

# Auction Schemes for Energy and Signal Cooperation in Two-Hop Networks

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**Abstract**—In this paper, we study a cooperative two-hop network with multiple sources and multiple relays where the energy required for the relays is transferred by the sources. In return, the relays transmit the sources' data, along with their own data, to the destination. We consider the setup where each node's objective is to maximize the amount of its own data delivered to the destination. We take a game theoretic approach and first model the selfish cooperation scenario with one source and one relay as a Stackelberg game where (i) the relay or (ii) the source is the leader. We demonstrate how the leader of the game takes advantage of its ability to compute the follower's optimal strategy to influence the follower and improve its own utility. In both cases, we also consider the case with multiple followers. We employ Vickrey auctions to model the inter-follower competition. We identify the winner of the auction in both cases and observe that the followers must compromise their individual utilities to win the auction. Consequently, the leader's utility turns out to be nondecreasing in the number of competing followers.

## I. INTRODUCTION

Cooperation improves wireless networking by utilizing the resources in a more efficient fashion. While traditionally understood as signal cooperation [1] where nodes forward other nodes' data to the destination, energy cooperation [2] where nodes transfer energy to other nodes with energy deficiency has also recently been considered as a new form of it. Energy cooperation and signal cooperation are undoubtedly effective, but they are not always straightforward to expect. In this work, we use game theory to investigate ways to simultaneously incentivize the nodes to participate in energy cooperation in exchange for signal cooperation and vice versa.

Signal cooperation in two-hop networks has been studied in [3] where the achievable rates for two-hop networks with multiple sources and relays are derived. Reference [2] has introduced energy cooperation to the two-hop relay channel where the source can wirelessly transfer energy to the relay. A two dimensional waterfilling interpretation of the throughput maximizing transmission and energy transfer policy has been identified. It has been shown that energy cooperation can improve the throughput. Reference [4] has studied the throughput maximization problem in a two-hop network where the relays can transfer energy to the sources as well. Reference [5] has studied the multiple access and two-way channels with energy

cooperation where energy transfer may take place between any two transmitters.

Another area of research on energy and signal cooperation that has recently gained attention is simultaneous wireless information and power transfer (SWIPT) where signal cooperation and energy cooperation are performed using power splitting, time sharing, and relay selection schemes [6]–[10]. References [11], [12] have studied energy transfer from the source to the relay in a two-hop network. Reference [13] has considered a cognitive radio setup where secondary users can use the nearby primary users' transmission for energy harvesting. References [14], [15], among others, have studied cellular networks where the users are wirelessly powered by the base station. Reference [16] has investigated the trade off between transmitting information and transferring energy over the same link from the relays to the destinations in a two-hop network. Reference [17] has provided an overview of SWIPT.

The previous efforts in improving the end-to-end throughput in multi-terminal networks using energy and signal cooperation together have assumed that all nodes are altruistic and are thus willing to cooperate. In practice, this may not be the case. For example, reference [18] has considered a cognitive radio setup where the secondary users are offered spectrum access in return for cooperative relaying of the primary user's messages, thereby incentivizing signal cooperation. We have recently studied a two-hop setup with one source and one relay, amplify-and-forward relaying, and the simplifying assumption that the energy transferred to the relay is used for the relay's own data only [19].

In this work, we study an energy harvesting two-hop network with multiple sources or multiple relays in a leader-follower game theoretic setting. As compared to [19], aside from considering competition with multiple sources or relays, we consider a more refined signal cooperation model with decode-and-forward relaying where the relay can superpose cooperative transmission with its own. We consider Stackelberg competition schemes where either the source or the relay node can be the leader of the game, to capture all practical scenarios that may be more fittingly modeled by each node's leadership. We formulate the Stackelberg game [20] and demonstrate how the leader chooses its strategy so that the follower's reaction improves the leader's utility as well. We employ a Vickrey auction among the followers

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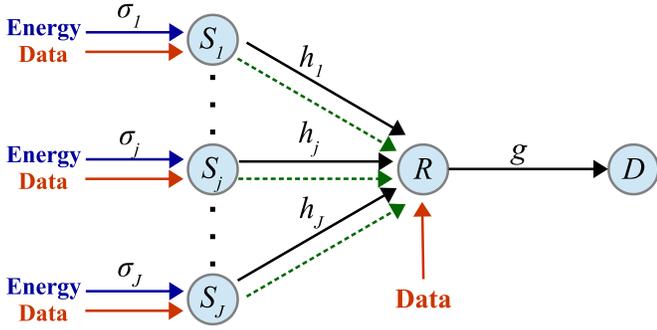


Fig. 1. The two-hop channel with multiple source nodes. Data links are shown in solid lines and energy transfers are shown in dashed lines.

and identify the winner of the auction in each case. We find that having an additional layer of competition by means of an auction improves the auctioneer's utility, and that the auctioneer's utility is nondecreasing in the number of bidders, i.e., followers.

