{N}$ and $\mathcal{M}' \subset \mathcal{M}$, there exists $m \in \mathcal{M}'$ such that $R_m \succ_n R_{m'}$ for all $m' \in \mathcal{M}' \setminus \{m\}$. Likewise, for all $m \in \mathcal{M}$ and $\mathcal{N}' \subset \mathcal{N}$, there exists $n \in \mathcal{N}'$ such that $T_n \succ_m T_{n'}$ for all $n' \in \mathcal{N}' \setminus \{n\}$.

Definition 3: Matching μ is stable if there exists no $(T_n, R_m) \in \mathcal{T} \times \mathcal{R}$ such that $\mu(T_n) \neq R_m$, but $R_m \succ_n \mu(T_n)$ and $T_n \succ_m \mu(R_m)$. In other words, there does not exist a transmitter-receiver pair that prefer each other and yet are not matched to each other, i.e., all nodes are satisfied by μ .

Definition 4: Stable matching μ is optimal for the transmitters (resp. the receivers) if the utility of T_n (resp. R_m) under

 μ is no less than its utility under any other stable matching μ' for all $n \in \mathcal{N}$ (resp. all $m \in \mathcal{M}$).

Although there may exist multiple stable matchings, the optimal matching must be unique, provided that it exists, due to the fact that all preference relations are strict. We next study the matching game given by $(\{\mathcal{T}, \mathcal{R}\}, \{\succ_n, \bar{\succ}_m\})$ and how energy cooperation impacts the resulting matchings. We consider the case where the transmitters propose to the receivers and note that our results can readily be extended to the case where the receivers propose.

B. A Matching Game

Initially, we assume that the transmitters have no knowledge of the other nodes' utility functions or the strategies available to them. However, T_n knows $h_{n,m}$ for all $m \in \mathcal{M}$. T_n 's best strategy is therefore to maximize its own utility, i.e.,

$$r_{n,m}^* = \underset{r_{n,m} \ge 0}{\arg \max} u_{n|m}(r_{n,m})$$
(9)

$$= \underset{r_{n,m} \ge 0}{\arg \max} \lambda_n r_{n,m} - \frac{\sigma_n}{h_{n,m}} \left(2^{2r_{n,m}} - 1 \right)$$
(10)

$$=\frac{1}{2}\log\left(\frac{\lambda_n h_{n,m}}{2\sigma_n \ln 2}\right) \tag{11}$$

where the trivial case of $r_{n,m}^* = 0$ when (11) turns out to be negative is omitted for brevity as will other trivial special cases in the sequel. At rate $r_{n,m}^*$, T_n 's utility is given as

$$u_{n|m}(r_{n,m}^*) = \frac{\lambda_n}{2} \log\left(\frac{\lambda_n h_{n,m}}{2\sigma_n \ln 2}\right) + \frac{\sigma_n}{h_{n,m}} - \frac{\lambda_n}{2\ln 2}.$$
 (12)

 T_n can use (12) to find its favorite receiver among any collection of receivers $\mathcal{R}' \subset \mathcal{R}$, and subsequently characterize its preference relation \succ_n . Note that (12) depends on receiver index m only through $h_{n,m}$ and it is convex in $\frac{\sigma_n}{h_{n,m}}$. Therefore, T_n 's favorite receiver in \mathcal{R}' is either R_{m_1} or R_{m_2} , whichever results in a larger utility for T_n where indices m_1 and m_2 are found as

$$m_1 = \underset{m: \mathcal{B}}{\arg\max} h_{n,m}, \tag{13}$$

$$m_2 = \underset{m: R_m \in \mathcal{R}'}{\arg\min} h_{n,m}.$$
 (14)

Starting with $\mathcal{R}' = \mathcal{R}$, T_n finds $R_m \succ_n R_{m'}$ for all $m' \in \mathcal{R}' \setminus \{R_m\}$, and next finds the second favorite receiver by setting $\mathcal{R}' = \mathcal{R} \setminus \{R_m\}$. Continuing in this fashion, preference relation \succ_n is identified for all $n \in \mathcal{N}$ (see Algorithm 1, lines 2–8 for a detailed description).