## II. STACKELBERG COMPETITION WITH THE RELAY AS THE LEADER

### A. System Model

Consider a half-duplex Gaussian two-hop network with  $J$  source nodes,  $S_j$ ,  $j \in \mathcal{J} \triangleq \{1, 2, \dots, J\}$ , a decode-and-forward relay node,  $R$ , and a destination node,  $D$ , as shown in Fig. 1. All nodes have data to transmit to the destination. The power gain is  $h_j$  from node  $S_j$  to  $R$ ,  $j \in \mathcal{J}$ , and  $g$  from  $R$  to  $D$ . The Gaussian noise variance of all links is assumed to be unity without loss of generality. The source nodes are not directly connected to the destination. All source and relay nodes are assumed selfish in the sense that each node wishes to maximize the amount of its own data delivered to the destination. Node  $S_j$  has energy available from an external source of energy at a price of  $\sigma_j$  per unit of energy,  $j \in \mathcal{J}$ . The relay node can harvest a fraction of the energy in the source's transmission at efficiency  $\eta \in [0, 1]$ .

The communication scheme is based on an auction between the source nodes. Node  $S_j$  bids average transmit power  $p_j \in [0, P_j]$  where  $P_j$  models a maximum average power constraint at node  $S_j$ ,  $j \in \mathcal{J}$ . Only the winning source will be given the chance to adjust its average transmit power, and transmit its data to the relay and subsequently to the destination. The auction scheme will be explained in the sequel, but suppose, for the moment, node  $S_{j^*}$  wins the auction, and settles on average transmit power  $p^* \in [0, P_{j^*}]$ . The relay chooses  $\delta \in [0, 1]$  denoting the fraction of the received signal that will be used for energy harvesting.

Without loss of generality, suppose a two-phase communication scheme with phases  $A$  and  $B$  of equal duration  $T$  is employed for the transmission of  $S_{j^*}$  and  $R$ , respectively. In phase  $A$ ,  $R$  listens to  $S_{j^*}$ 's transmission for  $(1-\delta)T$  seconds while  $S_{j^*}$  transmits at  $p^*/(1-\delta)$ , resulting in an average transmit power of  $p^*$ . During the remaining  $\delta T$  seconds of phase  $A$ ,  $S_{j^*}$  transmits at transmit power  $p^*/\delta$  while  $R$

harvests  $\eta h_{j^*} p^* T$ . In phase  $B$ , the relay uses the harvested energy to forward  $S_{j^*}$ 's data and send its own data to node  $D$ . For equal duration of phases and the winning bidder  $S_{j^*}$ , the utilities  $u_{S_j}$  for node  $S_j$ , and  $u_{R|j^*}$  for node  $R$  are

$$u_{S_{j^*}}(p^*, \delta) = \frac{1-\delta}{4} \log \left( 1 + h_{j^*} \frac{p^*}{1-\delta} \right) - \sigma_{j^*} p^*, \quad (1)$$

$$u_{S_j}(p_j, \delta) = 0, \quad j \in \mathcal{J} \setminus \{j^*\}, \quad (2)$$

$$u_{R|j^*}(p^*, \delta) = \frac{1}{4} \log(1 + \eta h_{j^*} g p^*) - \frac{1-\delta}{4} \log \left( 1 + h_{j^*} \frac{p^*}{1-\delta} \right). \quad (3)$$

The definition of the utilities dictates that  $\delta$  be chosen large enough so that  $u_{R|j^*}(p^*, \delta) \geq 0$ . In other words, if node  $R$  chooses a small  $\delta$  and cannot harvest sufficient energy to forward all of node  $S_{j^*}$ 's data, then  $u_{R|j^*}(p^*, \delta) < 0$ . Therefore, node  $R$  must limit the amount of data it receives from node  $S_{j^*}$  by increasing  $\delta$  accordingly so that it will have enough energy for node  $S_{j^*}$ 's data, and possibly for its own data. While the current formulation of the utilities allows node  $R$  to forward more of node  $S_{j^*}$ 's data than it can with the harvested energy, node  $R$  will never choose such a low  $\delta$  and obtain a negative utility, as will be demonstrated.

### B. Two-Hop Channel with One Source

We first formulate and solve a Stackelberg game with the relay node as the leader for the case with  $J = 1$ . Since there is only one source node, there is no need for an auction, and we thus have  $j^* = 1$  and  $p^* = p_1$ . In a Stackelberg game, the follower chooses a strategy that maximizes the follower's utility given the leader's strategy. That is, the leader and the follower play a sequential game where the follower must react to the leader's strategy optimally. The leader is capable of calculating the follower's best response to any leader strategy. The leader hence chooses a strategy that maximizes its own utility knowing how the follower will react [20].

Consider a Stackelberg game where  $R$  is the leader,  $S_1$  is the follower, the strategy spaces are  $[0, P_1]$  for  $S_1$  and  $[0, 1]$  for the relay, and the payoffs are  $u_{S_1}$  and  $u_{R|1}$  as in (1) and (3). Given any leader strategy  $\delta \in [0, 1]$ , the follower solves

$$p_1(\delta) = \arg \max_{p' \in [0, P_1]} u_{S_1}(p', \delta) \quad (4)$$

$$= \min \left\{ \max \left\{ \left( \frac{1}{4\sigma_1 \ln 2} - \frac{1}{h_1} \right) (1-\delta), 0 \right\}, P_1 \right\} \quad (5)$$

where  $p_1(\delta)$  is nonincreasing in  $\sigma_1$  and  $\delta$ , and nondecreasing in  $h_1$ . Node  $S_1$  reacts to a large  $\delta$  chosen by node  $R$  by lowering the average transmit power. This is because a larger  $\delta$  implies less time dedicated to improving node  $S_1$ 's utility, and the throughput it can attain can no longer compensate for the incurred energy cost. The leader knows this, i.e., node  $R$  can calculate  $p_1(\delta)$  for all  $\delta \in [0, 1]$ . The leader takes this information into account when choosing a  $\delta$ , and solves

$$\delta = \arg \max_{\delta' \in [0, 1]} u_{R|1}(p_1(\delta'), \delta'). \quad (6)$$

Before solving (6), let us take a closer look  $p_1(\delta)$  in (5). If  $\phi \triangleq \frac{1}{4\sigma_1 \ln 2} - \frac{1}{h_1} \leq 0$ , then  $p_1(\delta) = 0$  for all  $\delta \in [0, 1]$ . In this case, the objective of (6) is zero, and regardless of the choice of  $\delta$ , the total utility is zero. This results from the energy price  $\sigma_1$  of node  $S_1$  being too high, or the power gain to the relay  $h_1$  being too low, i.e., node  $S_1$  could not attain a positive utility even if it were allocated the entire transmission session.

Suppose now that  $\phi > 0$ , thus node  $S_1$  has incentive to transmit. In this case, we can restate  $p_1(\delta)$  as

$$p_1(\delta) = \begin{cases} P_1 & \text{if } \delta \in [0, \bar{\delta}] \text{ (Case 1)} \\ \phi(1 - \delta) & \text{if } \delta \in [\bar{\delta}, 1] \text{ (Case 2)} \end{cases} \quad (7)$$

where  $\bar{\delta} \triangleq 1 - \min\{P_1/\phi, 1\}$ . Using the piecewise description of  $p_1(\delta)$  in (7), we separate the feasible region of (6) into two regions  $[0, \bar{\delta}]$  and  $[\bar{\delta}, 1]$ , solve the problem in each region, and finally identify the optimal  $\delta$ .

- Case 1: We have  $p_1(\delta) = P_1$  for all  $\delta \in [0, \bar{\delta}]$ . The objective of (6) becomes

$$u_{R|1}(P_1, \delta) = \frac{1}{4} \log(1 + \eta h_1 g P_1) - \frac{1 - \delta}{4} \log\left(1 + h_1 \frac{P_1}{1 - \delta}\right) \quad (8)$$

and is strictly increasing in  $\delta$ . Therefore, no  $\delta \in [0, \bar{\delta}]$  can outperform  $\bar{\delta}$ , i.e., the maximizer of (6) lies in  $[\bar{\delta}, 1]$ .