For the receivers' preference relations, suppose R_m receives a proposal from all $T_n \in \mathcal{T}_m \subset \mathcal{T}$ where we define \mathcal{T}_m to be the set of all transmitters which have proposed to R_m with a rate offer. The ideal proposal for receiver m would maximize its utility, i.e.,

$$r_{n,m}^{\dagger} = \operatorname*{arg\,max}_{r_{n,m} \ge 0} \bar{u}_{m|n}(r_{n,m})$$
 (15)

$$= \underset{r_{n,m} \ge 0}{\arg \max} \, \bar{\lambda}_m r_{n,m} - \bar{\sigma}_m (c_m 2^{\alpha_m r_{n,m}} + \beta_m r_{n,m} + \gamma_m)$$
(16)

$$= \frac{1}{\alpha_m} \log\left(\frac{\bar{\lambda}_m/\bar{\sigma}_m - \beta_m}{c_m \alpha_m \ln 2}\right).$$
(17)

We observe that $\bar{u}_{m|n}(r_{n,m})$ is concave in $r_{n,m}$. Therefore, R_m finds its favorite among all proposals it has received from the transmitters in \mathcal{T}_m as the proposal of T_{n_1} or T_{n_2} , whichever results in a larger utility for R_m where indices n_1 and n_2 are identified as

$$n_1 = \underset{\substack{n: \ T_n \in \mathcal{T}_m, \\ r_n^* = m \le r_n^{\dagger}, m}}{\arg \max} r_{n,m}^*, \tag{18}$$

$$n_{2} = \arg \min_{\substack{n: \ T_{n} \in \mathcal{T}_{m}, \\ r_{n,m}^{*} > r_{n,m}^{\dagger}}} r_{n,m}^{*}.$$
(19)

Note that R_m can identify its preference relation \succ_m over \mathcal{T}_m using a similar procedure to the one described above for the transmitters, i.e., R_m starts with \mathcal{T}_m , finds its favorite transmitter in \mathcal{T}_m , removes this transmitter from \mathcal{T}_m , finds the second favorite transmitter and so on. However, as will be seen in Algorithm 1, our solution requires only the favorite proposal.

Now that matching game $(\{\mathcal{T}, \mathcal{R}\}, \{\succ_n, \bar{\succ}_m\})$ is fully characterized, we can identify the optimal matching for our setting. In order to accomplish this, we adopt the Deferred Acceptance Algorithm (DAA) proposed in [4] to our setting. It is shown in [4, Theorem 2] that DAA finds the unique stable matching that is optimal for the proposing nodes, in our case, the transmitters. In this algorithm, the transmitters first propose to their favorite receivers. Each receiver finds the one proposal that yields the largest receiver utility and rejects all others. In the next iteration, the rejected transmitters propose to their second favorite receivers and the receivers find the best proposal among all new proposals and the best proposal from the previous iteration. In this fashion, the receivers identify the best proposal for themselves, rejecting all others, but *defer* the acceptance of said proposal until they have seen all of their options.

In our implementation of this algorithm, we improve upon the resulting utilities by imposing that the transmitters refrain from proposing to receivers which yield negative utilities for them. Likewise, we require that receivers prefer being matched to themselves if the best proposal they receive results in a negative utility for them. This modification eliminates all matches which result in negative utilities while retaining those with positive utilities, and necessarily results in improved utilities for the whole network. In addition, this modification is in line with the selfish nature of the nodes in our model since they cannot be expected to tolerate negative utilities which they can easily improve by solitude. We provide the complete optimal solution of $(\{\mathcal{T}, \mathcal{R}\}, \{\succ_n, \bar{\succ}_m\})$, including the computation of preference relations and the Modified DAA, in Algorithm 1.

We next consider the same game under a different setting where each transmitter is provided with additional knowledge, i.e., the utility functions of the receivers, in order to facilitate competition among the transmitters. We solve the game for

Algorithm 1 Optimal solution μ of $(\{\mathcal{T}, \mathcal{R}\}, \{\succ_n, \overline{\succ}_m\})$.

II The transmitters identify their preference relations \succ_n . 1: for n = 1, 2, ..., N do

2: Initialize $\mathcal{R}' = \mathcal{R}$.

- 3: while $\mathcal{R}' \neq \emptyset$ do
- 4: Find R_{m_1} and R_{m_2} using (13) and (14).

5: Identify the favorite receiver as $R_m = R_{m_1}$ or R_{m_2} .

6: Update $\mathcal{R}' := \mathcal{R}' \setminus \{R_m\}.$

7: Update \succ_n such that $R_m \succ_n R_{m'}, \forall R_{m'} \in \mathcal{R}'$.

8: end while9: end for

// The Modified Deferred Acceptance Algorithm.