- Case 2: In this case,  $p_1(\delta) = \phi(1 - \delta)$  for all  $\delta \in [\bar{\delta}, 1]$ . The objective of (6) becomes

$$u_{R|1}(\phi(1 - \delta), \delta) = \frac{1}{4} \log(1 + \eta h_1 g \phi(1 - \delta)) - \frac{1 - \delta}{4} \log(1 + h_1 \phi) \quad (9)$$

and is maximized by

$$\delta = \min \left\{ \max \left\{ 1 - \frac{1}{\ln\left(\frac{h_1}{4\sigma_1 \ln 2}\right)} + \frac{1}{\eta g \left(\frac{h_1}{4\sigma_1 \ln 2} - 1\right)}, \bar{\delta} \right\}, 1 \right\}. \quad (10)$$

We can observe from (10) that as  $\sigma_1$  increases or  $h_1$  decreases, node  $R$  tends to choose a lower  $\delta$ . This follows from the fact that node  $R$  knows that such changes in  $\sigma_1$  and  $h_1$  will cause node  $S_1$  to lower  $p_1$ . Therefore, node  $R$  proactively lowers  $\delta$  so as to counteract the influence of  $\sigma_1$  and  $h_1$  on node  $S_1$ 's decision. This demonstrates how the leader uses its knowledge the follower's best reaction.

### C. Two-Hop Channel with Multiple Sources

In this subsection, we study the two-hop channel with multiple source nodes. We introduce an additional layer of competition by utilizing an auction scheme between the multiple source nodes where the auctioned item is the relay's signal cooperation. That is, only the winning source node can

deliver its data to the destination using the relay. We foresee that this will improve the relay's utility since the sources need to outbid each other in order to obtain positive utilities, and consequently, the winner will be a source that is most willing to compromise its own utility.

We employ a Vickrey auction between the sources where the bids are the relay utilities that can be obtained with the average transmit powers chosen by the sources. In a Vickrey auction, the highest bidder wins, but has to pay the price offered by the second highest bidder [21]. As a result, the bidders are encouraged to bid the maximum price they are willing to pay. This is a desirable property of Vickrey auctions, and it is particularly useful in this work since it results in an improvement in the relay's utility while attaining a positive utility for the source as well.

Each source node bids an average transmit power  $p_j$  given leader strategy  $\delta$ , and offers the leader utility  $u_{R|j}(p_j, \delta)$  that results from these strategies. Due to the truthful bidding property of the Vickrey auction, the sources are willing to increase their bids until they can no longer have a nonnegative utility. Thus, the bids can be computed for all sources by solving

$$\frac{1 - \delta}{4} \log\left(1 + h_j \frac{p_j}{1 - \delta}\right) - \sigma_j p_j = 0, \quad \forall j \in \mathcal{J}. \quad (11)$$

This equation has a solution, other than  $p_j = 0$ , for all sources which can be stated as

$$p_j = -\frac{1 - \delta}{4\sigma_j \ln 2} W\left(-\frac{4\sigma_j \ln 2}{h_j} e^{-\frac{4\sigma_j \ln 2}{h_j}}\right) - \frac{1 - \delta}{h_j} \quad (12)$$

for all  $j \in \mathcal{J}$  where  $W(\cdot)$  is the lower branch of the Lambert W function. We can rewrite (11) as

$$f(h_j p_j) \triangleq \left(\frac{h_j p_j}{1 - \delta}\right)^{-1} \log\left(1 + \frac{h_j p_j}{1 - \delta}\right) = 4 \frac{\sigma_j}{h_j} \quad (13)$$

for all  $j \in \mathcal{J}$ . Since  $x \log(1 + 1/x)$  is strictly increasing in  $x \geq 0$ ,  $f(h_j p_j)$  is strictly decreasing in  $h_j p_j$ . Thus, we can infer that  $h_j p_j$  increases as  $\sigma_j/h_j$  decreases. That is, the source node with the lowest  $\sigma_j/h_j$  value can provide the highest receive power at the relay. Hence, the sources that can buy energy at a lower price, or have a better link to the relay are willing to bid higher average receive powers at the relay. The winner of the auction is the source node that can provide the highest utility to the relay with its bid, i.e.,

$$j^* = \arg \max_{j \in \mathcal{J}} u_{R|j}(p_j, \delta) \quad (14)$$

$$= \arg \max_{j \in \mathcal{J}} (\log(1 + \eta g h_j p_j) - f(h_j p_j) h_j p_j) \quad (15)$$

where (15) follows from (11). Recall that  $-f(h_j p_j)$  is strictly increasing in  $h_j p_j$ . Thus, we have

$$j^* = \arg \min_{j \in \mathcal{J}} h_j p_j = \arg \min_{j \in \mathcal{J}} \frac{\sigma_j}{h_j}. \quad (16)$$

In other words, the winner of the auction is the source node with the highest received power at the relay, or equivalently, the one with the lowest  $\sigma_j/h_j$  ratio. Note that, the source

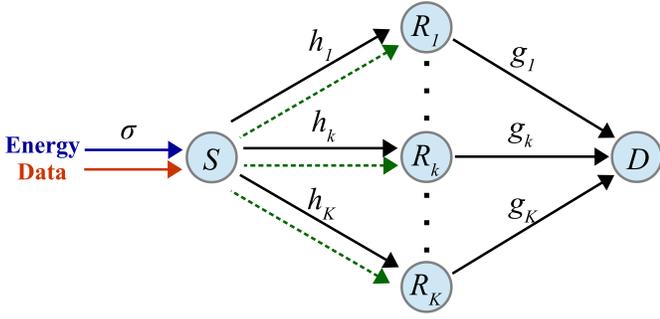


Fig. 2. The two-hop channel with multiple relay nodes. Data links are shown in solid lines and energy transfers are shown in dashed lines. Data arrivals at the relays are omitted.

nodes need not calculate their bids in order to determine who will win the auction. Recall that in the single source case,  $\delta$  turned out to be decreasing in  $\sigma/h$  as observed in (10). It is thus shown that with multiple sources, the source node with the lowest  $\sigma_j/h_j$  will agree with the largest  $\delta$  chosen by the relay, and therefore provide the largest utility to the relay. Consequently, the auction improves the auctioneer's payoff, in this case, the relay's utility.

Let  $S_{j^\dagger}$  be the runner up. Node  $S_{j^*}$  must provide at least  $u_{R_{j^\dagger}}(p_{j^\dagger}, \delta)$ . We know that  $h_{j^*}p_{j^*} \geq h_{j^\dagger}p_{j^\dagger}$ , and thus  $S_{j^*}$  can lower its transmit power to  $h_{j^\dagger}p_{j^\dagger}/h_{j^*}$  and provide the required relay utility. However, if  $p_{j^*}(\delta)$  computed as

$$p_{j^*}(\delta) = \arg \max_{p' \in [0, P_{j^*}]} u_{S_{j^*}}(p', \delta) \quad (17)$$

is larger than  $h_{j^\dagger}p_{j^\dagger}/h_{j^*}$ , then both  $S_{j^*}$  and  $R$  can have higher utilities if  $S_{j^*}$  lowers its power only to  $p_{j^*}(\delta)$ . Thus,

$$p^*(\delta) = \max\{p_{j^*}(\delta), h_{j^\dagger}p_{j^\dagger}/h_{j^*}\}. \quad (18)$$

That is, the Vickrey auction results in a minimum average power requirement at node  $S_{j^*}$ . Since the auctioneer, i.e., node  $R$ , would have to sacrifice its utility by lowering  $\delta$  in order to increase the average transmit power of the source node if there were only one source node, the auction results in an increase in the auctioneer's utility.

### III. STACKELBERG COMPETITION WITH THE SOURCE AS THE LEADER

#### A. System Model

The communication system we study in this section is similar to the one in Section II, except here, there are multiple relay nodes instead of multiple source nodes. We thus have a half-duplex Gaussian two-hop network with a source node,  $S$ ,  $K$  decode-and-forward relay nodes,  $R_k$ ,  $k \in \mathcal{K} \triangleq \{1, 2, \dots, K\}$ , and a destination node,  $D$ , as shown in Fig. 2. All nodes have data to transmit to node  $D$ , and they wish to maximize their individual throughputs. The power gain is  $h_k$  from  $S$  to  $R_k$ , and  $g_k$  from  $R_k$  to  $D$  with unit noise variance. Node  $S$  can purchase energy at price  $\sigma$ . Relay  $R_k$  can harvest energy from  $S$ 's transmission at efficiency  $\eta_k \in [0, 1]$ .