10: Initialize $\mathcal{R}_n = \mathcal{R}, \forall n \in \mathcal{N}; \mu(K) = K, \forall K \in \mathcal{T} \cup \mathcal{R}.$

11: Remove all R_m yielding $u_{n|m}(r_{n,m}^*) < 0$ from $\mathcal{R}_n, \forall n$.

12: while $\exists n \in \mathcal{N} : \mu(T_n) = T_n$ and $\mathcal{R}_n \neq \emptyset$ do

- 13: **for** n = 1, 2, ..., N **do**
- 14: **if** $\mu(T_n) = T_n$ and $\mathcal{R}_n \neq \emptyset$ **then**
 - T_n finds its favorite $R_m \in \mathcal{R}_n$ and proposes (11).

16: Update $\mathcal{R}_n := \mathcal{R}_n \setminus \{R_m\}.$

17: **end if**

15:

23:

24:

- 18: **end for**
- 19: **for** m = 1, 2, ..., M **do**
- 20: **if** $\mathcal{T}_m \neq \emptyset$ **then**

21: R_m finds its favorite $T_n \in \mathcal{T}_m \cup \{\mu(R_m)\}$ using (18) and (19).

22: **if** $\bar{u}_{m|n}(r^*_{n,m}) \ge 0$ then

Set
$$T'_m = \mu(R_m)$$
, and update $\mu(T'_m) = T'_m$.

Update $\mu(R_m) = T_n$, $\mu(T_n) = R_m$.

25: **end if**

26: **end if**

27: **end for**

28: end while

this case using a modified version of Algorithm 1 which again identifies an optimal matching.

C. A Matching Game with Energy Cooperation

Consider now that the transmitters are aware of the utility functions of the receivers. This additional knowledge allows them to tailor their proposals better to the needs of the receivers. In this setup, we consider the additional incentive of energy cooperation from the transmitters to their favorite receiver in order to promote their proposals over others. Note that this was not possible for the setting in Section III-B since the transmitters could not compute the ideal proposal for their favorite receiver, and therefore could not compete with each other directly. We incorporate energy cooperation into our model by modifying the utilities as

$$u_{n|m}(r_{n,m}, p_{n,m}) = \lambda_n r_{n,m} - \frac{\sigma_n}{h_{n,m}} \left(2^{2r_{n,m}} - 1 \right) - \sigma_n p_{n,m}$$
(20)

$$\bar{u}_{m|n}(r_{n,m}, p_{n,m}) = \bar{\lambda}_m r_{n,m} - \bar{\sigma}_m (c_m 2^{\alpha_m r_{n,m}} + \beta_m r_{n,m} + \gamma_m - p_{n,m} h_{n,m} \eta_m)$$
(21)

where $p_{n,m}$ is the amount of energy offered to R_m by T_n averaged over the duration of the time slot for consistency with other average quantities in our model.

For the receivers that receive multiple proposals, we employ a Vickrey auction [17] between the proposing transmitters to determine which one should be matched to the receiver. A Vickrey auction is a second price sealed bid auction where the bidder with the highest bid wins the auction, but pays the second highest bid only. Due to the second price property, the bidders are encouraged to bid their true valuations of the auction item, here the receiver with multiple proposals [18]. Consequently, the transmitters bid the highest receiver utility they can provide while ensuring that their own utilities are nonnegative. Since the winner has to provide the second highest bid only, it can modify its bid and obtain a positive utility for itself. Therefore, Vickrey auctions result in improved utilities for the auctioneers without necessitating vanishing utilities for the bidders.

 T_n first uses (12) to find its favorite receiver among any collection of receivers $\mathcal{R}' \subset \mathcal{R}$, and similarly generates its preference relation \succ_n . Note that the transmitter utilities at this point are the same as those in Section III-B since all $p_{m,n} = 0$ before the inter-transmitter competition by means of a Vickrey auction ensues. T_n can next compute its bid to its favorite receiver, say R_m , as

$$(r_{n,m}^*, p_{n,m}^*) = \arg\max_{(r_{n,m}, p_{n,m}) \ge 0} \bar{u}_{m|n}(r_{n,m}, p_{n,m})$$
(22a)

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a.t.
$$u_{n|m}(r_{n,m}, p_{n,m}) \ge 0.$$
 (22b)