We consider an auction between the relay nodes. The underlying communication scheme is again carried out in two phases. Node  $S$ 's strategy is the average transmit power  $p \in [0, P]$  where  $P$  is the maximum average power. Node  $R_k$  bids harvesting fraction  $\delta_k \in [0, 1]$ ,  $k \in \mathcal{K}$ . The winning relay will be able to adjust its fraction, and take part in the two-phase communication scheme. Suppose the winning relay is node  $R_{k^*}$ , and the adjusted harvesting fraction is  $\delta^* \in [0, 1]$ . Given that  $k^*$  is the winning bidder index, the resulting utilities  $u_{S|k^*}$  for node  $S$  and  $u_{R_{k^*}}$  for node  $R_{k^*}$  can be stated as

$$u_{S|k^*}(p, \delta^*) = \frac{1 - \delta^*}{4} \log \left( 1 + h_{k^*} \frac{p}{1 - \delta^*} \right) - \sigma p, \quad (19)$$

$$u_{R_{k^*}}(p, \delta^*) = \frac{1}{4} \log(1 + \eta_{k^*} h_{k^*} g_{k^*} p) - \frac{1 - \delta^*}{4} \log \left( 1 + h_{k^*} \frac{p}{1 - \delta^*} \right), \quad (20)$$

$$u_{R_k}(p, \delta_k) = 0, \quad k \in \mathcal{K} \setminus \{k^*\}. \quad (21)$$

#### B. Two-Hop Channel with One Relay

We consider the reciprocal Stackelberg game to Section II with the source node as leader. Consider first the case with only one relay, i.e.,  $K = 1$ . Given leader strategy  $p$ , the follower, node  $R_1$ , solves

$$\delta_1(p) = \arg \max_{\delta' \in [0, 1]} u_{R_1}(p, \delta'). \quad (22)$$

Since  $u_{R_1}(p, \delta_1)$  is increasing in  $\delta_1$ , it is immediate that the solution of (22) is  $\delta_1(p) = 1$  for all  $p > 0$ , and  $u_{R_1}(p, \delta) = 0$  for any  $\delta_1$  if  $p = 0$ . The leader knows  $\delta_1(p)$ , and solves

$$p = \arg \max_{p' \in [0, P]} u_{S|1}(p', \delta_1(p')). \quad (23)$$

If the leader picks a positive  $p$ , then the first term in its utility will be zero, but the second term will be negative. That is,  $u_{S|1}(p, \delta_1(p)) < 0$  for any  $p > 0$ , and  $u_{S|1}(p, \delta_1(p)) = 0$  for  $p = 0$ . Thus, the optimal strategy for the leader is to stop transmission, resulting in vanishing utilities for both nodes. Therefore, with a single relay in a Stackelberg setup with the source as the leader, the two players cannot agree on a strategy pair that yields positive utilities. However, positive utilities can be encouraged by introducing more relay nodes to the system as presented next.

#### C. Two-Hop Channel with Multiple Relays

Let us now consider a Vickrey auction where the auction item is the energy that can be harvested from the source's transmission, and the bids are the utilities for the source resulting from the harvesting fractions chosen by the relays. Similar to Section II-C, the relays are motivated to pay the maximum price they can pay. Since the relay utilities are increasing in  $\delta$ , each relay is willing to lower their  $\delta$  until the relay utility vanishes. That is, node  $R_k$  solves

$$\log(1 + \eta_k h_k g_k p) - (1 - \delta_k) \log \left( 1 + h_k \frac{p}{1 - \delta_k} \right) = 0 \quad (24)$$

for all  $k \in \mathcal{K}$ , which yields

$$\delta_k = \max \left\{ \left[ \frac{1}{\psi_k} W \left( -\frac{\psi_k}{h_k p} e^{-\frac{\psi_k}{h_k p}} \right) + \frac{1}{h_k p} \right]^{-1} + 1, 0 \right\} \quad (25)$$

for all  $k \in \mathcal{K}$  where  $\psi_k = \ln(1 + \eta_k h_k g_k p)$ . To determine the winner of the auction, one need not calculate  $\delta_k$ . The winner of the auction is the relay node that can provide the source with the highest utility. That is,

$$k^* = \arg \max_{k \in \mathcal{K}} u_{S|k}(p, \delta_k) \quad (26)$$

$$= \arg \max_{k \in \mathcal{K}} (\log(1 + \eta_k h_k g_k p) - 4\sigma p) \quad (27)$$

where we use (24) to arrive at (27). Since the source utility is strictly increasing in  $\eta_k h_k g_k$ , we have

$$k^* = \arg \max_{k \in \mathcal{K}} \eta_k h_k g_k. \quad (28)$$

In other words, the winner of the auction is the relay that can utilize its harvested energy most efficiently, and thus deliver the most data to the destination. Let  $R_{k^\dagger}$  be the runner up. Relay  $R_{k^*}$  must provide at least  $u_{S|k^\dagger}(p, \delta_{k^\dagger})$ . Since the winning relay's own utility is increasing in the harvesting fraction, it is optimal for  $R_{k^*}$  to provide exactly  $u_{S|k^\dagger}(p, \delta_{k^\dagger})$ . Thus, node  $R_{k^*}$  solves

$$\log(1 + \eta_{k^\dagger} h_{k^\dagger} g_{k^\dagger} p) = (1 - \delta^*) \log \left( 1 + h_{k^*} \frac{p}{1 - \delta^*} \right) \quad (29)$$

The unique solution to (29) can be computed as

$$\delta^* = \left[ \frac{1}{\psi_{k^\dagger}} W \left( -\frac{\psi_{k^\dagger}}{h_{k^*} p} e^{-\frac{\psi_{k^\dagger}}{h_{k^*} p}} \right) + \frac{1}{h_{k^*} p} \right]^{-1} + 1. \quad (30)$$

Since  $\eta_{k^*} h_{k^*} g_{k^*} \geq \eta_{k^\dagger} h_{k^\dagger} g_{k^\dagger}$  due to (28), the right-hand side of (29) is at least  $\log(1 + \eta_{k^\dagger} h_{k^\dagger} g_{k^\dagger} p)$  when  $\delta^* = \delta_{k^*}$ , and it decreases to 0 as  $\delta^*$  increases to 1. Thus, a unique solution exists, and unless  $\eta_{k^*} h_{k^*} g_{k^*} = \eta_{k^\dagger} h_{k^\dagger} g_{k^\dagger}$ , the follower can also have a positive utility while delivering the required leader utility. The leader can now calculate  $p$  using (23). As a final remark, we note that no Stackelberg game with the source as the leader results in a positive utility for any node. By introducing competition between the relays by means of an auction, we obtain positive utilities.

#### IV. NUMERICAL RESULTS

In this section, we evaluate the utilities that result from the equilibria found in Sections II and III versus the number of competing nodes in the network. The simulation setup is with a 1 MHz bandwidth, noise density equal to  $10^{-19}$  W/Hz, and the power gain between two nodes that are  $d$  meters apart computed as  $-100$  dB/ $d^3$ . The harvesting efficiencies, maximum average powers, and energy prices are drawn uniformly from  $\beta_\eta[0, 1]$ ,  $\beta_p[0, 100]$  mW, and  $\beta_\sigma[0, 0.1]$  bps/W, respectively. We vary  $\beta_\eta, \beta_p, \beta_\sigma \in [0, 1]$  vary for each simulation.

Fig. 3 shows the leader and follower utilities for the setup in Section II-C with one relay and  $J$  sources versus the number of sources in the network. The distance from  $R$  to  $D$  is set to be 5 m, and the distance between each  $S_j$  and  $R$  is uniform

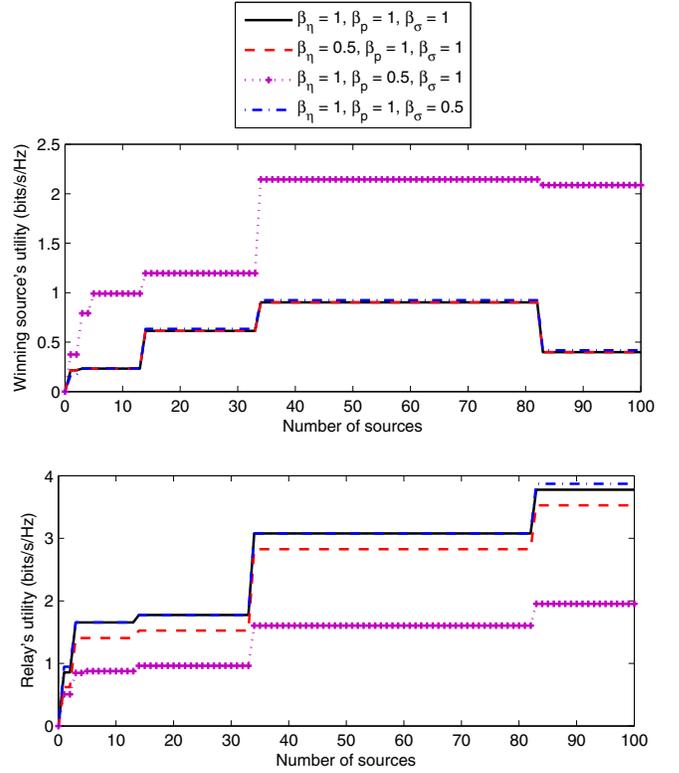


Fig. 3. The utilities in a two-hop network with  $J$  sources versus  $J$  for the setup in Section II. The utility curves are redrawn for lower harvesting efficiency, maximum power, and energy price values.