We solve (22) by first solving it in $p_{n,m}$ for any $r_{n,m}$. We observe that $\bar{u}_{m|n}(r_{n,m}, p_{n,m})$ is increasing in $p_{n,m}$ for a given $r_{n,m}$ and $u_{n|m}(r_{n,m}, p_{n,m})$ is decreasing in $p_{n,m}$. In other words, $p_{n,m}$ must be as large as possible while constraint (22b) is satisfied. Therefore, we have

$$p_{n,m}^*(r_{n,m}) = \frac{\lambda_n r_{n,m}}{\sigma_n} - \frac{1}{h_{n,m}} \left(2^{2r_{n,m}} - 1\right)$$
(23)

which guarantees that constraint (22b) is satisfied for any $r_{n,m}^*$. Problem (22) becomes

$$r_{n,m}^* = \arg\max_{r_{n,m} \ge 0} \bar{u}_{m|n}(r_{n,m}, p_{n,m}^*(r_{n,m}))$$
(24)

which is a convex problem with a unique maximizer. Here, we define

$$\psi_{n,m} \triangleq \frac{1}{\ln 2} \left(\frac{\bar{\lambda}_m}{\bar{\sigma}_m} - \beta_m + \frac{h_{n,m}\eta_m\lambda_n}{\sigma_n} \right).$$
(25)

The unique optimal solution of (24) is identified as the $r_{n,m}^*$ value that satisfies

$$c_m \alpha_m 2^{\alpha_m r_{n,m}^*} + 2\eta_m 2^{2r_{n,m}^*} = \psi_{n,m}.$$
 (26)

In general, (26) is a nonlinear equation, in fact, an exponential polynomial equation [19] which can be solved numerically. Note that when α_m is an integer, (26) reduces to a polynomial equation. For the special case of $\alpha_m = 0$, i.e., linear processing

cost for the receivers, the solution of (26) is found as

$$r_{n,m}^* = \frac{1}{2} \log\left(\frac{\psi_{n,m}}{2\eta_m}\right) \tag{27}$$

and for the special case of $\alpha_m = 2$, the solution of (26) is found as

$$r_{n,m}^* = \frac{1}{2} \log \left(\frac{\psi_{n,m}}{2(c_m + \eta_m)} \right).$$
(28)

This completes the characterization of all bids $(r_{n,m}^*, p_{n,m}^*)$ received by R_m . Suppose R_m has received proposals from all $T_n \in \mathcal{T}_m \subset \mathcal{T}$. R_m then finds the best proposal as

$$(r_{n^{\dagger},m}^{*}, p_{n^{\dagger},m}^{*}) = \arg\max_{\substack{(r_{n,m}^{*}, p_{n,m}^{*}):\\T_{n} \in \mathcal{T}_{m}}} \bar{u}_{m|n}(r_{n,m}^{*}, p_{n,m}^{*})$$
(29)

and the runner-up as

$$(r_{n^{\ddagger},m}^{*}, p_{n^{\ddagger},m}^{*}) = \underset{\substack{(r_{n,m}^{*}, p_{n,m}^{*}):\\T_{n} \in \mathcal{T}_{m} \setminus \{T_{n^{\ddagger}}\}}}{\arg \max} \quad \bar{u}_{m|n}(r_{n,m}^{*}, p_{n,m}^{*})$$
(30)

which are optimization problems with finite feasible sets. Finally, R_m identifies $T_{n^{\dagger}}$ as its favorite transmitter which has to provide only $\bar{u}_{m|n}(r^*_{n^{\dagger},m}, p^*_{n^{\dagger},m})$, which is necessarily less than $\bar{u}_{m|n}(r^*_{n^{\dagger},m}, p^*_{n^{\dagger},m})$. Thus, $T_{n^{\dagger}}$ can lower $p^*_{n^{\dagger},m}$ to provide $\bar{u}_{m|n}(r^*_{n^{\dagger},m}, p^*_{n^{\dagger},m})$ only and obtain a positive utility for itself as well.

In order to solve $(\{\mathcal{T}, \mathcal{R}\}, \{\succ_n, \overleftarrow{\succ}_m\})$ for an optimal matching in this case, we modify Algorithm 1 to incorporate the inter-transmitter competition, which we model as a Vickrey auction, into our solution. The generation of preference relations \succ_n remains the same. What is different from Algorithm 1 is that the transmitters must use (22) to compute their proposals, or in this case, their bids, as opposed to (11). In addition, the receivers identify their favorite among all the bids they have received using (29) and (30) as opposed to (18) and (19).