on  $[0, 10]$  m. As more sources are added to the system, the sources face more intense competition whereas the relay has more options to choose from. Moreover, since the distances between the sources and the relay, and the power prices at the sources are random, the added source may have a larger power gain to the relay or may be able to obtain energy at a lower price. Therefore, the relay's utility is nondecreasing in the number of sources in the system. However, the winning source's utility is not monotone since the addition of new sources with random distances and random energy prices can impact the source's utility in any direction. Note that the new sources added to the system during the constant portions of both utilities cannot win the auction or have the second best offer, thus their involvement does not impact the utilities. The utility curves are also drawn for lower harvesting efficiencies, maximum powers, and energy prices. A lower harvesting efficiency results in a lower relay utility, but it does not impact the winning source's utility. A lower maximum power constraint helps the source and causes a lower utility at the relay since it limits the relay's ability to influence the source to increase the transmit power. In addition, a lower energy price results in higher bids from all sources, and thus a higher relay utility. It also results in a lower energy cost at the source.

Fig. 4 shows the leader and follower utilities for the setup in Section III-C with one source and  $K$  relays versus the number

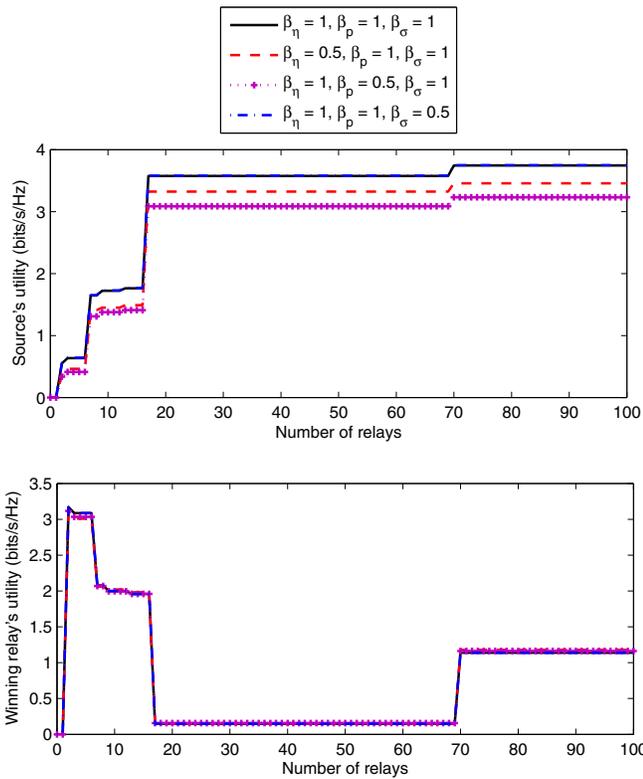


Fig. 4. The utilities in a two-hop network with  $K$  relays versus  $K$  for the setup in Section III. The utility curves are redrawn for lower harvesting efficiency, maximum power, and energy price values.

of relays in the network. The distance from  $S$  to  $D$  is set to be 10 m, and the relays are placed uniformly on the line between  $S$  and  $D$ . Similarly, the auctioneer's utility is nondecreasing in the number of bidders whereas the auction winner's utility is not monotone. The source's utility is increasing in the harvesting efficiency when the source is the leader of the game. With a higher  $\eta$ , the source can encourage node  $R$  to use the additional harvested energy for forwarding node  $S$ 's data. While the source can adjust the average transmit power and in turn the harvesting fraction at the relay in accordance with the changes in the energy price, a lower maximum power constraint at the source results in a lower source utility. This is because with a lower maximum power, the source has a smaller feasible set, and hence its utility potentially decreases.

## V. CONCLUSION

In this paper, we have studied signal and energy cooperation in two-hop wireless networks. We have considered a scenario where the relay is offered energy by the source and is in exchange asked to forward the source's data to the destination. We have studied selfish nodes that aim to maximize their individual utilities, with the objective of properly incentivizing them to take part in signal and energy cooperation. We have formulated Stackelberg games with the relay or the source as the leader. We have shown that the leader can influence the follower's decision so as to have a higher individual utility. We

have shown that the leader's utility can be improved further by introducing more followers into the network and employing a Vickrey auction. We have observed that auctions result in more energy transferred to the relay by the sources, or more data forwarded by the relays. Moreover, the inter-source or the inter-relay competition becomes more intense as there are more bidders added to the system, and thus the leader utility is improved even further.

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