IV. NUMERICAL RESULTS

In this section, we present simulation results for the games in Section III-B and Section III-C. We consider a simulation setup of N transmitters and M receivers uniformly placed on a 100 m × 100 m square with a 1 MHz band for each orthogonal link, carrier frequency 900 MHz, noise density 10^{-19} W/Hz, and Rayleigh fading. Consequently, the mean fading level between two nodes which are d m apart is computed as $-40 \text{ dB}/d^2$ [20], [21]. For processing costs, we assume $c_m = 5 \text{ mW}$, $\alpha_m = 2 \text{ (bps)}^{-1}$, $\beta_m = 5 \text{ mW/bps}$, and $\gamma_m = 50 \text{ mW}$ for all receivers [15], [16], [22]. In addition, σ_n and $\bar{\sigma}_m$ are uniform in [0, 0.1] bps/W, η_m is uniform in [0, 1], $\lambda_n = 1$, and $\bar{\lambda}_m = 1$ for all nodes. We average our results over 1000 realizations of this setup.

Fig. 2 shows the sum rate of the network resulting from our solution for the game in Section III-B divided by the number of matched transmitters. Here, we vary N and M from 0 to 50, and redraw the curves for a direct application of DAA without our modification. As can be seen, our modification





Fig. 2. Average rate per matched transmitter versus N and M for the game in Section III-B.

results in an improvement in the average rate of the network as compared to vanilla DAA since our solution does not allow any transmitter-receiver pairs to be matched with each other unless said matching results in non-negative utilities for both nodes. As we add more transmitters to the network, the receivers are presented with a larger selection of proposals to choose from. Likewise, the addition of more receivers into the network may result in a new favorite receiver for each transmitter, improving their best option. In other words, larger N and M yields more options for both sides and better matches. As a result, the average rate is increasing in the number of transmitters and the number of receivers in the network.

We repeat this experiment for the game in Section III-C with energy cooperation and present our findings in Fig. 3. We observe similar phenomena for this case and note the larger average rate values as compared to Fig. 2. This additional improvement is due to the competition between the transmitters which results from the Vickrey auction we employ for this case. The transmitters are more inclined to compromise their own utilities so that they can propose better offers to their favorite receivers, which yields an overall improvement in the resulting rates.

Figs. 4 and 5 show the average number of proposals that must be presented and considered before our solution converges to an optimal matching for the games in Sections III-B and III-C, respectively. Here, we normalize the number of proposals by NM which is the maximum number of proposals and thus corresponds to the worst case scenario. As can be seen, our solution requires a smaller number of proposals as compared to DAA since in our solution, the transmitters automatically eliminate receivers which yield negative utilities whereas they may propose to such receivers in DAA. We observe that both our solution and DAA are efficient in the

Fig. 3. Average rate per matched transmitter versus N and M for the game in Section III-C.

sense that the addition of more receivers into the system results in a lower number of proposals per receiver required for convergence. Note that without normalization, the number of proposals is increasing in both N and M since the nodes may choose to explore the new options made available to them by the new additions. Lastly, we observe that the game in Section III-C with energy cooperation requires a smaller number of proposals on average than the one in Section III-B without energy cooperation. This is due to the fact that with energy cooperation, the transmitters can propose better offers to their favorite receivers. Hence, they are more likely to be matched to their favorite receivers and do not need to propose to their second favorite receivers and so on, which results in a lower number of proposals required to converge to a stable matching.

V. CONCLUSION

In this paper, we have considered a wireless ad hoc network composed of N transmitters and M receivers. We have studied a communication scenario where the transmitters collect data which they can deliver to the receivers. We have taken into account the energy consumption of the entire network by modeling the transmission and decoding costs at the transmitters and receivers appropriately, bearing in mind the fact that energy is often not free which may influence the nodes' decisions regarding their operation. We have formulated a matching game between the transmitters and the receivers, and provided analytical expressions for each node's optimal decision with respect to its individual utility. We have next introduced another medium of competition by employing a Vickrey auction among the transmitters. We have shown that the transmitters can offer energy cooperation to the receivers to obtain better matches. We have observed that energy coop-



Fig. 4. The normalized number of proposals before an optimal matching is found versus N for the game in Section III-B.

eration lets the transmitters provide additional incentive to the receivers and results in larger rates for the network.

Future directions include many-to-one games, i.e., college admission games for multiple access and broadcast scenarios, and bidirectional energy transfer where the receivers can transfer energy to the transmitters as well. In addition, it is left as future work to extend our model to one with interference between the transmitter-receiver pairs which is envisioned to lower the rates, but also provide an additional source of energy for the receivers.

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Fig. 5. The normalized number of proposals before an optimal matching is found versus N for the game in Section III-C.

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